Baxter-Sklyanin representation for the interaction of reggeized gluons

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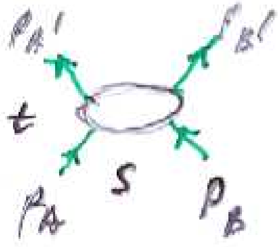
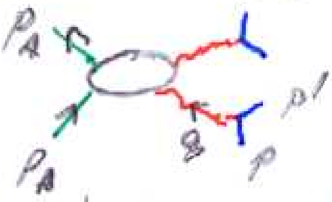
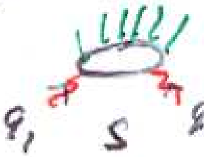
Regge kinematics: $s=(p_A+p_B)^2 \gg -t \sim m^2$



Applications: DIS at small $x = \frac{Q^2}{2pq}$

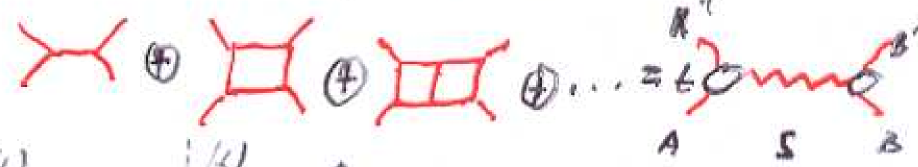


$\gamma^*\gamma^* \to$ hadrons



$s \gg |q_1||q_2|$

Gluon reggeization:

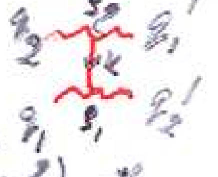


$$A(s,t) = \frac{1}{t}\Gamma^c_{A'A}\left(\left(\frac{s}{-t}\right)^{j(t)} - \left(\frac{-s}{-t}\right)^{j(t)}\right)\Gamma^c_{B'B}$$

Regge trajectory: $j(t) = 1 + \frac{g^2}{8\pi^2}N_c\,\omega_1(t) + O(g^4)$

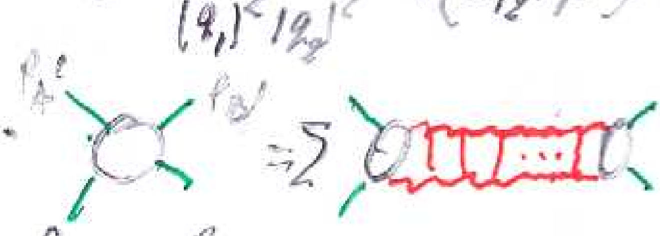
$$\omega_1 = -\ln\frac{-t}{\mu^2}$$

Reggeon interactions



$= -\frac{g^2}{8\pi^2}V_{12}$,

$$V_{12} = \frac{q_1 q_2^*}{|q_1|^2|q_2|^2}\ln(|\rho_{12}|^2\mu^2)\,q_1^* q_2 + h.c.,\quad H_{12} = T_{12}+V_{12},\quad T_{12} = \ln\frac{|p_1|^2}{\mu^2}+\ln\frac{|p_2|^2}{\mu^2}$$



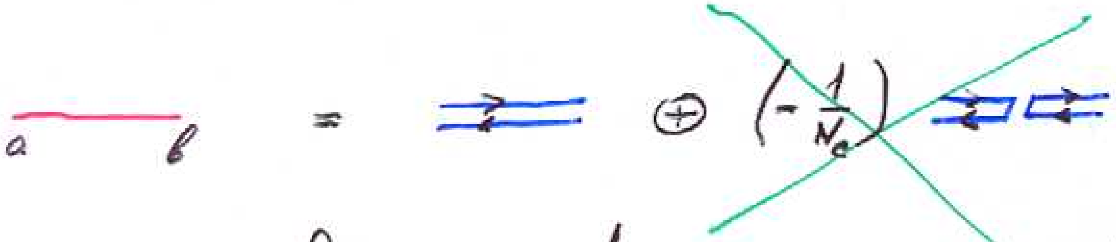
$\sim c\, s^{1+\Delta_{12}}$, $\Delta_{12} = -\frac{g^2 N_c}{8\pi^2}E_{12}$

BFKL equation: $E\,\Psi(\rho_1,\rho_2;\rho_0) = H_{12}\Psi(\rho_1,\rho_2;\rho_0)$

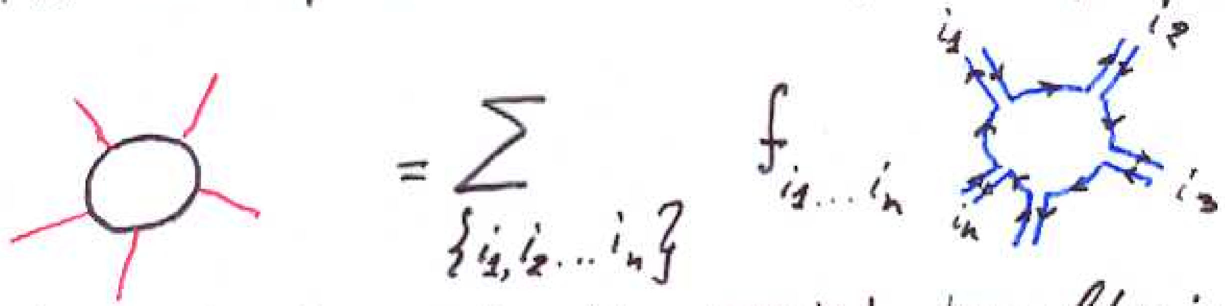
Möbius invariant solution: $\Psi(\rho_1,\rho_2;\rho_0) = \left(\frac{\rho_{12}}{\rho_{10}\rho_{20}}\right)^m \left(\frac{\rho_{12}^*}{\rho_{10}^*\rho_{20}^*}\right)^{\tilde m}$

$m = \frac12 + i\nu + \frac n2$, $\tilde m = \frac12 + i\nu - \frac n2$, $\min E = -8$.

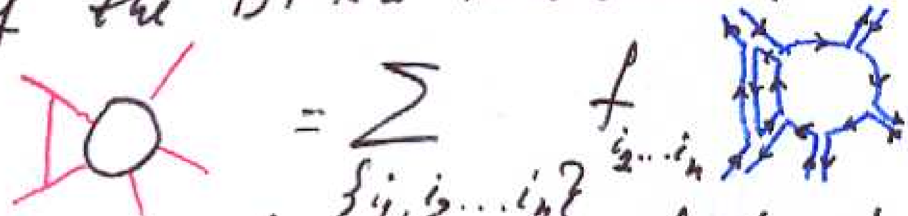
② High energy scattering in the multi-colour QCD



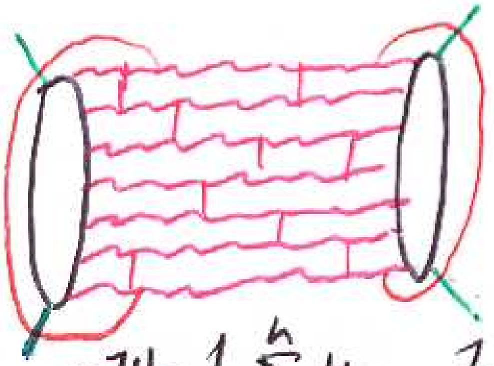
The wave function of n reggeized gluons



Colour structure of the BFKL Hamiltonian

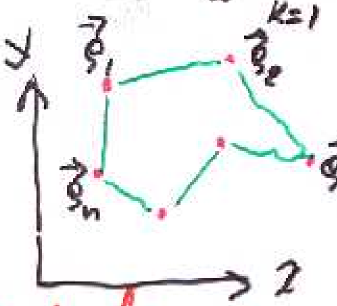


Neighbouring gluons in the octet state → factor $\frac{1}{2}$

$\sum$  $\sim i^{n-1} s^{1+\Delta}$

$$\Delta = -\frac{g^2}{8\pi^2} N_c E$$

$E\Psi = \frac{1}{2}\sum_{k=1}^{n} H_{k,k+1}\Psi$ – only neighbouring gluons interac



finite chains string

$$\rho_k = x_k + i y_k\,, \quad \rho_k^* = x_k - i y_k$$

$$H_{k,k+1} = \ln|p_k|^2 + \ln|p_{k+1}|^2 + \frac{1}{p_k}\frac{1}{p_{k+1}^*}\ln|\rho_{k,k+1}|^2 p_k p_{k+1}^* + \frac{1}{p_k^*}\frac{1}{p_{k+1}}\ln|\rho_{k,k+1}|^2 p_k^* p_{k+1} + 4\gamma$$

③ Möbius invariance, holomorphic factorization, duality symm[etry]

