Instanton Model of light hadrons: from Low to High Energies.

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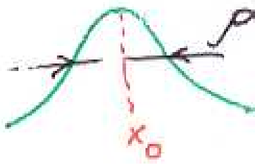
1. Instanton Model of QCD vacuum and Light Mesons
2. $\pi\gamma^*\gamma^*$ form factor: from Anomaly to twist expansion.
3. Instanton corrections to Sudakov F.F.

Instanton Vacuum of QCD /BPST/

$$\begin{cases} D_\mu G_{\mu\nu} = 0 \\ -i\hat{D}\,\Psi = \varepsilon \Psi \end{cases}$$

Classical Gluon field solution, Instanton

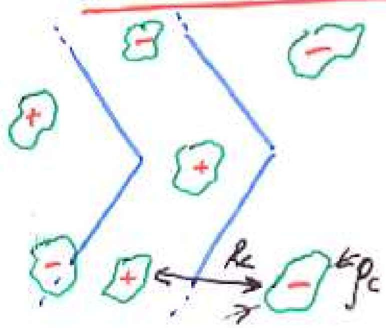
$$\frac{\tau^a}{2} A^{\pm a}_\mu (x) = U^{ab} \frac{1}{g} \tau^a \eta^{\pm b}_{\mu\nu} \frac{(x-x_0)_\nu}{(x-x_0)^2+\rho^2} \frac{\rho^2}{(x-x_0)^2}$$

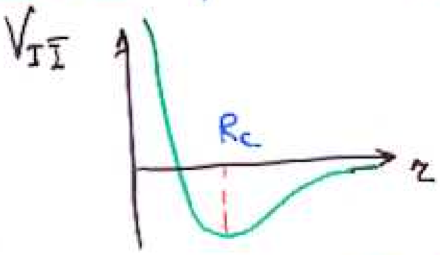


Quark zero mode ($\varepsilon=0$) solution

$$\psi^{\pm}(x) = \frac{\sqrt{2}}{\pi} \frac{\rho}{[\rho^2+(x-x_0)^2]^{3/2}} \chi^{\pm}$$

Instanton Interacting Liquid /Shuryak/



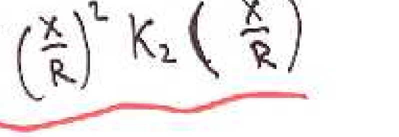


$$\begin{cases} \lambda_g^2 = 1\text{–}1.5\,\mathrm{GeV}^2 \\ \lambda_q^2 = 0.5\text{–}0.6\,\mathrm{GeV}^2 \end{cases}$$

ρ_c - mean size $\approx 0.3\,\mathrm{fm}$

$n_c \sim 1/R_c^4$ – effective density $\approx 1\,\mathrm{fm}^{-4}$

Constrained Instantons. /A.D., S.Mikhailov/ 97'99



$$A^{\pm a}_\mu(x) = \frac{1}{g} \eta^a_{\mu\nu} \frac{x_\nu}{x^2+\rho^2(x^2)} \frac{\rho^2(x^2)}{x^2}$$

$$\rho^2(0) = \rho^2 \qquad \rho^2(x^2\to\infty) \sim e^{-(|x|/R)^\alpha}$$

$$\rho^2(x^2) = \rho^2 \left(\frac{x}{R}\right)^2 K_2\left(\frac{x}{R}\right)$$

$$\eta^{a\pm}_{\mu\nu} = \begin{cases} \varepsilon^{aij} & \mu=i \quad \nu=j \\ \mp\delta^{ai} & \mu=4 \quad \nu=i \\ \pm\delta^{aj} & \mu=j \quad \nu=4 \end{cases}$$

Distribution functions in Vacuum.

(A) Gluon Field Strength Correlator

$$D^{\mu\nu,\rho\sigma}(x) = \langle 0 | T(G^a_{\mu\nu}(x)\, \tilde{E}^{ab}(x,0)\, G^b_{\rho\sigma}(y)) | 0 \rangle =$$

$$= \frac{1}{24} \underline{\langle 0 | G^2 | 0 \rangle} \Big\{ (\delta_{\mu\rho}\delta_{\nu\sigma} - \delta_{\mu\sigma}\delta_{\nu\rho}) [\underline{D(x^2)} + \underline{D_1(x^2)}] +$$

$$+ (x_\mu x_\rho \delta_{\nu\sigma} - x_\mu x_\sigma \delta_{\nu\rho} + x_\nu x_\sigma \delta_{\mu\rho} - x_\nu x_\rho \delta_{\mu\sigma}) \cdot \underline{\frac{\partial D_1(x^2)}{\partial x^2}} \Big\}$$

$$D(x^2) = D(0) - \frac{x^2}{8} \underline{\lambda_g^2} + \dots$$

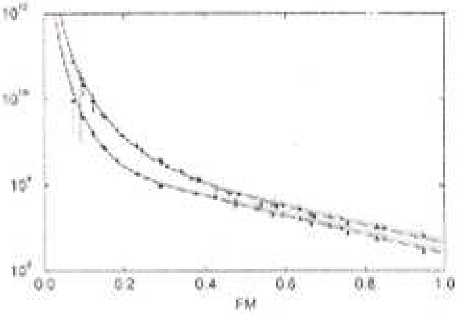


Fig.1 $D_{\|}^L/a^4 = \mathcal{D} + \mathcal{D}_1 + x^2 \partial\mathcal{D}_1/\partial x^2$ and $D_\perp/a^4 = \mathcal{D} + \mathcal{D}_1$ versus x. The lines correspond to the best fit to eq.(19),(20).

