

# Proton Structure Functions

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- **Introduction**
- **high  $Q^2$ , electro-weak effects**
- **QCD fits**
- **low  $x$  behaviour of  $F_2$**
- **Conclusion**

# Introduction

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proton (and n) best known hadronic particle

e.g.  $p$  charge agrees to  $\sim 10^{-21}$  with that of electron,  
mass is known better  $10^{-7}$

Internal properties less well known

e.g. charge radius on the % level

Internal hadronic structure

probed in hard interactions, described since the seventies by quark and gluon densities (pdfs)  
explored in particular by lepton nucleon scattering

⇒ One of the main activities at HERA  
approaching few % level

Important as

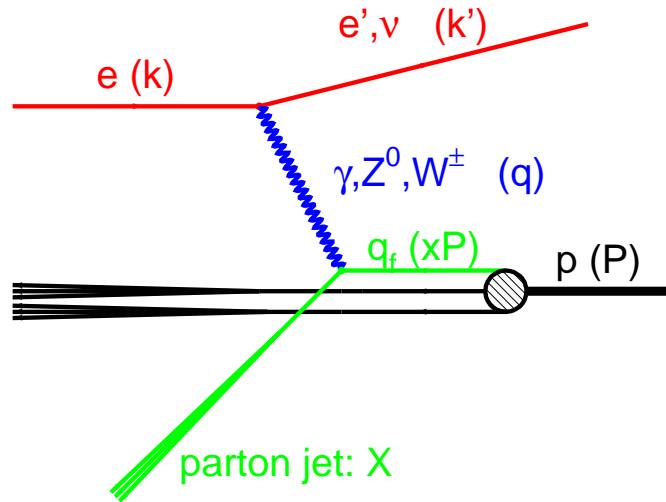
- pdfs not yet predicted by QCD
- pdfs needed to predict other reactions, e.g.  $\bar{p}p$
- Due to constraints by QCD, important testing ground for theory

GPDs, non-collinear pdfs, correlations, spin density not accessed in inclusive DIS, not discussed here

# Inclusive DIS

$ep \rightarrow eX$  (NC)

$ep \rightarrow \nu_e X$  (CC)



$$Q^2 = -q^2$$

4-momentum transfer

$$x = Q^2 / 2(P \cdot q)$$

$p$  momentum fraction of parton

$$y = (P \cdot q) / (P \cdot k)$$

inelasticity

$$NC \quad d^2\sigma_{NC}^{\pm} / dx dQ^2 = \frac{2\pi\alpha^2}{xQ^4} [Y_+ \cdot \tilde{F}_2 \mp Y_- \cdot x\tilde{F}_3 - y^2 \cdot \tilde{F}_L] \equiv \frac{2\pi\alpha^2}{xQ^4} \tilde{\sigma}_{NC}^{\pm}$$

$$Y_{\pm} = 1 \pm (1 - y)^2$$

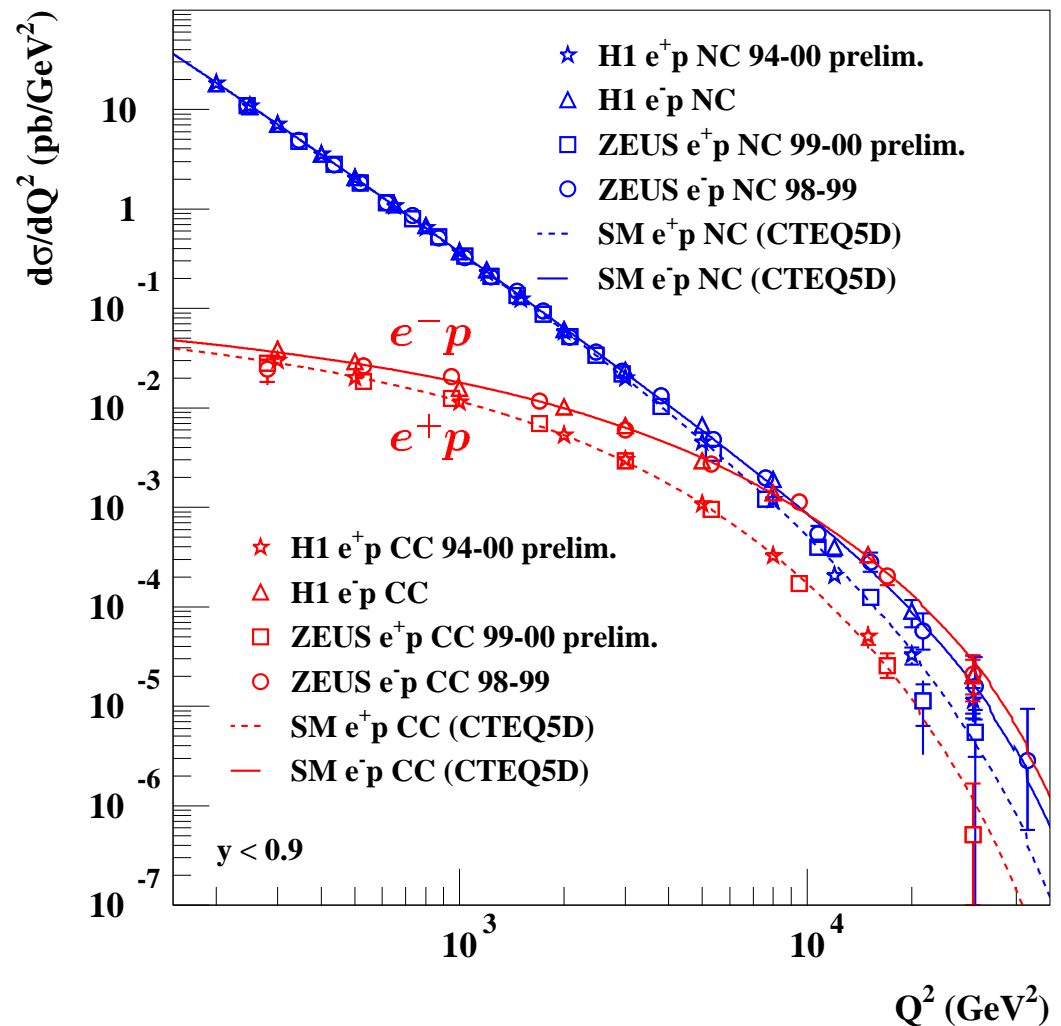
$\tilde{F}_2$ ,	dominating contribution,	in leading order QCD	$\sim x \sum_q (q + \bar{q})$
$x\tilde{F}_3$ ,	in particular $\gamma Z$ interference,	significant at large $Q^2 \gtrsim M_Z^2$	$\sim x \sum_q (q - \bar{q})$
$\tilde{F}_L$ ,	longitudinal contribution,	important only at large $y$ ,	zero in LO QCD

$$CC \quad d^2\sigma_{CC}^{\pm} / dx dQ^2 = \frac{G_F^2}{2\pi x} \left( \frac{M_W^2}{Q^2 + M_W^2} \right)^2 \cdot \tilde{\sigma}_{CC}^{\pm}$$

$$LO \quad \tilde{\sigma}_{CC}^+ = x[(\bar{u} + \bar{c}) + (1 - y)^2(d + s)] \quad \tilde{\sigma}_{CC}^- = x[(u + c) + (1 - y)^2(\bar{d} + \bar{s})]$$

