CORRELATIONS BETWEEN <p_T> AND JET MULTIPLICITIES FROM THE BFKL POMERON

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HIGH-ENERGY HADRONIC COLLISIONS :

→ STRONG CORRELATIONS ARE OBSERVED EXPERIMENTALLY BETWEEN THE $\langle p_T \rangle$ AND THE MULTIPLICITIES OF THE PRODUCED PARTICLES.

→ $\langle p_T \rangle$ GROWS WITH MULTIPLICITY.

SEVERAL EXPLANATIONS → IT IS ASSUMED THAT WITH ONLY ONE HARD COLLISION (ONE PAIR OF NON-INTERACTING STRINGS) THERE ARE NO CORRELATIONS BETWEEN $\langle p_T \rangle$ AND MULTIPLICITY.

THIS ASSUMPTION CAN BE TESTED WITHIN THE BFKL DYNAMICS, WHICH PRESENTS A DETAILED DESCRIPTION OF PARTICLE (JET) PRODUCTION AT HIGH ENERGIES.

- OUR AIM: TO SEE WHETHER THERE EXIST CORRELATIONS BETWEEN $\langle p_T \rangle$ AND THE NUMBER OF PRODUCED JETS IN THE HARD POMERON DESCRIBED BY THE BFKL CHAIN OF INTERACTING REGGEIZED GLUONS.

THIS STUDY IS RELATED TO THAT BY KWIECINSKI, LEWIS & MARTIN (PRD 54 (1996) 6664) WHO CALCULATED THE EXCLUSIVE PROBABILITIES TO OBSERVE A GIVEN NUMBER OF PRODUCED JETS FROM THE EXCHANGED HARD POMERON.

FORMALISM USED BY KLM

LET $f(y,k)$ BE THE AMPUTATED BFKL AMPLITUDE,

WHERE : $y \longrightarrow$ RAPIDITY.

$k \longrightarrow$ TWO-DIMENSIONAL TRANSVERSE MOMENTUM OF THE VIRTUAL (REGGEIZED) GLUON.

$\dfrac{f(y,k)}{k^2} \longrightarrow$ UNINTEGRATED GLUON DISTRIBUTION.

THE BFKL EQUATION FOR f :

$$\boxed{f(y,k) = f^{(0)}(y,k) + \overline{\alpha}_s \cdot \int_0^y dy_1 \cdot \int \frac{d^2 k_1}{\pi q^2} \cdot \left(\frac{k^2}{k_1^2} f(y_1,k_1) - f(y,k) \cdot \theta(k^2 - q^2) \right)}.$$

HERE : $\overline{\alpha}_s = \dfrac{3 \cdot \alpha_s}{\pi}$.

$q = k - k_1 \longrightarrow$ THE TRANSVERSE MOMENTUM OF THE EMITTED (REAL) GLUON.

$f^0(y,k) \longrightarrow$ THE IMPACT FACTOR OF THE TARGET.

TO SUPPRESS THE PHISICALLY UNKNOWN INFRARED DOMAIN AND MAKE THE EQUATION NUMERICALLY TRACTABLE, THE INTEGRATION OVER k_1 IS CONSTRAINED TO THE INTERVAL:

$$Q_0 \left(1\ \mathrm{GeV}/c\right) < k_1 < Q_f \left(100\ \mathrm{GeV}/c\right)$$

→ DEFINITION : AN OBSERVABLE JET IS A REAL GLUON WITH $q^2 \geq \mu^2$.

THIS DEFINITION SPLITS THE INTEGRATION OVER MOMENTA INTO TWO PARTS.

→ TWO PARTS INTO THE INTEGRATION KERNEL:

- K_R (RESOLVED ONE) → EMITTED GLUONS WITH $q^2 > \mu^2$.
- K_{UV} (UNRESOLVED ONE) → IT COMBINES EMISSION OF GLUONS WITH $q^2 < \mu^2$ AND THE SUBSTRACTION TERM.

