Oscillating H_q

Intermittency Indices

and

QCD

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Charged Particle Multiplicity

Multiplicity distribution, P_n is often analysed in terms of factorial moments, F_q, their cumulants, K_q,

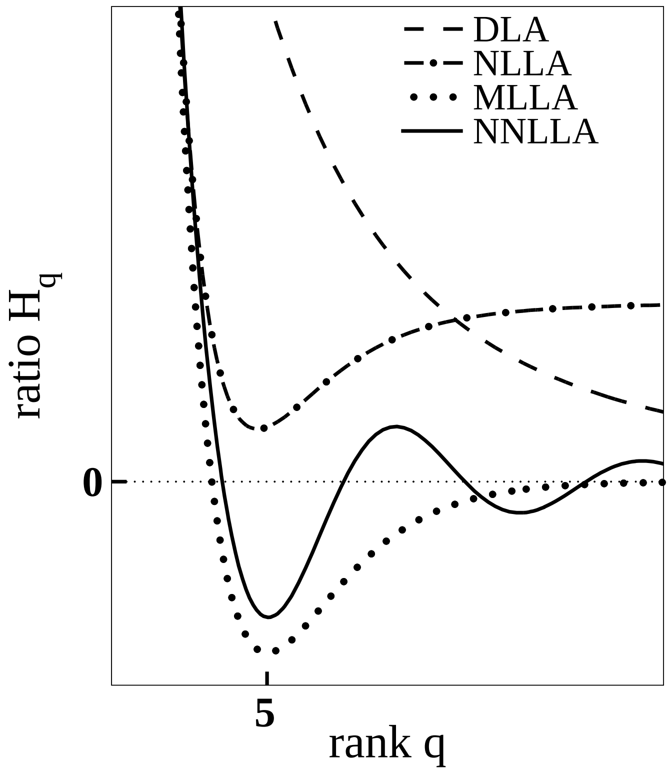
$$F_q = \frac{\sum_{n=q}^{\infty} \frac{n!}{(n-q)!} P_n}{\left(\sum_{n=1}^{\infty} n\, P_n\right)^q}$$

$$K_q = F_q - \sum_{m=1}^{q-1} \binom{q-1}{m} K_{q-m} F_m$$

and their ratio $$H_q = \frac{K_q}{F_q}$$

- H_q are calculable in pQCD
- assuming LPHD, can compare with H_q for charged particles

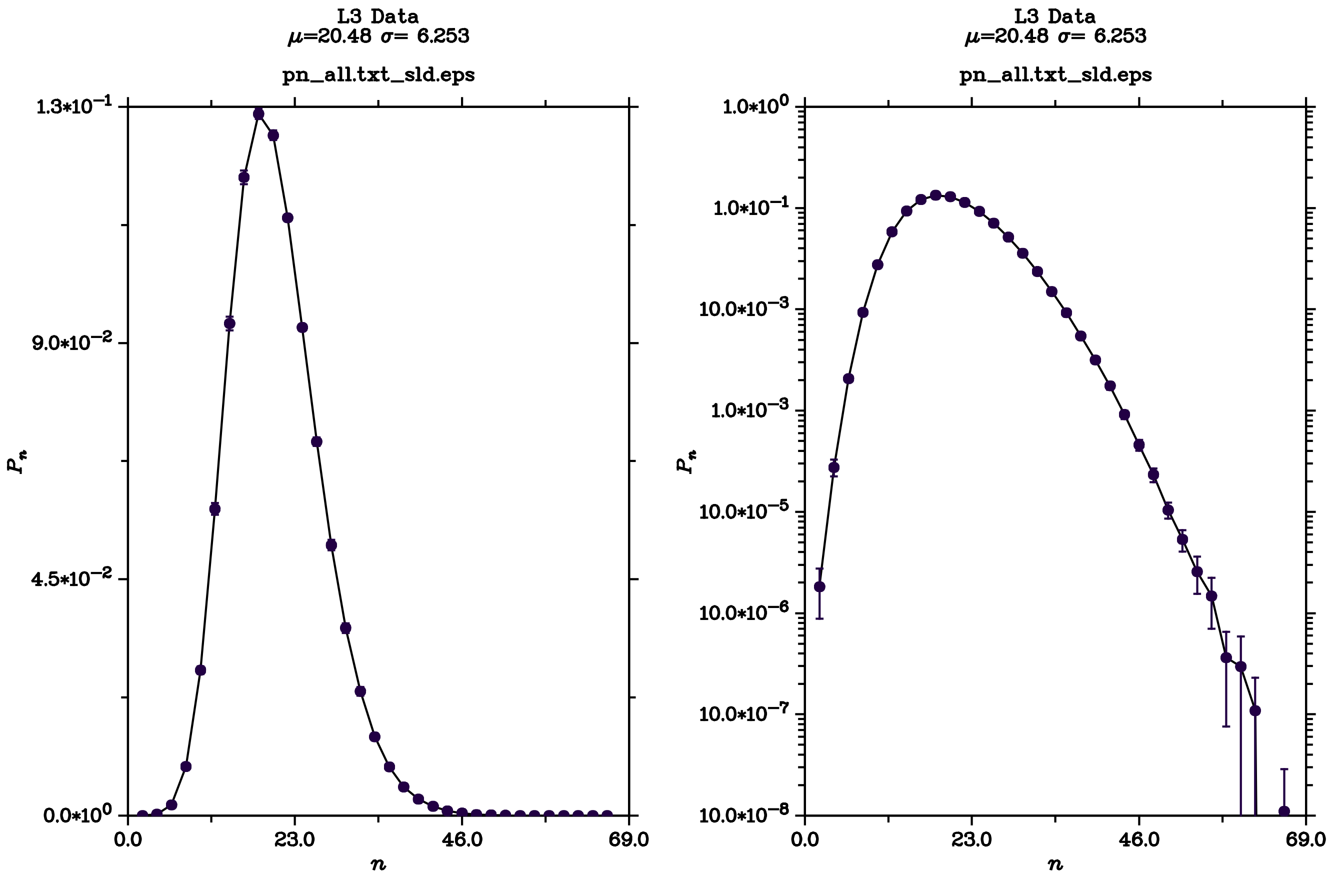
SLD observed the behavior predicted by NNLLA, *i.e.*, min. + oscillations (P.L. B371 (1996) 149)