$$\langle G^2 \rangle \approx 32\pi^2 n_c \approx 0.5\,\mathrm{GeV}^4$$

$$\lambda_g^2 = \frac{\langle G^a_{\mu\nu} \tilde{D}^2 G^a_{\mu\nu} \rangle}{\langle G^2 \rangle} \approx \frac{24}{5} \frac{1}{\rho_c^2} \approx 1.2\,\mathrm{GeV}^2$$

$$\to \langle g G^3 \rangle - \langle g^4 (\bar{q} \gamma_\mu \tfrac{\lambda^a}{2} q)^2 \rangle$$

(B) Quark Field Correlator

$$M(x) = \langle 0 | \bar{q}(0)\, E(0,x)\, q(x) | 0 \rangle = \langle \bar{q} q \rangle\, \underline{S(x^2)}$$

$$S(x^2) = 1 - \frac{x^2}{8} \lambda_q^2 + \dots$$

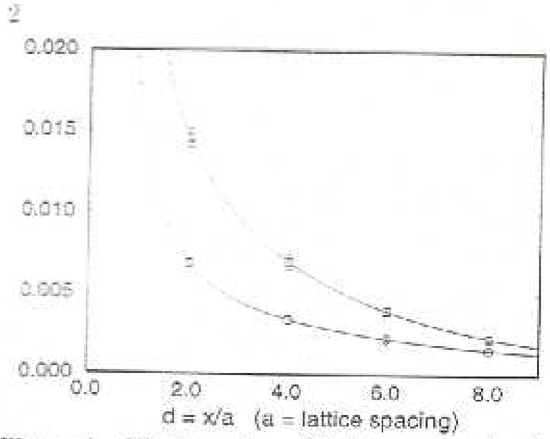


Figure 1. The function $a^3 C_0(x)$ versus the distance d, for the full-QCD case at $\beta = 5.35$ and quark masses $a \cdot m_q = 0.01$ (circles) and $a \cdot m_q = 0.02$ (squares). The curves correspond to our best fits [Eq. (6)].

$$\lambda_q^2 = \frac{\langle \bar{q} D^2 q \rangle}{\langle \bar{q} q \rangle} \approx \frac{2}{\rho_c^2} \approx 0.5\,\mathrm{GeV}^2$$

$$\to \langle \bar{q}\, i g G^a_{\mu\nu} \tfrac{\lambda^a}{2} \sigma^{\mu\nu} q \rangle$$

1. $D \gg D_1$
2. $\lambda_g^2 \simeq \lambda_{1,g}^2$
3. $\lambda_g^2 \gg \lambda_q^2$ $\qquad \dfrac{\lambda_g^2}{\lambda_q^2} = 2.4$

/A.D., Mikhailov, Esaybegyan/
Instanton 97', 99'

/Lattice: 96', 98'-02'
M. D'Elia, A. Di Giacomo, E. Meggiolaro, H. Panagopoulos/

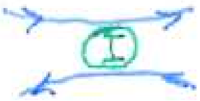
Nonlocal Effective Instanton induced Action

$$S = S_0 + S_{4q}$$

$$S_0 = \int d^4x\, d^4y\; \delta(x-y)\, \bar{Q}(x,X)\, i\hat{\partial}_y\, Q(X,y)$$

$$S_{4q} = \frac{1}{2} G_I \int d^4X \int \prod_{i=1}^{4} d^4x_i\; K_I(x_1,x_2,x_3,x_4)\,\cdot$$

$$\cdot \Big\{ \sum_j \big[\bar{Q}_R(X-x_1,X)\,\Gamma_j\, Q_L(X, X+x_3)\big]\big[\bar{Q}_R(X-x_2,X)\,\Gamma_j\, Q_L(X,X+x_4)\big] + (R \leftrightarrow L) \Big\}$$



$$\Gamma_j \times \Gamma_j = 1\times 1 - \tau^a \times \tau^a \;;\; \frac{1}{2(2N_c-1)}\left(\sigma_{\mu\nu}\times\sigma_{\mu\nu} - \tau^a\sigma_{\mu\nu}\times\tau^a\sigma_{\mu\nu}\right)$$

$$Q_{R,L}(x,y) = \frac{1\pm\gamma_5}{2}\, Q(x,y)$$

Gauged Quark Field

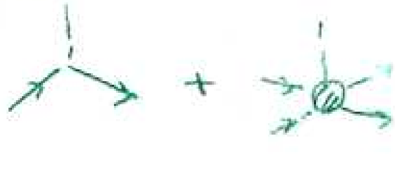
$$Q(x,y) = P\exp\left\{-i\int_x^y dz^\mu\, \Lambda^a_\mu(z)\,\frac{\tau^a}{2}\right\} q(y)$$

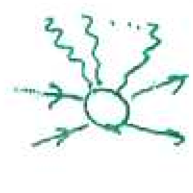
$$\Lambda^a_\mu(z) = V^a_\mu(z) + A^a_\mu(z)\gamma_5 + \ldots$$

$$K_I(x_1,x_2,x_3,x_4) = f(x_1)\, f(x_2)\, f(x_3)\, f(x_4)$$

in separable approximation

By Gauging Quark Fields the conserved currents consistent with Ward-Takahashi Identities are generated



