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"Analysis of self-similar solutions and asymptotic properties of hadron production in relativistic nuclear collisions."

Self-similarity is a special symmetry of solutions which means that the change in the scales of independent variables can be compensated by the self-similarity transformation of other dynamical variables. This results in the reduction of the number of the variables which any physical law depends upon.

The methods based on symmetry of solutions consist of the following:

1. The parameters describing the problem - the space of basic parameters - are selected.
2. The symmetry of this space is detected or guessed, and the corresponding invariants are determined.
3. The laws of Nature are treated as relations between the invariants.
4. Additional principles - the CDP, the intermediate asymptotics, the hypothesis of analyticity of physical laws, local self-similarity - are applied.

In the case of relativistic nuclear physics, the basic parameters include:

the cross sections (quantities derived from them),

the invariant dimensionless intervals in the 4-velocity space

$$b_{ik} = -(U_i - U_k)^2 = 2\cdot[(U_i U_k) - 1] = 2\cdot\left[\frac{E_i \cdot E_k - \vec{p}_i \cdot \vec{p}_k}{m_i \cdot m_k} - 1\right] = 2\cdot\left[ch\rho_{ik} - 1\right]$$

The general form of the self-similar solution for inclusive hadron production processes:

$$E\frac{d^3\sigma}{d^3p} \doteq C_1 A_1^{\alpha(X_1)} A_2^{\alpha(X_2)} f(\Pi)$$

C_1 - constant determining the dimensionality of the invariant differential cross section.

A –the atomic numbers of the colliding nuclei,

α & f - the functions determined experimentally

The similarity parameter:

$$\Pi = \frac{1}{2}\left(X_1^2 + X_2^2 + 2X_1X_2\gamma_{12}\right)^{1/2},$$

$$\gamma_{ij} = u_i u_j = P_i P_j / M_i M_j .$$

X_1 & X_2 - the fractions of the 4-momentum needed to produce the detected particle (the effective number of nucleons participating in the reaction).

The procedure for determining X_1 , X_2 & Π consists of determining the minimum of Π by using the energy-momentum conservation law in the form:

$$(X_1 m_0 u_1 + X_2 m_0 u_2 - M_3 u_3)^2 = (X_1 m_0 u_1' + X_2 m_0 u_2' + \sum_{k=4} M_k u_k)^2$$

Hypothesis! It's possible to neglect the relative motion of all the remaining undetected particles -

$$2\sum_{k>1} (\gamma_{kl} - 1) M_k M_l \ .$$

The relation between X_1 & X_2:

$$X_1 X_2 (\gamma_{12} - 1) - X_1\left(\frac{M_3}{m_0}\gamma_{13} + \frac{M_4}{m_0}\right) - X_2\left(\frac{M_3}{m_0}\gamma_{23} + \frac{M_4}{m_0}\right) = \frac{M_4^2 - M_3^2}{2m_0^2}$$

The quantitative form of the solution:

$$E\frac{d^3\sigma}{d^3p} = C_1 A_1^{1/3 + X_1/3} A_2^{1/3 + X_2/3} \exp\left(-\Pi/C_2\right)$$

C2=0.125±0.002

C1=19000[mb GeV^{-2} c^3 sr^{-1}].

The correlation depletion principle.

For the case $b_{ik} \to \infty$:

$$W(b_{\alpha k}, b_{\alpha\beta}, b_{\beta k}, \ldots) \to W^{\alpha} \cdot W^{\beta} \ldots$$

$$W(b_{\alpha k}, b_{\alpha\beta}, b_{\beta k}) \to \frac{1}{b_{\alpha\beta}^{n}} W^{\alpha}(b_{\alpha\beta} \frac{b_{\beta k}}{b_{\alpha\beta}})$$

$$M^2 = (p_{jet}^{\alpha} + p_{jet}^{\beta})^2$$

$M^2 = m^2 b_{\alpha\beta}$, $dN/MdM = const/M^{2n}$ $n=3$

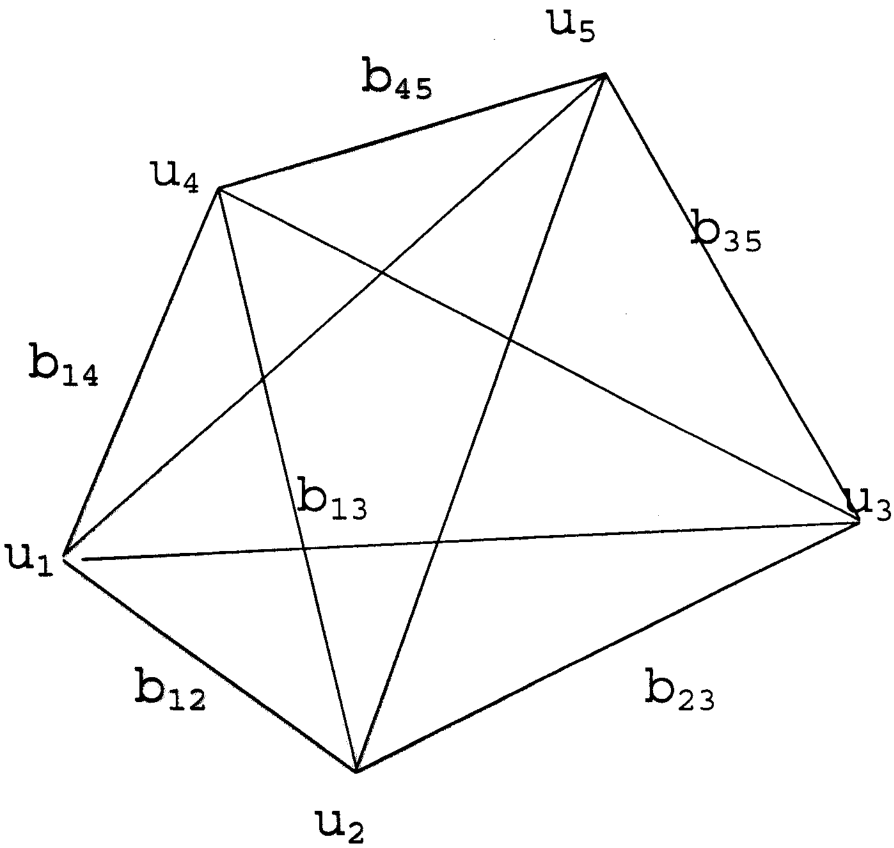
$$\frac{dN}{db_{\alpha\beta}} = \frac{A}{b^{n}{}_{\alpha\beta}}$$