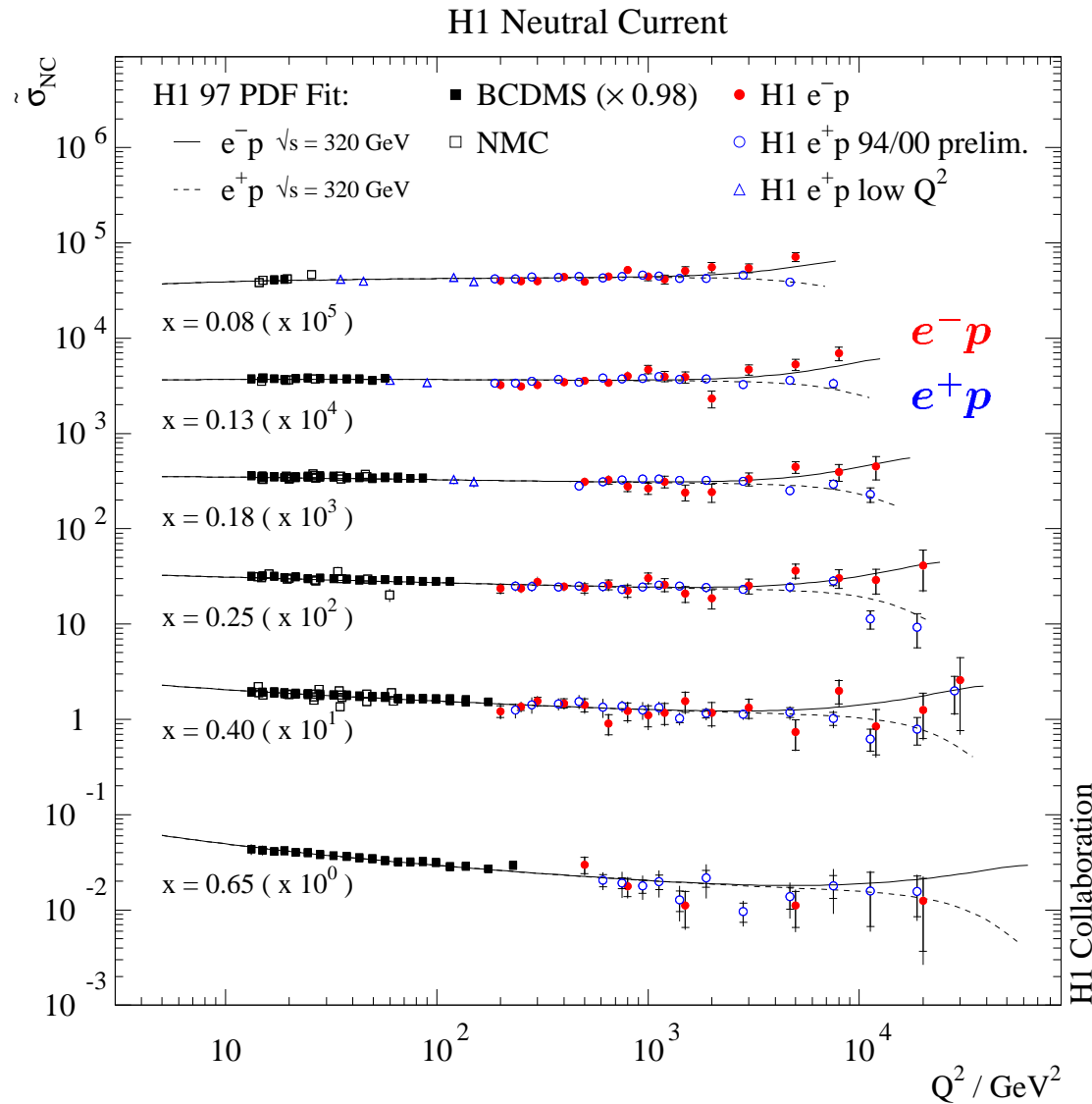
# $d\sigma/dQ^2$ NC vs. CC and $e^-p$ vs. $e^+p$

- ◇ H1 and ZEUS data consistent
- ◇ NC  $\sigma(e^-p) > \sigma(e^+p)$   
 $\gamma Z$  interference
- ◇ CC  $\sigma(e^-p) > \sigma(e^+p)$   
 $\sim xu(x) \sim (1-y)^2 d(x)$
- ◇ data well described by SM in range  
where NC  $d\sigma/dQ^2$  varies by 7 orders



$Q^2 \gtrsim M_Z^2, M_W^2 \Rightarrow \sigma_{CC} \approx \sigma_{NC}$ , illustration of electro-weak unification

# NC reduced cross section $\tilde{\sigma}$ at high $x$



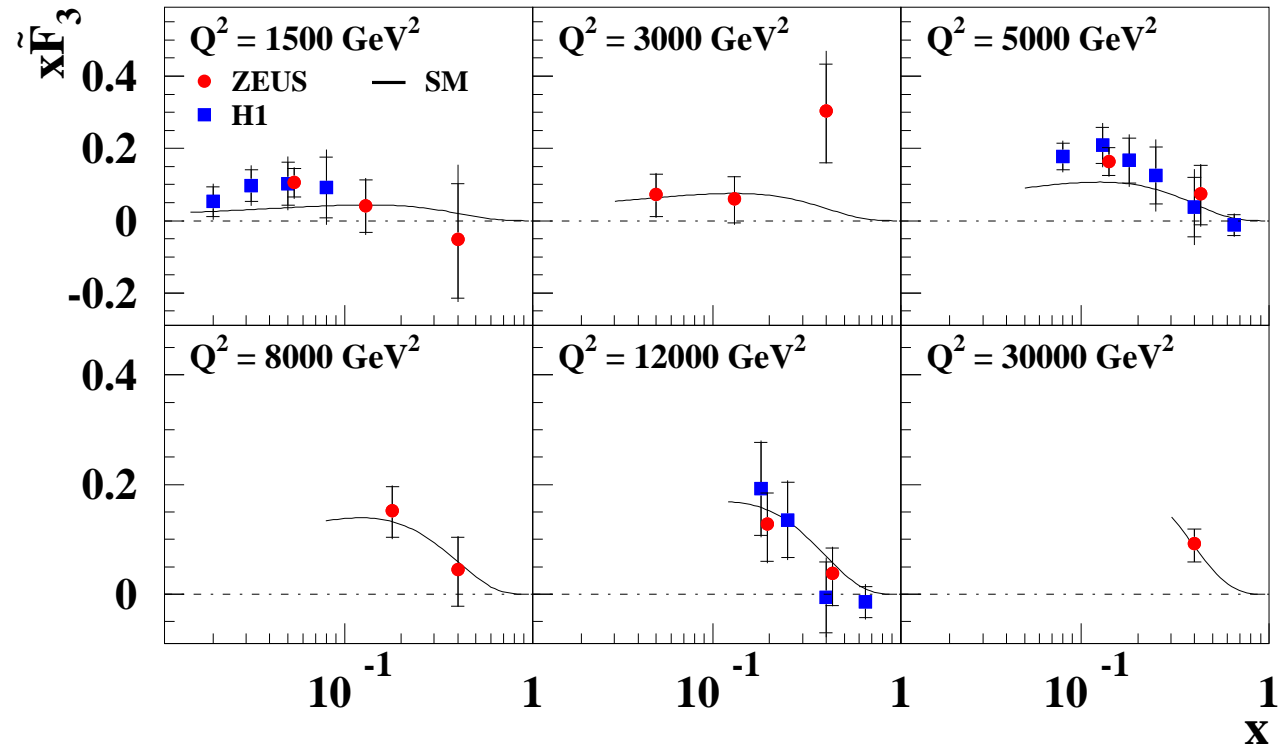
- data consistent within errors  
with fixed target results
- electro-weak effects in NC  
visible at highest  $Q^2$