EXPLICITLY:

$$\left(K_R f\right)(k) = \bar{\alpha}_s \cdot k^2 \cdot \int \frac{d^2 k_1}{\pi q^2 k_1^2} \cdot \theta\left(q^2 - \mu^2\right) \cdot f\left(k_1\right)$$

$$\left(K_{UV} f\right)(k) = \bar{\alpha}_s \cdot k^2 \cdot \int \frac{d^2 k_1}{\pi q^2 k_1^2} \left(\theta\left(\mu^2 - q^2\right) \cdot f\left(k_1\right) - \frac{k_1^2}{k^2} \theta\left(k^2 - q^2\right) \cdot f(k)\right)$$

→ EXCLUSIVE PROBABILITIES TO PRODUCE n JETS ARE OBTAINED BY INTRODUCING n OPERATORS K_R BETWEEN THE GREEN'S EQUATIONS WITH KERNEL K_{UV}.

EXCLUSIVE PROBABILITIES TO PRODUCE n JETS

IF ONE WRITES

$$f(y) = \sum_{n=0} f_n(y)$$

CONTRIBUTION TO THE FULL GLUON DISTRIBUTION FROM THE PRODUCTION OF n JETS.

THEN ONE GETS A RECURSIVE RELATION

$$f_n(y) = \int_0^y dy\, k(y-y_1) \cdot f_{n-1}(y) \quad (*)$$

WHERE

$$k(y) = e^{y \cdot k_{UV}} \cdot k_R$$

THIS RELATION ONE TO SUCCESSIVELY CALCULATE THE RELATIVE PROBABILITIES TO PRODUCE $n = 0, 1, 2, \dots$ JETS STARTING FROM THE NON-JET CONTRIBUTION

$$f_0(y) = e^{y k_{UV}} f^{(0)}(0) + \int_0^y dy_1 \cdot e^{(y-y_1) \cdot k_{UV}} \cdot \frac{d f^{(0)}(y_1)}{d y_1}$$

$f^{(0)}(y, k)$ IS CHOSEN TO VANISH AT $y = 0$.

THE DISTRIBUTIONS $f_n(y,k)$ ARE NOT OBSERVABLES.

- PHYSICAL PROBABILITIES ARE OBTAINED BY CONVOLUTING f_n WITH THE GLUON DISTRIBUTION IN THE PROJECTILE (THE PROJECTILE IMPACT FACTOR).

- PERTURBATIVE QCD ⟶ VIRTUAL PHOTON AS A PROJECTILE.
- HADRONIC COLLISIONS ⟶ UNPERTURBATIVE IMPACT FACTOR.

IN BOTH CASES THE EXCLUSIVE PROBABILITIES TO OBSERVE n JETS ARE GIVEN BY

$$\longrightarrow P_n(y) = \frac{\int (d^2k/k^4)\, h(k)\, f_n(y,k)}{\int (d^2k/k^4)\, h(k)\, f(y,k)}$$

WHERE $h(k)$ IS THE IMPACT FACTOR OF THE PROJECTILE.

BOTH IMPACT FACTORS, $f^{(0)}(k)$ OF THE TARGET AND $h(k)$ OF THE PROJECTILE, SHOULD VANISH AS $k \to 0$.

- VIRTUAL PHOTON IMPACT FACTOR BY NIKOLAEV & ZAKHAROV Z. PHYS. C 49, 607 (1991).

- HADRONIC IMPACT FACTOR:

$$f^{(0)}(k) = k^2 \cdot e^{-k^2/Q_0^2}$$

→ WE ARE INTERESTED IN THE $\langle q \rangle_m$ OF THE OBSERVED JETS, PROVIDED THEIR NUMBER m IS FIXED.