For $20 < b_{\alpha\beta} < 10^5$ $n = 3 \pm 0.2$

$$\gamma_{ij} = u_i u_j = P_i P_j / M_i M_j$$

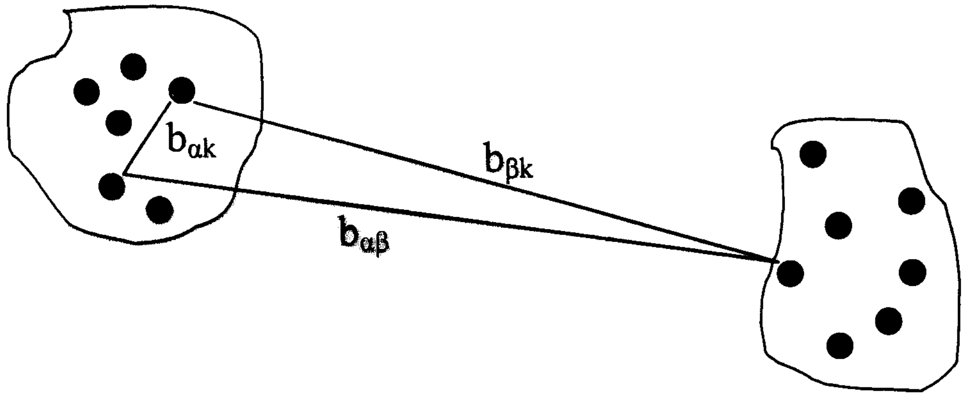
$$\Delta N = \sigma \cdot j_{01} \cdot n_{02} \cdot \Delta V \cdot \Delta t,$$

$$j_{01} \cdot n_{02} = n_{01} \cdot n_{02} \cdot \gamma_{01} \cdot \beta_{01} = n_{01} \cdot n_{02} \cdot \sqrt{\gamma_{01}^2 - 1}.$$



Clusters **are the sets of points U_k in the 4-velocity space, average distance between which $b_{\alpha k} = -(V_\alpha - U_k)^2$ is much smaller than the average distance between the clusters $b_{\alpha\beta} = -(V_\alpha - V_\beta)^2$.**

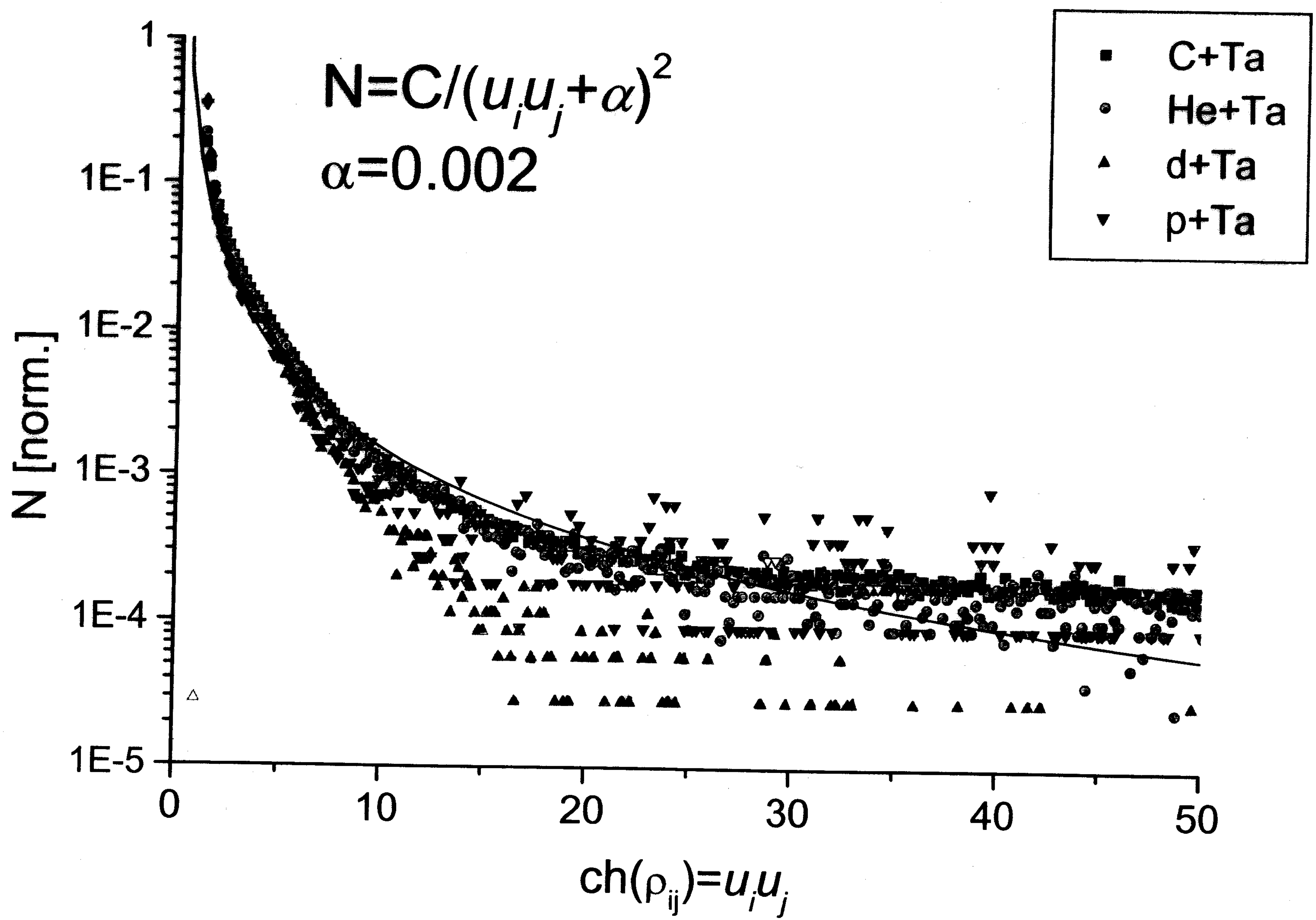
$$V_\alpha = \frac{\sum U_k^\alpha}{\sqrt{(\sum U_k^\alpha)^2}}, \quad V_\beta = \frac{\sum U_j^\beta}{\sqrt{(\sum U_j^\beta)^2}} ,...$$