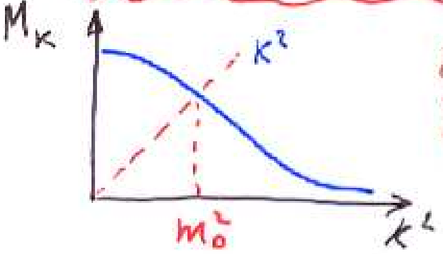


Quark-Pion-Photon Dynamics

Due to SBCS the Dynamical Quark Mass appear

$$\boxed{S^{-1}(k) = \hat{k} - M(k^2)} \xrightarrow[k^2\to\infty]{} \hat{k} - m_c$$

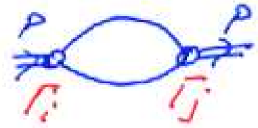
$$\boxed{M(k^2) = \frac{1 - \mathrm{sgn}(k^2 - m_0^2)\sqrt{1 - k^2 Q^2(k^2)}}{Q(k^2)}}$$



$$Q(k^2) = \frac{1}{2N_c} F_{4d}\left\{\langle 0|\bar{q}(x)\, P\exp[ig\int_0^x dz_\mu A^\mu(z)]\, q(0)|0\rangle\right\}$$

4d Fourier tr-n

Polarization operator $J_{ij}(p^2) =$



enters scattering Matrix $T = \frac{G}{1 - GJ}$ bound state pole $\qquad g^2_{\pi q} = \frac{dJ}{dp^2}\Big|_{p^2 = M_\pi^2}$

Quark-Pion Vertex

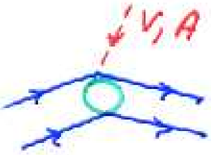
$V_\pi^a(k,k') =$

$$= [g_\pi - \tilde{g}_\pi \frac{\hat{p}}{|p|}]\, i\gamma_5 \tau^a \frac{\sqrt{M_k M_{k'}}}{f_\pi}$$

Conserved Currents

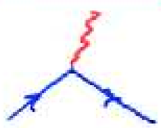
$j_\mu(x) =$

$+$

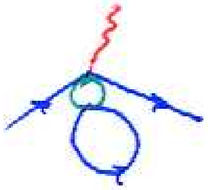


Quark-Photon Vertex

$\Gamma_\mu(k,k') =$



$+$

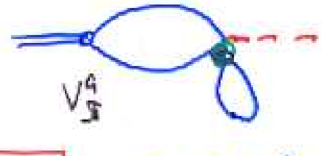


$$= eQ\left[\gamma_\mu - (k+k')_\mu \frac{M_k - M_{k'}}{k^2 - k'^2}\right] \xrightarrow[k^2\to\infty]{} eQ\gamma_\mu$$

Pion Decay Constant

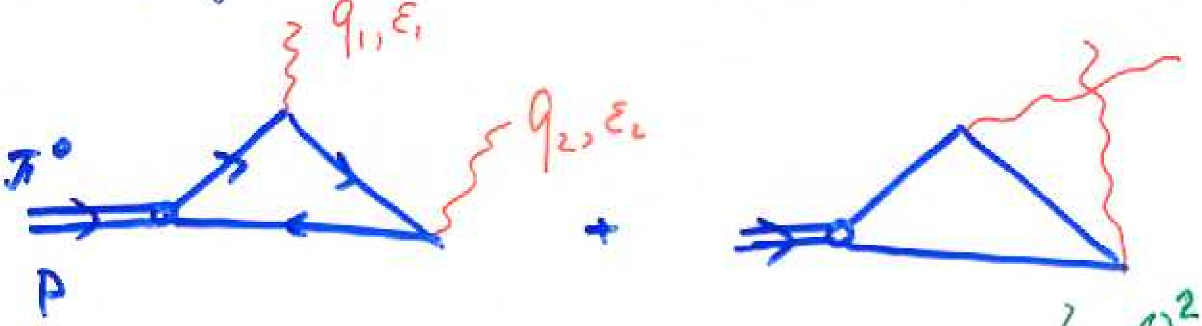
$f_\pi \sim$

$+$



$$\boxed{f_\pi = \frac{1}{g_\pi} M(0)}$$ Goldberger-Treiman relation

$\pi\gamma^*\gamma^*$ Form Factor



$$\mathcal{M}\left(\gamma^*(q_1,\varepsilon_1)\gamma^*(q_2,\varepsilon_2)\to\pi(P)\right) = \left\{ \begin{array}{l} Q^2 = -(q_1^2+q_2^2) \geq 0 \\ \omega = \dfrac{q_1^2-q_2^2}{q_1^2+q_2^2} \end{array} \right.$$

$$= e^2 \varepsilon_{\mu\nu\alpha\beta}\, \varepsilon_1^\mu\, \varepsilon_2^\nu\, q_1^\alpha\, q_2^\beta\, T(q_1^2; q_2^2; P^2)$$

In the chiral limit $P^2 = m_\pi^2 = 0$

$$T(0,0,0) = \frac{1}{4\pi^2 f_\pi} \quad \text{Axial Anomaly}$$

In the symmetric asymptotic limit

$$T\left(\tfrac{1}{2}q^2, \tfrac{1}{2}q^2, m_\pi^2\right) \underset{-q^2\to\infty}{=} \left\{ \begin{array}{ll} \dfrac{4}{3}\dfrac{f_\pi}{Q^2}\left(1+\dfrac{\Delta^2}{Q^2}\right) & \text{pQCD} \\ (M_q^2/2\pi^2 f_\pi)\,\ln(Q^2/M_q^2)/Q^2 & \text{Quark triangle model} \end{array} \right.$$