SINCE THE MOMENTUM q OF THE EMITTED GLUON IS HIDDEN INSIDE THE KERNEL K_R, TO FIND AN AVERAGE OF ANY QUANTITY $\phi(q)$ ONE HAS TO CHANGE K_R TO THE KERNEL K_{av} DEFINED BY

$$\boxed{\left(K_{av} f\right)(k) = \bar{\alpha}_s k^2 \int \frac{d^2 k_1}{\pi q^2 k_1^2} \theta\left(q^2 - \mu^2\right) \cdot \phi(q) \cdot f(k_1)}$$

→ WITH m JETS ONE HAS TO:

- SUBSTITUTE ONE OF THE m OPERATORS K_R WHICH GENERATE THE JETS BY K_{av}.
- TAKE A SUM OF ALL SUCH SUBSTITUTIONS, AND DIVIDE BY m.
- INTEGRATE OVER ALL MOMENTA OF THE VIRTUAL GLUON k MULTIPLIED BY THE PROJECTILE IMPACT FACTOR.
- NORMALIZE THE RESULT TO THE TOTAL PROBABILITY TO HAVE m JETS.

• HOW TO FORMALIZE THIS RECIPE?

→ INTRODUCE A GENERALIZED OPERATOR IN THE VIRTUAL GLUON MOMENTUM SPACE

$$K_1(y) = e^{yK_{UV}}\left[K_R + K_{QV}\right]$$

LET THE FUNCTION $F(y,k)$ OBEY THE EQUATION

$$F(y) = f_0(y) + \int_0^y dy_1\, K_1(y-y_1)\cdot F(y_1)$$

→ ONE CAN SPLIT THE FUNCTION F INTO A SUM OF CONTRIBUTIONS F_{nm} CORRESPONDING TO THE ACTION OF n OPERATORS K_1, OUT OF WHICH $m = 0, 1, \dots, n$, ARE OPERATORS K_{QV}:

$$\boxed{F(y) = \sum_{n=0} \sum_{m=0}^{n} F_{nm}(y).}$$

$$F_{n0} = f_n$$

WE ARE INTERESTED IN THE CONTRIBUTION $F_{n1} \equiv g_n$ WHICH CONTAINS A SINGLE OPERATOR K_{QV}.

→ THE AVERAGE VALUE OF INTEREST IS DETERMINED BY

$$\boxed{\left\langle \phi(q) \right\rangle_n = \frac{1}{n} \frac{\int (dk^2/k^4)\, h(k)\, g_n(y,k)}{\int (dk^2/k^4)\, h(k)\, f_n(y,k)}}$$

AGAIN ONE EASILY SETS UP A RECURSION RELATION FOR g_m:

$$\boxed{g_m = \int_0^y dy_1\, K(y-y_1)\, g_{m-1}(y_1) + \int_0^y dy_1 \cdot e^{(y-y_1)K_{uv}} \cdot K_{av}\, f_{m-1}(y_1) \qquad (**)}$$

WITH THE INITIAL CONDITION $g_0(y) = 0$.

TOGETHER WITH $(*)$, THIS RELATION ALLOWS ONE TO CALCULATE THE FUNCTION g_m FOR $m = 1, 2, \dots,$ AND THEN TO FIND THE DESIRED AVERAGES.

THE CONCRETE CHOICE OF $\phi(q)$ IS RESTRICTED BY THE CONDITION OF CONVERGENCE AT LARGE q:

$$\phi(q) < q^2, \quad \text{AS } q \to \infty$$

WE MAKE A NATURAL CHOICE $\phi(q) = q$:

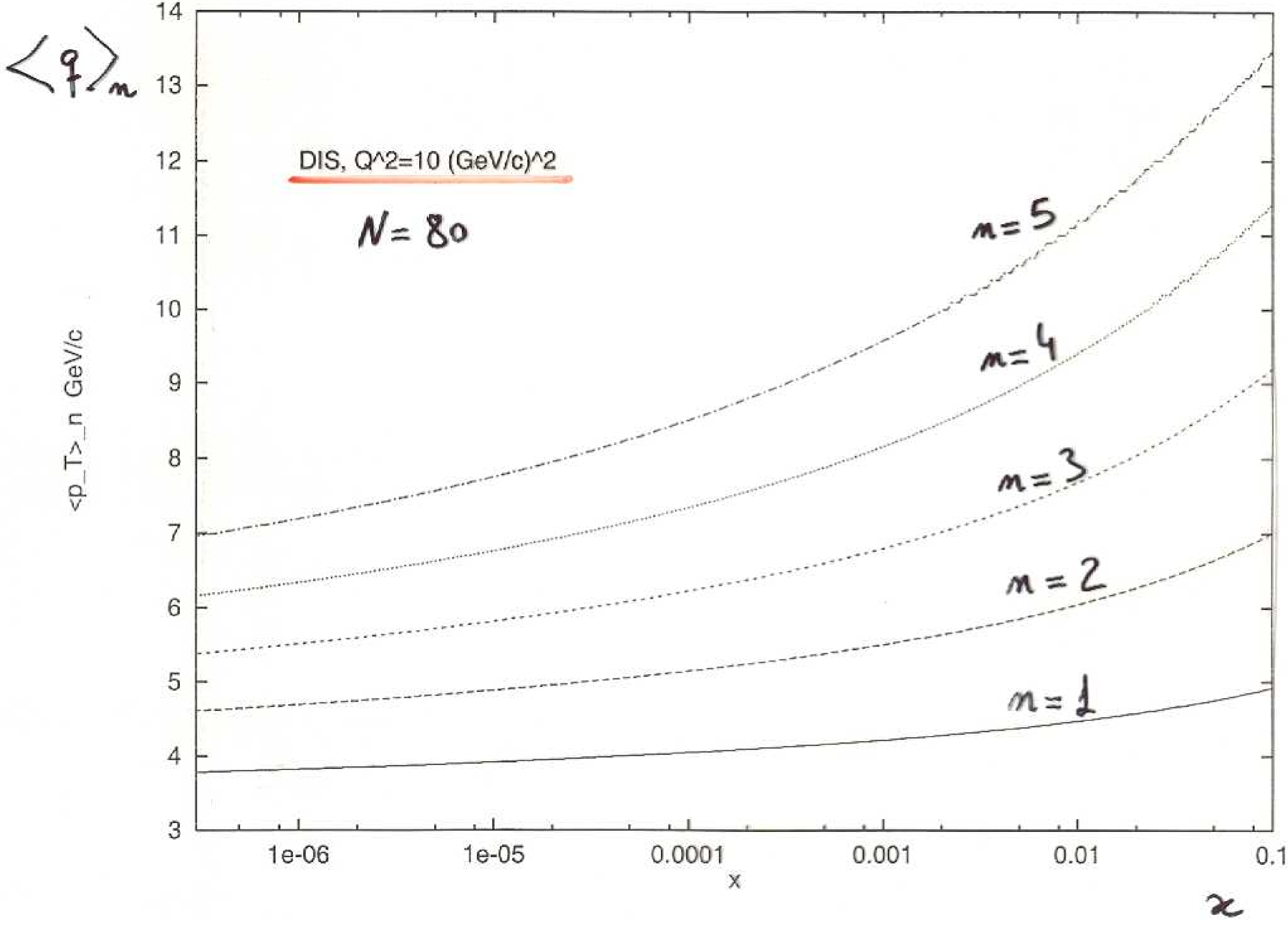
$$\boxed{(K_{av} f)(k) = \bar{\alpha}_s \cdot k^2 \int \frac{d^2 k_1}{\pi \cdot k_1^2\, q}\, \theta(q^2 - \mu^2)\, f(k_1)}$$

WITH THIS CHOICE THE ANGULAR INTEGRATION CAN BE DONE ANALITICALLY.