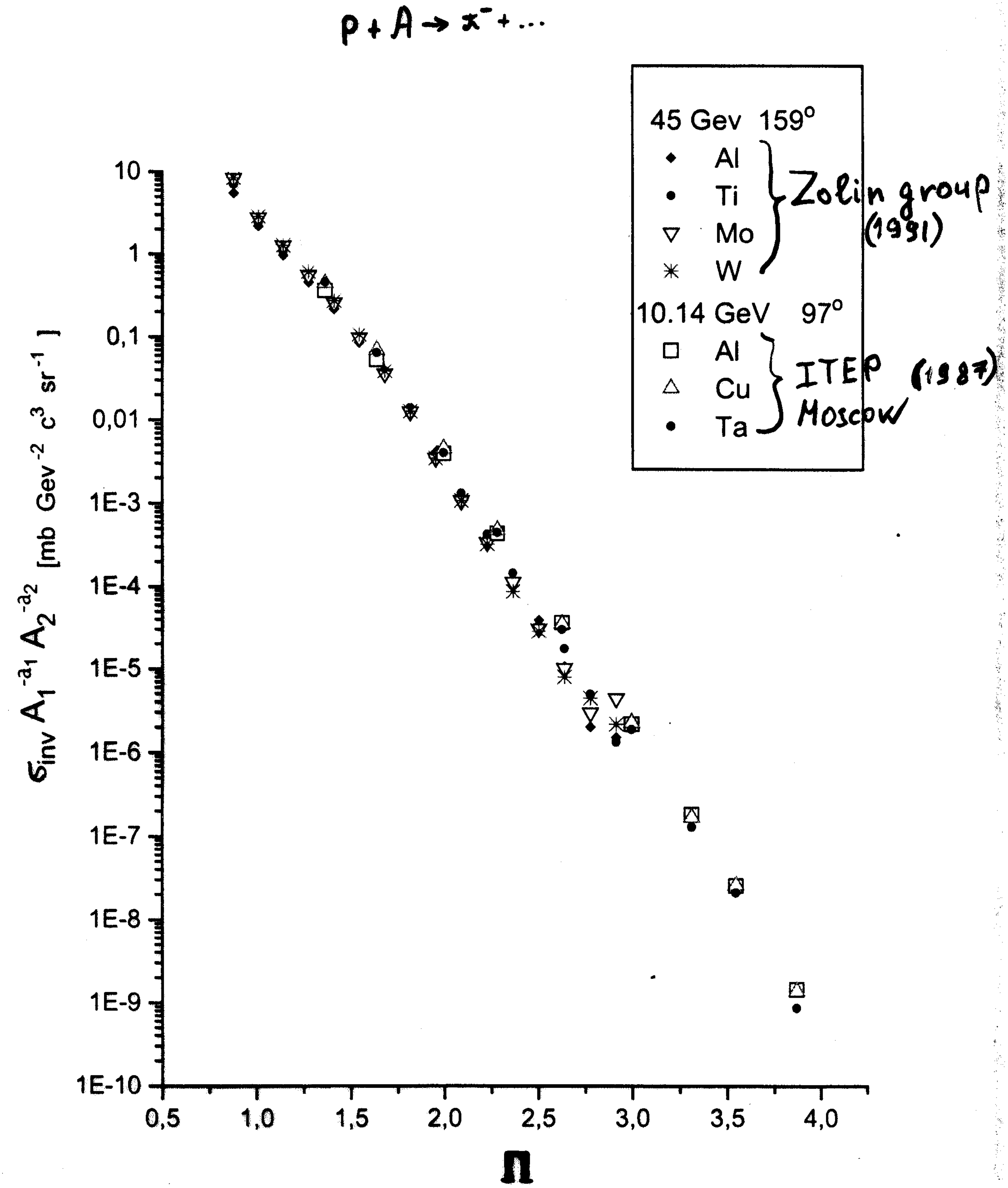
$\frac{b_{\beta k}}{b_{\alpha\beta}} \rightarrow x_k$, where x_k is the light cone variable.

In factorization of distributions when relative velocity tends to infinity, there remains the dependence on the direction to the infinite point. This anisotropy bears purely geometrical character. While passing to non-relativistic approximation, i.e. from the Lobachevsky to Euclidean geometry, this dependence degenerates into isotropy.