Multiplicity Distribution

• L3 preliminary $e^+e^- \to Z \to$ hadrons

Unfold the mult. dist. using iterative Bayesian method with a detector response matrix determined from MC



Events: L3: 1 M SLD: 87 K

H_q

From P_n calculate H_q

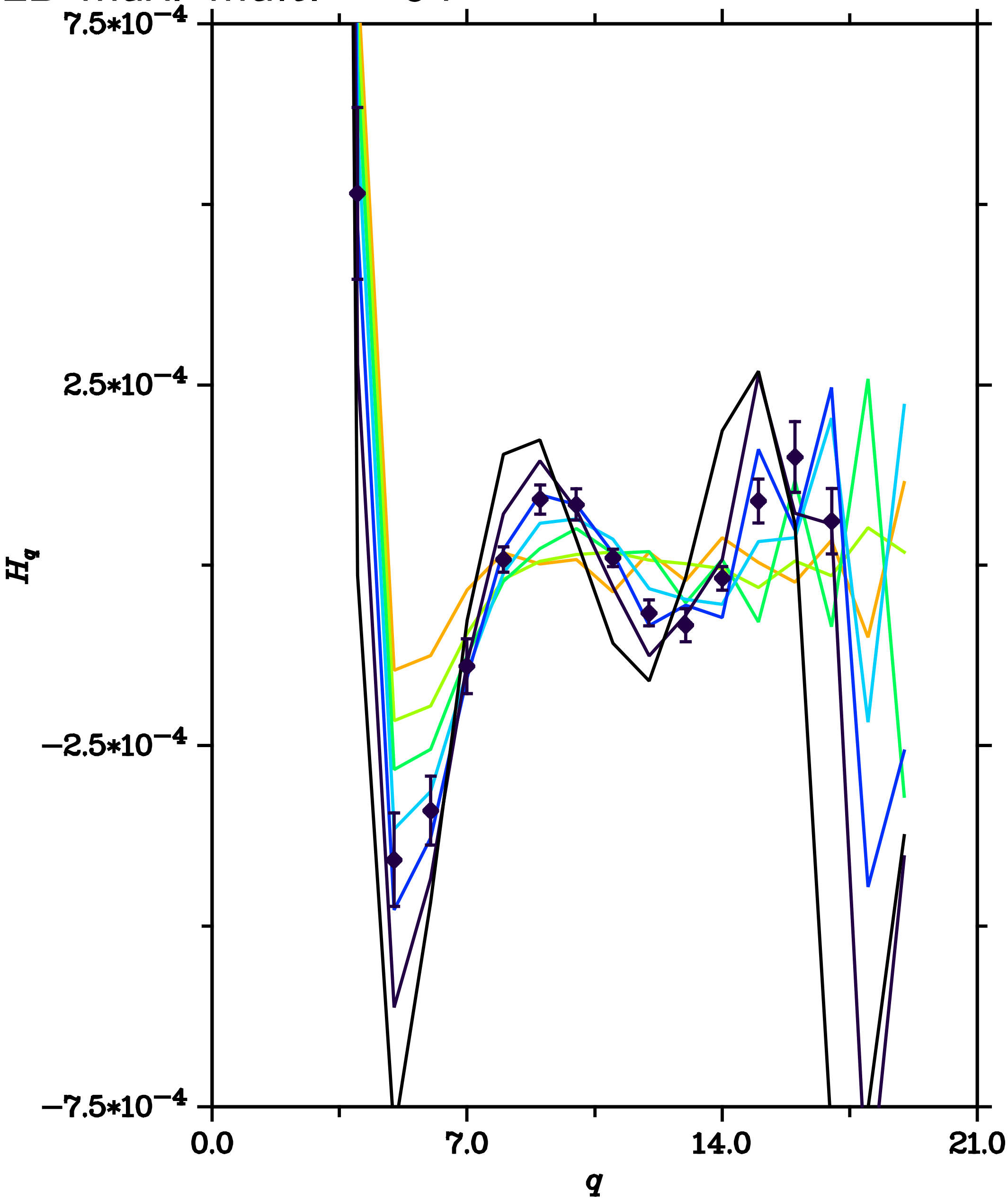
- Stat. err. from propagation of covariance matrix of unfolded mult. dist.
- Syst. err. from
  - variation of selection cuts (largest)
  - JETSET/ARIADNE for unfolding
- F_q is very sensitive to tails of dist.
  Truncate mult. dist. where errors become too large
  $\frac{\delta P_n}{P_n} > 0.5 \quad \Longrightarrow \quad$ events with $N_{\text{ch}} > 53$ rejected

