Three-Body Forces in Multiparticle Dynamics: New Sites of Fundamental Dynamics

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Abstract

A manifestation of the three-body forces in multiparticle dynamics is discussed.

CONTENTS

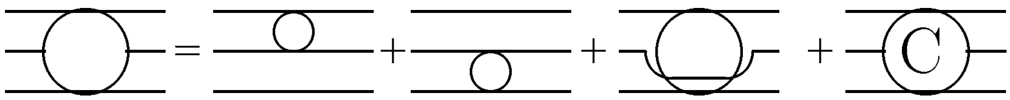
INTRODUCTION

The three-body forces appear as a result of consistent consideration of the three-body problem in the framework of local QFT.

Using the LSZ or Bogoljubov reduction formulae in quantum field theory we can easily obtain the following cluster structure for $3 \to 3$ scattering amplitude

$$\mathcal{F}_{123} = \mathcal{F}_{12} + \mathcal{F}_{23} + \mathcal{F}_{13} + \mathcal{F}_{123}^{C}$$

where $\mathcal{F}_{ij}, (i, j = 1, 2, 3)$ are $2 \to 2$ scattering amplitudes, $\mathcal{F}_{123}^{C}$ is called the connected part of the $3 \to 3$ scattering amplitude.



In the framework of single-time formalism in quantum field theory (adopted in Sov. J. TMF **83**, 247 (1990)) we construct the $3 \to 3$ off energy shell scattering amplitude $T_{123}(E)$ with the same (cluster) structure

$$T_{123}(E) = T_{12}(E) + T_{23}(E) + T_{13}(E) + T_{123}^{C}(E).$$

The three particle interaction quasipotential $V_{123}(E)$ is related to the off energy shell $3 \to 3$ scattering amplitude $T_{123}(E)$ by the LS-transformation

$$T_{123}(E) = V_{123}(E) + V_{123}(E) G_0(E) T_{123}(E).$$

There exsists the same transformation between two particle interaction quasipotentials V_{ij} and off energy shell $2 \to 2$ scattering amplitudes T_{ij}

$$T_{ij}(E) = V_{ij}(E) + V_{ij}(E) G_0(E) T_{ij}(E).$$

In QFT a three particle interaction quasipotential has the structure:

$$V_{123}(E) = V_{12}(E) + V_{23}(E) + V_{13}(E) + V_0(E).$$

$V_0(E)$ **is the three-body forces quasipotential, it represents the defect of three particle interaction quasipotential over the sum of two particle quasipotentials and describes the true three-body interactions. Three-body forces quasipotential is an inherent connected part of total three particle interaction quasipotential which cannot be represented by the sum of pair quasipotentials.**

The three-body forces scattering amplitude is related to the three-body forces quasipotential by the LS-transformation

$$T_0(E) = V_0(E) + V_0(E) G_0(E) T_0(E).$$

Global analyticity of the three-body forces.

Let us introduce the following useful notations

$$< p_1'p_2'p_3'|S-1|p_1p_2p_3 >= 2\pi i\delta^4(\sum_{i=1}^{3} p_i' - \sum_{j=1}^{3} p_j)\mathcal{F}_{123}(s;\hat{e}',\hat{e}),$$

$$s = (\sum_{i=1}^{3} p_i')^2 = (\sum_{j=1}^{3} p_j)^2, \quad \hat{e}',\hat{e} \in S_5,$$

$$\cos\omega = \hat{e}'\cdot\hat{e},$$

$$T_0 \mid_{on\,energy\,shell} = \mathcal{F}_0,$$

$$\mathcal{F}_0(s;\hat{e}',\hat{e}) = \mathcal{F}_0(s;\eta,\cos\omega),$$

η are all other variables.