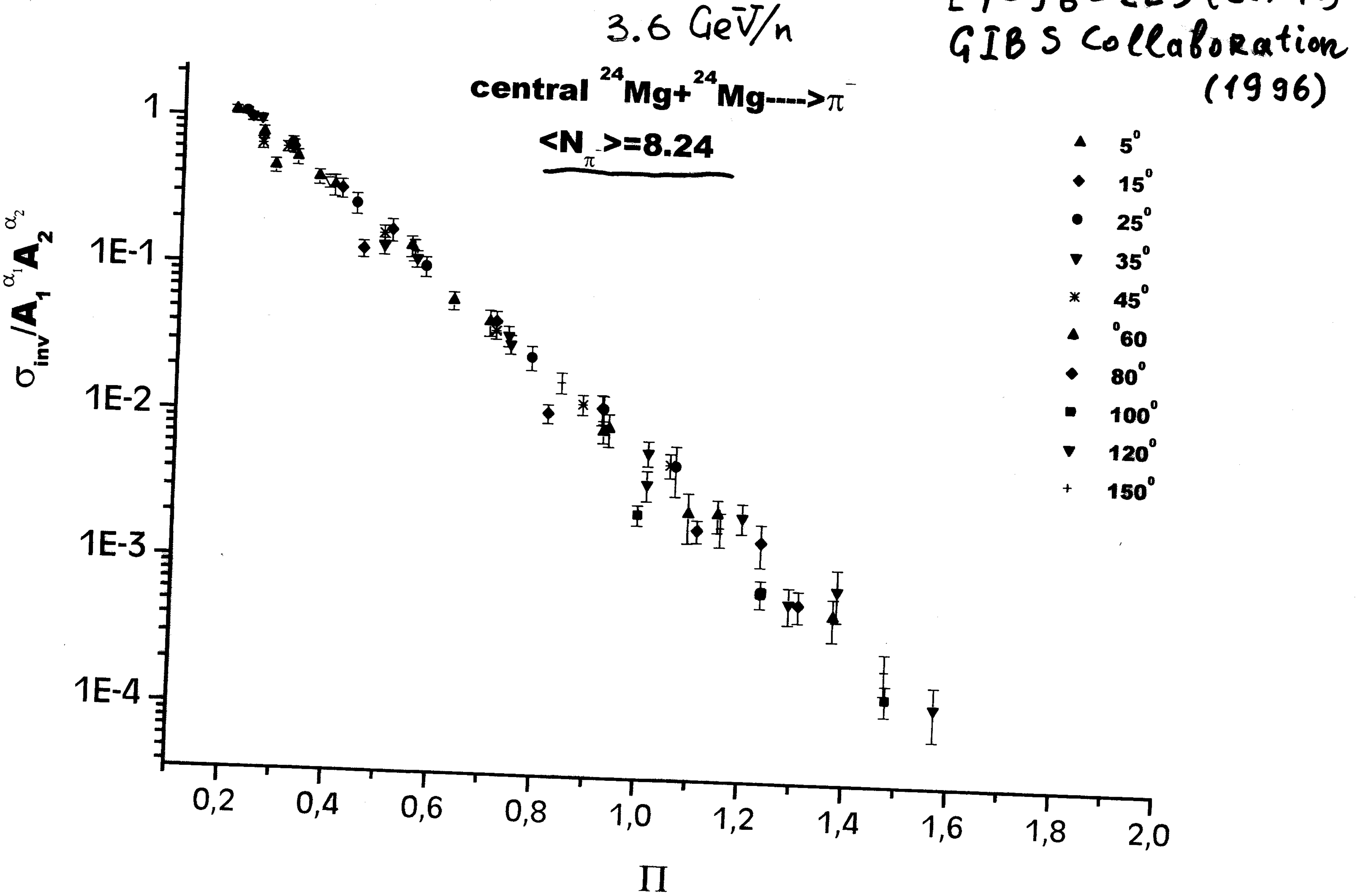
This note indicates groundlessness of attempts to reveal quasi-stationary objects in multiple particle production on the basis of isotropy of decays in their rest system. In relativistic dynamics such decays are always anizotropic.

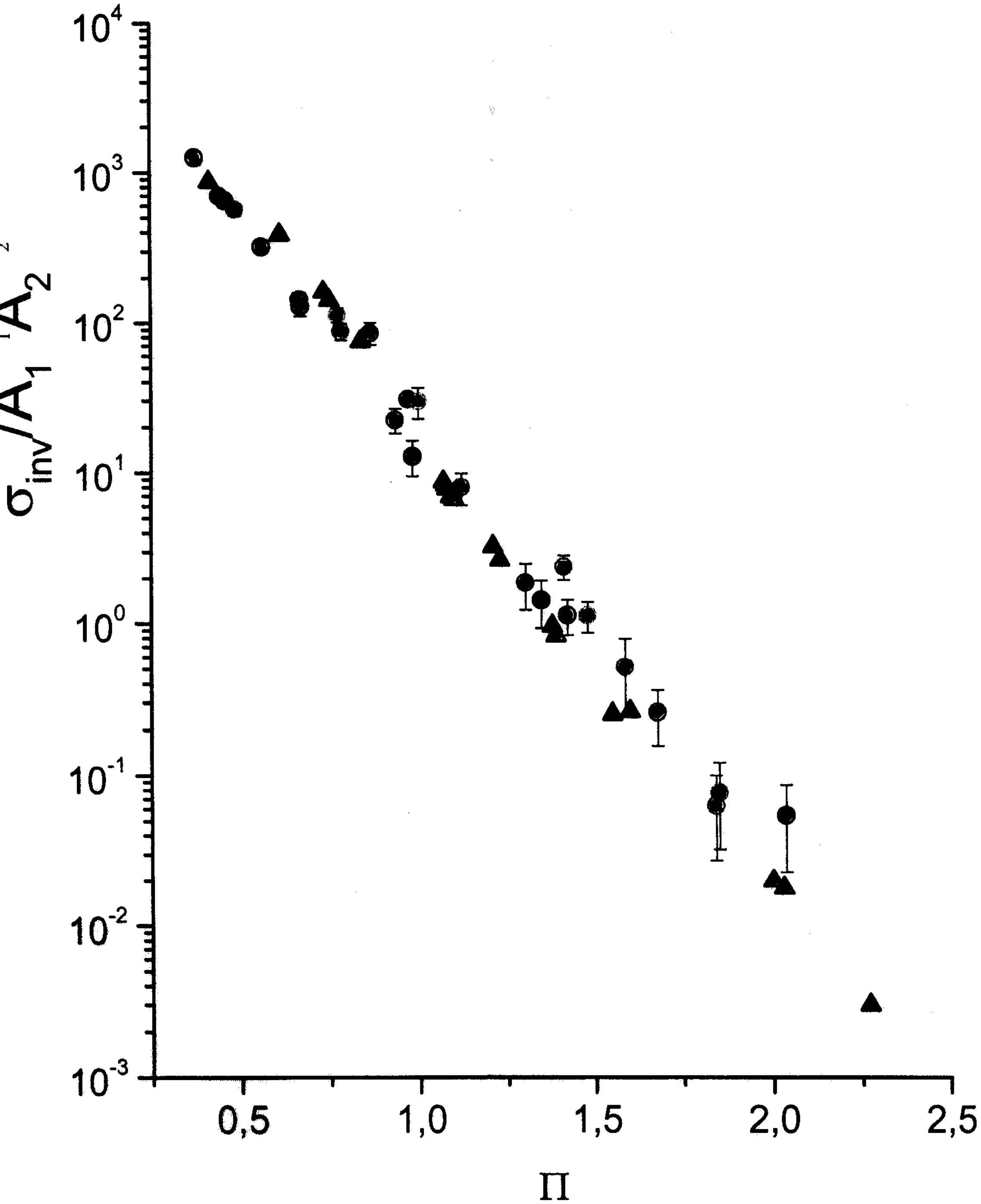


N=C/(u_i u_j+α)^2
α=0.002
C+Ta
He+Ta
d+Ta
p+Ta
N [norm.]
1
1E-1
1E-2
1E-3
1E-4
1E-5
0
10
20
30
40
50
ch(ρ_ij)=u_i u_j



p + A → π⁻ + ...
45 Gev 159°
Al
Ti
Mo
W
Zolin group (1991)
10.14 GeV 97°
Al
Cu
Ta
ITEP (1987) Moscow
$\sigma_{inv} A_1^{-a_1} A_2^{-a_2}$ [mb Gev^{-2} c^3 sr^{-1}]
10
1
0,1
0,01
1E-3
1E-4
1E-5
1E-6
1E-7
1E-8
1E-9
1E-10
0,5
1,0
1,5
2,0
2,5
3,0
3,5
4,0
n