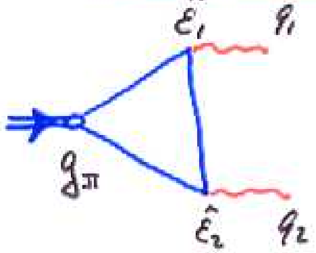
In the asymmetric asymptotic limit

$$T(q^2; 0, 0) \underset{-q^2\to\infty}{=} \left\{ \begin{array}{ll} J^{(2)}\dfrac{f_\pi}{Q^2} + J^{(4)}\dfrac{f_\pi}{Q^4} & \text{pQCD} \\ \dfrac{M_q^2}{4\pi^2 f_\pi}\;\dfrac{\ln^2(Q^2/M_q^2)}{Q^2} & \text{quark triangle loop} \end{array} \right.$$

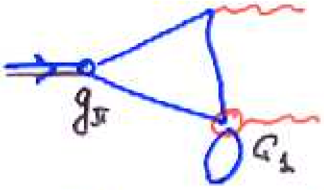
$$J = \frac{4}{3}\int_0^1 \frac{dx}{x}\, \varphi_\pi^A(x) = \left\{ \begin{array}{ll} 2 & \text{in asymptotic pQCD} \\ 10/3 & \text{CZ} \end{array} \right.$$

$\pi^0 \to \gamma\gamma$ Decay (Axial Anomaly)

Ⓐ G_1 sector



$$= 4i\,\varepsilon_{\alpha\beta\mu\nu}\, q_1^\alpha q_2^\beta \varepsilon_1^\mu \varepsilon_2^\nu \cdot \frac{g_\pi}{M_q}\int\frac{d^4k}{(2\pi)^4}\,\frac{M_k(-2M_k^2+2k^2M_k')}{(k^2+M_k^2)^3}$$



$$= \cdot\,\frac{g_\pi}{M_q}\int\frac{d^4k}{(2\pi)^4}\,\frac{M_k\; 2k^2M_k'}{(k^2+M_k^2)^3}$$

The sum of 2 integrals after changing variable

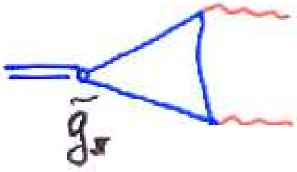
$$t=\frac{M^2(k^2)}{k^2} \longrightarrow \int_0^\infty \frac{dt}{(1+t)^3} = \frac{1}{2}$$

and the total contribution is

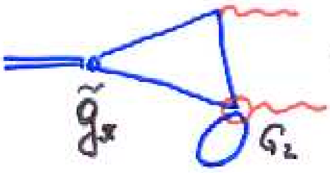
$$\cdot\,\frac{g_\pi}{M_q}\,\frac{1}{4\pi^2} \overset{GTR}{=} \cdot\, \boxed{\frac{1}{4\pi^2 f_\pi}}$$

The result required by Axial Anomaly independent of shape of nonlocality. !

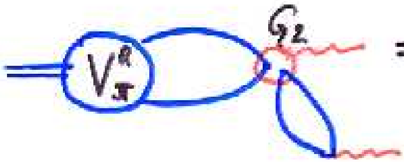
Ⓑ G_2 sector



$$=\cdot\,\frac{\tilde{g}_\pi}{m_\pi}\,\frac{1}{M_q}\int\frac{d^4k}{(2\pi)^4}\,\frac{M_k}{(k^2+M_k^2)^3}\,(-2M_k^2)$$