⇒ evaluate  $x F_3$

# Results on $x\tilde{F}_3$

$$x\tilde{F}_3 \approx x\tilde{F}_3^{\gamma Z} \sim 2u_v + d_v$$

$x\tilde{F}_3$  measured at large  $Q^2$   
consistently with SM



once precisely measured,

$x\tilde{F}_3$  interesting consistency check for  $d_v$  density from NC  $ep$  only

( $u(x)$  much better known)

# Recent QCD Analyses of $F_2$ data

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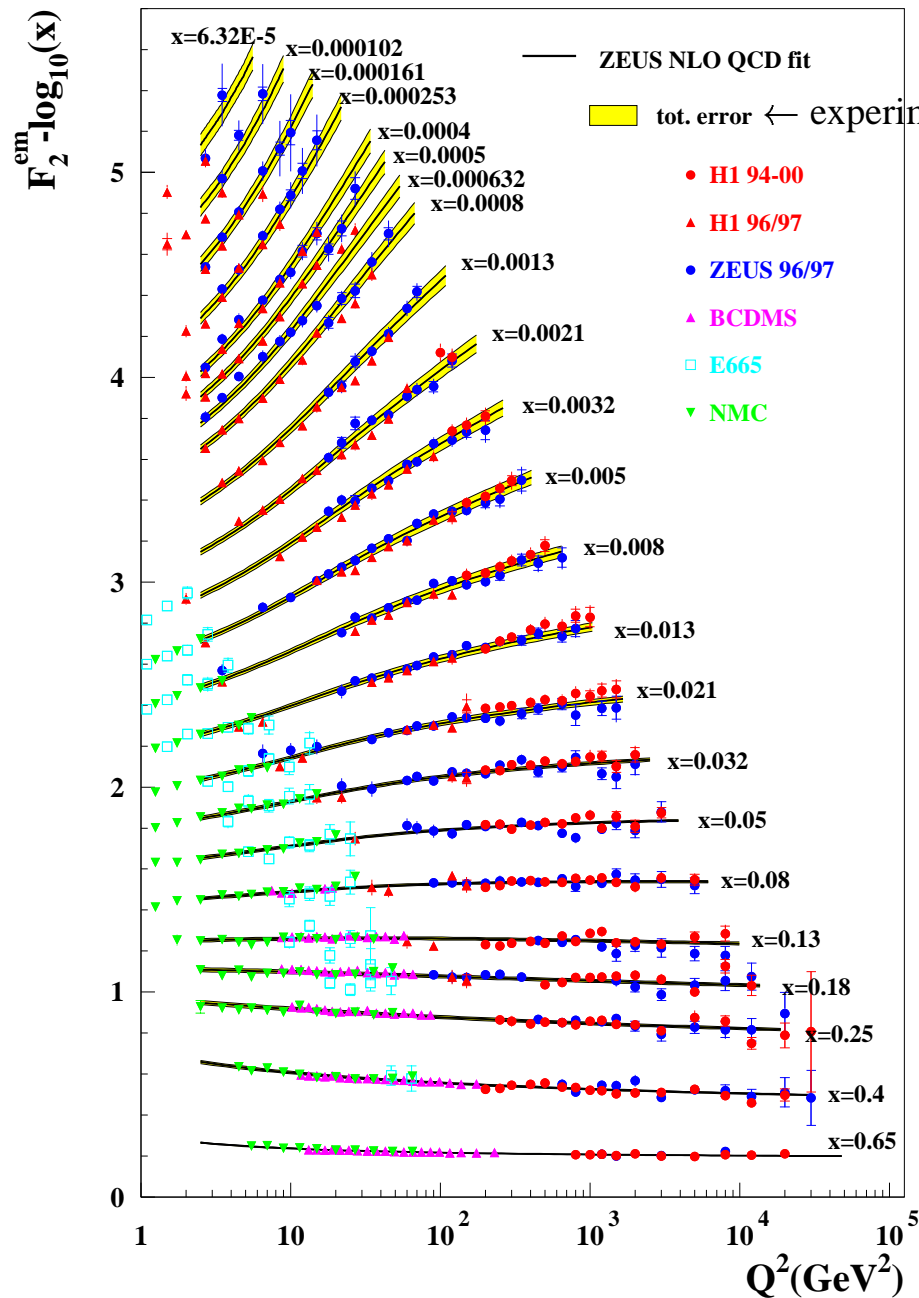
- Procedure :**
- parametrisation of pdfs at starting scale  $Q_0^2$
  - $Q^2$  dependence by DGLAP pQCD evolution in NLO
  - pdf parameters at  $Q_0^2$  determined by fits to measured  $F_2$  at  $Q^2 > Q_{min}^2$

- Approaches differ mainly in :**
- amount of data used
  - parametrisations at  $Q_0^2$
  - treatment of heavy quarks
  - treatment of systematics

H1 NC	H1 NC,CC	ZEUS NC
Eur.Phys.J.C21(2001)3	ICHEP02, 978	DESY-02-105
<hr/>		
<b>other experiments used in main fit</b>		
BCDMS ( $\mu p$ )	$(\mu p, \mu d)$	BCDMS, NMC ( $\mu p, \mu d$ ), E665( $\mu p, \mu d$ ), CCFR( $\nu Fe$ )
<b>fit ted distributions</b>		
<b><i>ep</i> valence and sea terms</b>	$u + c, \bar{u} + \bar{c}, d + s, \bar{d} + \bar{s}, g$	$u_v(x), d_v(x), S(x), \bar{d} - \bar{u}, g$
$Q_{min}^2$ [GeV <sup>2</sup> ]	3.5	2.5
main aim	$\alpha_s, g(x)$	pdfs, $\alpha_s$

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## ZEUS NLO fit compared with HERA and fixed target NC data



- ▽ **ZEUS and H1 NLO fits describe used data well**  
( $\sim 10^{-4} < x < 0.65$ )
- ▽ **DGLAP QCD fit follows strong rise**  
at small  $x$  driven in the fits by  $g(x)$
- ▽ **Is this approach at small  $x$  really good enough?**  
(neglected  $\ln 1/x$  terms important?)
- ▽ **or parametrisations too flexible ?**  
(e.g. ZEUS fit 11 parameters)?