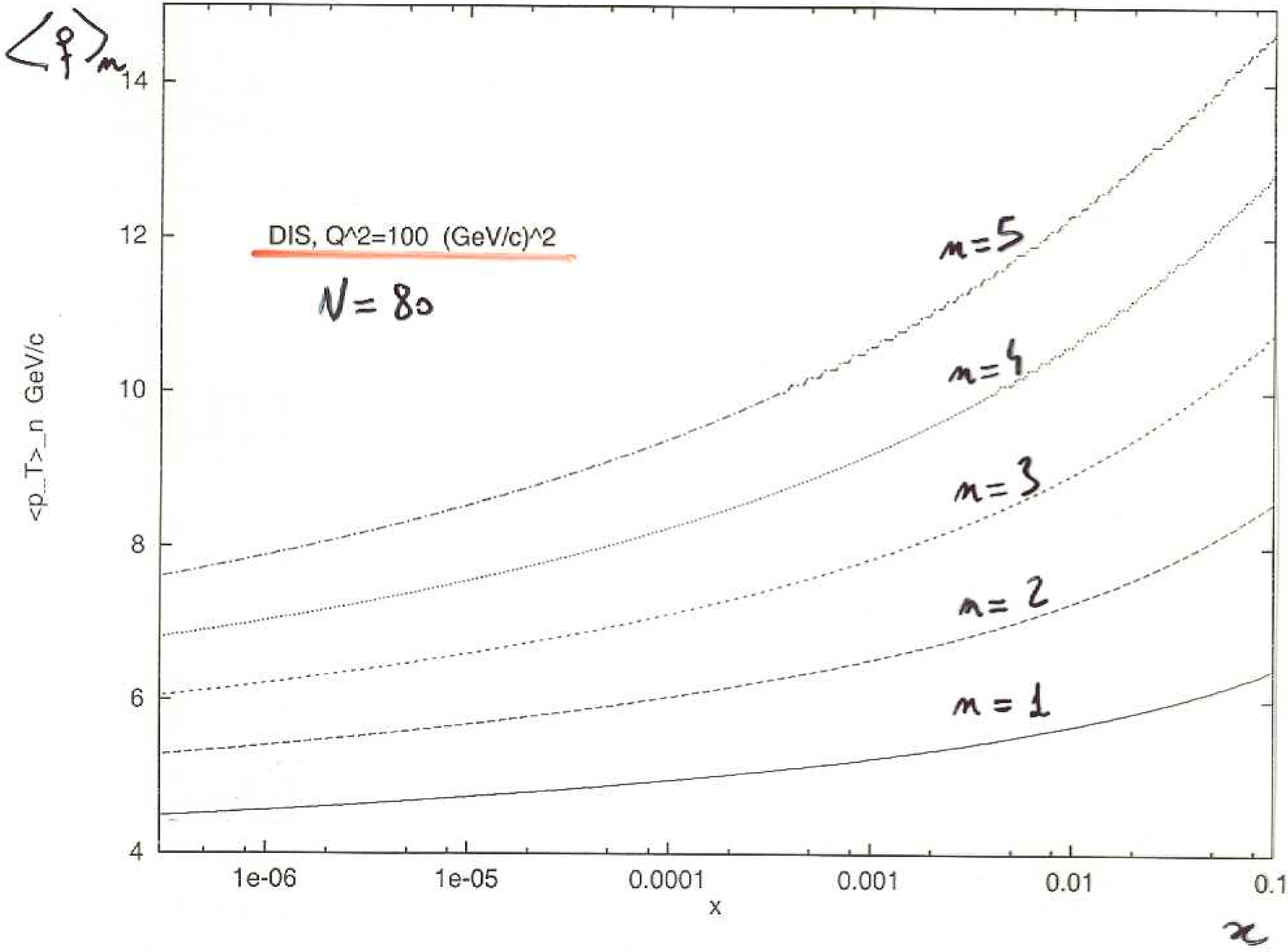
WE DEFINED OUR JETS BY TAKING $\mu = 2\,\text{GeV}/c$.

WE HAVE CALCULATED f_m AND g_m FROM $(*)$ AND $(**)$ UP TO $m = 5$ AND $y = 15$. TO DISCRETIZE THE KERNELS WE HAVE USED THE EXPANSION IN N CHEBYSHEV POLINOMIALS