We will assume that for physical values of the variable s and fixed values of η the amplitude $\mathcal{F}_0(s;\eta,\cos\omega)$ is an analytical function of the variable $\cos\omega$ in the ellipse $E_0(s)$ with the semi-major axis

$$z_0(s) = 1 + \frac{M_0^2}{2s}$$

and for any $\cos\omega \in E_0(s)$ and physical values of η it is polynomially bounded in the variable s. M_0 is some constant having mass dimensionality.

$$\boxed{\text{GLOBAL ANALYTICITY}}\ \&\ \boxed{\text{UNITARITY}}$$

$$\Downarrow$$

$$\boxed{\text{GENERALIZED ASYMPTOTIC BOUNDS}}$$

The generalized asymptotic bound for $O(6)$-invariant three-body forces scattering amplitude looks like

$$Im\,\mathcal{F}_0(s;\cos\omega = 1) < \text{Const}\, s^{3/2}(\frac{\ln s/s_0'}{M_0})^5 = \text{Const}\, s^{3/2}R_0^5(s)$$

where $R_0(s)$ is the effective radius of the three-body forces

$$R_0(s) = \frac{\Lambda_0}{\Pi(s)} = \frac{r_0}{M_0}\ln\frac{s}{s_0'}, \quad \Pi(s) = \frac{\sqrt{s}}{2}, \quad s\to\infty.$$

THREE-BODY FORCES IN SINGLE DIFFRACTION DISSOCIATION

The formula connecting one-particle inclusive cross section with the three-body forces scattering amplitude looks like

$$\boxed{2E_N(\vec{\Delta})\frac{d\sigma_{hN\to NX}}{d\vec{\Delta}}(s,\vec{\Delta}) = -\frac{(2\pi)^3}{I(s)} Im\mathcal{F}_0^{scr}(\bar{s};-\vec{\Delta},\vec{\Delta},\vec{q};\vec{\Delta},-\vec{\Delta},\vec{q})},$$

$$Im\mathcal{F}_0^{scr}(\bar{s};-\vec{\Delta},\vec{\Delta},\vec{q};\vec{\Delta},-\vec{\Delta},\vec{q}) = Im\mathcal{F}_0(\bar{s};-\vec{\Delta},\vec{\Delta},\vec{q};\vec{\Delta},-\vec{\Delta},\vec{q}) -$$
$$-4\pi\int d\vec{\Delta}'\frac{\delta\left[E_N(\vec{\Delta}-\vec{\Delta}')+\omega_h(\vec{q}+\vec{\Delta}')-E_N(\vec{\Delta})-\omega_h(\vec{q})\right]}{2\omega_h(\vec{q}+\vec{\Delta}')2E_N(\vec{\Delta}-\vec{\Delta}')}\times$$
$$Im\mathcal{F}_{hN}(\hat{s};\vec{\Delta},\vec{q};\vec{\Delta}-\vec{\Delta}',\vec{q}+\vec{\Delta}')Im\mathcal{F}_0(\bar{s};-\vec{\Delta},\vec{\Delta}-\vec{\Delta}',\vec{q}+\vec{\Delta}';\vec{\Delta},-\vec{\Delta},\vec{q}),$$

$$E_N(\vec{\Delta}) = \sqrt{\vec{\Delta}^2+M_N^2},\quad \omega_h(\vec{q}) = \sqrt{\vec{q}^{\,2}+m_h^2},$$
$$I(s) = 2\lambda^{1/2}(s,m_h^2,M_N^2),\quad \hat{s} = \frac{\bar{s}+m_h^2-2M_N^2}{2},$$
$$\bar{s} = 2(s+M_N^2)-M_X^2,\quad t=-4\vec{\Delta}^2.$$

A simple model for the three-body forces

$$Im\,\mathcal{F}_0(s;\vec{p}_1,\vec{p}_2,\vec{p}_3;\vec{q}_1,\vec{q}_2,\vec{q}_3) = f_0(s)\exp\Big\{-\frac{R_0^2(s)}{4}\sum_{i=1}^{3}(\vec{p}_i-\vec{q}_i)^2\Big\},$$

where $f_0(s)$, $R_0(s)$ are model parameteric functions of s, gives the following result for the one-particle inclusive cross section in the region of diffraction dissociation

$$\frac{s}{\pi}\frac{d\sigma_{hN\to NX}}{dtdM_X^2} = \frac{(2\pi)^3}{I(s)}\chi(\bar{s})Im\mathcal{F}_0(\bar{s};-\vec{\Delta},\vec{\Delta},\vec{q};\vec{\Delta},-\vec{\Delta},\vec{q})$$
$$= \frac{(2\pi)^3}{I(s)}\chi(\bar{s})f_0(\bar{s})\exp\Big[\frac{R_0^2(\bar{s})}{2}t\Big]$$

where

$$\chi(\bar{s}) = \frac{\sigma_{hN}^{tot}(\bar{s}/2)}{2\pi[B_{hN}(\bar{s}/2)+R_0^2(\bar{s})]}-1.$$