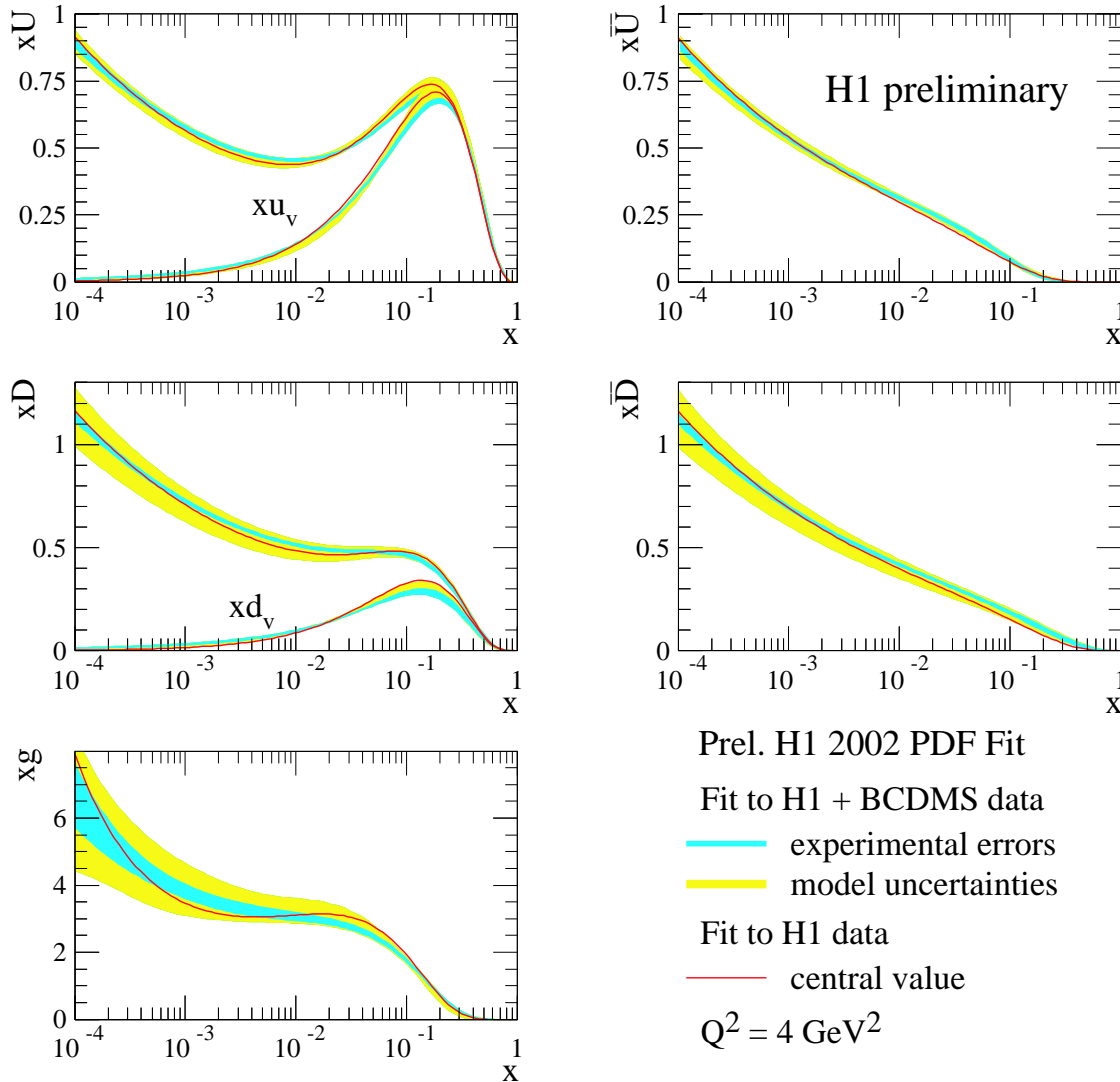
**In any case**

**data consistent with DGLAP evolution of pdfs**



# PDFs from H1 2002 Fit (prel.) fitting H1 (NC + CC) + BCDMS ( $\mu p, \mu d$ )

H1 Parton Distributions



Determination of

$$\begin{aligned}
 U &= u + c & \bar{U} &= \bar{u} + \bar{c} \\
 D &= d + s & \bar{D} &= \bar{d} + \bar{s} \\
 u_v &= U - \bar{U} & d_v &= D - \bar{D} \\
 g & & &
 \end{aligned}$$

**fitting in total 13 free parameters**

$\alpha_s = 0.1185$  fixed

**massless quarks**

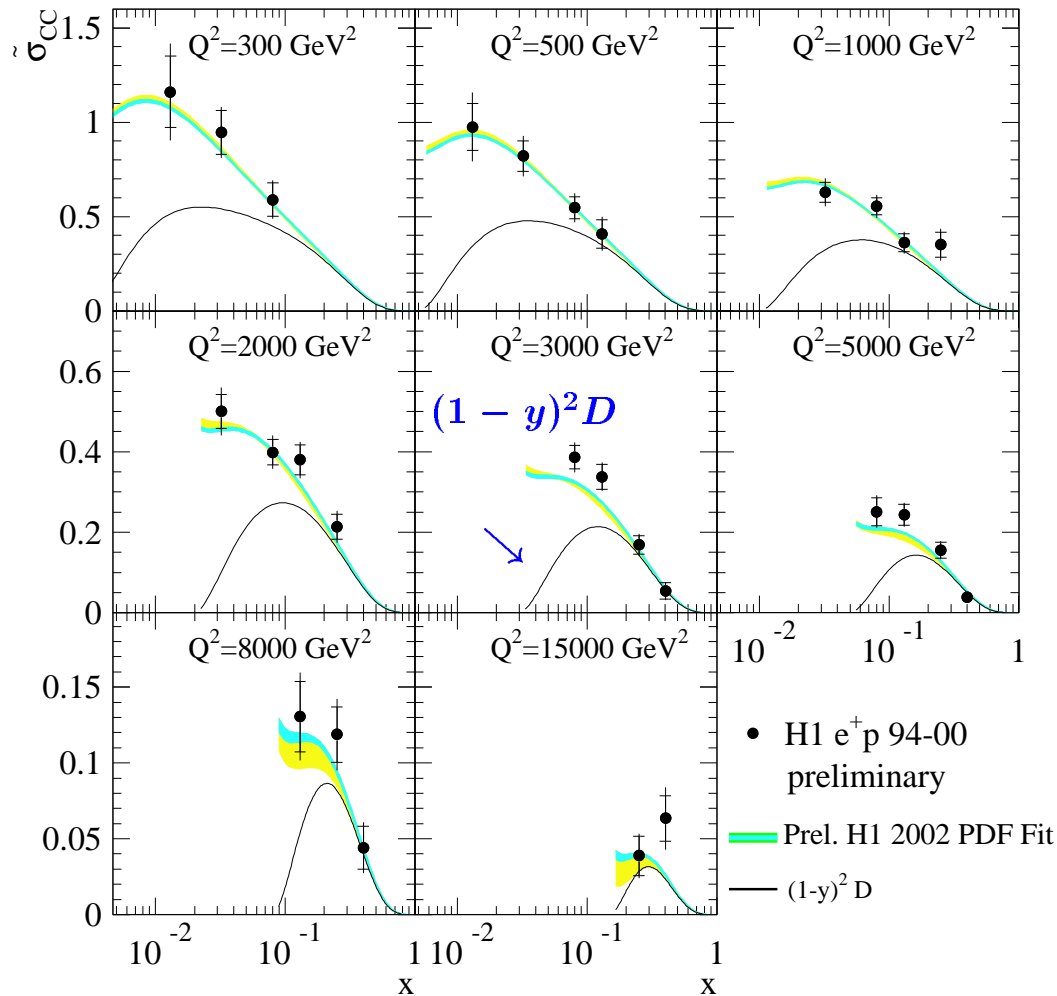
**assuming  $xb(x, Q^2)$  for  $Q^2 < m_b^2$**