Effect of truncation

lines: L3 preliminary truncated at

67 (no trunc.), 57, 55, 53, 51, 49, 47

points: SLD max. mult. = 54



Truncation increases amplitudes

Positions of extrema shift to lower q

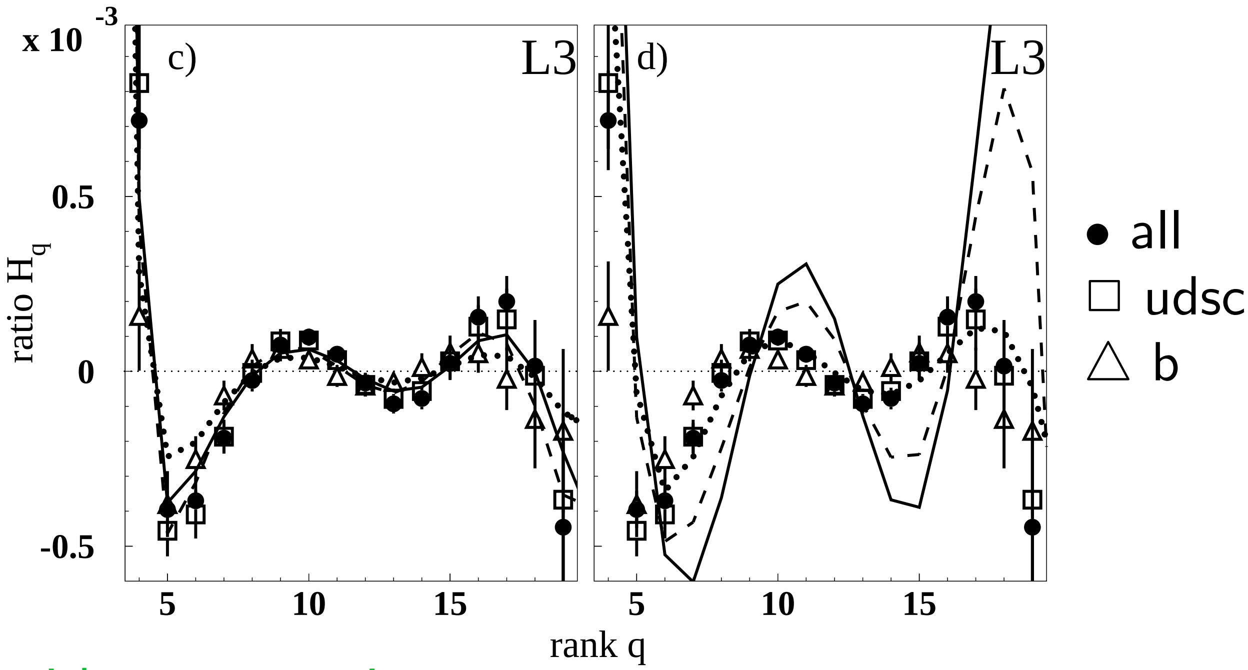
L3 ≈ SLD if truncate near SLD max. mult.

H_q

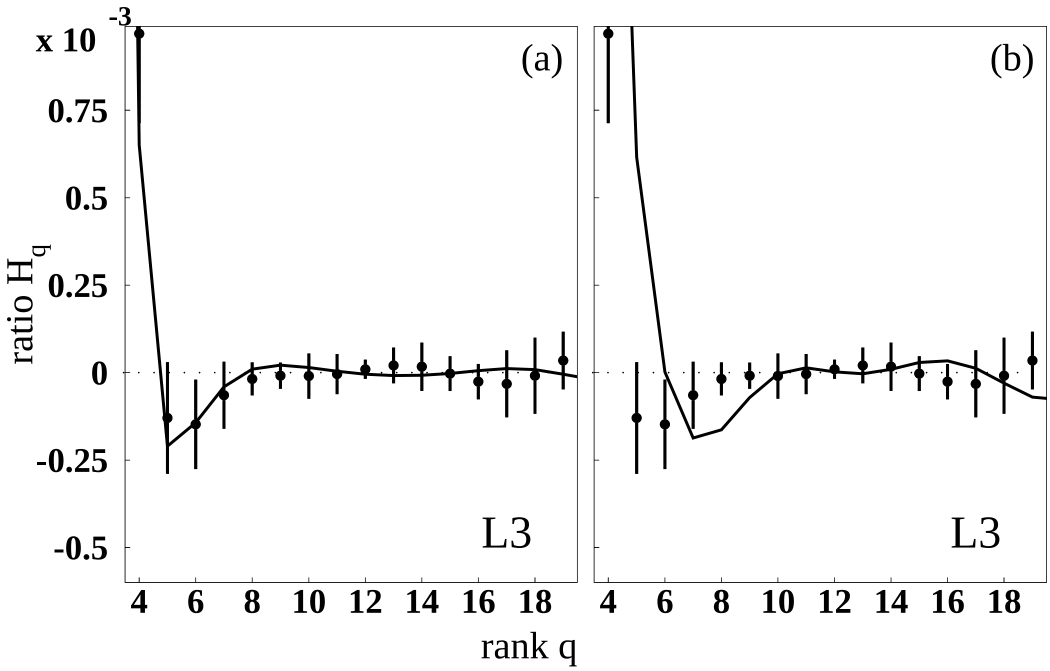
L3 preliminary compared to

JETSET HERWIG

with truncation at 53



with no truncation



No oscillations

First minimum ?