If we take the usual parameterization for one-particle inclusive cross section in the region of diffraction dissociation

$$\frac{s}{\pi}\frac{d\sigma}{dtdM_X^2} = A(s.M_X^2)\exp[b(s,M_X^2)t]$$

then we obtain for the quantities A and b

$$A(s,M_X^2) = \frac{(2\pi)^3}{I(s)}\chi(\bar{s})f_0(\bar{s}), \quad b(s,M_X^2) = \frac{R_0^2(\bar{s})}{2}.$$

It is remarkable fact:

the effective radius of three-body forces is related to the slope of diffraction cone for inclusive diffraction dissociation processes in the same way as the effective radius of two-body forces is related to the slope of diffraction cone in elastic scattering processes.

Moreover it follows from the expressions

$$R_0(\bar{s}) = \frac{r_0}{M_0}\ln \bar{s}/s_0', \quad \bar{s} = 2(s+M_N^2) - M_X^2$$

that

the slope of diffraction cone for inclusive diffraction dissociation processes at fixed energy decreases with the growth of missing mass.

This property agrees qualitatively well with the experimentally observable picture.

There is a very important relation

$$\boxed{A(s,M_X^2) = \frac{\bar{s}M_N[R_0^2(\bar{s})+R_d^2]^{3/2}}{(2\pi)^{3/2}I(s)}\delta\sigma^{inel}(\bar{s})}.$$

SCATTERING FROM DEUTERON

The total cross-section in the scattering from deuteron can be expressed by the formula

$$\sigma_{hd}^{tot}(s) = \sigma_{hp}^{tot}(\hat{s}) + \sigma_{hn}^{tot}(\hat{s}) - \delta\sigma(s),$$

where $\sigma_{hd}, \sigma_{hp}, \sigma_{hn}$ are the total cross-sections in scattering from deuteron, proton and neutron,

$$\delta\sigma(s) = \delta\sigma^{el}(s) + \delta\sigma^{inel}(s) =$$
$$= 2\sigma^{el}(s)a^{el}(x_{el}) + 2\sigma_{sd}^{ex}(s)a^{inel}(x_{inel}),$$
$$\sigma^{el}(s) \equiv \frac{\sigma_{hN}^{tot\,2}(s)}{16\pi B_{el}(s)}, \quad a^{el}(x_{el}) = \frac{x_{el}^2}{1+x_{el}^2},\ x_{el}^2 \equiv \frac{2B_{el}(s)}{R_d^2} = \frac{R_2^2(s)}{R_d^2},$$
$$a^{inel}(x_{inel}) = \frac{x_{inel}^2}{(1+x_{inel}^2)^{3/2}}, \qquad x_{inel}^2 \equiv \frac{R_3^2(s)}{R_d^2} = \frac{2B_{sd}(s)}{R_d^2}.$$

The total single diffractive dissociation cross-section $\sigma_{sd}^{ex}(s)$ is defined by the following equation

$$\sigma_{sd}^{\varepsilon}(s) = \pi\int_{M_{min}^2}^{\varepsilon s}\frac{dM_X^2}{s}\int_{t_-(M_X^2)}^{t_+(M_X^2)} dt\frac{d\sigma}{dtdM_X^2},$$

where

$$\varepsilon = \varepsilon^{ex} = \sqrt{2\pi}/2M_N R_d.$$

We supposed that at high energies

$$\sigma_{hp}^{tot} = \sigma_{hn}^{tot} = \sigma_{hN}^{tot}, \quad B_{el}^{hp} = B_{el}^{hn} = B_{el}.$$

The first term generalizes the known Glauber correction

$$\delta\sigma^{el}(s) = \delta\sigma_G(s) = \frac{\sigma_{hN}^{tot\,2}(s)}{4\pi R_d^2}, \quad x_{el}^2 << 1,$$

the second term in is totally new and comes from the contribution of the three-body forces to the hadron-deuteron total cross section.