**H1-only fit is very consistent  
with fit including BCDMS for  
reduction of error bands at large  $x$**

# Sensitivity to d quarks

distinguish *u* and *d* quarks at high *x* in CC

H1 Charged Current



$$e^+p \rightarrow \bar{\nu}_e X$$

$$\sigma \sim x[(\bar{u} + \bar{c}) + (1 - y)^2(d + s)]$$

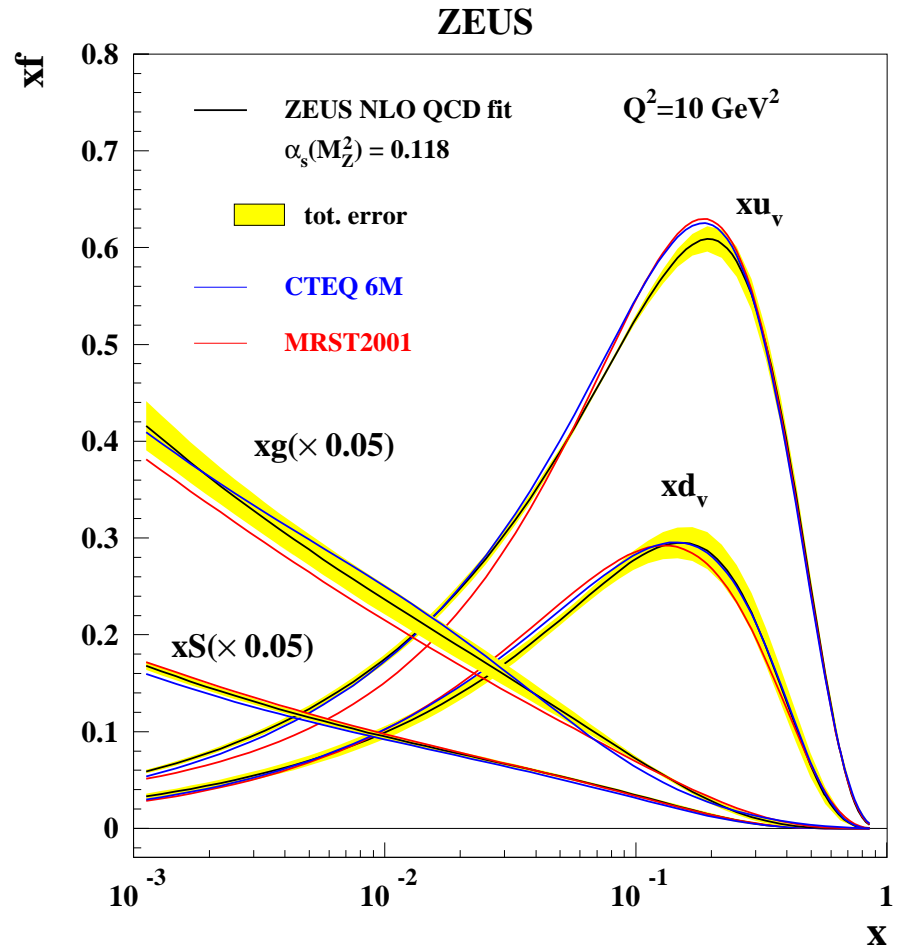
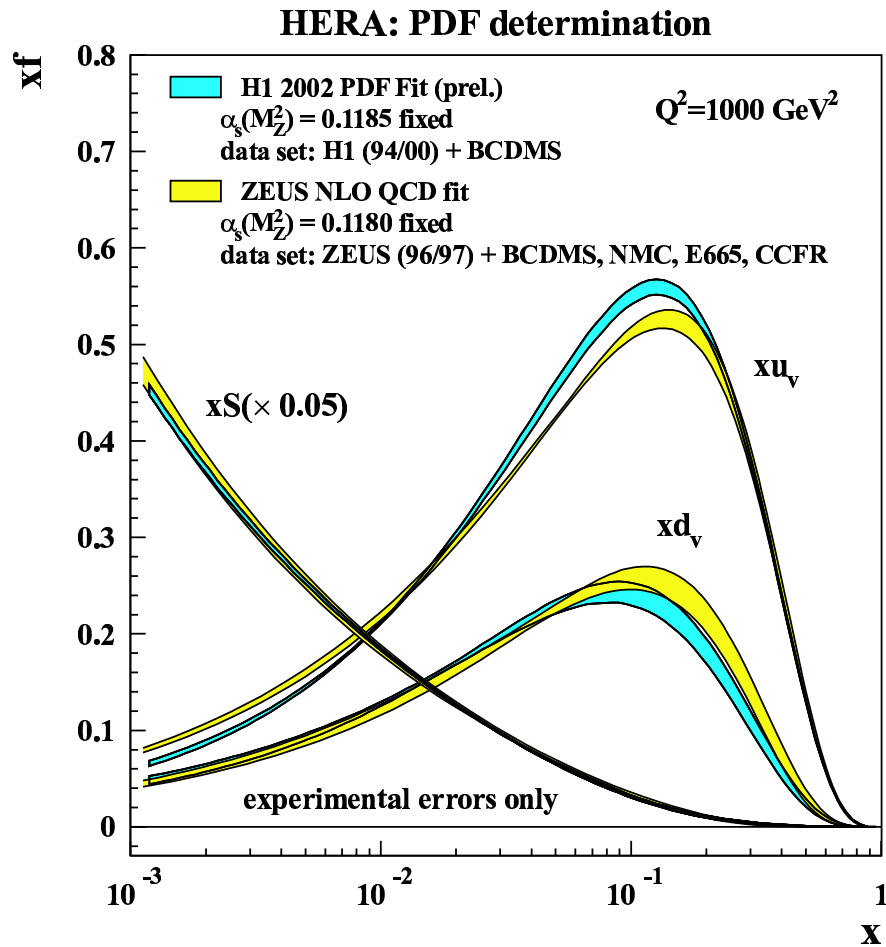
$$e^-p \rightarrow \nu_e X$$

$$\sigma \sim x[(u + c) + (1 - y)^2(\bar{d} + \bar{s})]$$

$(1 - y)^2 D$  dominates at  $x \gtrsim 0.1$

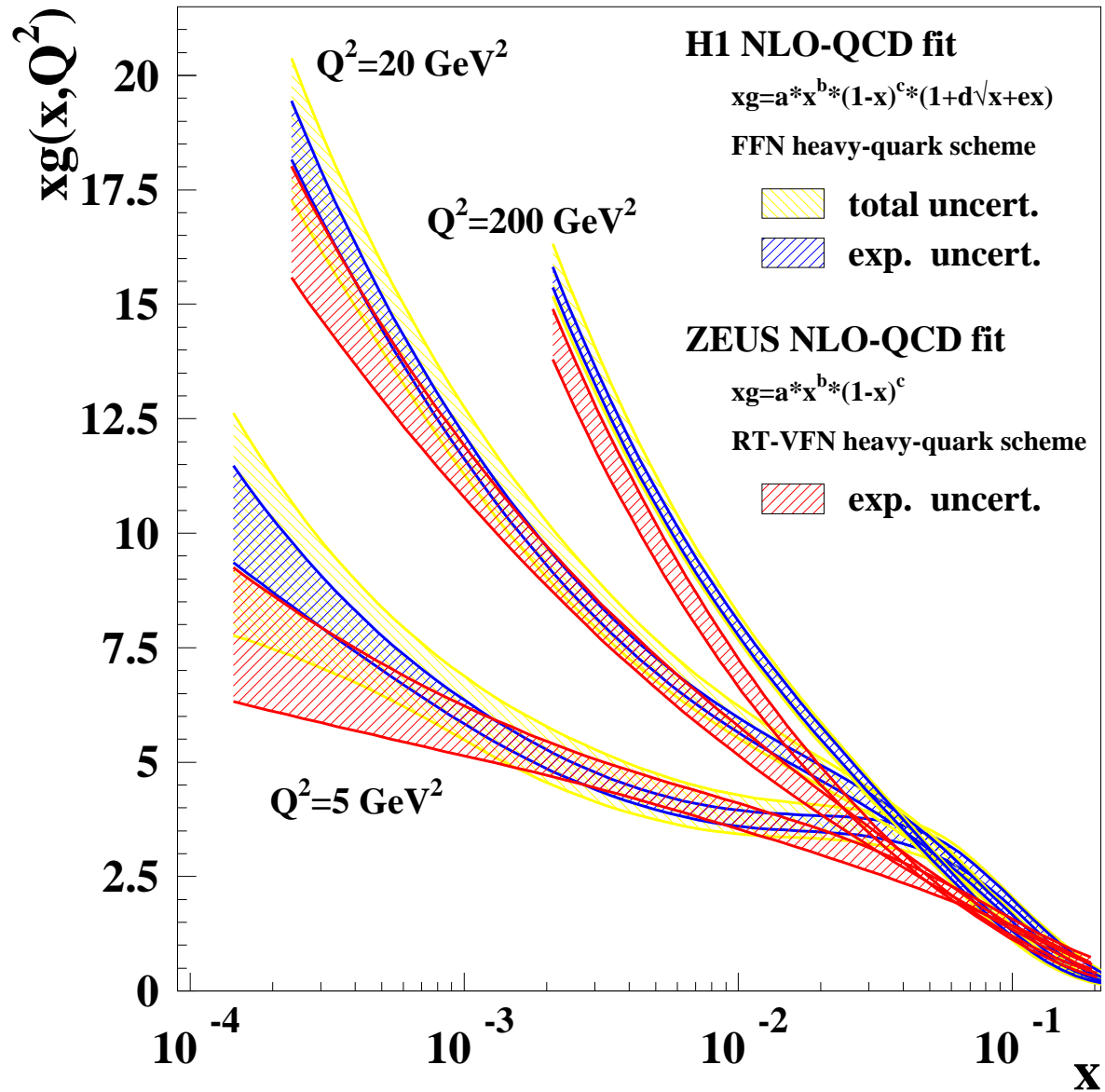
**HERA data begin to constrain *d* quark without nuclear corrections**