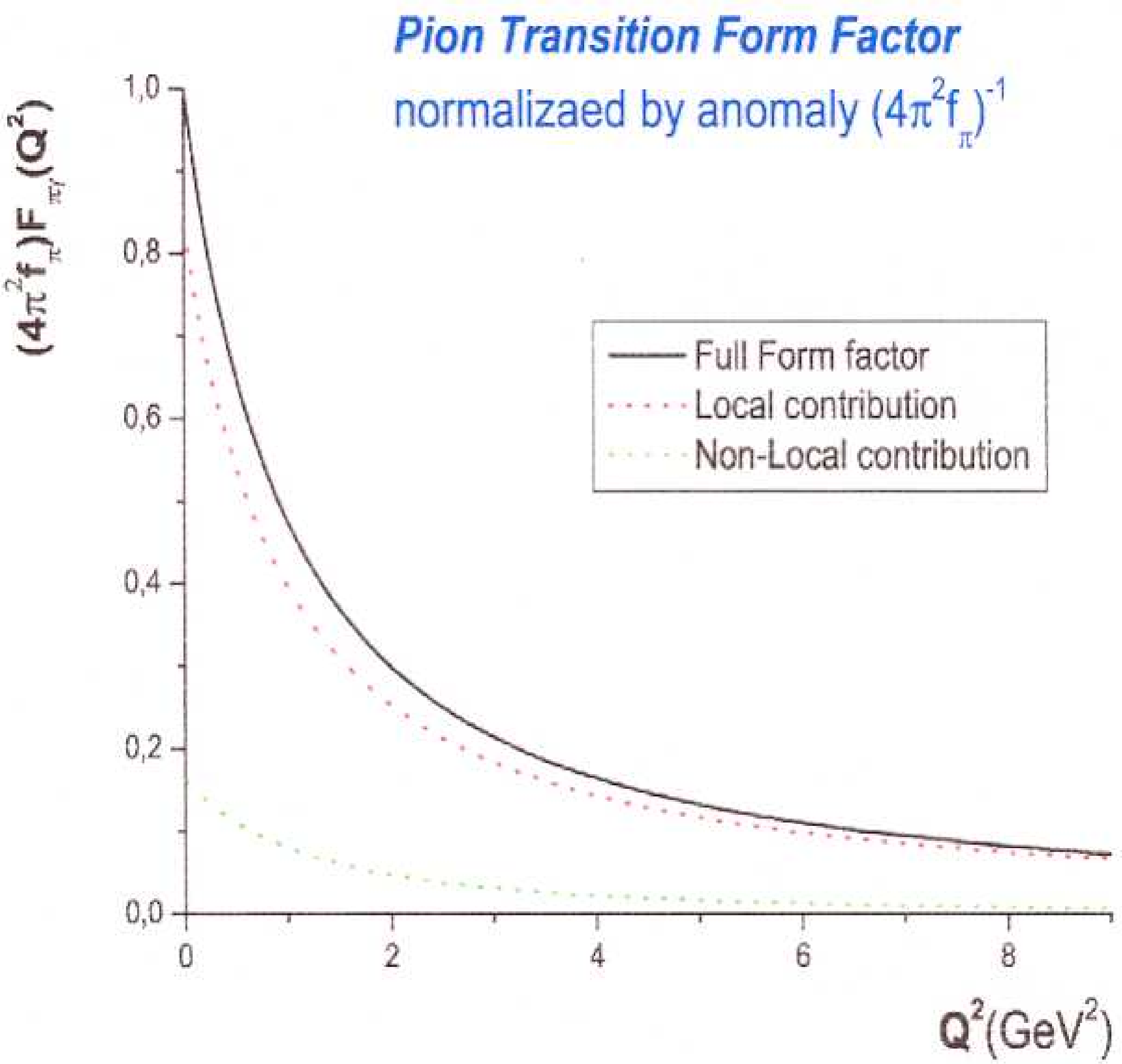


$$=\cdot\,\frac{\tilde{g}_\pi}{m_\pi}\,\frac{1}{M_q}\int\frac{d^4k}{(2\pi)^4}\,\frac{M_k}{(k^2+M_k^2)^3}\,(4M_k k^2 M_k')$$



$$=\cdot\,\frac{\tilde{g}_\pi}{m_\pi}\,\frac{1}{M_q}\int\frac{d^4k}{(2\pi)^4}\,\frac{k^2M_k'}{(k^2+M_k^2)^2}$$

The sum of these contributions is 0 !



Pion Transition Form Factor
normalizaed by anomaly $(4\pi^2 f_\pi)^{-1}$
$(4\pi^2 f_\pi)F_{\pi\gamma}(Q^2)$
1,0
0,8
0,6
0,4
0,2
0,0
0
2
4
6
8
$Q^2(GeV^2)$
Full Form factor
Local contribution
Non-Local contribution

Distribution Functions in Pion

on light cone $z^2 \to 0$

$$\langle \pi^0(p) | \bar{u}(z) \gamma_\mu \gamma_5 E(z,0)\, u(0) | 0 \rangle =$$

$$= -i f_\pi \int_0^1 dx\, e^{ixp\cdot z} [\underbrace{\varphi_\pi(x)}_{\text{Twist-2}} + \underbrace{z^2 g_1(x)}_{\text{Twist-4}}] +$$

$$+ f_\pi \left(z_\mu - \frac{z^2 p_\mu}{p\cdot z}\right) \int_0^1 dx\, e^{ixp\cdot z} \underbrace{g_2(x)}_{\text{Twist-4}}$$

$f_\pi = 92$ MeV

$$\langle \pi^0(p) | \bar{u}(z) \gamma_\mu \gamma_5\, g_s G_{\alpha\beta}(vz)\, u(0) | 0 \rangle =$$

0 vz z

$$= f_\pi \Big\{ \Big[p_\beta \Big(g_{\alpha\mu} - \frac{z_\alpha p_\mu}{z\cdot p}\Big) - p_\alpha \Big(g_{\beta\mu} - \frac{z_\beta p_\mu}{z\cdot p}\Big)\Big] \int D\alpha_i\, \underbrace{\varphi_\perp(\alpha_i)}_{\text{Twist-4}}\, e^{ip\cdot z(\alpha_1 + v\alpha_3)}$$

$$+ \frac{p_\mu}{p\cdot z}(p_\alpha z_\beta - p_\beta z_\alpha) \int D\alpha_i\, \underbrace{\varphi_\parallel}_{\text{Twist-4}}\, e^{ip\cdot z(\alpha_1 + v\alpha_3)} \Big\}$$

$$\langle \pi^0(p) | \bar{u}(z) \gamma_\mu\, g_s \tilde{G}_{\alpha\beta}(vz)\, u(0) | 0 \rangle = \ldots$$

$D\alpha_i = d\alpha_1 d\alpha_2 d\alpha_3\, \delta(1 - \Sigma\alpha_i)$

/A.S.Gorsky 85'–89'/
/V.Braun, I.Filyanov 89'–90'/

pQCD Asymptotic of $\pi\gamma^*\gamma^*$ FF.

$$F^{\gamma^*\pi}_{pQCD}(q_1^2; q_2^2) = \frac{2 f_\pi}{3} \left\{ \int_0^1 \frac{dx\, \varphi_\pi(x)}{\underbrace{q_1^2 \bar{x} + q_2^2 x}_{\sim Q^{-2}}} - \int_0^1 \frac{dx\, \varphi^{(4)}(x)}{\underbrace{(q_1^2 \bar{x} + q_2^2 x)^2}_{\sim Q^{-4}}} \right\}$$