GLOBAL STRUCTURE OF pp AND $p\bar{p}$ TOTAL CROSS SECTIONS

Recently (see Proceedings of VIIIth Blois Workshop (World Scientific, Singapore, 2000), pp. 109-118; hep-ph/9909531, hep-ph/9911533.) a simple theoretical formula describing the global structure of pp and $p\bar{p}$ total cross-sections in the whole range of energies available up today has been derived. The fit to the experimental data with the formula was made, and it was shown that there is a very good correspondence of the theoretical formula to the existing experimental data.

$$\sigma_{p\bar{p}}^{tot}(s) = \sigma_{asmpt}^{tot}(s)\left[1 + \frac{c}{\sqrt{s-4m_N^2}R_0^3(s)}\left(1 + \frac{d_1}{\sqrt{s}} + \frac{d_2}{s} + \frac{d_3}{s^{3/2}}\right)\right]$$

$$R_0^2(s) = \left[0.408740044\sigma_{asmpt}^{tot}(s)(mb) - B(s)\right](GeV^{-2}),$$

$$\sigma_{asmpt}^{tot}(s) = 42.0479 + 1.7548\ln^2(\sqrt{s}/20.74),$$

$$B(s) = 11.92 + 0.3036\ln^2(\sqrt{s}/20.74),$$

$$d_1 = (-12.12 \pm 1.023)GeV, \quad d_2 = (89.98 \pm 15.67)GeV^2,$$

$$d_3 = (-110.51 \pm 21.60)GeV^3, \quad c = (6.655 \pm 1.834)GeV^{-2}.$$

$$\sigma_{pp}^{tot}(s) = \sigma_{asmpt}^{tot}(s)\times$$

$$\left\{1 + \left[\left(\frac{c_1}{\sqrt{s-4m_N^2}R_0^3(s)} - \frac{c_2}{\sqrt{s-s_{thr}}R_0^3(s)}\right)(1+d(s))\right]_{s>s_{thr}} + Res(s)\right\},$$

$$d(s) = \sum_{k=1}^{8}\frac{d_k}{s^{k/2}}, \quad Res(s) = \sum_{i=1}^{8}\frac{C_R^i s_R^i {\Gamma_R^i}^2}{\sqrt{s(s-4m_N^2)}[(s-s_R^i)^2 + s_R^i {\Gamma_R^i}^2]},$$

$$\boxed{s_{thr} = (3.5283 \pm 0.0052)GeV^2}.$$

It turned out there is a very good correspondence of the theory to all existing cosmic ray experimental data as well.

The predicted values for σ_{tot}^{pp} obtained from theoretical description of all existing accelerators data are completely compatible with the values obtained from cosmic ray experiments.

(hep-ph/0108118)

New "threshold", which is near the elastic one, looks like a manifestation of a new unknown particle:

$$\sqrt{s_{thr}} = 2m_p + m_{\mathcal{L}}, \qquad m_{\mathcal{L}} = 1.833\, MeV$$

Diproton resonances.

$m_R(MeV)$	$\Gamma_R(MeV)$	$Refs.$	$C_R(GeV^2)$
1937 ± 2	7 ± 2	[1]	0.058 ± 0.018
$1947(5) \pm 2.5$	8 ± 3.9	[2]	0.093 ± 0.028
1955 ± 2	9 ± 4	[1]	0.158 ± 0.024
1965 ± 2	6 ± 2	[1]	0.138 ± 0.009
1980 ± 2	9 ± 2	[1]	0.310 ± 0.051
1999 ± 2	9 ± 4	[1]	0.188 ± 0.070
2008 ± 3	4 ± 2	[1]	0.176 ± 0.050
$2027 \pm ?$	$10 - 12$		0.121 ± 0.018
2087 ± 3	12 ± 7	[1]	-0.069 ± 0.010
2106 ± 2	11 ± 5	[1]	-0.232 ± 0.025
$2127(9) \pm 5$	4 ± 2	[1]	-0.222 ± 0.056
$2180(72) \pm 5$	7 ± 3	[1]	0.131 ± 0.015
$2217 \pm ?$	$8 - 10$		0.112 ± 0.031
2238 ± 3	22 ± 8	[1]	0.221 ± 0.078
2282 ± 4	24 ± 9	[1]	0.098 ± 0.024