# Comparison of different fits

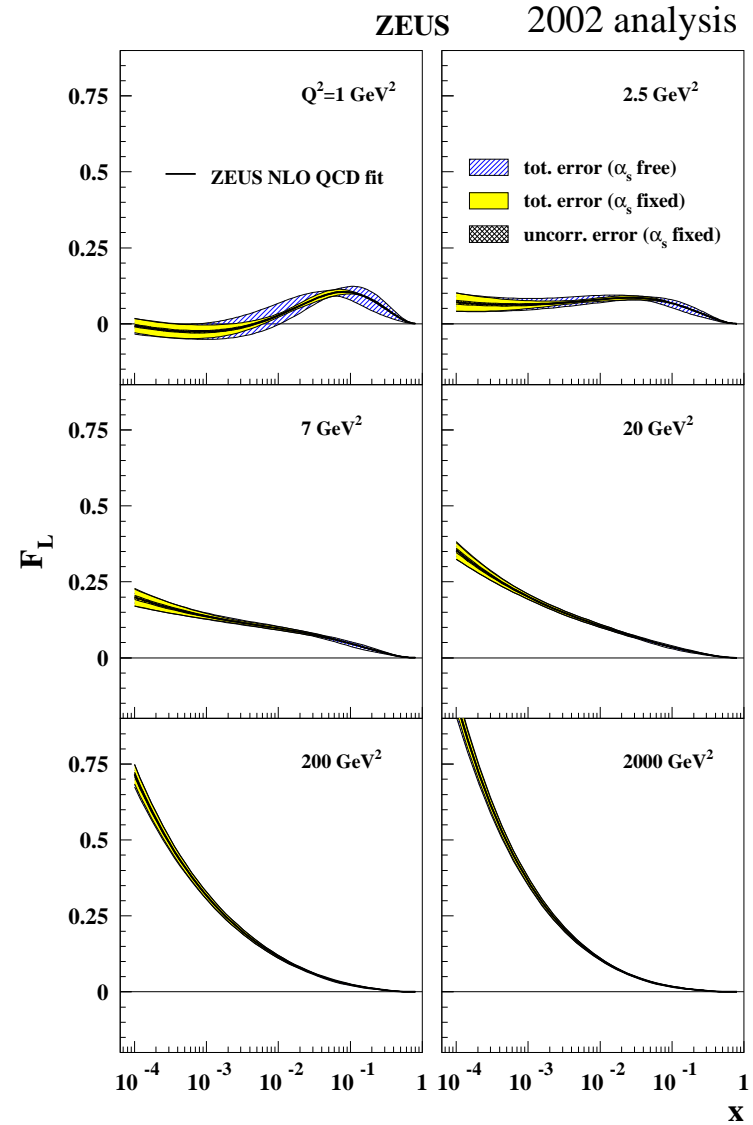
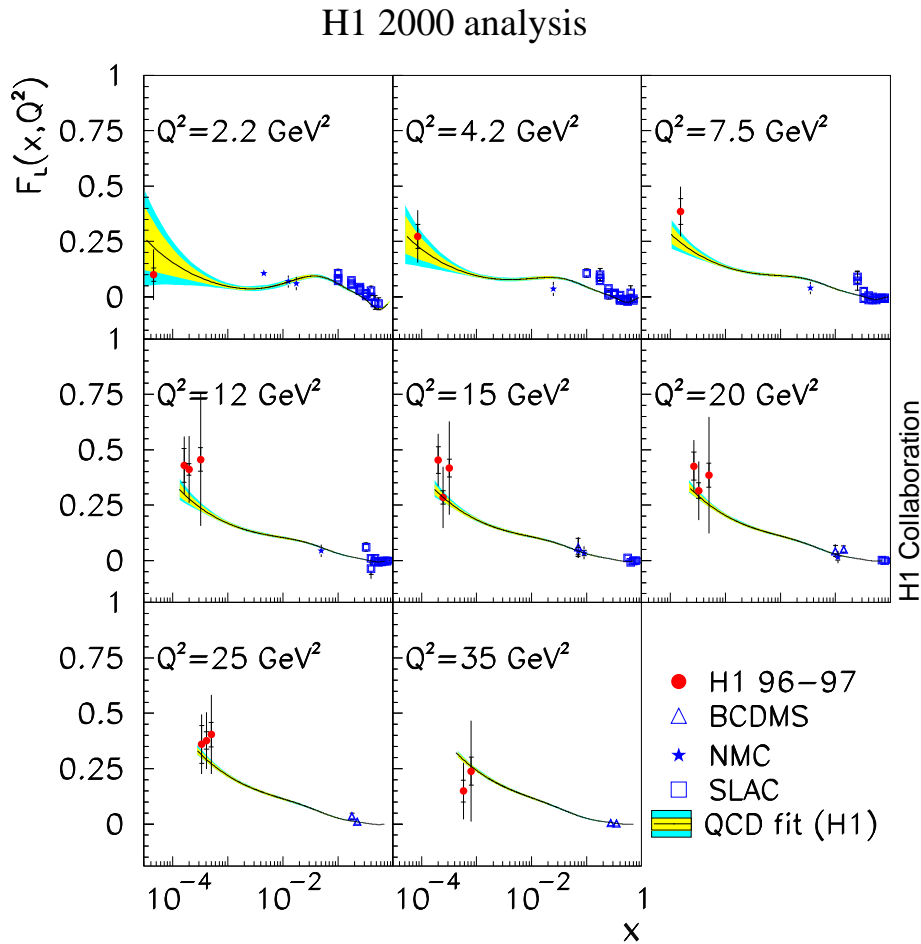


**pdfs from inclusive DIS fits by ZEUS and H1 in reasonable agreement among themselves and with global fits**

# H1+ZEUS



# $F_L$ determinations and predictions



at small  $x$   $F_L \sim \alpha_s x g(x)$  (approx.)