$\bar{x} = 1 - x$

$$\varphi_\pi^{AS}(x) = 6 x \bar{x}, \qquad \varphi_\pi^{AS(4)}(x) = \frac{8}{9} \delta^2 [30 x^2 \bar{x}^2]$$

$$\langle \pi(p) | \bar{d} \gamma^\mu \gamma_5 u | 0 \rangle = i\sqrt{2} f_\pi p_\mu$$

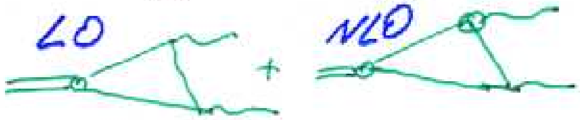
$$\delta^2 = \frac{\langle \pi(p) | \bar{d}\, g_s \tilde{G}_{\alpha\mu} \gamma_\alpha p_\mu\, u | 0 \rangle}{\langle \pi(p) | \bar{d} \gamma_\mu \gamma_5 p_\mu u | 0 \rangle} \approx 0.2\ \text{GeV}^2$$

/NSVZ 84'
ChZZ 83'/

Twist-4 Asymptotics

/A.D. 02/

$$F_{\pi\gamma^*\gamma^*}(Q^2;\omega)\Big|_{Q^2\to\infty} = J^{(2)}\frac{1}{Q^2} + J^{(4)}\frac{1}{Q^4} + O(Q^{-6}) + O\left(\frac{\alpha_s}{\pi}\right)$$



$$J^{(2)}(\omega) = \frac{4}{3} f_\pi \int_0^1 dx\, \frac{\varphi_\pi^{(2)}(x)}{1-\omega^2(2x-1)^2}$$

$$J^{(4)}(\omega) = \frac{8}{3} f_\pi \Delta^2 \int_0^1 dx\, \frac{1+\omega^2(2x-1)^2}{\left[1-\omega^2(2x-1)^2\right]^2}\, \varphi_\pi^{(4)}(x)$$

The leading Q^{-2} behaviour comes from the local part of γqq vertex, and NLO is from nonlocal piece.

$$\varphi_\pi^{(2)}(x) = \frac{N_c}{4\pi^2 f_\pi^2} \int_{-\infty}^{\infty} \frac{d\lambda}{2\pi} \int_0^\infty du\, \frac{\left[x M^{3/2}(u+i\lambda\bar{x}) M^{1/2}(u-i\lambda x) + (x \leftrightarrow \bar{x})\right]}{\left[u-i\lambda x + M^2(u-i\lambda x)\right]\left[u+i\lambda\bar{x}+M^2(u+i\lambda\bar{x})\right]}$$

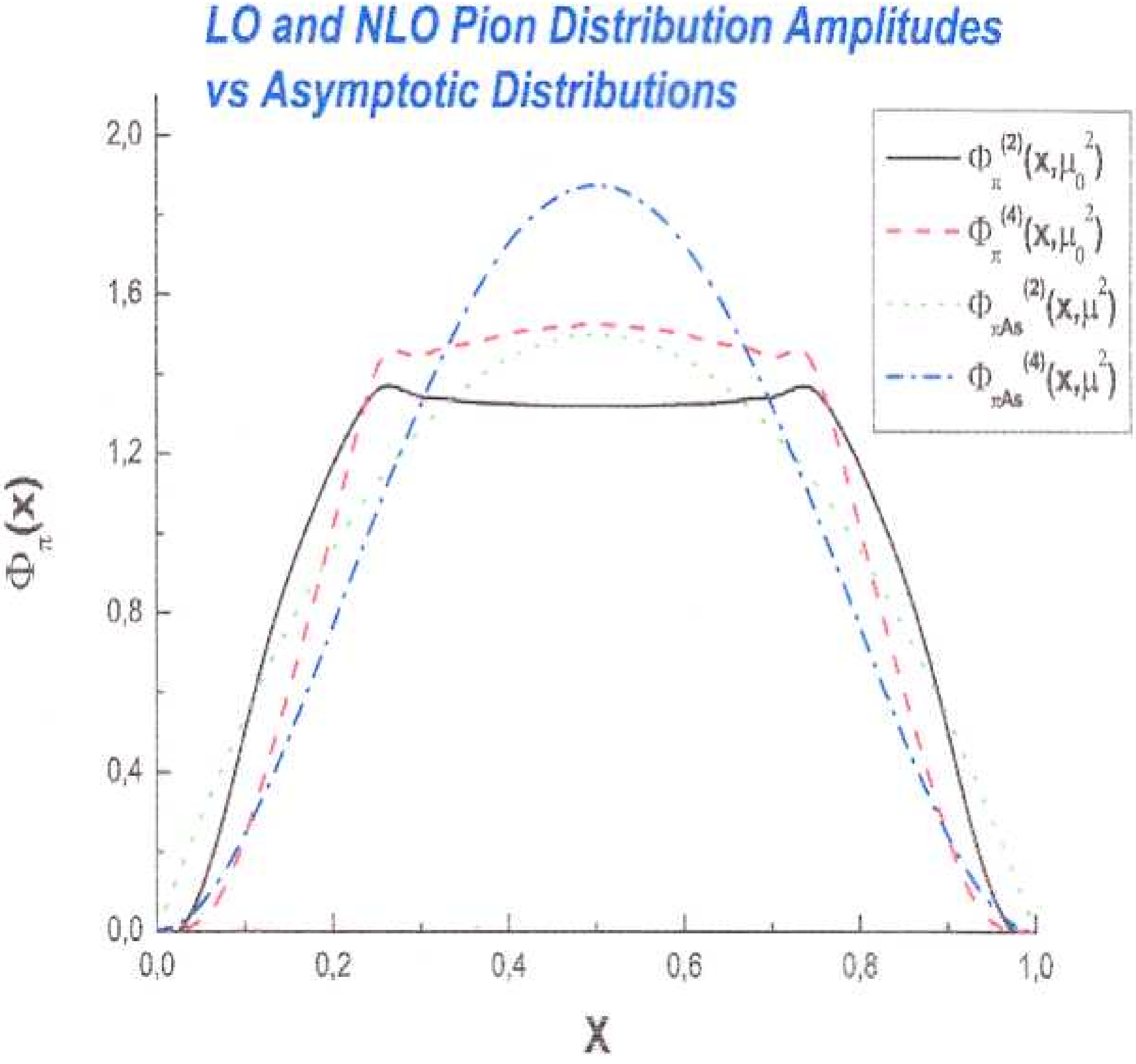
$$\varphi_\pi^{(4)}(x) = \frac{N_c}{4\pi^2 f_\pi^2} \int_{-\infty}^{\infty} \frac{d\lambda}{2\pi} \int_0^\infty du\, \frac{\left[M^{3/2}(u+i\lambda\bar{x}) M^{1/2}(u-i\lambda x) + (x \leftrightarrow \bar{x})\right]}{\left[u-i\lambda x + M^2(u-i\lambda x)\right]\left[u+i\lambda\bar{x}+M^2(u+i\lambda\bar{x})\right]}$$