E1-96-223 (JINR)
GIBS Collaboration
(1996)
3.6 GeV/n
central ^{24}Mg+^{24}Mg---->π^-
<N_{π^-}>=8.24
5°
15°
25°
35°
45°
60°
80°
100°
120°
150°
$\sigma_{inv}/A_1^{\alpha_1}A_2^{\alpha_2}$
1
1E-1
1E-2
1E-3
1E-4
0,2
0,4
0,6
0,8
1,0
1,2
1,4
1,6
1,8
2,0
Π



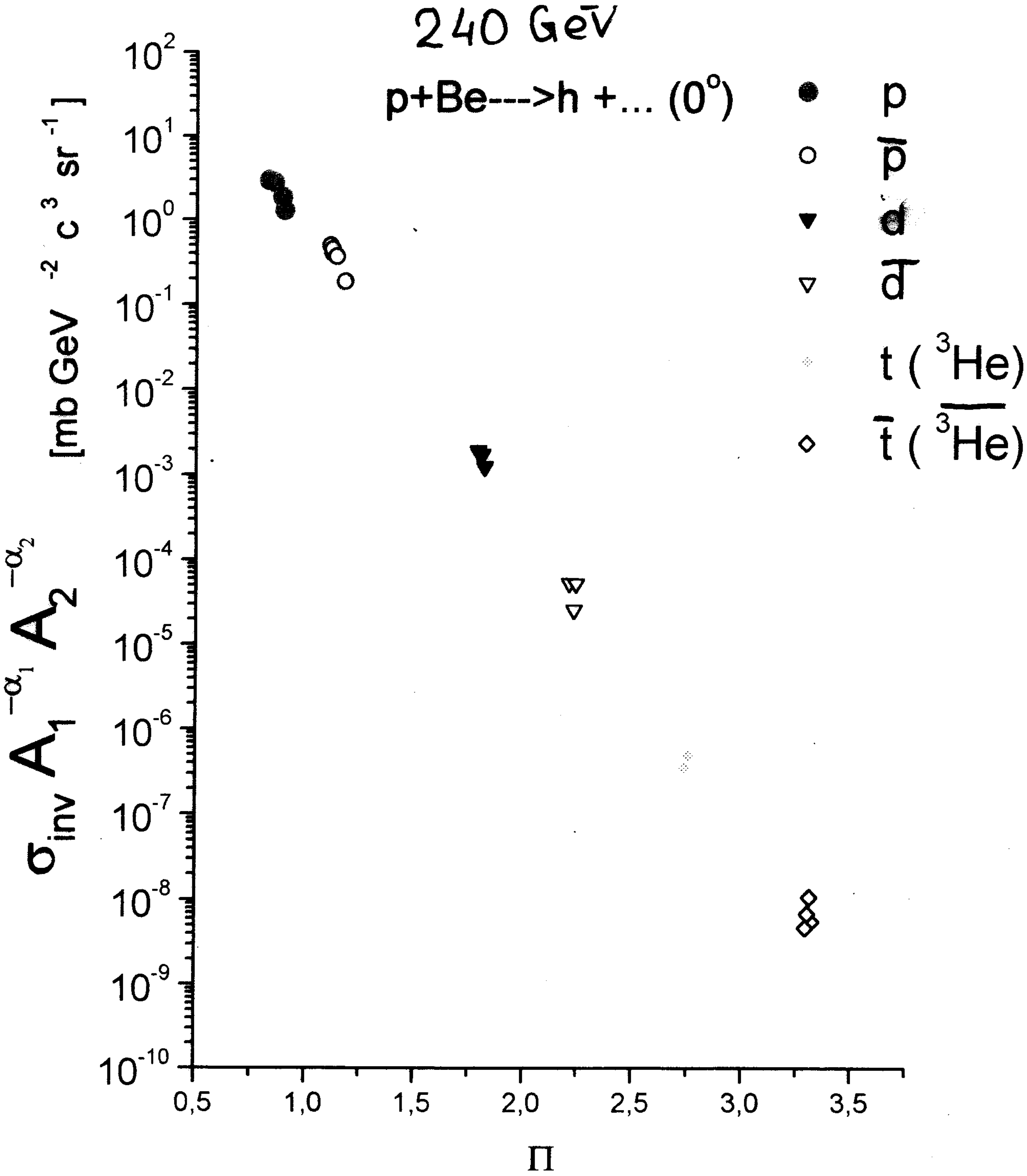
π^-

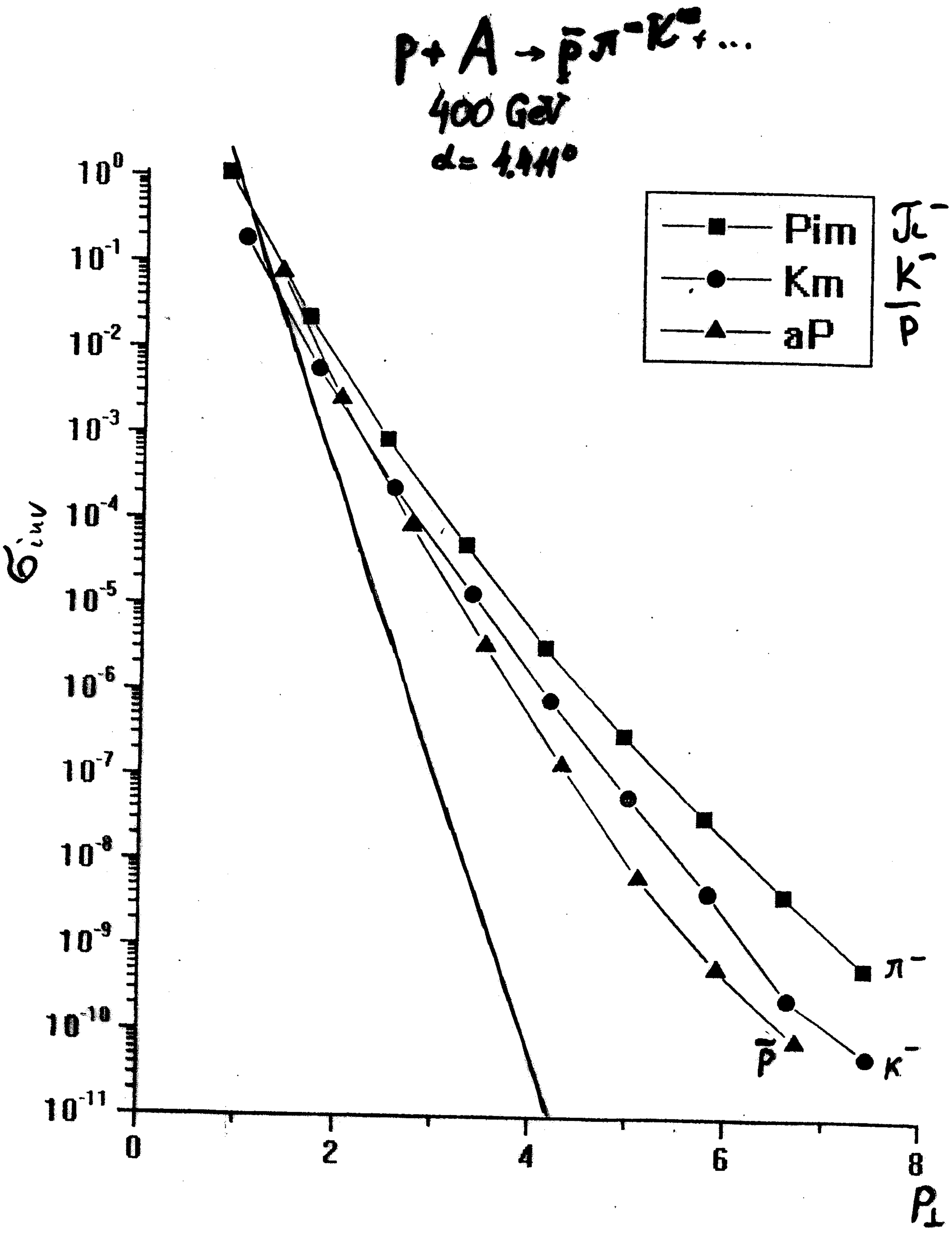
● $^{12}C+^{181}Ta$ 3.65 GeV/n