Möbius transformation: $\rho_k \to \frac{a\rho_k + b}{c\rho_k + d}$; a,b,c,d - complex

H is invariant, therefore Ψ is an eigenfunction of two Casimir operators (L.L. (1986)), $\boxed{\begin{array}{l} m = \frac{1}{2} + i\nu + \frac{n}{2} \\ \tilde{m} = \frac{1}{2} + i\nu - \frac{n}{2} \end{array}}$

$$M^2 \Psi_{m,\tilde{m}} = m(m-1)\Psi_{m,\tilde{m}}, \quad M^{*2}\Psi_{m,\tilde{m}} = \tilde{m}(\tilde{m}-1)\Psi_{m,\tilde{m}}$$

principal series of unitary repr[esentations]

$$M^2 = \left(\sum_{k=1}^{n} \vec{M}_k\right)^2 = -\sum_{k<k'} \rho_{k,k'}^2 \frac{\partial}{\partial\rho_k}\frac{\partial}{\partial\rho_{k'}}, \quad \rho_{k,k'} = \rho_k - \rho_{k'}$$

Eigenfunction for 2 gluons $\Psi_{m,\tilde{m}} = \left(\frac{\rho_{12}}{\rho_{10}\rho_{20}}\right)^m \left(\frac{\rho_{12}^*}{\rho_{10}^*\rho_{20}^*}\right)^{\tilde{m}}$

Eigenvalue of BFKL Hamiltonian. (L.L. (1989))

$$E_{m,\tilde{m}} = \varepsilon_m + \varepsilon_{\tilde{m}}, \quad \varepsilon_m = \psi(m) + \psi(1-m) - 2\psi(1)$$

$$\psi(x) = \frac{\Gamma'(x)}{\Gamma(x)}$$

Holomorphic separability of H: (1989)

$$H = \frac{1}{2}(h + h^*), \quad h = \sum_k \left[\ln p_k + \ln p_{k+1} + \frac{1}{p_k}\ln(\rho_{k,k+1}) p_k + \frac{1}{p_{k+1}}\ln(\rho_{k,k+1}) p_{k+1} \ldots\right]$$

Holomorphic factorization of $\Psi_{m,\tilde{m}}$

$$\Psi_{m,\tilde{m}} = \sum_{r,s} C_{r,s}\, \Psi_m^{(r)}(\rho_1, \rho_2 \ldots \rho_n)\, \Psi_{\tilde{m}}^{(s)}(\rho_1^*, \rho_2^*, \ldots \rho_n^*)$$

$C_{r,s}$ are found from the single-valuedness of $\Psi_{m,\tilde{m}}$

Duality symmetry (1999) $p_k \to \rho_{k,k+1} \to p_{k+1} \to \ldots$ and the transposition of operators multiplication

(4) Integrals of motion and Heisenberg spin model

Normalization conditions

$$\|\Psi\|_1^2 = \int \prod_{k=1}^{n} d^2\rho_k \; \Psi^* \prod_{k=1}^{n} |p_k|^2 \Psi$$

$$\|\Psi\|_2^2 = \int \prod_{k=1}^{n} \frac{d^2\rho_k}{|\rho_{k,k+1}|^2} \, |\Psi|^2$$

compatible with the hermicity of H:

$$H^+ = \prod_{k=1}^{n} |p_k|^2 \, H \left(\prod_{k=1}^{n} |p_k|^2\right)^{-1}$$

$$H^+ = \left(\prod_{k=1}^{n} |\rho_{k,k+1}|^2\right)^{-2} H \prod_{k=1}^{n} |\rho_{k,k+1}|^2$$

Therefore $[q_n, h] = 0$, $q_n = \rho_{12}\rho_{23}\ldots\rho_{n1}\, p_1 p_2 \ldots p_n$

There are many integrals of motion (L.L. (1993))

Generating function - transfer matrix

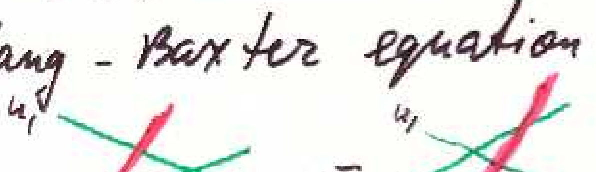
$$T(u) = \mathrm{tr}\,[L_1(u) L_2(u) \ldots L_n(u)] = \sum_{r=0}^{n} u^{n-r} q_r \,, \quad [q_r, h] = 0$$
$$[q_r, q_s] = 0$$

$$L_k(u) = \begin{pmatrix} u + \rho_k p_k, & -p_k \\ \rho_k^2 p_k, & u - \rho_k p_k \end{pmatrix}$$

Monodromy matrix

$$t(u) = L_1(u) L_2(u) \ldots L_n(u) =$$  $$= \begin{pmatrix} j_0(u) + j_3(u), & j_-(u) \\ j_+(u), & j_0(u) - j_3(u) \end{pmatrix}$$

Yang - Baxter equation (L.L.(1993)), Bethe anzatz



- bilinear relations for $j_r(u)$

h - Hamiltonian of Heisenberg spin model (integrable) 

⑤ Baxter equation in conjugated space

Monodromy matrix in the conjugated space (K., F. (1995))

$$\tilde{t}(u) = \tilde{L}_n(u) \dots \tilde{L}_1(u) = \begin{pmatrix} A(u) & B(u) \\ C(u) & D(u) \end{pmatrix}$$

$$\tilde{L}_k(u) = \begin{pmatrix} u + p_k \rho_k, & -p_k \\ p_k \rho_k^2, & u - p_k \rho_k \end{pmatrix}$$

Pseudo-vacum state:

$$C(u)|0\rangle = 0 \rightarrow \boxed{|0\rangle = \prod_{k=1}^{n} \frac{1}{\rho_k^2}}, \quad m = n$$

(non-physical)

Other states:

$$\boxed{|u_1, u_2 \dots u_z\rangle = B(u_1)\, B(u_2) \dots B(u_z)\, |0\rangle}$$

Due to the Yang-Baxter equation they are eigenstates of $T(u) = A(u) + D(u)$ if and only if $u_1, u_2, \dots, u_z$ satisfy the Bethe equations.

Generating functions of solutions of Bethe equations

$$Q(\lambda) = \prod_{t=1}^{z} (\lambda - u_t)$$

Baxter equation:

Bethe equations: $$\frac{(\lambda+i)^n Q(\lambda+i) + (\lambda-i)^n Q(\lambda-i)}{Q(\lambda)} = \Lambda^{(n)}(\lambda)$$

$\Lambda^{(n)}(\lambda)$ - polynomial coinciding with eigenvalue of the transfer matrix $T(u)$. Is it possible to find non-polynomial solutions $Q(\lambda)$ of the Baxter equation?

⑥ Sklyanin's representation for the wave function

$$\Psi(\vec{\rho}_1, \vec{\rho}_2 \ldots \vec{\rho}_n) = Q(\vec{\lambda}_1)\, Q(\vec{\lambda}_2) \ldots Q(\vec{\lambda}_{n-1})\, \Psi_0(\vec{\rho}_1, \ldots \vec{\rho}_n)$$

$$\Psi_0(\vec{\rho}_1, \vec{\rho}_2, \ldots \vec{\rho}_n) = \prod_{k=1}^{n} \frac{1}{|\rho_k|^4},$$

$$\Lambda^{(n)}(\lambda)\, Q(\lambda) = (\lambda+i)^n Q(\lambda+i) + (\lambda-i)^n Q(\lambda-$$

$$\Lambda^{(n)}(\lambda) = \sum_{k=0}^{n} \lambda^{n-k} q_k, \quad q_0 = 2,\ q_1 = 0$$

$$q_2 = m(m-1)$$

$$\boxed{Q(\vec{\lambda}) = \sum C_{st}\, Q^{(s)}(\lambda)\, Q^{(t)}(\lambda^*)}$$ – holomorphic factorization

λ_r – operators being zeroes of $B(u)$:

$$B(\lambda_r) = 0$$

$$[B(u), B(v)] = 0 \longrightarrow [\lambda_r, \lambda_s] = 0$$

What are eigenvalues of λ_r ?

$$\boxed{\lambda_r = \sigma_r + \frac{N_r}{2} i, \quad \lambda_r^* = \sigma_r - \frac{N_r}{2} i}$$