H1 determinations consistent with pQCD expectation

**More data desirable, important consistency check**

# $\alpha_s$

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In central H1 and ZEUS fits  $\alpha_s(M_Z^2)$  is fixed

Special fits with  $\alpha_s(M_Z^2)$  as free parameter yield

**H1 analysis 2000**

$$\alpha_s(M_Z^2) = 0.1150 \pm 0.0017(\text{exp}) \begin{matrix} +0.0009 \\ -0.0005 \end{matrix}(\text{model})$$

**ZEUS analysis**

$$\alpha_s(M_Z^2) = 0.1166 \pm 0.0008(\text{uncorr.}) \pm 0.0032(\text{corr.}) \pm 0.0036(\text{norm.}) \pm 0.0018(\text{model})$$

**uncorr. systematics      corr. systematics      normalisation of exps.**

world average (PDG 2000) :  $\alpha_s(M_Z^2) = 0.1185 \pm 0.0020$

**theoretical error:**

**splitting terms not yet available in next to NLO for inclusive DIS**

**uncertainty of  $\approx \pm 0.005$  estimated by change of renormalisation scale by factor 4**

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**results consistent and very competitive  
will improve with NNLO**

# The Rise of $F_2$ towards low $X$

strong rise of  $F_2$  since long discussed in QCD frame:

1974: (De Rujula, Glashow, Politzer, Treiman, Wilczek, Zee)

2000

( 1994: (Ball, Forte) )

double asymptotic limit: rise faster

than any power of  $\log(x)$ , nearly like power in  $x$ .

1976 - 1978 (Balitsky, Fadin, Kuraev, Lipatov)

BFKL theory expects power behaviour  $F_2 \sim x^{-\lambda}$

1981, 1983 (Gribov, Levin, Ryskin)

1986 (Mueller, Qiu)

saturation effects due to increasing gluon densities

Recent years (Buchmüller, Gehrmann, Hebecker ;

Golec-Biernat, Wüsthoff ;.....)

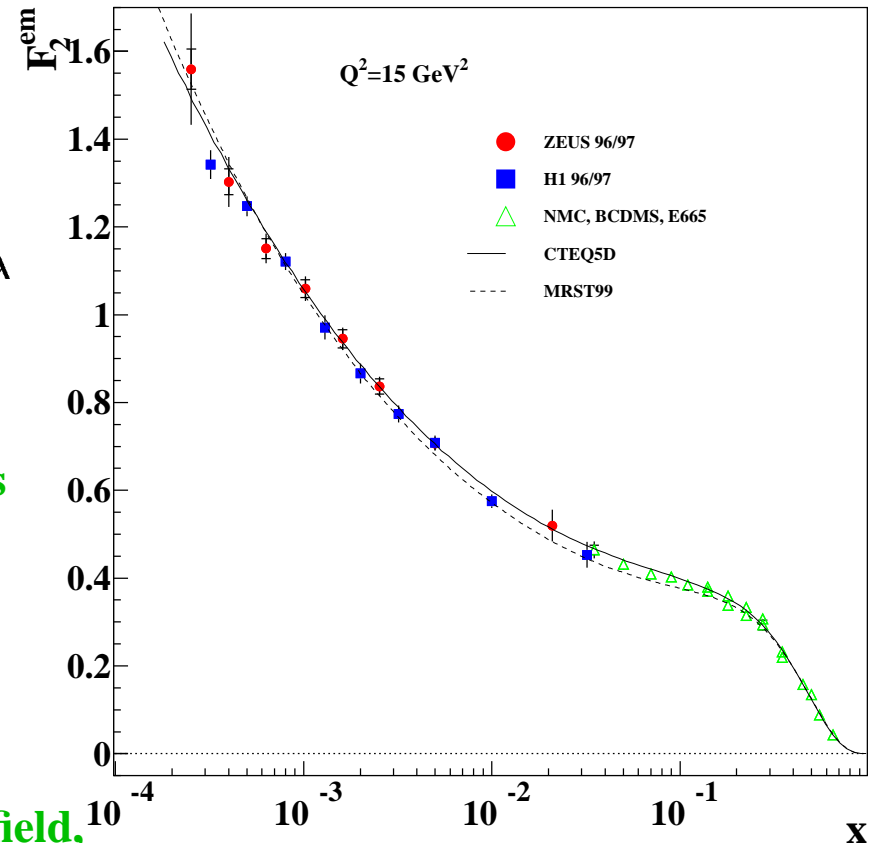
discussion of colour dipole models

$\gamma^* \rightarrow q\bar{q}, q\bar{q}g$   $\times$  dipole- $p$  cross section

non-perturbative interaction with proton colour field,

saturation at large radii, i.e. small  $Q^2, p_t^2$ ,

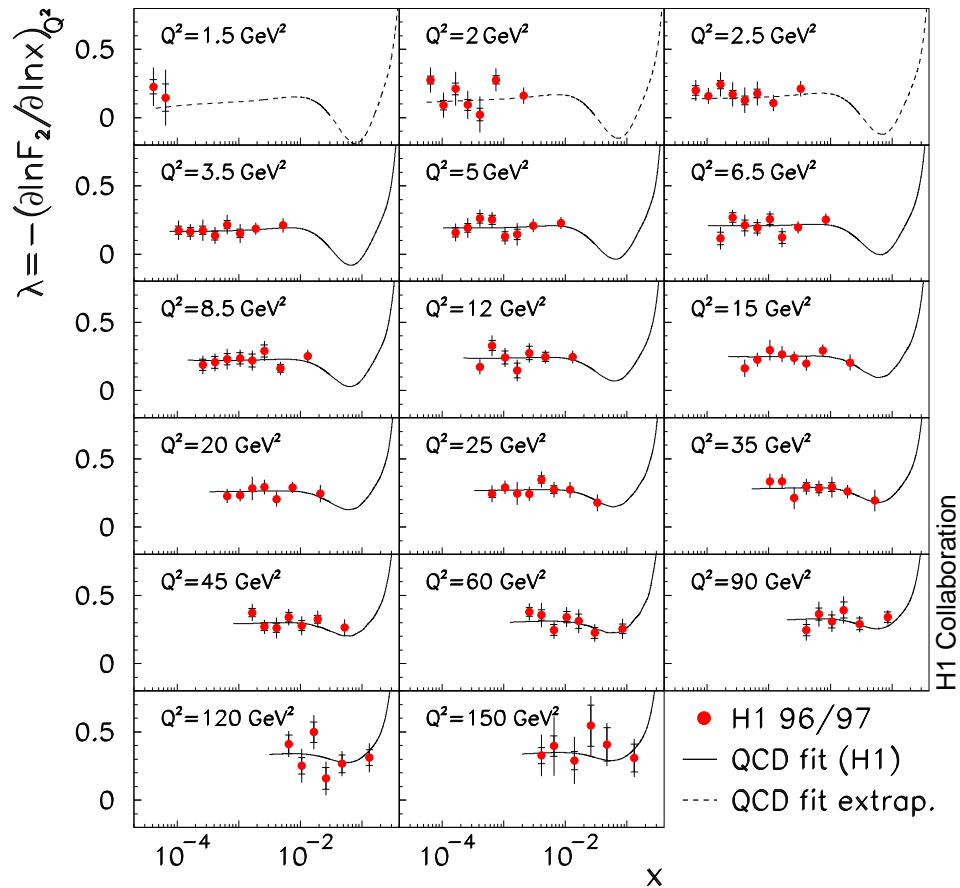
reached at smaller  $x_{Bj}$  already at smaller radii, i.e. at larger  $Q^2, p_t^2$ .