$10^0, 35^0, 45^0, 60^0, 100^0, 120^0$

▲ $^{20}Ne+^{64}Cu, ^{119}Sn, ^{209}Bi$ 1.5-1.9 GeV/n

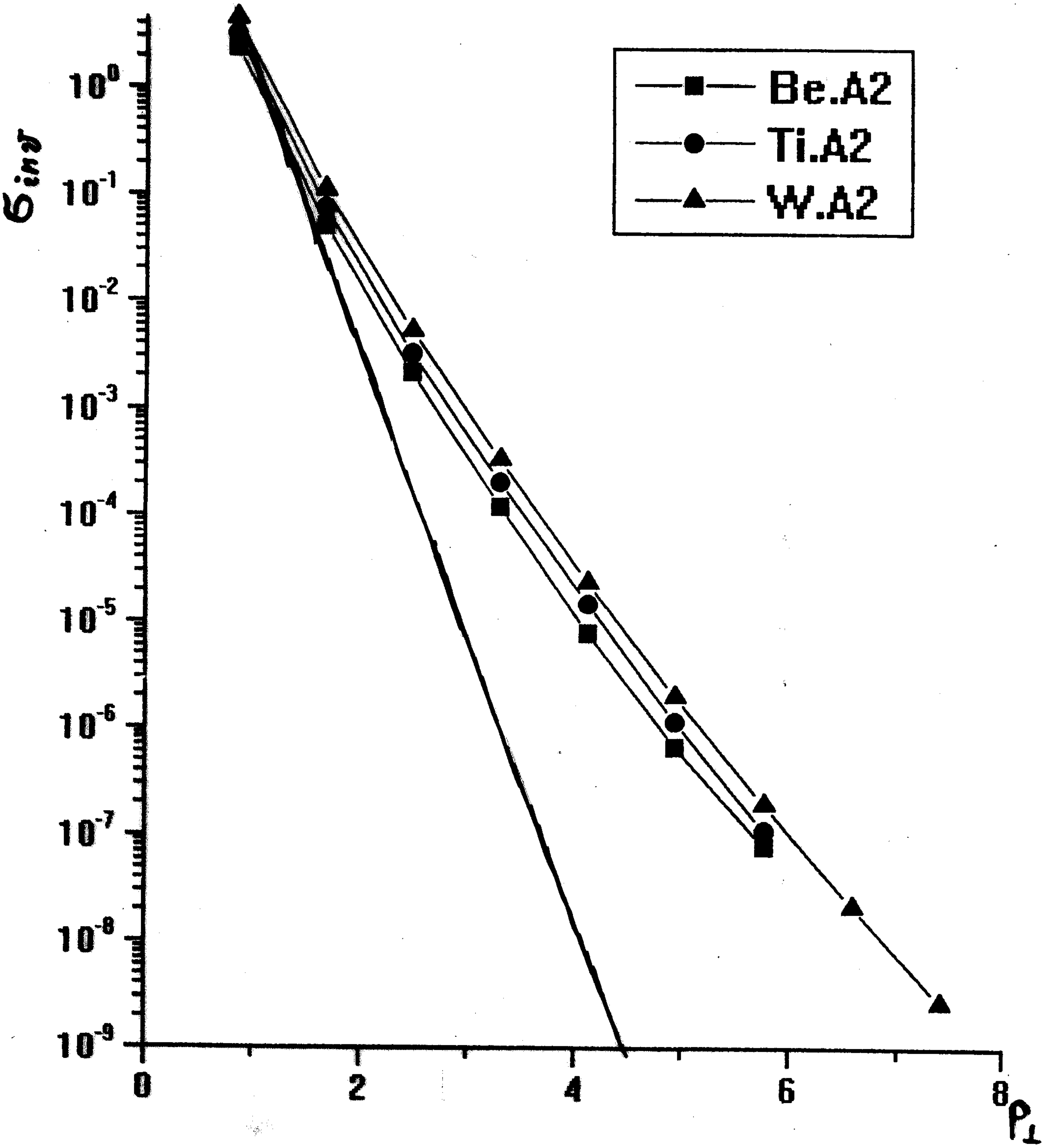
$^{58}Ni+^{58}Ni$ 1.7-1.9GeV/n 0^0


240 GeV
p+Be--->h +... (0°)
p
p̄
d
d̄
t (³He)
t̄ (³He̅)
σinv A1^-α1 A2^-α2 [mb GeV^-2 c^3 sr^-1]
10^2
10^1
10^0
10^-1
10^-2
10^-3
10^-4
10^-5
10^-6
10^-7
10^-8
10^-9
10^-10
0,5
1,0
1,5
2,0
2,5
3,0