$N_r = 0, \pm 1, \pm 2, \ldots$; σ_r - real

It can be obtained by the unitary transformation to the representation where $\vec{P} = \sum_{k=1}^{n} \vec{p}_k$ and $\vec{\lambda}_r$ are diagonal (H. de Vega, L.L (2001))

$$\Psi_{\vec{p}, \vec{\lambda}_1, \ldots \vec{\lambda}_{n-1}} = \int \prod_{k=1}^{n} d^2\rho_k\, U_{\vec{p}, \vec{\lambda}_1, \ldots \vec{\lambda}_{n-1}}(\vec{\rho}_1, \vec{\rho}_2 \ldots \vec{\rho}_n)\, \Psi(\vec{\rho}_1, \vec{\rho}_2 \ldots \vec{\rho}_n)$$

$$\hat{B}(u)\, U = B(u)\, U, \quad \hat{B}^* U = B^*(u)\, U$$

7 Pomeron and Odderon in the Baxter-Sklyanin representation

Fourier transformation to the momentum space

$$\Psi(\vec p_1,\vec p_2,\dots \vec p_n) = \int \prod_{r=1}^{n} d^2\rho_r\, e^{i \vec p_r \vec\rho_r}\,\Psi(\vec\rho_1,\vec\rho_2,\dots \vec\rho_n)$$

New variables: $P=\sum_{r=1}^{n} p_r = 1$

$$y_1 = \ln\frac{p_1}{1-p_1},\quad y_2 = \ln\frac{p_1+p_2}{1-p_1-p_2},\ \dots,\ y_{n-1} = \ln\frac{1-p_n}{p_n}$$

For two particles

$$\boxed{\Psi_{\vec p,\vec\lambda} = \int \frac{d^2k}{|k(1-k)|^2}\left(\frac{k}{1-k}\right)^{i\lambda^*}\left(\frac{k^*}{1-k^*}\right)^{i\lambda}\varphi(\vec k,\vec 1-\vec k)}$$
$$1 \ ''$$

$$\varphi(\vec k,\vec 1-\vec k) = \int d^2k'\left[\frac{k'^*(1-k'^*)}{k^*-k'^*}\right]^{m-1}\left[\frac{k'(1-k')}{k-k'}\right]^{\tilde m-1}$$

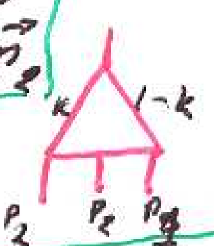
Hypergeometric functions

$$\Psi_{1,\vec\lambda} = |\lambda|^2\, Q(\vec\lambda),\quad Q(\vec\lambda) = Q(\lambda,m)\,Q(\lambda^*,\tilde m) + (-1)^n Q(-\lambda,m)\,Q(-\lambda^*,\tilde m)$$

Baxter function: $Q(\lambda,m) = {}_3F_2(-i\lambda+1,\,2-m,\,1+m;\,2,2;\,1)$

For three particles

$$\boxed{\Psi_{\vec 1,\vec\lambda_1,\vec\lambda_2} = \int d^2t\,\frac{d^2z}{|z(1-z)|^2}\,U_{\vec\lambda_1,\vec\lambda_2}(\vec t,\vec z)\,\Psi(\vec p_1,\vec p_2)\quad \vec p_3 = 1-\vec p_1-\vec p_2}$$



$$t = \ln\frac{p_1(p_1+p_2)}{(p_2+p_3)\,p_3},\quad z = \frac{p_1\,p_3}{(p_2+p_3)(p_1+p_2)},$$

$$U_{\vec\lambda_1,\vec\lambda_2}(\vec t,\vec z) = e^{i\vec t(\vec\lambda_1+\vec\lambda_2)}\,U_{\vec\lambda_1,\vec\lambda_2}(\vec z),\quad \boxed{U_{\vec\lambda_1,\vec\lambda_2}(z) = \chi_{\lambda_1\lambda_2}(z^*)\,\chi_{\lambda_1^*\lambda_2^*}(z) - \chi_{\lambda_2\lambda_1}(z^*)\,\chi_{\lambda_2^*\lambda_1^*}(z)}$$

$$\chi_{\lambda_1\lambda_2}(z^*) = (z^*)^{i\frac{\lambda_1-\lambda_2}{2}}\,F(-i\lambda_2,\,i\lambda_1;\,1+i(\lambda_1-\lambda_2);\,z^*)$$

Addison Wesley, Cambridge,Mass 1954.

[KW] K. G. Wilson and J. Kogut, The Renormalization Group and the ε Expansion, Phys. Rep. (Sect C of Phys Lett.), 12, 75–200 (1974).

[MS] P. K. Mitter and B. Scoppola: Renormalization Group Approach to Interacting Polymerised Manifolds, Comm. Math. Phys. 209 207–261 (2000).

[S] M. Shub: Global Stability of Dynamical Systems, Springer-Verlag, New York, 1987.

(8) **Solution of the Baxter equation and energy of the composite state**

H is known in $\vec{p}$ representation with the use of the unitary transformation to the Baxter-Sklyanin representation we have an expression for H acting on $\Psi_{\vec{1},\vec{\lambda}_1,\vec{\lambda}_2\ldots\vec{\lambda}_{n-1}}$

$$\Psi_{1,\vec{\lambda}_1\ldots\vec{\lambda}_{n-1}} \sim Q(\vec{\lambda}_1)Q(\vec{\lambda}_2)\ldots Q(\vec{\lambda}_{n-1})\,\mathcal{R}_0(\vec{\lambda}_1\ldots\vec{\lambda}_{n-1})$$

$$Q(\vec{\lambda}) = \sum_{r,s} C_{rs}\, Q^{(r)}(\lambda)\, Q^{(s)}(\lambda^*), \quad \boxed{E = \frac{H\Psi}{\Psi}\Big|_{\substack{\lambda_1\to i\\ \lambda_1^*\to i\\ \lambda_k\to 0}}}$$

$Q^{(r)}(\lambda)$ - meromorphic functions with singularities (poles) at $\lambda = ik$, $k = 0, \pm 1, \ldots$

The simplest solution

$$Q^{(n-1)}(\lambda) = \sum_{k=0}^{\infty}\left[\frac{a_k}{(\lambda-ik)^{n-1}} + \frac{b_k}{(\lambda-ik)^{n-2}} + \cdots + \frac{z_k}{\lambda-ik}\right]$$

$$\boxed{E_{m,\tilde{m}} = \mathcal{E}_m + \mathcal{E}_{\tilde{m}}}, \quad \boxed{\mathcal{E}_m = \frac{b_1}{a_1} + n = b_0 - \frac{\mu_{n-1}}{\mu_n}}$$

$q_r = i\mu_r$, $a_k, \ldots, z_k$ satisfy the recurrence relations expressing them through $a_0, b_0, \ldots z_0$

$a_0 = 1$ and $\boxed{b_0, \ldots z_0 \text{ are fixed by } Q^{(n-1)}(\lambda)\big|_{\lambda\to\infty} \leq \frac{c}{\lambda^{n-1}}}$

($Q(\lambda)$ is a solution of the Baxter eq. at $\lambda\to\infty$)

9 Quantization of the integrals of motion

The Baxter equation in the real form: $\boxed{\lambda = i x}$

$$\Omega(x,\vec{\mu})\,Q(x,\vec{\mu}) = (x+1)^n Q(x+1,\vec{\mu}) + (x-1)^n Q(x-1,\vec{\mu})$$

$$\Omega(x,\vec{\mu}) = \sum_{k=0}^{n} (-1)^k \mu_k \, x^{n-k}\,, \qquad \boxed{q_k = i^k \mu_k} \text{ - eigenvalues} \qquad \boxed{\mu_k \text{ - real!!}}\,, k>2$$

$$\mu_0 = 2\,,\ \mu_1 = 0\,,\ \mu_2 = m(m-1)$$

Auxiliary functions $r = 1, 2, \dots n-1$

$$f_r(x,\vec{\mu}) = \sum_{\ell=0}^{\infty} \left[\frac{\tilde{a}_\ell(\vec{\mu})}{(x-\ell)^r} + \frac{\tilde{b}_\ell(\vec{\mu})}{(x-\ell)^{r-1}} + \dots + \frac{\tilde{g}_\ell(\vec{\mu})}{x-\ell} \right]$$

Due to the Baxter equation all residues are known if

$$a_0(\vec{\mu}) = 1, \quad \tilde{b}_0(\vec{\mu}) = \dots = \tilde{h}_0(\mu) = 0$$

Solutions

$$Q^{(t)}(x,\vec{\mu}) = \sum_{r=1}^{t} C_r^{(t)} f_r(x,\vec{\mu}) + \beta^{(t)}(\vec{\mu}) \sum_{r=1}^{n-1-t} C_r^{(n-1-t)} f_r(-x,\vec{\mu}^S)$$

$$t = 0, 1, 2, \dots, n-2, n-1\,, \qquad \mu_k^S = (-1)^k \mu_k \text{ - symmetry } \left\{ \begin{array}{l} x \to -x \\ \vec{\mu} \to \vec{\mu}^S \end{array} \right\}$$