The nonlocal model fixes the absolute normalization of asymptotics.

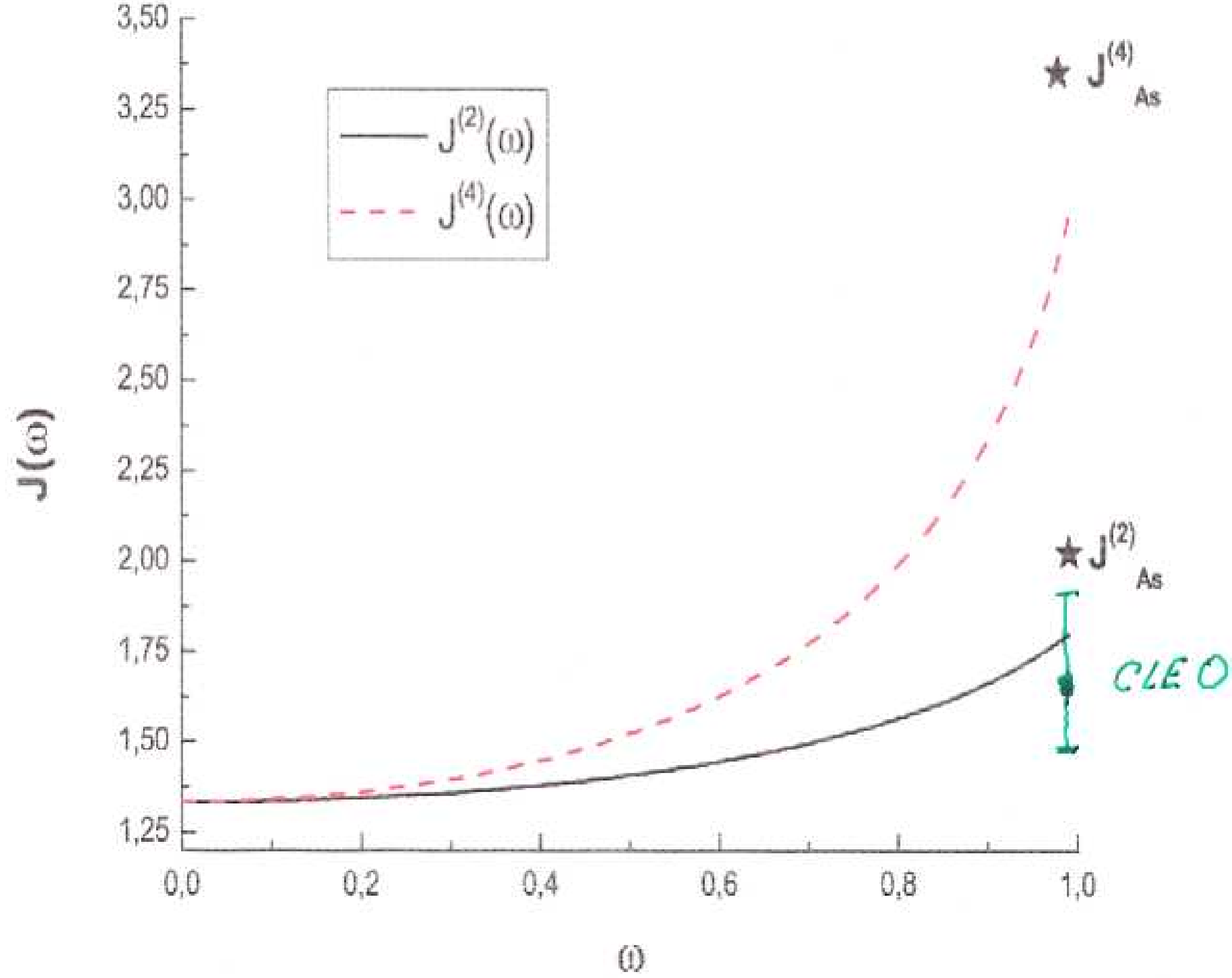
The asymptotic coefficient has been measured by CLEO collab. for process $\pi\gamma^*\gamma$ ($\omega = 1$)

