Analysis of multiparticle dynamics from Two Stage Model at e^+e^--annihilation into hadrons

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2. Two Stage Model (TSM)
3. Dynamics of Multiparticle production (MP)
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e^+e^--annihilation 14 - 189 GeV

QCD + phenomenological model hadronization

Statistical methods

$\boxed{\text{TSM}}$ Kuvshinov V.I. Kokoulina E.S.
Acta Phys. Pol. B13 (1982) 533.
Proc. 6th Int. Conf. on HEP Problems. (1981)
Proc. Seminar NPCS (1991, 1995, 1999, 2002)

I stage quark-gluon cascade

II stage hadronization

Multiplicity distributions (MD) first stage

Giovannini A. Nucl. Phys. B161 (1979) 429

$\left\{\begin{array}{l}\text{QCD evolution parameter } Y \sim \ln s \\ \text{Markov branching process}\end{array}\right.$ (1)

gluon fission + quark bremsstrahlung

MD for quark jet from Giovannini

$$P_m^p(s) = \frac{K_p(K_p+1)\ldots(K_p+m-1)}{m!}\left(\frac{K_p}{K_p+\bar{m}}\right)^{K_p}\left(\frac{\bar{m}}{K_p+\bar{m}}\right)^m \qquad (2)$$

K_p, $\bar{m}$ - parameters

$\bar{m}$ - the average multiplicity of gluons

Generating function for MD $Q^p(s,z) = \sum_m P_m^p z^m$

$$Q^p(s,z) = \left(1 + \frac{\bar{m}}{K_p}(1-z)\right)^{-K_p}, \qquad (3)$$

$$P_m^p = \frac{1}{m!}\frac{\partial^m Q^p(s,z)}{\partial z^m}.$$

Second stage:

Supernarrow binomial distributions ($f_2 < 0$ ...)

$$P_n^H = C_{N_p}^n \left(\frac{\bar{n}_p^h}{N_p}\right)^n \left(1 - \frac{\bar{n}_p^h}{N_p}\right)^{N_p - n} \qquad (4)$$

generation function

$$Q_p^H = \left(1 + \frac{\bar{n}_p^h}{N_p}(z-1)\right)^{N_p}, \quad (p = q, g) \qquad (5)$$

$\bar{n}_p^h$ - average multiplicity of hadrons formed from parton on the second stage

N_p - maximum number of hadrons which can be formed from parton on this stage

If Hypothesis of soft discoloration is right then we add stage of hadronization to parton stage by means factorization

$$P_n(s) = \sum_m P_m^P \, P_n^H(m, s) \qquad (6)$$

MD in e^+e^--annihilation are determined by convolution of two stages:

$$P_n(s) = \sum P_m^P \frac{\partial^n}{\partial z^n} \left(Q^H\right)^{2+m} \Big|_{z=0} \qquad (7)$$

We do the following simplification for second stage:

$$\frac{\bar{n}_q^h}{N_q} \approx \frac{\bar{n}_g^h}{N_g} \qquad (N \equiv N_q),\ (\bar{n}_q^h \equiv \bar{n}^h)$$

We introduce parameter $\alpha = \frac{N_g}{N_q}$ for distingiishing between hadrons jets created from q and g on this stage

Then
$$Q_q^H = \left(1 + \frac{\bar{n}}{N}(z-1)\right)^N \qquad (\ )$$
$$Q_g^H = \left(1 + \frac{\bar{n}}{N}(z-1)\right)^{\alpha \cdot N} \qquad (\ )$$

(2) and (5) → (6), differentiating on z → MD

For comparison with experemental data we used

$$P_n(s) = \Omega \sum_{m=0}^{M_g} P_m^P \, C_{(2+\alpha \cdot m)N}^{n} \left(\frac{\bar{n}^h}{N}\right)^n \left(1 - \frac{\bar{n}^h}{N}\right)^{(2+\alpha \cdot m)N - n} \qquad (10)$$

Ω - normalized factor

M_g - maximal number of possible gluons created on the first stage

P_m^P - (2).

Results of comparison of model expression (10) to experimental data are represented in Table and figures

MD in TSM describe well data from 14 to 189 GeV ($\chi^2 \sim 1$)

$\boxed{\text{Dynamics of MP}}$ In accordanc with TSM

cascade stage:

$\bar{m}$ - has tendency to rise (insignificant deviations) considerable decrease at 161 GeV

K_p - values K_p are changed insignificantly ~ 10

physical meaning from thermodynamical model is temperature $K_p^{-1} = T_0 + 1/c \cdot E$ (11)

E - energy spended on creation of new particles

hadronization:

N - we can't reveal steady energy rise or fall for it big N_g points to predominance of hadrons formed from gluon jets in hadron multiplicity. small N_g points to essential contribution of quark; Average $N \sim 16$.

$\bar{n}^h$ - the tendency to weak rise with big scatter. The average value of $\bar{n}^h \sim 5$-6 in the research region

α - it's almost constant $\sim .2$

We can determine analogous parameter for gluon jet

$N_g = \alpha N$ and $\bar{n}_g^h = \alpha \bar{n}^h$.

These parameters remain constants without considerable deviations:

$N_g \sim 3$ and $\bar{n}_g^h \sim 1$.

From this results we can affirm about universality of hadronization!

$\alpha < 1$ - hadronization of gluon jets are more soft than quark ones.

The ratio $\frac{\bar{n}^h}{N}$ determines the probability of formation of hadron from parton. It is increased to 50 GeV, then there are big variations in KEK-region and it is almost constant in higher energies and $\sim .4$.

$\Omega \simeq 2$.