The coefficients $C_r^{(t)}$ satisfy the system of linear equations obtained from

$$\boxed{Q^{(t)}(x,\vec{\mu})\Big|_{x\to\infty} \lesssim \frac{c}{x^{n-1}}}$$

Linear relations: Similar to recurrence relations for orthogonal polynomials

$$\boxed{\begin{array}{l} \left[\delta^{(r)}(\vec{\mu}) + \pi\cot(\pi x)\right] Q^{(r)}(x,\vec{\mu}) = Q^{(r+1)}(x,\vec{\mu}) + d^{(r)}(\vec{\mu})\, Q^{(r-1)}(x,\vec{\mu}) \\ \varepsilon_m^{(r)} \text{ does not depend on } r \to \boxed{\delta^{(r)}(\vec{\mu}) = 0} \text{ (quantization of } \vec{\mu}) \end{array}}$$

(9a) Spectrum of eigenvalues of integrals of motion

There are several arguments, that for Odderon ($n=3$) the eigenvalues $q_3 = i\mu$ are pure imaginary

For example, the holomorphic hamiltonian h can be considered as symmetric on the functions $f(z, z^*)$, $z = \frac{\rho_{12}\rho_{30}}{\rho_{10}\rho_{30}}$ and z^* - real

$$\int d^2 z \, |f(z, z^*)|^2 \xrightarrow[\text{anti-Wick rotation}]{} \int \frac{dz\,dz^*}{2} (f(z, z^*))^2$$

$$\boxed{\int dz dz^* f h f = \int dz\, dz^* (h^T f^*) f}$$

But we have for large μ for holomorphic energy

$$\boxed{\varepsilon = \ln \mu + 3\gamma + \left[\frac{3}{448} + \frac{13}{120}(m-\tfrac{1}{2})^2 - \frac{1}{12}(m-\tfrac{1}{2})\right]\frac{1}{\mu^2} + \ldots}$$

Therefore μ is real or pure imaginary [for real $m(m-1)$]

Another argument for reality of $\vec{\mu}$:

$\boxed{E = \frac{H_4}{4}}$, there are two different limits:

$\lambda, \lambda^* \to i$ and $\lambda, \lambda^* \to -i$

We have 2 different values for energy:

$$\boxed{E_{m,\tilde{m}} = \varepsilon_m(\vec{\mu}) + \varepsilon_{\tilde{m}}(\vec{\mu}^{s*}) = \varepsilon_m(\vec{\mu}^s) + \varepsilon_{\tilde{m}}(\vec{\mu}^*)}$$

If there is no accidental degeneracy for real $m(m-1)$ we have: $\boxed{\vec{\mu}^* = \vec{\mu}}$ or $\vec{\mu}^* = \vec{\mu}^s$, $\boxed{\mu_k^s = (-1)^k \mu_k}$

(10) Intercepts of the composite states of reggeized gluons

$$\boxed{\sigma^{(n)} \sim s^{\Delta^{(n)}}}, \quad \boxed{\Delta^{(n)} = -\frac{g^2}{8\pi^2} N_c E_{m,\tilde{m}}}, \quad E_{m,\tilde{m}} = \varepsilon_m + \varepsilon_{\tilde{m}}$$

Pomeron $(n=2)$:

BFKL

$$\boxed{\begin{aligned} \varepsilon_m &= \psi(m) + \psi(1-m) - 2\psi(1) \\ \min E_{m,\tilde{m}} &= E_{\frac{1}{2},\frac{1}{2}} = -8 \ln 2 \end{aligned}}$$

$$m = \tfrac{1}{2} + i\nu + \tfrac{n}{2}, \quad \tilde{m} = \tfrac{1}{2} + i\nu - \tfrac{n}{2}$$

$$\sigma^{(2)} \sim s^{\Delta}, \quad \Delta = \frac{g^2}{\pi^2} N_c \ln 2$$

Odderon $(n=3)$. Janik, Wosiek $\rightarrow$

$$\boxed{\begin{aligned} &\text{For } \mu \neq 0 \ \ \min E > 0 \\ &E_1 = 0.49434 \ \text{ for } \mu_1 = 0.20526 \end{aligned}}$$

$$\boxed{\begin{aligned} &\text{But for } \mu = 0, \ \ \min E\big|_{n=1} = 0, \ \ \sigma_{pp} - \sigma_{p\bar{p}} \sim \text{Const}/(\ln s)^{1/2} \\ &(\text{G. Vacca, J. Bartels, L.L (2000)}) \end{aligned}}$$

Quarteton $(n=4)$. $\min E$ is obtained for $\mu_3 = 0$, $\mu_4 \neq 0$

$$\boxed{\min E = -1.34832 \ \text{ for } \left\{ \begin{aligned} &\mu_4 = 0.15359 \\ &\mu_3 = 0 \\ &m = \tilde{m} = \tfrac{1}{2} \end{aligned} \right.}$$

However, for $n=1$ we have

$$\boxed{\min E = -2.0799 \ \text{ for } \mu_4 = 0.12167}$$

for $n=2$ we have

$$\boxed{\min E\big|_{n=2} = -5.863 < E_{pom} = -5.545}$$

(11) Anomalous dimensions of quasi-partonic operators

Partonic (twist-2) operators

$$\boxed{O^{j}_{\dots} = n^{\mu_1} n^{\mu_2} \dots n^{\mu_j} \mathrm{tr}\, G_{\rho\mu_1} D_{\mu_2} \dots D_{\mu_{j-1}} G_{\rho\mu_j}}$$

$n_\mu = q_\mu + x p_\mu \;,\; n_\mu^2 = 0$

$$\langle p | O^j_{\dots} | p \rangle \sim \exp\left(\int_{Q_0^2}^{Q^2} \gamma_j (\alpha_s(Q'^2)) \, d \ln Q'^2 \right)$$

$$\gamma_j(\alpha) = \sum_{k=1}^{\infty} c_j^{(k)} \left(\frac{\alpha N_c}{\pi}\right)^k , \quad \omega = j - 1$$

$$\boxed{\gamma_j \Big|_{j \to 1} = \frac{\alpha N_c}{\pi \omega} - 4\,\zeta(3) \left(\frac{\alpha N_c}{\pi \omega}\right)^4 + \dots}$$

can we calculate γ for higher-twist operators

Next-to-leading BFKL corrections (FL) $\sim \frac{\alpha^2}{\omega}$

Simplest quasi-partonic operators (L. Gribov, E. Levin, M. Ryskin)

$$Q^j = \prod_{r=1}^{p} O^{j_r}_{\dots} \;,\; j = \sum_{r=1}^{p} j_r = p + \omega \;,\; \omega = \sum_{r=1}^{p} \omega_r$$

Total dimension $\Gamma = p - \gamma \;,\; \gamma = \sum_{r=1}^{p} \gamma(\omega_r)$

In the Regge regime $\omega_0 = p\, \omega_{BFKL}$

In the deep-inelastic regime $\gamma = p\, \omega^{(-1)}\left(\frac{\omega}{p}\right)$. For $\omega \to 0$:

$$\boxed{\gamma = \frac{\alpha_s N_c}{\pi \omega} p^2}$$

(Mandelstam cuts) E. Levin, J. Bartels...