- JETSET agrees well with data; HERWIG less well

Questions

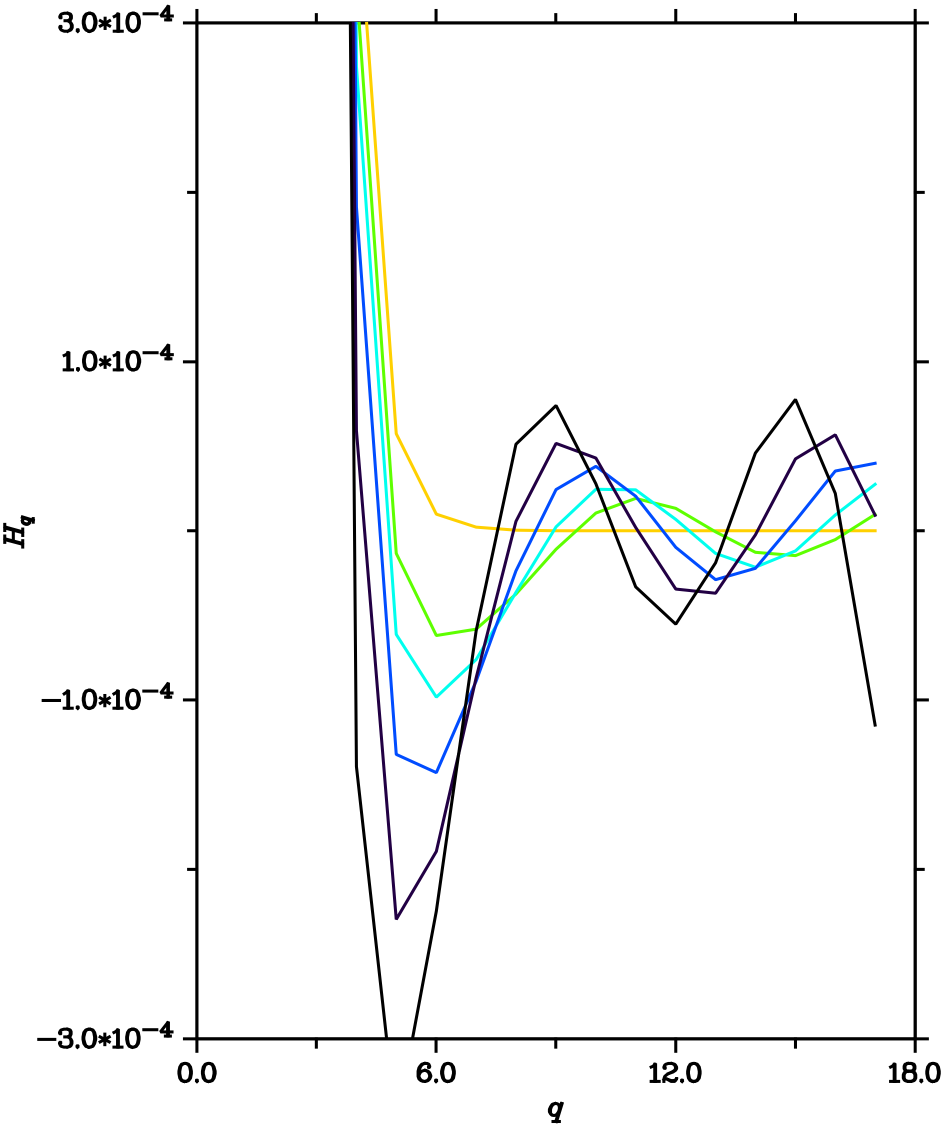
H_q appear to be biased estimators
because of "natural truncation" of tail
but are asymptotically unbiased — tail well-measured

- From SLD (87K events) to L3 (1M events)
  amplitudes of oscillations decrease (if no truncation).
  What is the asymptotic limit?
  Do the oscillations and first minimum disappear?

- Is what we see only a consequence of truncation?

Negative Binomial Distribution

NBD with same μ, σ as data (20.48, 6.253)
does not describe data, but is simple
Truncate at 500, 55, 53, 51, 49, 47



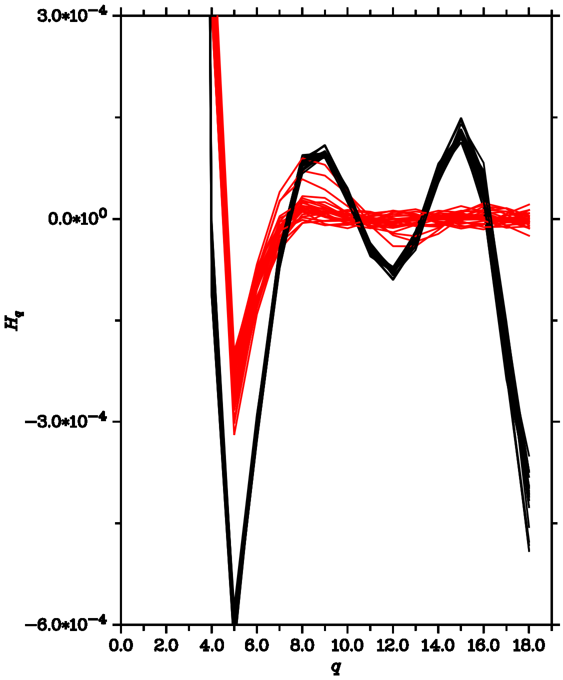
Analytically, NBD has no oscillations ≈ trunc. at 500
Truncations like in data produce first minimum and oscillations, rather like in data.

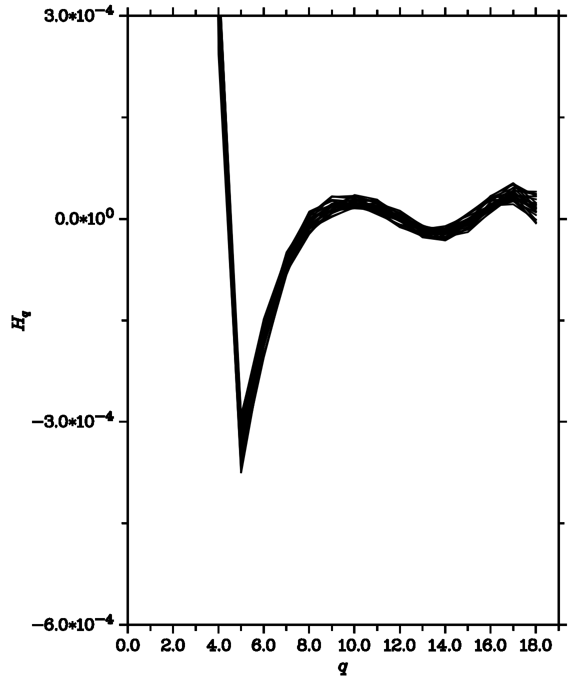
PYTHIA

PYTHIA *does* describe the data

25 samples of 1M Z decays:

Truncation: None, 47 53





Similarly to data, NBD:

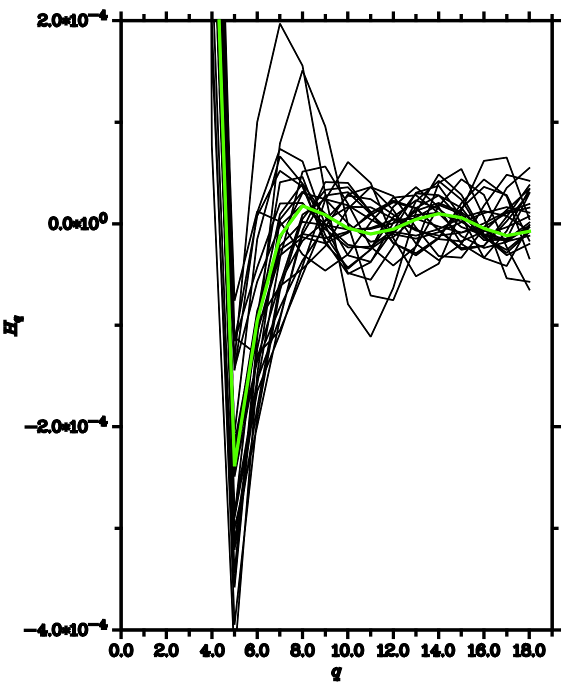
- Truncation increases amplitudes
- Truncation shifts positions of maxima

PYTHIA

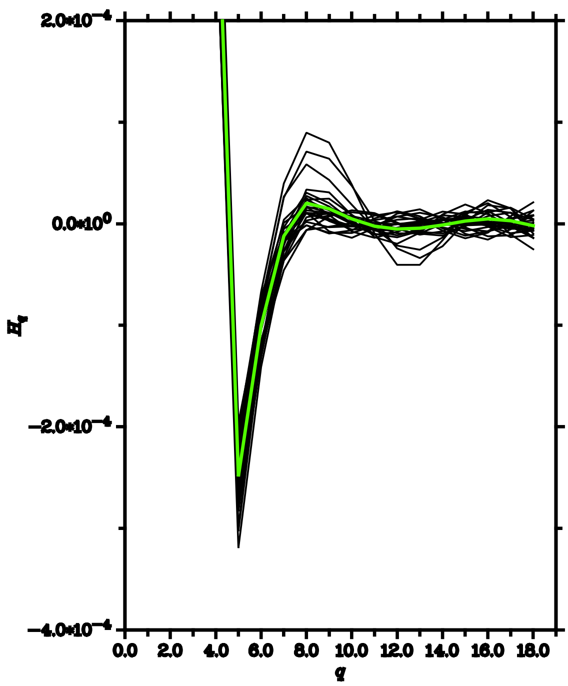
But is H_q with no truncation biased?

25 samples of Z decays, No truncation

100K events (SLD)



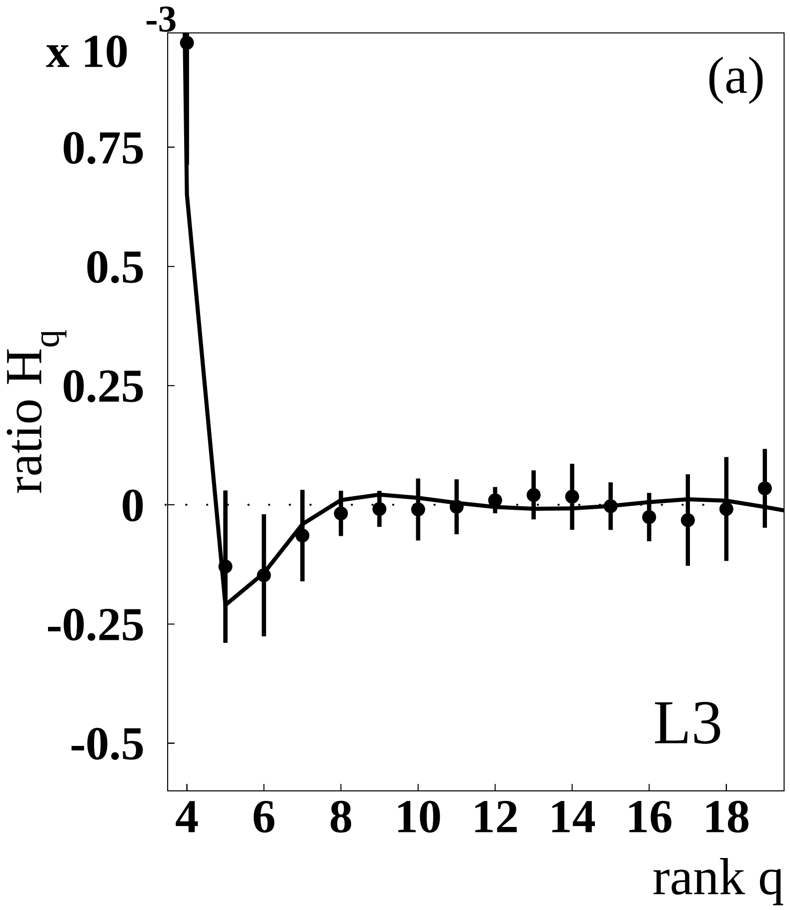
1M events (L3)