3,5
Π



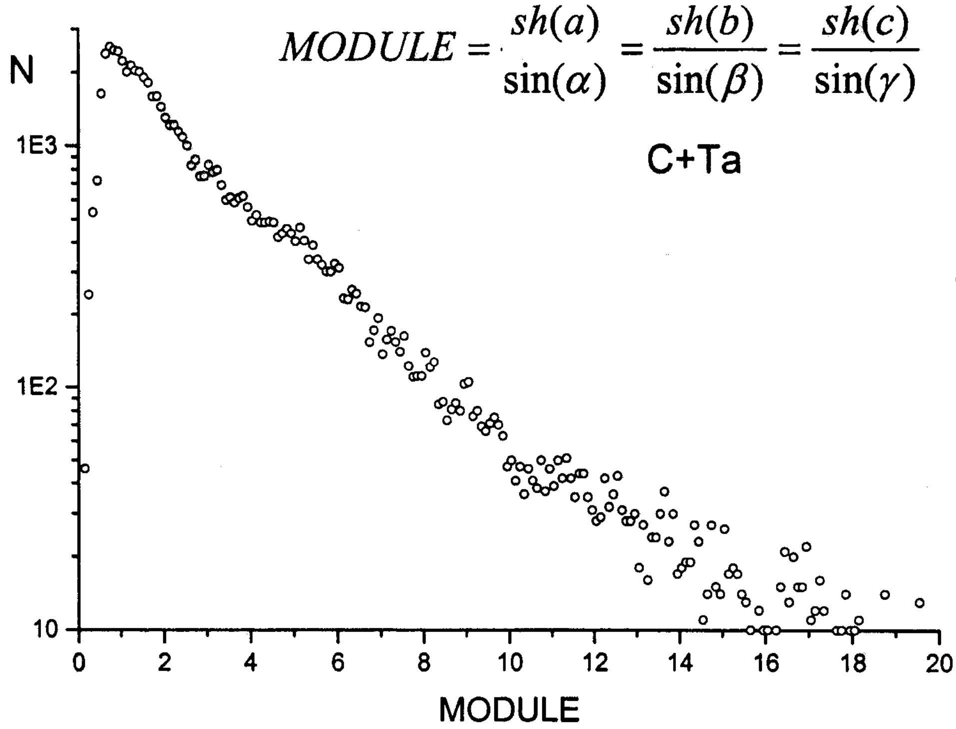
p + A → p̄ π⁻ K⁻ + ...
400 GeV
α = 1.411°
Pim π⁻
Km K⁻
aP p̄
σ_inv
10^0
10^-1
10^-2
10^-3
10^-4
10^-5
10^-6
10^-7
10^-8
10^-9
10^-10
10^-11
0
2
4
6
8
P⊥
π⁻
K⁻
p̄

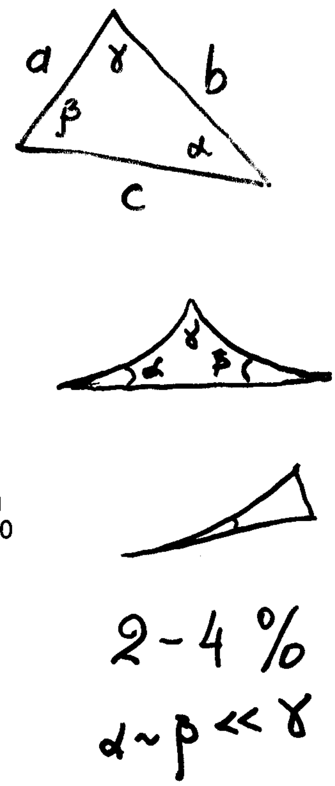
$p + A \to \pi^-$



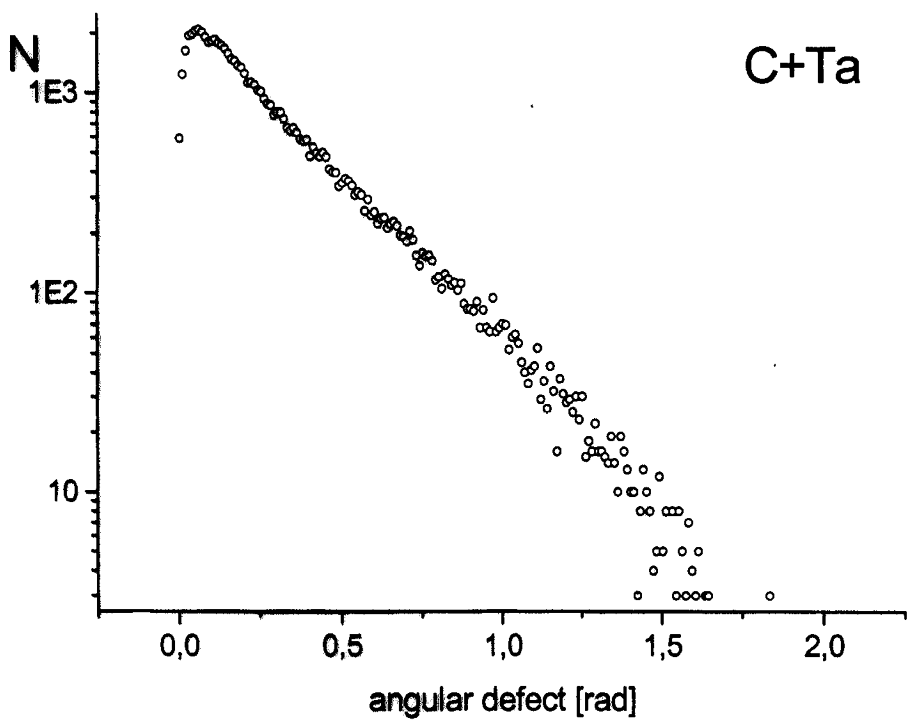
$$A^{\alpha(x)} \xrightarrow{?} A^{\alpha(x, m_\perp)}$$