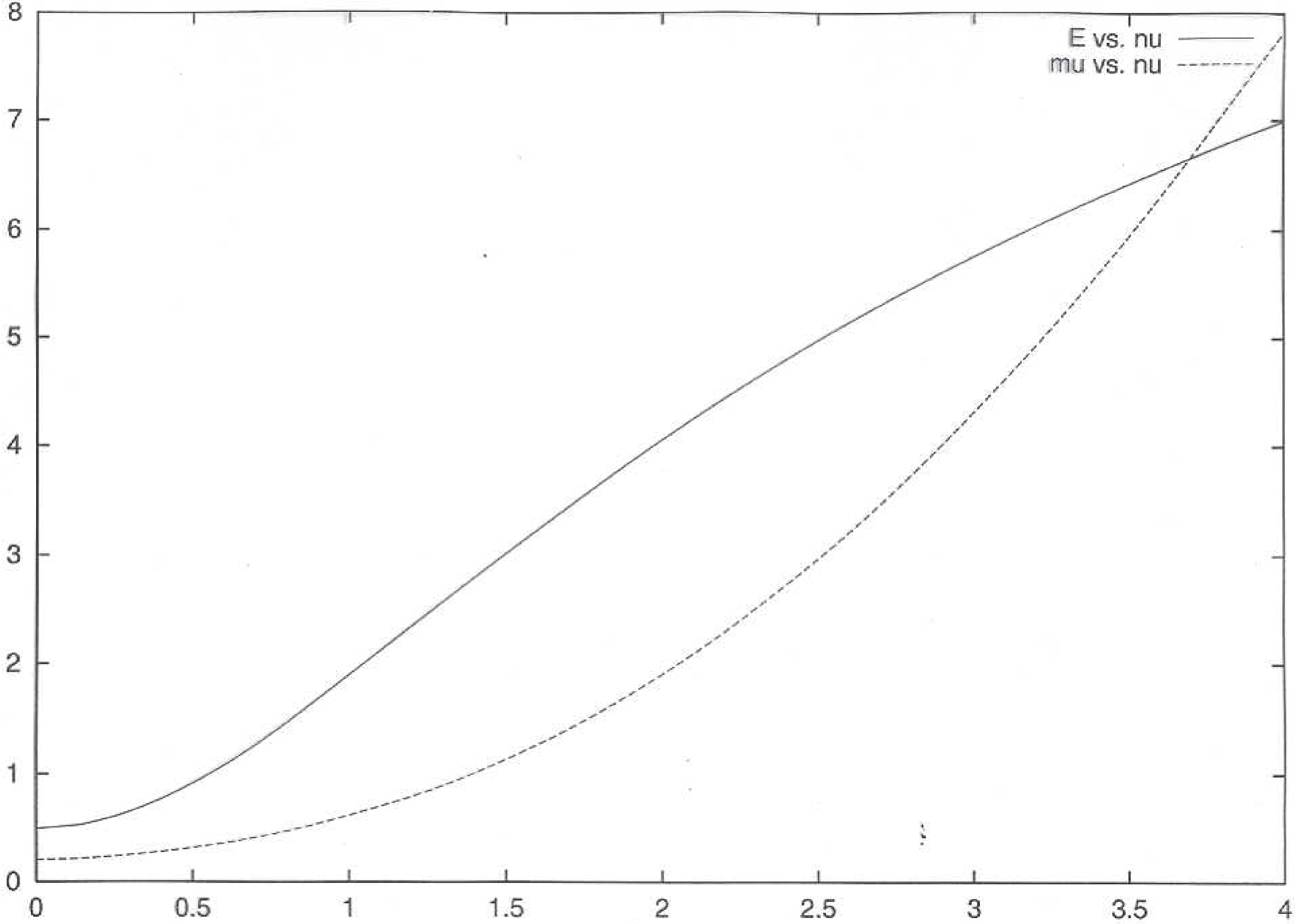
For irreducible quasi-partonic operators:

$$\boxed{\frac{m + \tilde{m}}{2} = \frac{n}{2} - \gamma^{(n)}} \;,\; \boxed{\gamma^{(n)} = c^{(n)} \frac{\alpha_s N_c}{\omega}}$$

We calculated $c^{(3)}$ and $c^{(4)}$

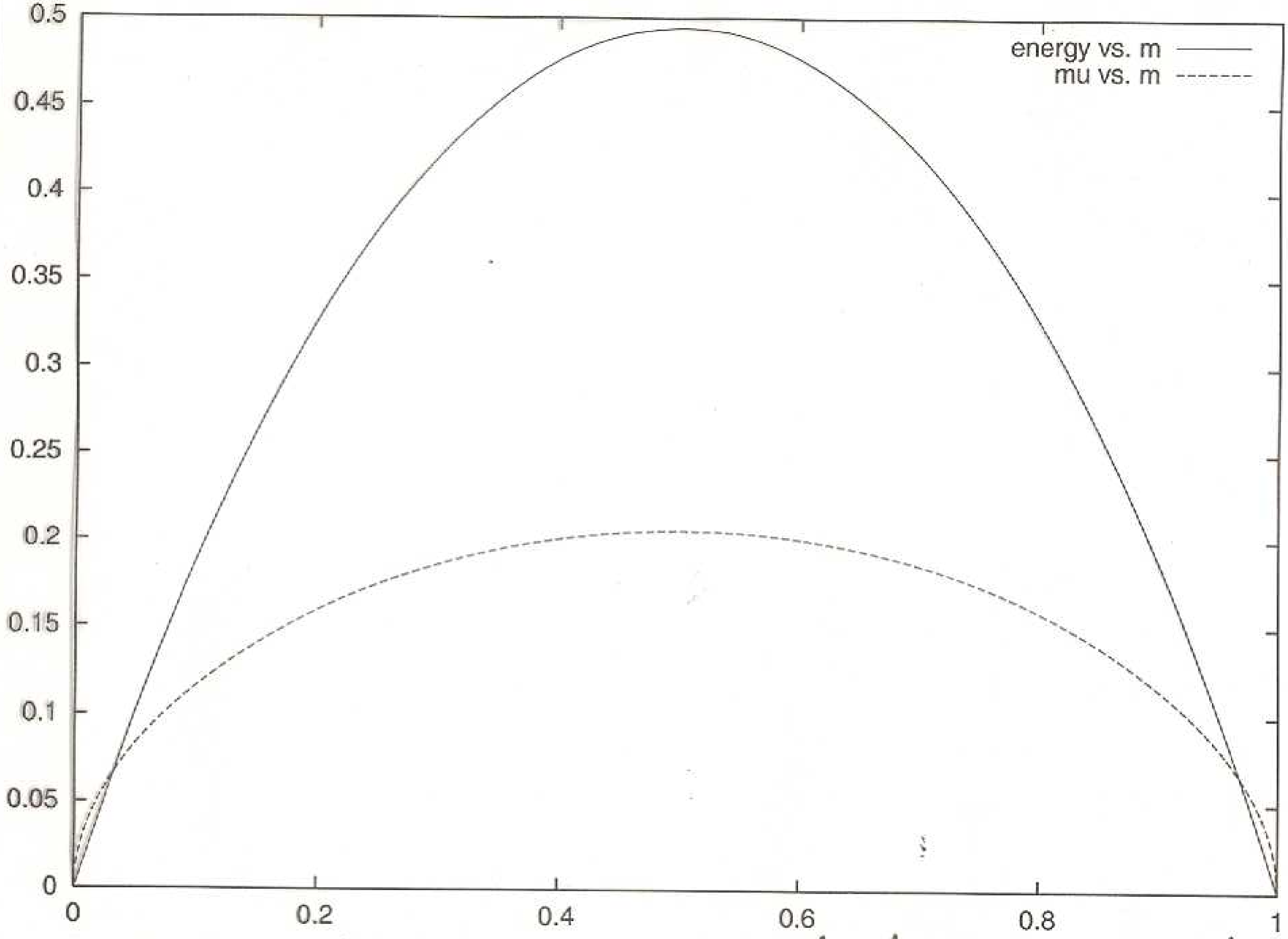
and all functions $\boxed{\gamma^{(n)}\left(\frac{\alpha_s}{\omega}\right)}$.