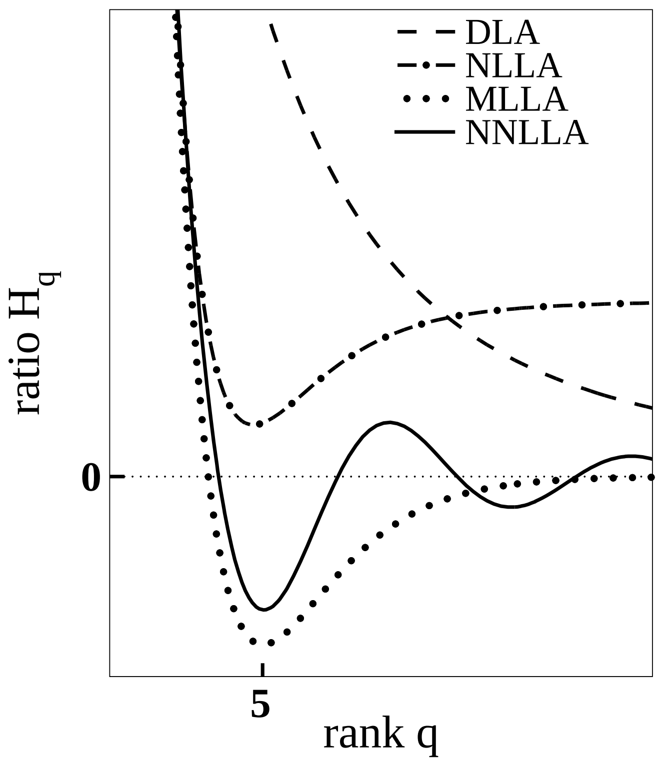


First minimum:	Bias is very small
Oscillations:	Bias on amplitudes is small at low q
	but increases with q
	Period of oscillations increases
	and amplitude decreases
	with number of events

Conclusions H_q

Use data with
NO Truncation:





Data are not like DLA

First minimum at 5

No evidence
for further oscillations

Cannot distinguish
between
MLLA and NNLLA
behavior

Intermittency indices, ϕ_q

To measure ϕ_q:

1. Partition phase space (3-dim) in M bins
2. Measure F_q as function of M
3. Fit $F_q = b_q M^{\phi_q}$ (*) $M = M_{\mathrm{y}} M_{\mathrm{p_t}} M_{\varphi}$

We need:

a. Choose a coordinate system, *e.g.*, y, p_{t}, φ

b. Does (*) hold with same number of bins in each coordinate, $M_{\mathrm{y}} = M_{\mathrm{p_t}} = M_{\varphi}$?

yes - isotropic fluctuations - self-similar fractal

no - anisotropic fluctuations - self-affine fractal

If yes, from QCD, expect (Dremin & Dokshitzer)

$$\Phi_q = 1 - \frac{\frac{\phi_q}{3} + 1}{q}$$

to increase with q to a maximum
at same q where H_q has first minimum
$\phi_q/3$ since D&D use $F_q = b_q {M_{\mathrm{y}}}^{\phi_q}$

Coordinate system

To measure fluctuations from QCD,
coordinate system must not depend on QCD

$e^+e^- \rightarrow q\bar{q}$ is cylindrically symmetric about $q\bar{q}$ axis
$q\bar{q}$ axis determined by Electro-Weak, not by QCD
Approximate $q\bar{q}$ axis by Thrust axis.
Use cylindrical coordinate system: y, p_t, φ
Do NOT choose origin of φ as, *e.g.*, Major
but choose origin at random for each event
— "Random Frame"

Use nearly entire phase space:

$$-5 < y < 5 \qquad 0.1 < p_t < 3\,\text{GeV} \qquad -\pi < \varphi < \pi$$

Use bins with same mean multiplicity

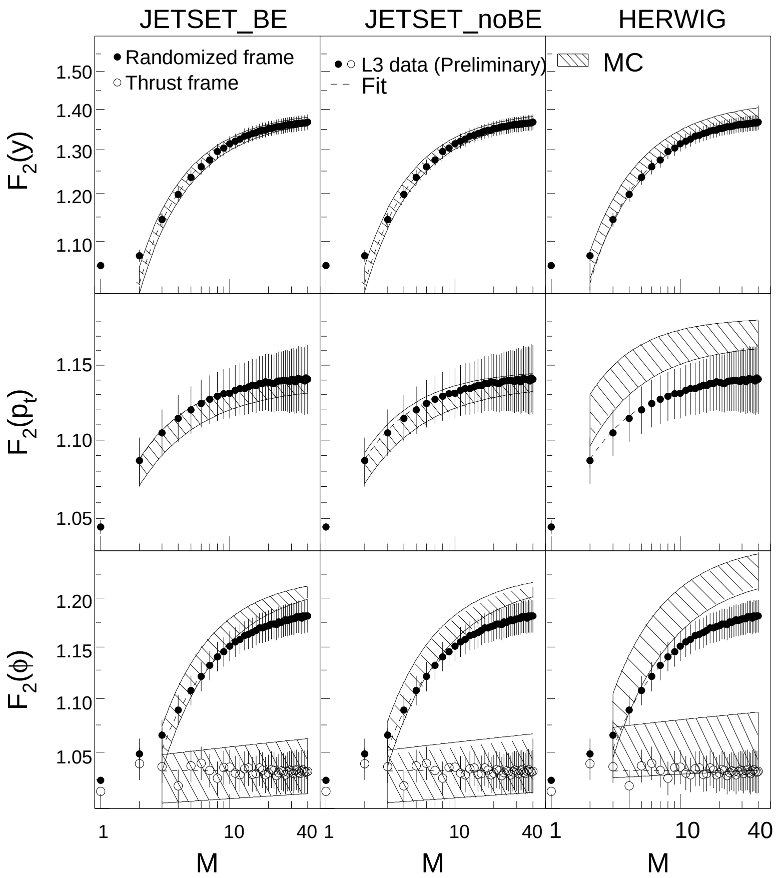
Also correct, via MC, from Random Frame to $q\bar{q}$ Frame

Self-similar or Self-affine?