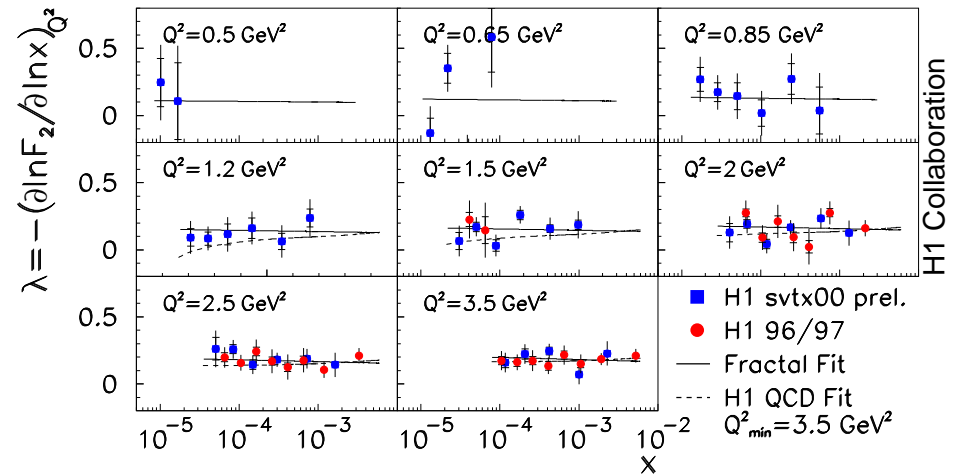
Precision of present data allows to study rise of  $F_2$  locally

# $x$ dependence of $\lambda = -(\partial \ln F_2 / \partial \ln x)_{Q^2}$

nominal vertex data



shifted vertex data (+ nominal)



fixed  $Q^2$ ,  $x < 0.01$  :  $\lambda \approx \text{const}$

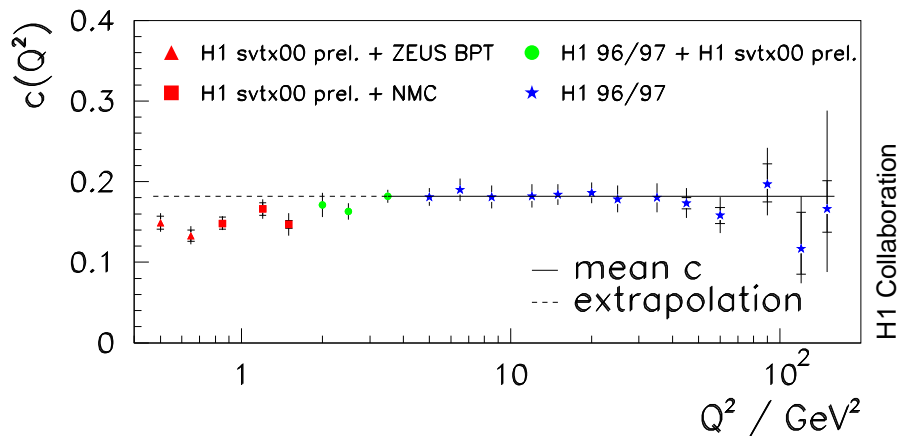
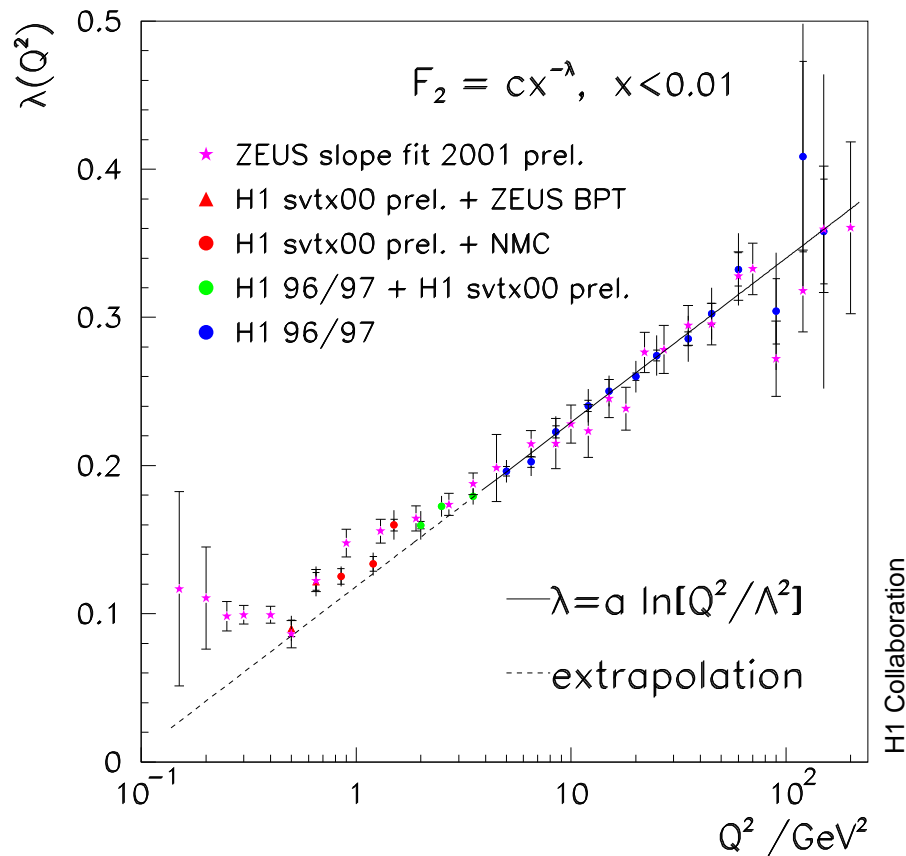
$$\Rightarrow F_2 = c(Q^2) \cdot x^{-\lambda(Q^2)}$$

no taming of rise visible yet for  $0.5 \lesssim Q^2 \lesssim 150 \text{ GeV}^2$



# Transition to small $Q^2$

H1 combined with NMC and ZEUS



for  $Q^2 \lesssim 3 \text{ GeV}^2$  :

deviation from log-dependence,

decrease of  $c$

$$\sigma_{tot}^{\gamma^*P} = 4\pi\alpha^2/Q^2 F_2 \sim x^{-\lambda}/Q^2$$

$$s = W^2 \sim Q^2/x$$

**Hadronic interactions at high energy:**

**Regge theory:**  $\sigma_{tot} \sim s^{\alpha_P(0)-1}$

$$\alpha_P(0) - 1 \approx 0.08 \quad (\text{Donnachie, Landshoff})$$

→ expect

$$F_2 \sim x^{-(\alpha_P(0)-1)} \approx x^{-0.08}, \quad \lambda \approx 0.08$$

$Q^2 \lesssim 1 \text{ GeV}^2$  :

rise compatible with soft hadronic interactions

## Conclusions

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- **Data on inclusive  $e^{+-}p$  scattering much improved in recent years**
- **High  $Q^2$  NC and CC interactions consistent with QCD and EW expectations**
- **pQCD fits, based on DGLAP evolution of pdfs describe data very well**
- **pdfs with uncertainties given and high precision  $\alpha_s$  determined**
- **at  $x \lesssim 0.01$  data consistent with  $F_2 \sim x^{-\lambda}$ , no damping of rise yet visible**
- **at low  $Q^2$  ( $Q^2 \lesssim 1 \text{ GeV}^2$ ) rise similar as in soft hadronic interactions**