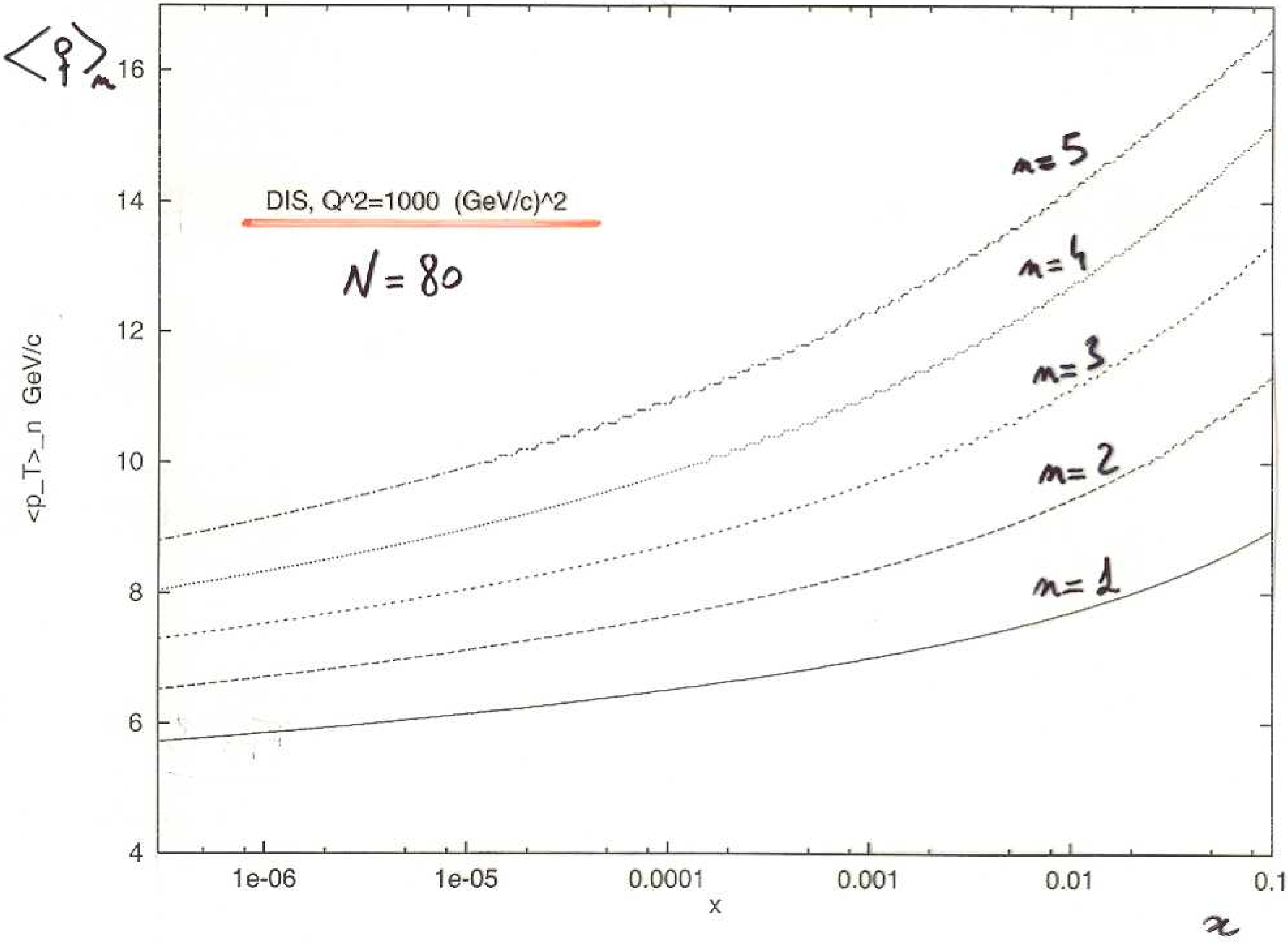
RESULTS



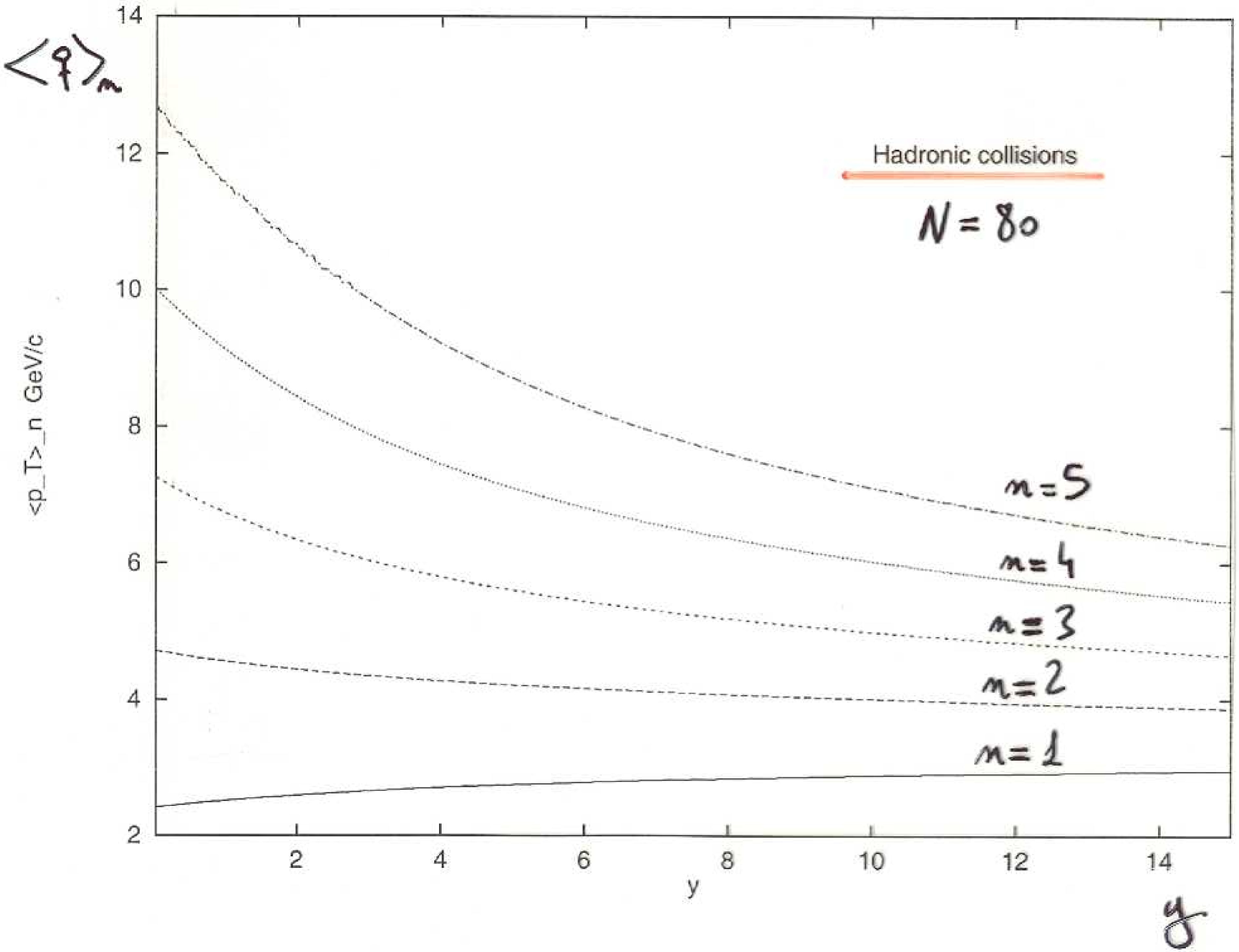
$\langle q \rangle_m$
<p_T>_n GeV/c
14
13
12
11
10
9
8
7
6
5
4
3
DIS, Q^2=10 (GeV/c)^2
$N = 80$
$m=5$
$m=4$
$m=3$
$m=2$
$m=1$
1e-06
1e-05
0.0001
0.001
0.01
0.1
x
x



$\langle q \rangle_m$
<p_T>_n GeV/c
14
12
10
8
6
4
DIS, Q^2=100 (GeV/c)^2
N = 80
m=5
m=4
m=3
m=2
m=1
1e-06
1e-05
0.0001
0.001
0.01
0.1
x
x



$\langle q \rangle_m$
<p_T>_n GeV/c
16
14
12
10
8
6
4
DIS, Q^2=1000 (GeV/c)^2
$N = 80$
$m=5$
$m=4$
$m=3$
$m=2$
$m=1$
1e-06
1e-05
0.0001
0.001
0.01
0.1
x
x



Hadronic collisions
N = 80
<p_T>_n GeV/c
<q>_n
n=5
n=4
n=3
n=2
n=1
y
14
12
10
8
6
4
2

→ AS ONE OBSERVES, IN ALL CASES $\langle q \rangle_m$ STRONGLY GROWS WITH m AT ALL RAPIDITIES. THE GROWTH IS APPROXIMATELY LINEAR

$$\langle p_T \rangle_m \simeq a(y, Q^2) + b(y) \cdot m$$

WHERE BOTH a AND b DEPEND ON RAPIDITY y, BUT, AT FIXED y, b IS UNIVERSAL.