Look at F_2 in 1 dimension for y, p_t, φ

Fit $F_2(M) = A - BM^{-\gamma}$

Self-Similar if $\gamma_y = \gamma_{p_\mathrm{t}} = \gamma_\varphi$



$\gamma_y = 0.992 \pm 0.014 \pm 0.028$

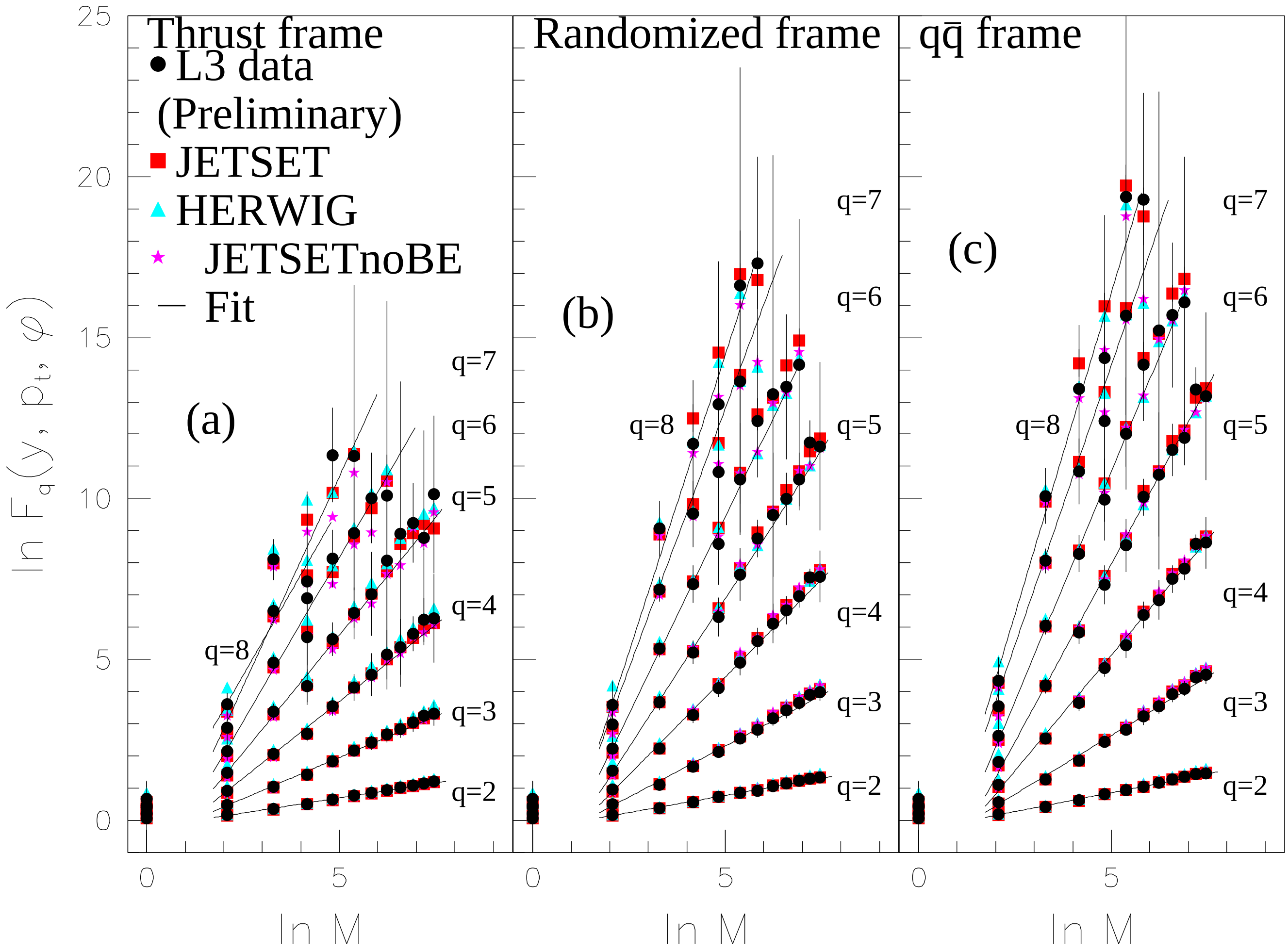
$\gamma_{p_\mathrm{t}} = 0.986 \pm 0.022 \pm 0.032$

$\gamma_\varphi = 0.993 \pm 0.024 \pm 0.033$

Conclusion: Self-Similar – use same M in each direction

Note much reduced fluctuations in φ if Major defines origin
JETSET and JETSET with no BE agree well with data;
HERWIG less well