<u>Odderon</u>



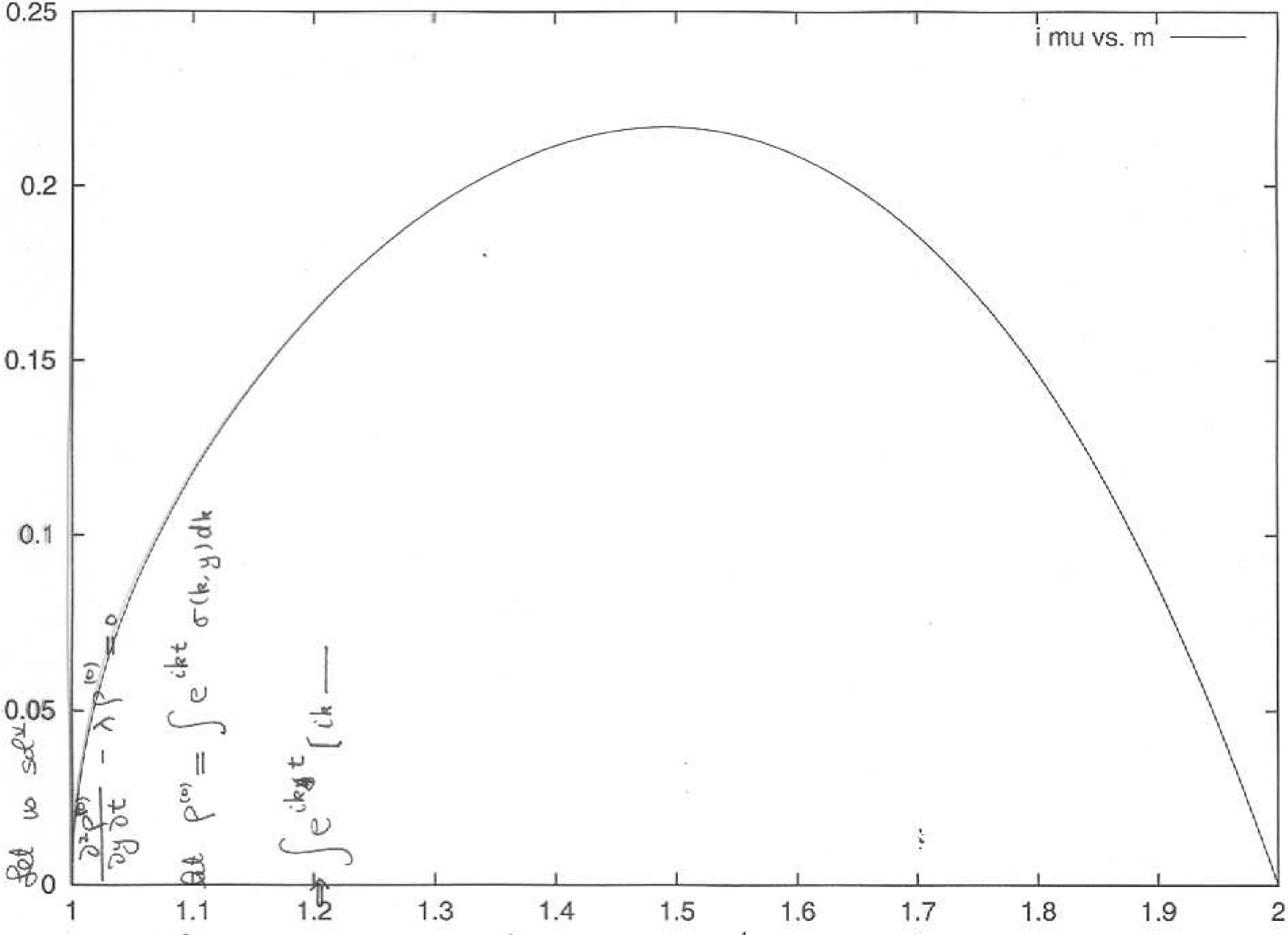
Energy and μ as function of real ν for $m=\frac{1}{2}+i\nu$

Odderon



ENERGY (LORENTZ SPIN) and μ as functions of $m = \frac{1}{2} + i\nu$ for real $m = \gamma$

Odderon



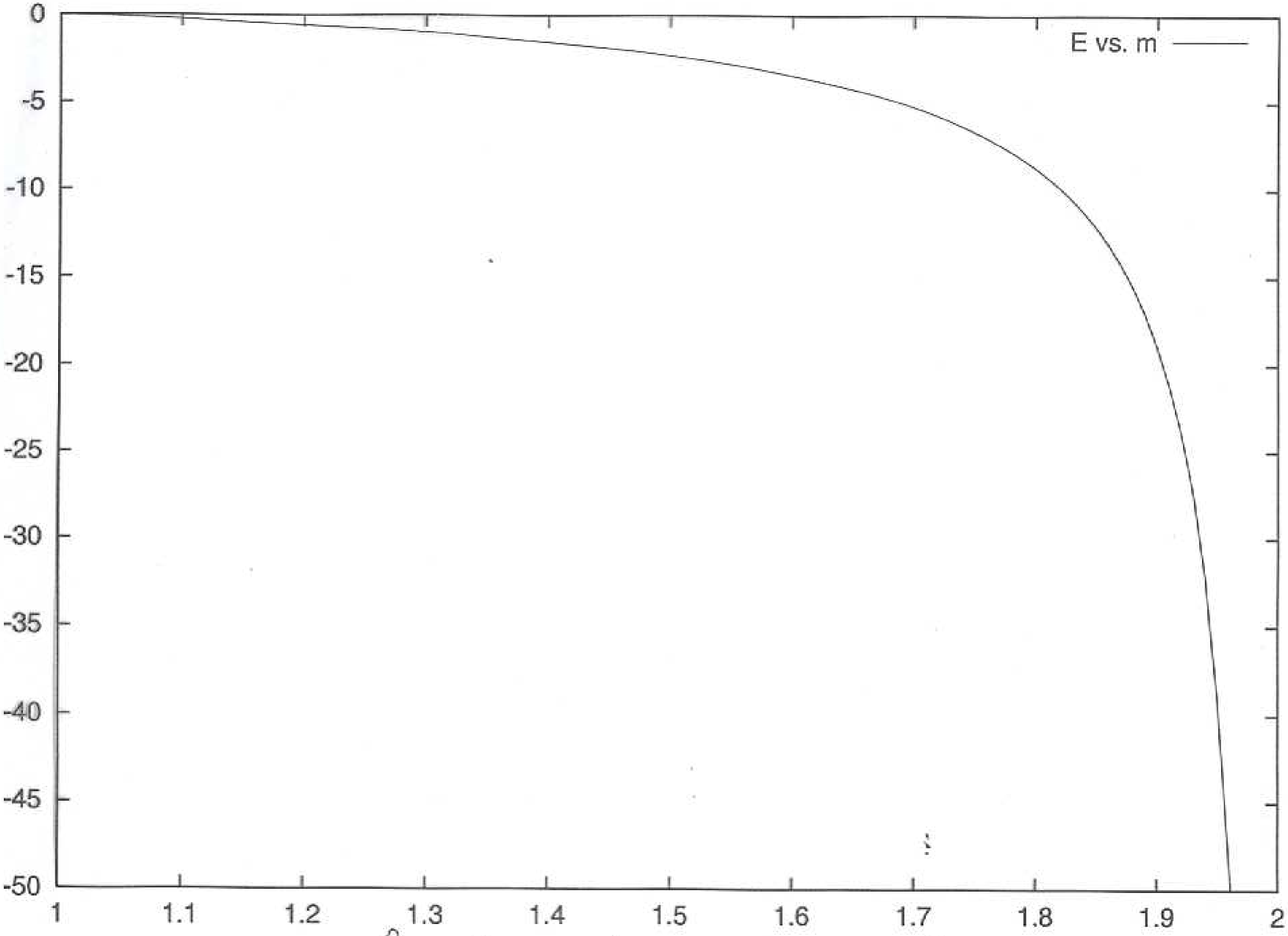
Set ω sol^{n} $\frac{\partial^2 \rho^{(0)}}{\partial y \partial t} - \lambda \rho^{(0)} = 0$

Set $\rho^{(0)} = \int e^{ikt} \sigma(k,y)\, dk$

$\Rightarrow \int e^{ikt} [ik$ —

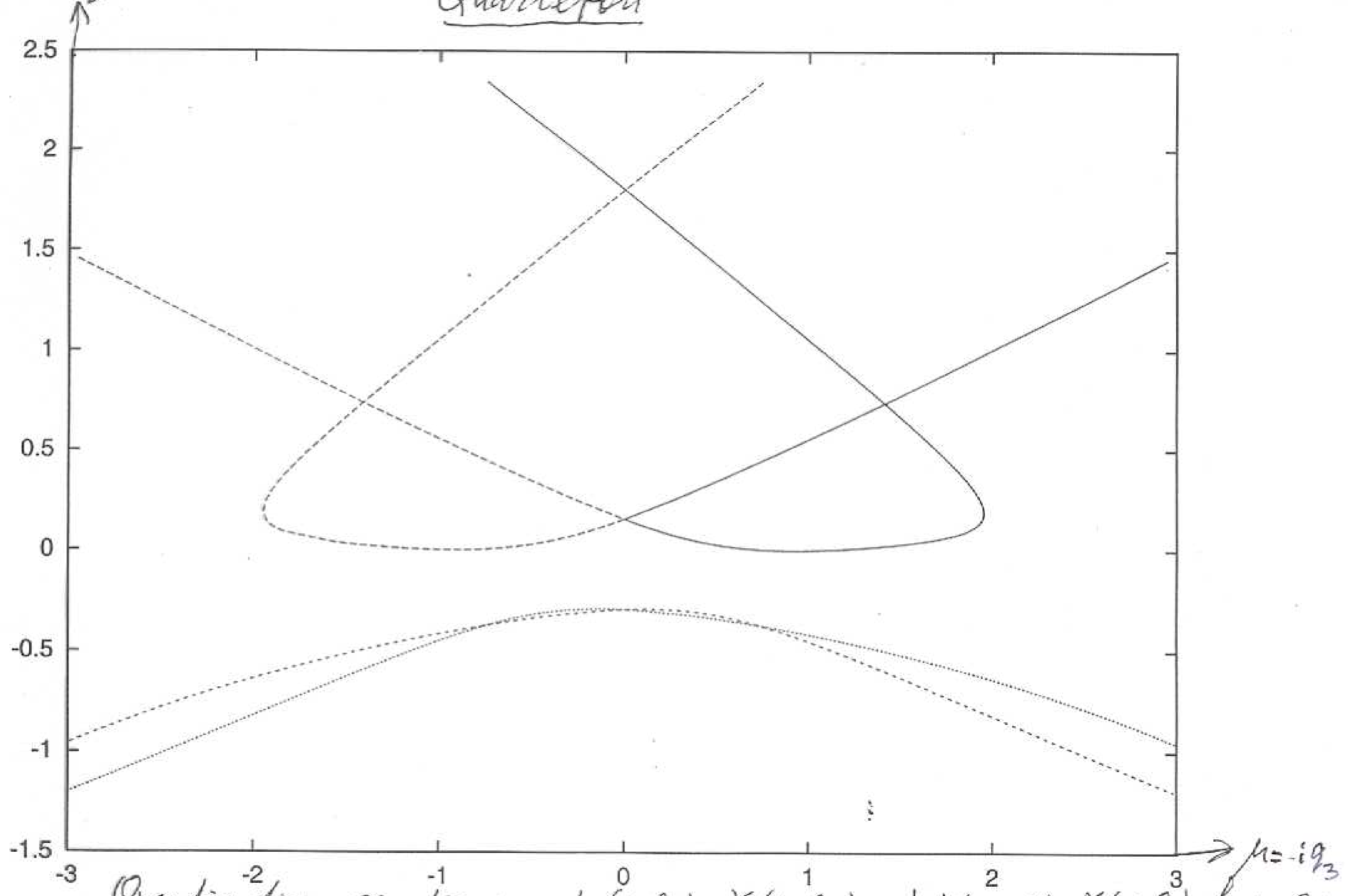
$\mathrm{Im}\,\mu$ as a function of m for real $m = \frac{1}{2} + i\nu + \frac{n}{2}$