- THE SLOPE $b(y)$ FALLS WITH y: IT IS EQUAL TO 1.25 GeV/c AT $y = 7.5$ AND 0.8 GeV/c AT $y = 15$, SO THAT AT ULTRAHIGH ENERGIES ONE MAY EXPECT THAT $\langle q \rangle_m$ WILL BECOME INDEPENDENT OF m.

- THE DEPENDENCE ON Q^2 RESULTS IS RATHER WEAK. IN THE BFKL MODEL AT LOW x THE BULK OF THE Q^2 DEPENDENCE IS SEPARATED IN AN OVERALL FACTOR WHICH CANCELS BETWEEN THE NUMERATOR AND DENOMINATOR WHEN COMPUTING THE AVERAGES.

WE FIND THAT THE AVERAGES $\langle q \rangle_m$ GO DOWN WITH RAPIDITY FOR ALL $m \geq 2$. THIS IS QUITE UNEXPECTED, SINCE IN BFKL AN OVERALL AVERAGE $\langle q \rangle$ RAPIDLY GROWS WITH y. IT SEEMS THAT THIS GROWTH IS TOTALLY EXPLAINED BY THE GROWTH OF THE AVERAGE NUMBER OF JETS $\langle m \rangle$.

DISCUSSION

- EMISSIONS OF HIGH-p_T JETS IN γ^*-HADRON COLLISIONS SEEM TO BE A SUITABLE PLACE TO SEE THE BFKL SIGNATURE.

- WE OBTAIN IN SUCH EMISSIONS STRONG POSITIVE CORRELATIONS BETWEEN $\langle p_T \rangle$ AND THE NUMBER OF JETS ALREADY FOR A SINGLE POMERON EXCHANGE.

  THIS INDICATES THAT THE PREDICTED CORRELATIONS DO NOT REQUIRE MULTIPLE RESCATTERINGS NOR POMERONIC INTERACTIONS, BUT THAT THEY ARE ALREADY PRESENT IN THE BASIC MECHANISM OF JET PRODUCTION.

- IF WE EXTRAPOLATE OUR RESULT TO ALL n AND ENERGIES WE WOULD GET A RELATION BETWEEN THE OVERALL AVERAGES

$$\langle p_T \rangle \propto \langle n \rangle$$

HOWEVER, THIS RELATION DOES NOT HOLD IN THE BFKL MODEL AT ASYMPTOTIC ENERGIES, WHEN $\langle p_T \rangle$ GROWS MUCH FASTER THAN $\langle n \rangle$.

THUS OUR RESULTS CANNOT BE VALID FOR ALL n AND ENERGIES AND REFER PRECISELY TO THE VALUES OF n AND ENERGIES FOR WHICH THE CALCULATIONS WERE DONE.

- THE UNEXPECTED RESULT THAT $\langle p_T \rangle_n$ AT FIXED $n \geq 2$ FALL WITH ENERGY IS NOT DRAMATIC BUT QUITE APPRECIABLE AT ENERGIES AT WHICH WE CAN EXPECT THE BFKL POMERON TO BE RELEVANT $(y > 10)$.

AT PRESENT WE DO NOT HAVE ANY PLAUSIBLE EXPLANATION OF THIS PHENOMENON, WHICH DESERVES FURTHER INVESTIGATION INCLUDING HIGHER y AND/OR n.

→ WE HOPE THAT THIS EFFECT CAN BE TESTED EXPERIMENTALLY AS A POSSIBLE SIGNATURE OF THE BFKL POMERON.