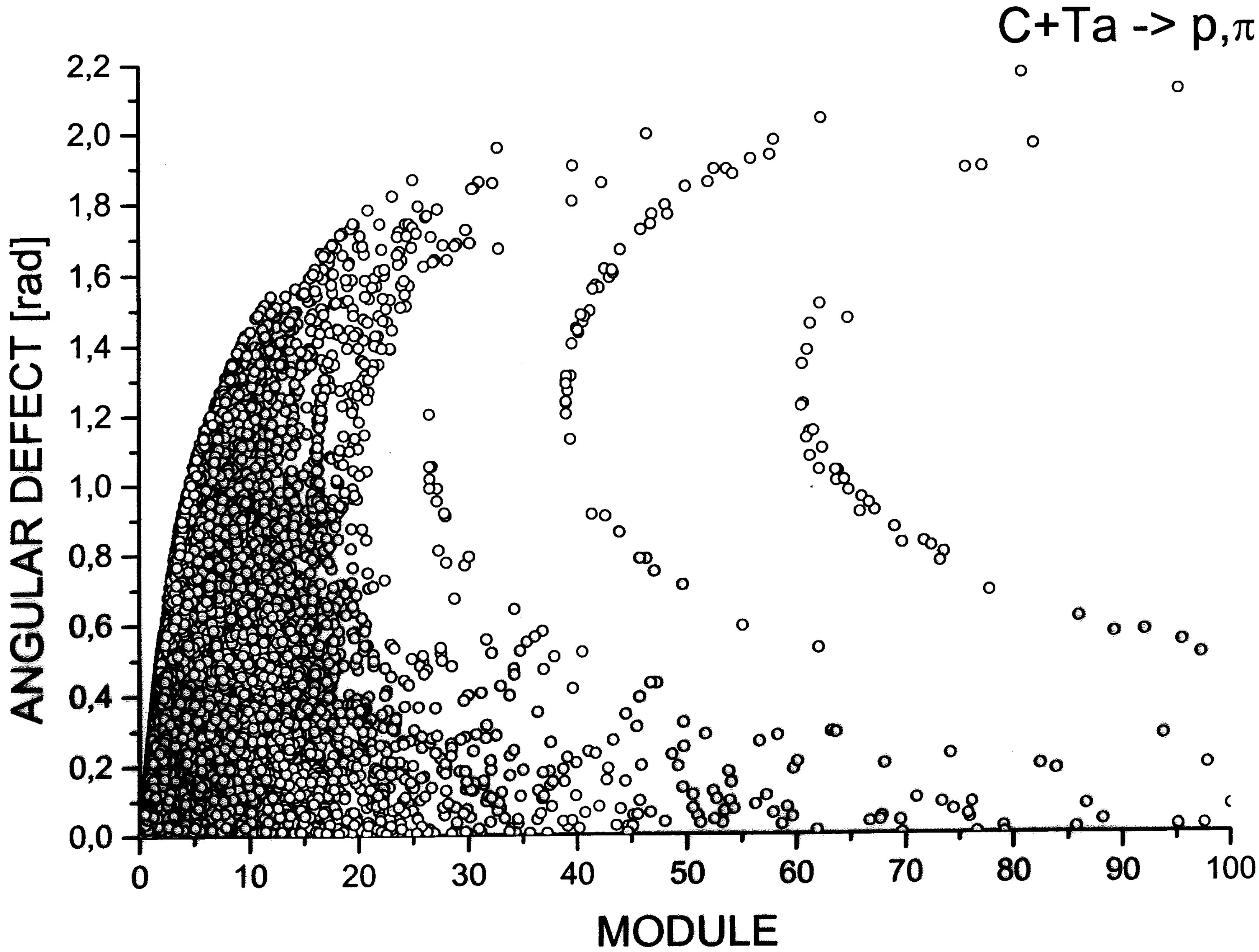
$$MODULE = \frac{sh(a)}{\sin(\alpha)} = \frac{sh(b)}{\sin(\beta)} = \frac{sh(c)}{\sin(\gamma)}$$





$$DEFECT=\pi-\alpha-\beta-\gamma$$




C+Ta -> p,π
ANGULAR DEFECT [rad]
2,2
2,0
1,8
1,6
1,4
1,2
1,0
0,8
0,6
0,4
0,2
0,0
0
10
20
30
40
50
60
70
80
90
100
MODULE

CONCLUSION

- **The self-similar solution**

$$E\frac{d^3\sigma}{d^3p} = C_1 A_1^{\alpha(X_1)} A_2^{\alpha(X_2)} f(\Pi)$$

quantitatively describes the inclusive spectra of hadron production for transverse momenta below 2GeV/c. In order to describe the A-dependences of the cross sections for transverse momenta higher than 2GeV/c it is necessary to introduce additional functions of X and $m_\perp$.

- **The calculated effective numbers of nucleons X_1 and X_2 give the universal parameterization of the inclusive cross sections of hadron production.**

- **The analysis of the experimental data show that the cross sections as functions of the distance between the centers of jets – clusters – in the 4-velocity space have the power-law form:**

$$\frac{dN}{db_{\alpha\beta}} = \frac{A}{b_{\alpha\beta}{}^{n}} \quad , \qquad n \approx 3 \pm 0.3$$

- **The distribution of relative velocities of pairs of particles and the shape of the inclusive spectra are independent on multiplicity.**
- **The investigation of geometric configurations in the 4-velocity space is a promising instrument of relativistically invariant analysis of experimental data on multiple particle production.**