Odderon



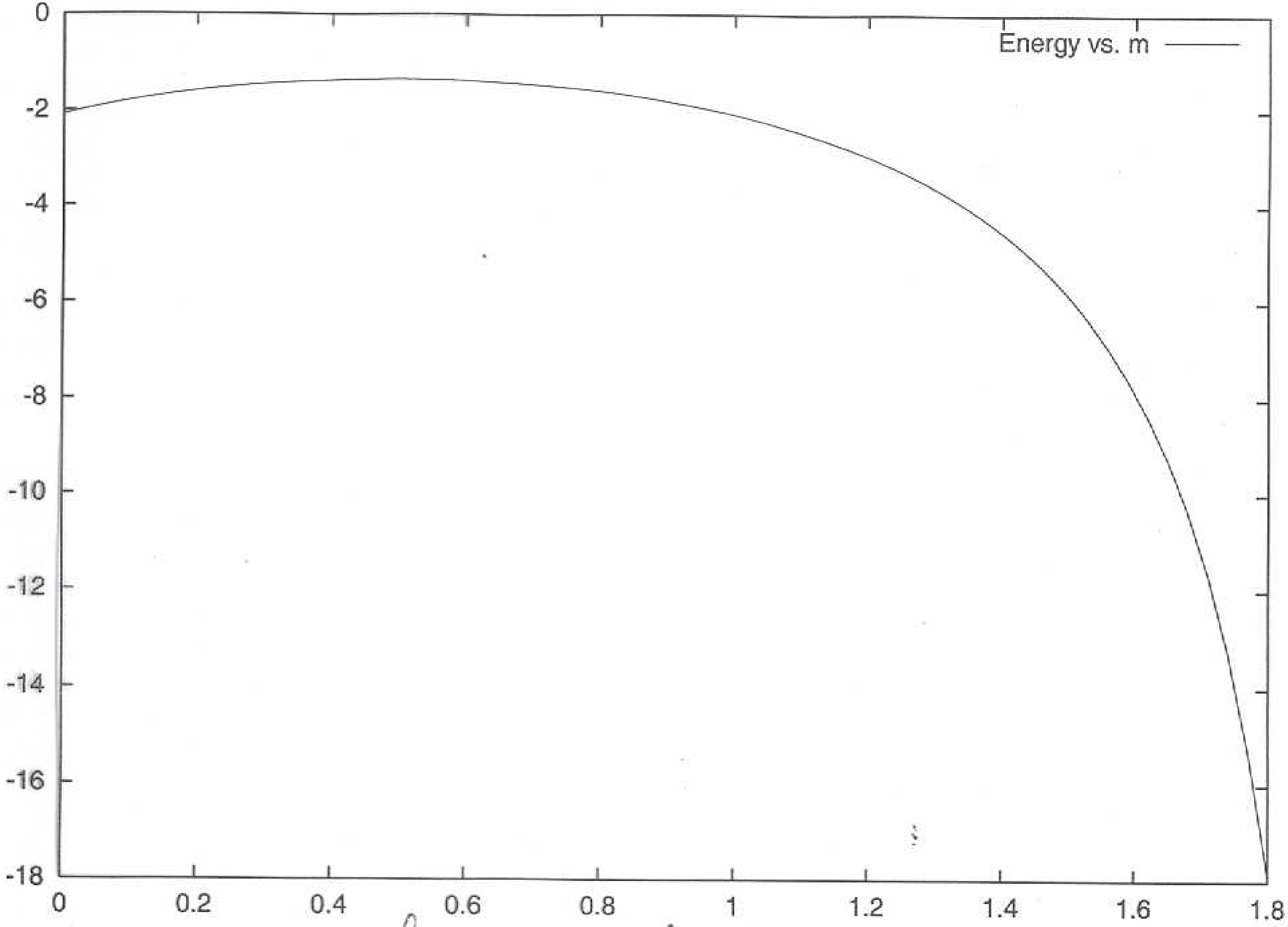
Energy as a function of $m = \frac{1}{2} + i\nu$ for real $m = -\gamma + 1$

Quartefon



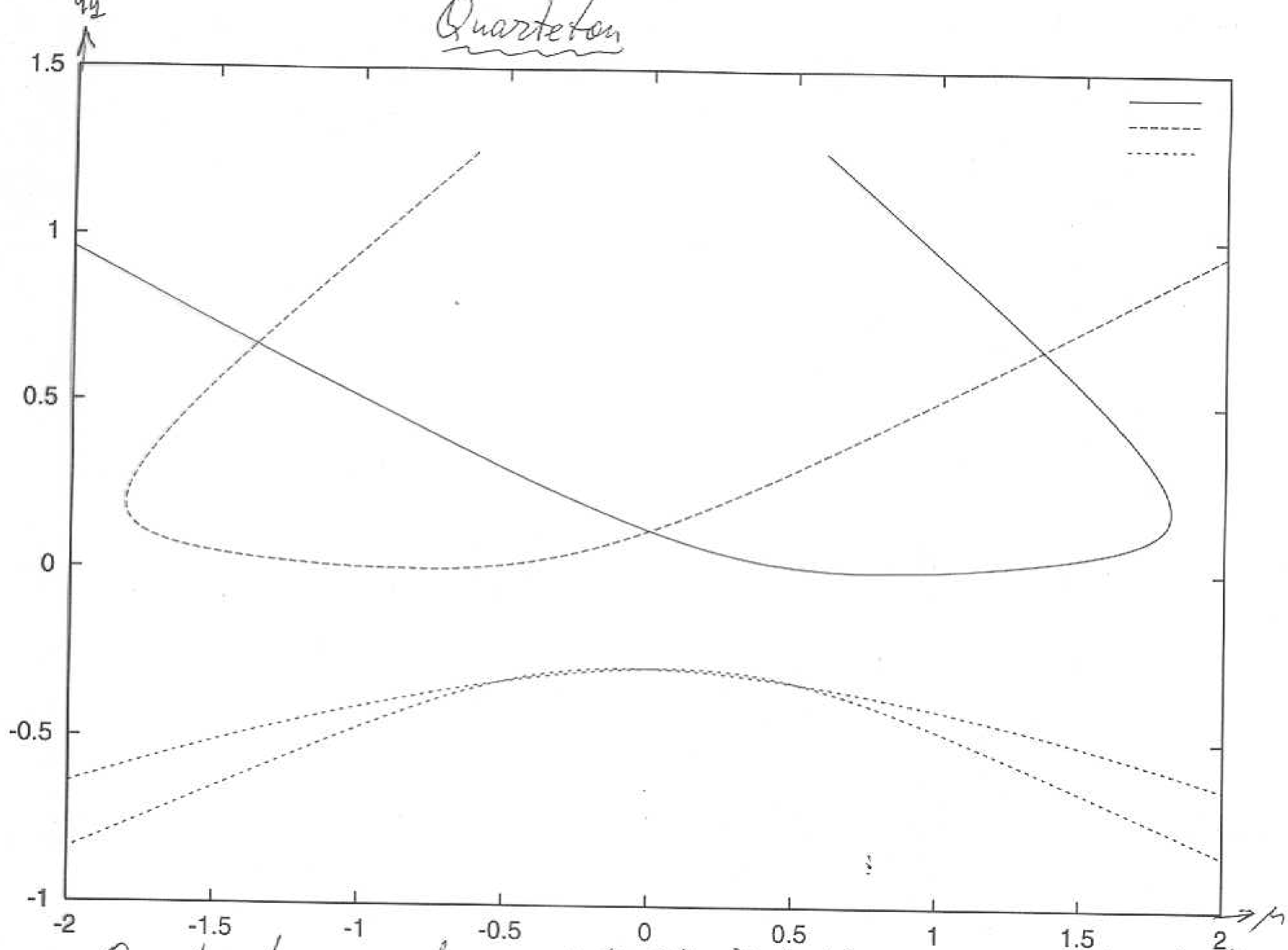
Quantization equations $\alpha_1(\mu, q_4) = \gamma_1(\mu, q_4)$ and $\alpha_1(-\mu, q_4) = \gamma_1(-\mu, q_4)$ for $m = \tilde{n} = \frac{1}{2}$