Three Dimensions



(a) Note reduced fluctuations in φ if Major defines origin

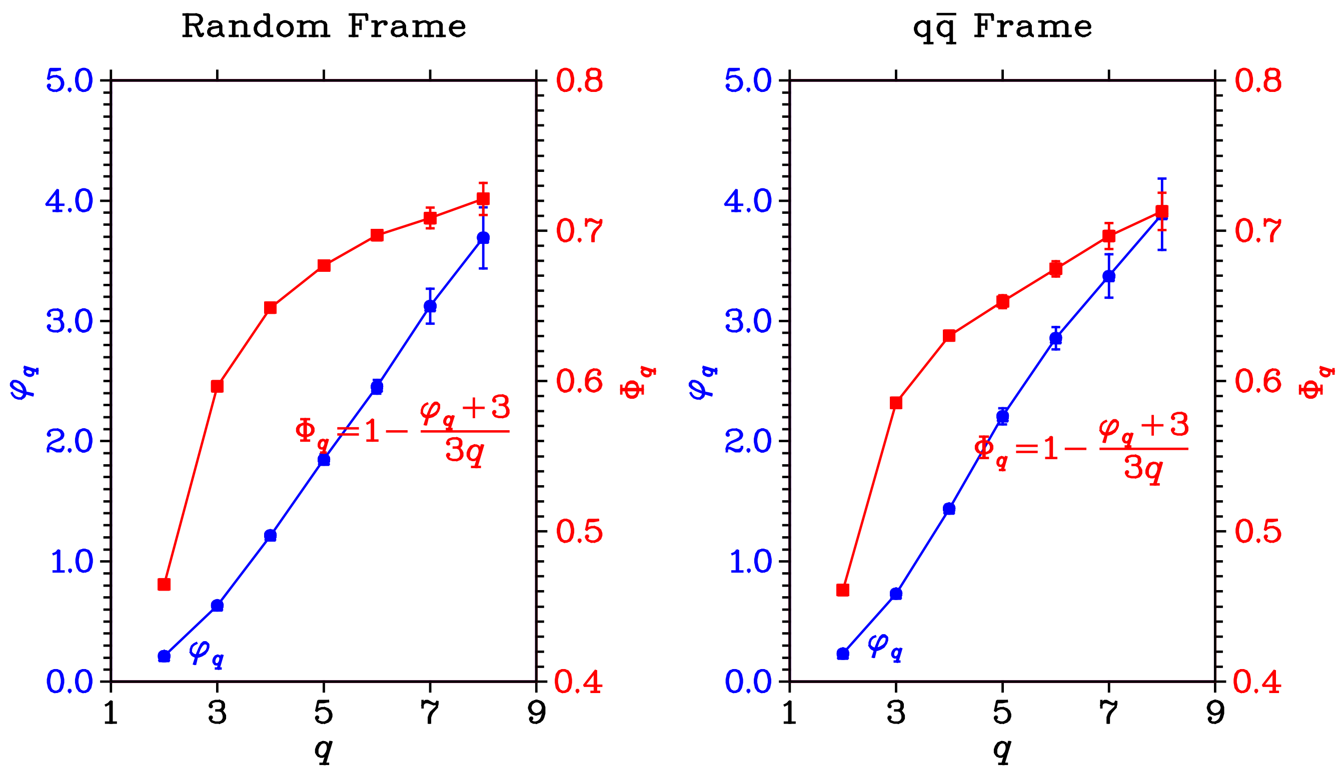
(b) Random origin for φ

(c) Correct (using JETSET) for difference Thrust/$q\bar{q}$ axis

JETSET, HERWIG agree with data

ϕ_q in Three Dimensions and Φ_q

Fit $F_q = b_q M^{\phi_q}$,

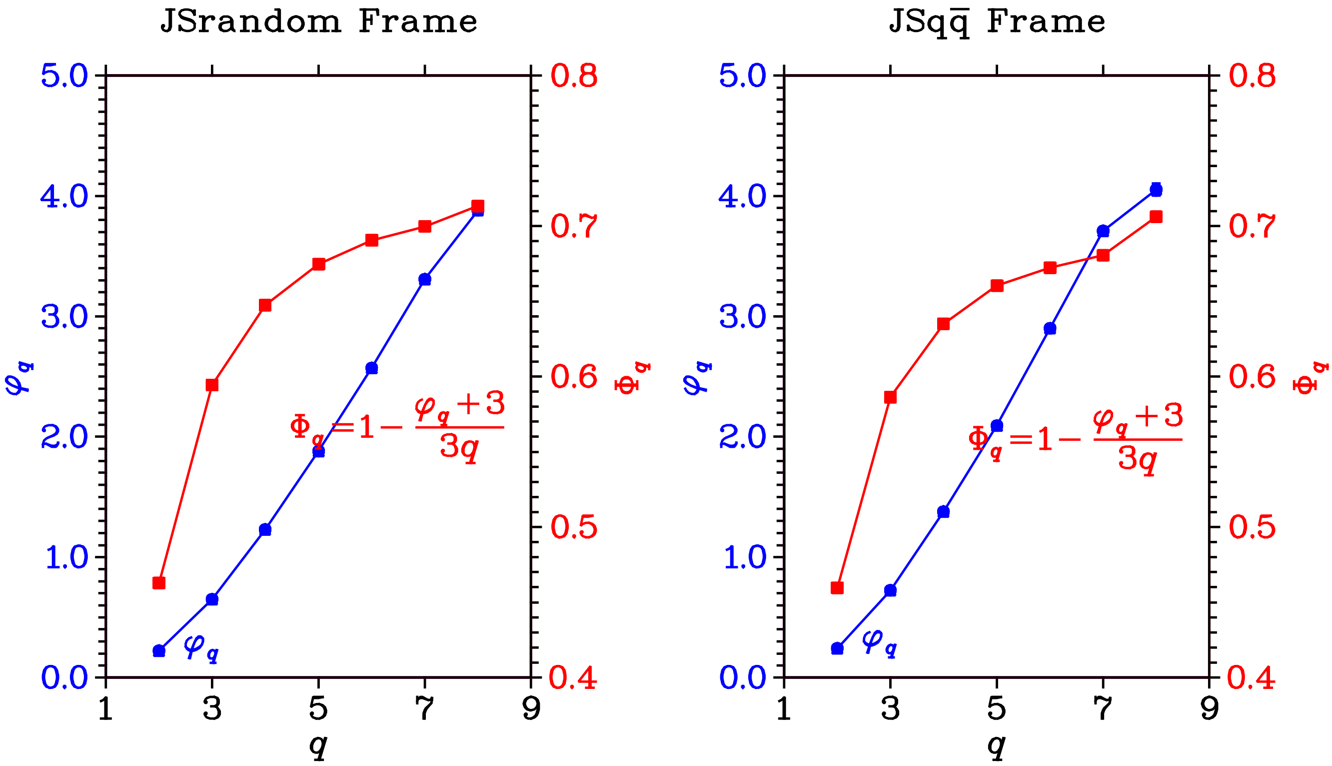


Does Φ_q have a max.?

If so, at $q > 7$

ϕ_q in Three Dimensions – JETSET

Fit $F_q = b_q M^{\phi_q}$,



Does Φ_q have a max.?

If so, at $q > 7$

Also in JETSET

Conclusions

- H_q
  - Data are not like DLA
  - First minimum at 5
  - Marginal evidence that it is negative
  - No evidence for oscillations
  - more like MLLA than NNLLA
- ϕ_q
  - dynamical fluctuations are isotropic self-similar fractal
  - If Φ_q has max., at $q > 7$
- Question to theorists:
  no oscillations (or at least not confirmed),
  Φ_q max at $q > 7$
  Is this a problem?
- JETSET agrees well everywhere