$\boxed{\text{Oscillations of moments}}$ in TSM

Generation function for MD (10)

$$Q(s,z) = Q^g\left(Q^H(z)\right)^m Q_q^2(z) \qquad (12)$$

We calculate F_q and K_q using (12)

$$F_q = \frac{1}{[\bar{n}(s)]^q} \frac{\partial^q Q}{\partial z^q}\bigg|_{z=0} \qquad (13)$$

$$K_q = \frac{1}{[\bar{n}(s)]^q} \frac{\partial^q \ln Q}{\partial z^q}\bigg|_{z=0} \qquad (14)$$

and the ratio

$$H_q = K_q / F_q \qquad (15)$$

(12) → logarithming → expanding to series in power on Q_q^H → (14) →

$$K_q = \left[K_p \sum_{m=1}^{\infty} \alpha \cdot m(\alpha \cdot m - 1/N) \dots (\alpha \cdot m - (q-1)/N) \left(\frac{\bar{m}}{\bar{m}+K_p}\right)^m \frac{1}{m} + 2(-1)^{q+1} \frac{(q-1)!}{N^{q-1}}\right] \left(\frac{\bar{n}^h(s)}{\bar{n}(s)}\right)^q \qquad (16)$$

$\bar{n}(s)$ - average multiplicity of hadrons in e^+e^--annihilation

For F_q we get

$$F_q = \sum_{m=0}^{\infty} (2+\alpha \cdot m)\left(2+\alpha m - \frac{1}{N}\right)\dots\left(2+\alpha \cdot m - \frac{q-1}{N}\right) P_m^g \left(\frac{\bar{n}^h}{\bar{n}(s)}\right)^q \qquad (17)$$

The sought-for ratio will be

$$H_q = \frac{K_p \sum_{m=1}^{\infty} \alpha \cdot m\left(\alpha \cdot m - \frac{1}{N}\right)\dots\left(\alpha m - \frac{q-1}{N}\right)\left(\frac{\bar{m}}{\bar{m}+K_p}\right)^m \cdot \frac{1}{m} + 2(-1)^{q+1}\frac{(q-1)!}{N^{q-1}}}{\sum \left(2+\alpha m\right)\left(2+\alpha \cdot m - \frac{1}{N}\right)\dots\left(2+\alpha m - \frac{q-1}{N}\right) P_m^g} \qquad (18)$$

The comparison with experimental data shows that (18) well describes ratio H_q ($\chi^2 \sim 2$). It is seen minimum at $q=5$ near Z^0.

H_q before Z^0 may oscillate in sign with period ~ 2, changing sign with parity q.

At more high energies the period is increased to 4 and higher.

It can be explained of more developed cascade of partons and non-integer values of hadronization parameters N and N_g.

Conclusions

TSM does not contradict to the experimental data on MD and the oscillations ratio of factorial moments.

TSM offer concreted physical picture of MP in high energy e^+e^--annihilation.

Table 1. Parameters of TSM.

$\sqrt{s}$ GeV	$\overline{m}$	k_p	N	$\overline{n}^h$	α	Ω	χ^2
14	.08	2.4×10^8	27.7	2.87	.97	2.	2.75
22	3.01	4.91	20.2	4.34	.2	2.	1.29
34.8	6.58	6.96	12.5	4.1	.195	2.	240
43.6	10.3	48.3	5.16	2.31	.444	2.	5.36
50	7.48	1.3	24.6	6.14	.1	2.09	1.97
52	11.5	1.	24.8	6.16	.104	2.46	2.51
55	8.6	$6. \times 10^4$	17.	4.	.26	2.2	124
56	9.81	8.23	6.51	3.73	.273	2.02	1.29
57	11.3	14.4	4.	2.76	.385	2.	1.95
60	8.92	9.	9.31	4.2	.254	1.99	2.97
60.8	9.52	6.68	7.	4.12	.246	2.02	2.98
61.4	10.4	1.	21.1	6.38	.108	2.29	1.76
91.4	10.9	7.86	11.2	4.8	.226	2.	1.16
133	12.0	2.99	17.1	6.5	.14	2.01	4.2
161	3.47	20.2	10.4	6.23	.55	1.99	4.45
172	20.1	9.11	9.17	4.34	.195	1.98	6.86
183	13.2	1.48	54.6	8.9	.086	2.06	2.56
189	15.1	6.9	11.6	5.15	.215	2.01	2.37

3 Dynamics of multiparticle production

We will analyse dynamics of MP which corresponds to values of parameters of TSM (Table 1). We will begin from a cascade stage. This stage is described by two parameters: $\overline{m}$ and k_p.

The average multiplicity of gluons $\overline{m}$ formed on fission stage has tendency to rise. It is changed from ~ 0.1 at 14GeV to ~ 20 at 183 Gev. But we can see certain insignificant deviation from this direction at $\sqrt{s}$=50−61.4GeV, at 183 GeV and considerable decrease to ~ 3.5 at 161GeV. It may be explained by passing over energy threshold of creating of heavy quarks.

It followes from QCD that in particular case ($B = 0$) the parameter k_p that equal to ratio $2\tilde{A}/A \to 1$. Values k_p are changed insignificantly. They are remained ~ 10 at almost all energies. There is some physical senses of this parameter. One of the most interesting from them is temperature T [10]: $T = k_p^{-1}$. From thermodynamical models we can also obtain the following connection

$$k_p^{-1} = T_0 + 1/cE, \tag{16}$$

where T_0 is the temperature of system before interaction, c - thermal capacity, E - energy spended on creationg new particles [10]. In this sense we can make assumptions: temper-