Quarteton



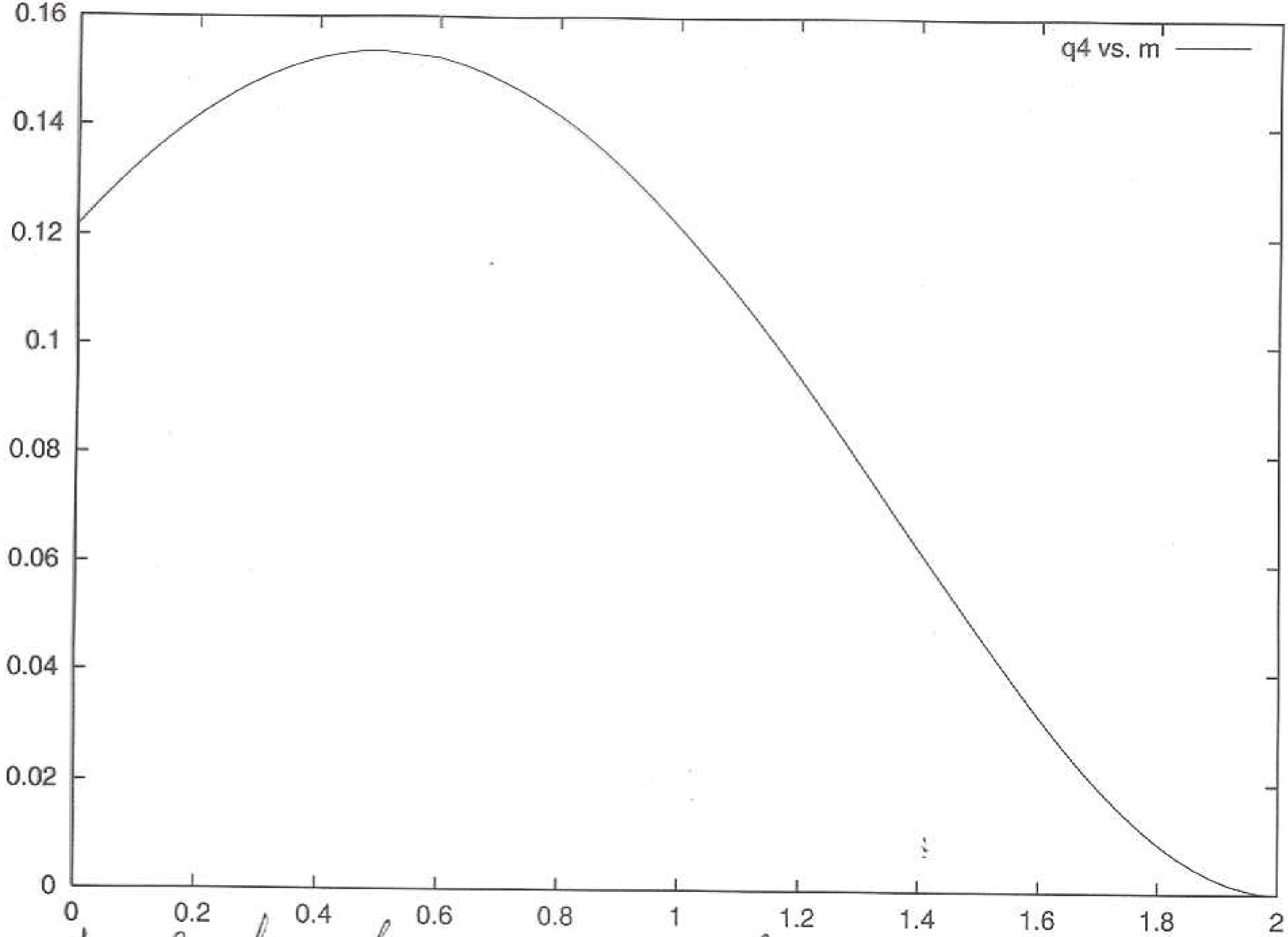
Energy as a function of real $m \le 1.8$ for $n = 0$

Quarteton



Quantization equations $\alpha_1(\mu, q_y) = \gamma_1(\mu, q_y)$; $\alpha_1(-\mu, q_y) = \gamma_1(-\mu, q_y)$ for $m = 0$, $\tilde{m} = 1$

<u>Quarteton</u>



Integral of motion q_4 as a function of real m for $\mu=0$

conformal spin	conformal weights	energy eigenvalues pomeron	energy eigenvalues odderon	energy eigenvalues quarteton
$n=0$	$m=\widetilde{m}=1/2$	$-8\ \ln 2=-5.545177$	0.49434	-1.34832
$n=1$	$m=1,\ \widetilde{m}=0$	0	0	-2.0799
$n=2$	$m=3/2,\ \widetilde{m}=-1/2$	$8(1-\ln 2)=2.45482$	imaginary μ	-5.863
$n=3$	$m=2,\ \widetilde{m}=-1$	4	2	4

TABLE 1. Conformal spins, conformal weights and corresponding lowest energy eigenvalues for the pomeron, odderon and quarteton states. The odderon state with imaginary μ is discarded as non-physical. The pomeron and quarteton states with $n=3$ have the same energy and are related presumably by the duality symmetry.

As displayed in Table 1 the intercept for the quarteton ground state possessing the conformal spin $n=2$ is larger than that for the BFKL Pomeron. This result is not very suprising because four reggeized gluons clustered into two pomerons have an even larger intercept. A large intercept for the state with the conformal spin 2 may lead to such unphysical results as negative cross sections. But it is known that the unitarization of scattering amplitudes is not solved within a framework where the number of reggeons is fixed.

$$\sigma_n \sim s^{\Delta_n}, \quad \Delta_n = -\frac{\alpha_s}{2\pi} N_c E_n$$

Chromody-

(12) Concluding remarks

1. Remarkable properties of the Reggeon Calculus in the perturbative QCD

a) Möbius invariance $\rho \to \frac{a\rho+b}{c\rho+d}$

b) Holomorphic factorization $H = h + h^*$

c) Duality symmetry $p_1 \to \rho_{12} \to p_2 \to \cdots$

d) Integrals of motion q_r, $r = 0, 2 \ldots n$

e) Relation with the Heisenberg spin model (non-compact) with $\vec{L}\,(\rho\partial, \partial, -\rho^2\partial)$

2. Baxter equation has the meromorphic solutions $Q(\lambda)$. Energy is expressed through the ratio of the residues at $\lambda = i$

3. Baxter - Sklyanin representation
$$\Psi = Q(\vec{\lambda}_1)\,Q(\vec{\lambda}_2) \ldots Q(\vec{\lambda}_{n-1})\,\Omega_0$$

4. Quantization of energy (intercepts)
$E_{m,\tilde{m}} = E_m + E_{\tilde{m}}$, E_m is the same for all $Q^{(r)}(\lambda)$
$\vec{\lambda}$ are also quantized and real.

5. $\sigma^{(n)} \sim s^{\Delta_n}$, $\Delta_3 = 0$, $\Delta_4 > \Delta_2$ (for conformal spin 2)
strong polarization effects

6. Is it possible to calculate the Regge trajectories? Möbius invariance should be broken.