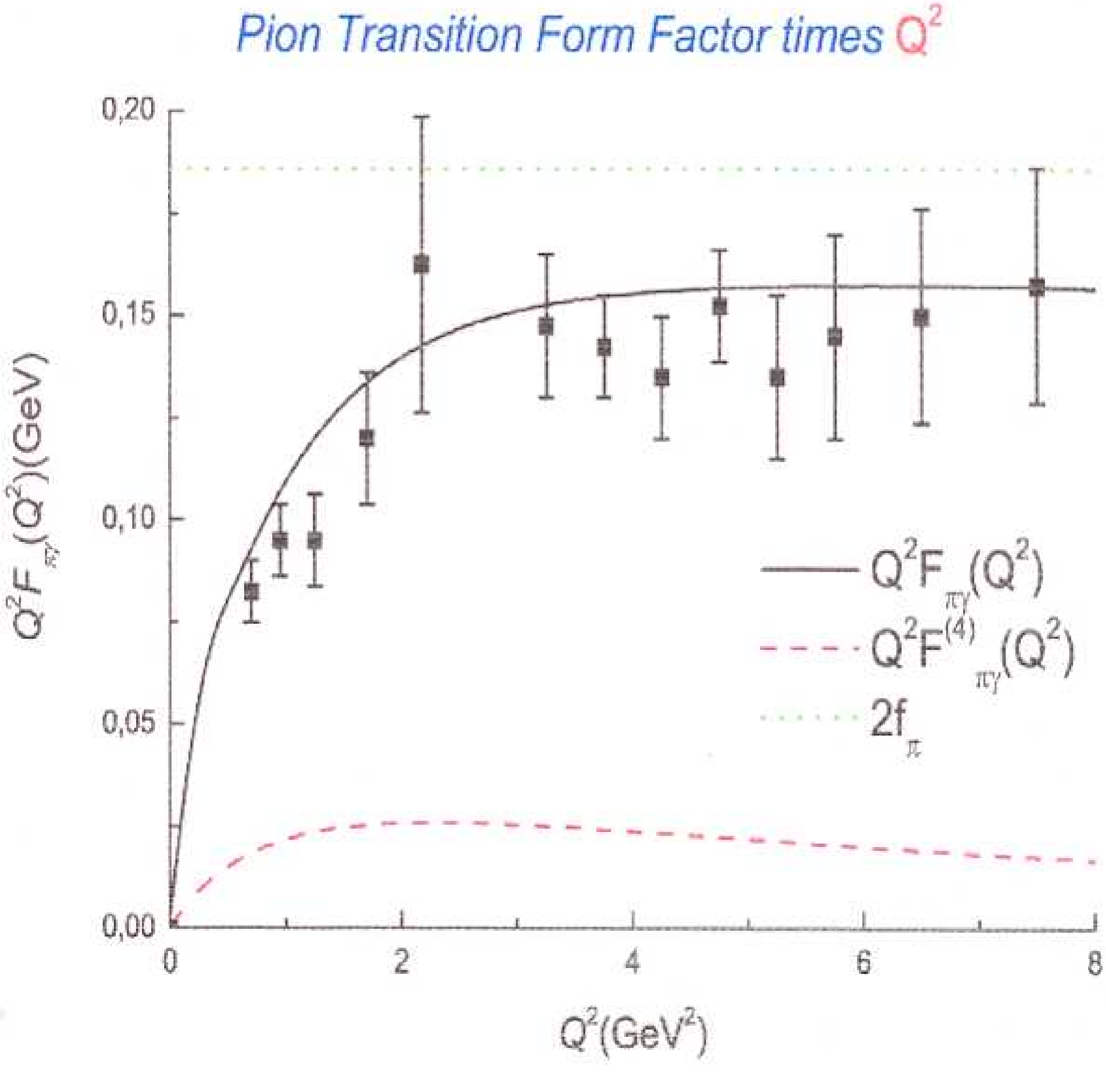
$J^{(2)}(\omega=1) = 1.6 \pm 0.3$ /CLEO, 98/

$J^{(2)}(\omega=1) = 1.8$ /Model/


LO and NLO Pion Distribution Amplitudes
vs Asymptotic Distributions
$\Phi_{\pi}^{(2)}(x,\mu_0^2)$
$\Phi_{\pi}^{(4)}(x,\mu_0^2)$
$\Phi_{\pi As}^{(2)}(x,\mu^2)$
$\Phi_{\pi As}^{(4)}(x,\mu^2)$
$\Phi_{\pi}(x)$
2,0
1,6
1,2
0,8
0,4
0,0
0,0
0,2
0,4
0,6
0,8
1,0
X

LO and NLO Asymptotic Coefficients





Pion Transition Form Factor times Q^2
0,20
0,15
0,10
0,05
0,00
Q^2F_πγ(Q^2)(GeV)
0
2
4
6
8
Q^2(GeV^2)
Q^2F_πγ(Q^2)
Q^2F^(4)_πγ(Q^2)
2f_π

Conclusions.

1. Instanton model is microscopic model of QCD vacuum
2. It describes the nonlocal structure of vacuum.
3. The model is responsible for description of parameters of nonperturbative QCD: condensates, distribution functions, etc.
4. At high energy the instanton exchange models the soft processes.