$\mathcal{L}$-PARTICLE AND KALUZA-KLEIN WORLD

The input space-time is a $(4+d)$-dimensional space $\mathcal{M}_{(4+d)}$

$$\mathcal{M}_{(4+d)} = M_4 \times K_d$$

with the factorizable metric

$$ds^2 = \mathcal{G}_{MN}(z)dz^M dz^N = g_{\mu\nu}(x)dx^\mu dx^\nu + \gamma_{mn}(x,y)dy^m dy^n,$$

$z^M = \{x^\mu, y^m\}$, $M = 0,1,\ldots,3{+}d$, $\mu = 0,1,2,3$, $m = 1,2,\ldots,d)$, $z \in \mathcal{M}_{(4+d)}$, $x \in M_4$, $y \in K_d$.

Let us consider the (4+d)-dimensional model of scalar field

$$S = \int d^{4+d}z\sqrt{-\mathcal{G}}\left[\frac{1}{2}\left(\partial_M\Phi\right)^2 - \frac{m^2}{2}\Phi^2 + \frac{G_{(4+d)}}{4!}\Phi^4\right],$$

$$\mathcal{G} = \det|\mathcal{G}_{MN}|.$$

Let Δ_{K_d} be the Laplace operator on K_d (with the characteristic size R), and $Y_n(y)$ are ortho-normalized eigenfunctions

$$\Delta_{K_d}Y_n(y) = -\frac{\lambda_n}{R^2}Y_n(y).$$

d-dimensional torus T^d with equal radii R is an especially simple example of K_d; here the eigenfunctions and eigenvalues look like

$$Y_n(y) = \frac{1}{\sqrt{V_d}}\exp\left(i\sum_{m=1}^{d} n_m y^m/R\right), \quad \lambda_n = |n|^2,$$

$$|n|^2 = n_1^2 + n_2^2 + \ldots n_d^2, \quad n = (n_1, n_2, \ldots, n_d), \quad -\infty \le n_m \le \infty,$$

n_m are integer numbers, $V_d = (2\pi R)^d$ is the volume of the torus. Let us wright a harmonic expansion for the field $\Phi(z)$

$$\Phi(z) = \Phi(x,y) = \sum_n \phi^{(n)}(x)Y_n(y).$$

The coefficients $\phi^{(n)}(x)$ are called KK excitations or KK modes. Substitution of harmonic expansion into S and integration gives

$$S = \int d^4x\sqrt{-g}\left\{\frac{1}{2}\left(\partial_\mu\phi^{(0)}\right)^2 - \frac{m^2}{2}(\phi^{(0)})^2 + \frac{g}{4!}(\phi^{(0)})^4 + \right.$$

$$\left. + \sum_{n\neq 0}\left[\frac{1}{2}\left(\partial_\mu\phi^{(n)}\right)\left(\partial^\mu\phi^{(n)}\right)^* - \frac{m_n^2}{2}\phi^{(n)}\phi^{(n)*}\right] + \frac{g}{4!}(\phi^{(0)})^2\sum_{n\neq 0}\phi^{(n)}\phi^{(n)*}\right\}$$

$$+\ldots.$$

For the masses of the KK modes one obtains

$$\boxed{m_n^2 = m^2 + \frac{\lambda_n}{R^2}},$$

and reduction formula relating the coupling constants looks like

$$\boxed{g = \frac{G_{(4+d)}}{V_d}}.$$

Let us apply the Kaluza-Klein approach to our case. We assume that $\mathcal{L}$-particle is related to the first KK excitation in the diproton system. So, we put as

$$\sqrt{s_{thr}} = 2m_p + m_{\mathcal{L}} = 2\sqrt{m_p^2 + \frac{1}{R^2}},$$

then

$$\frac{1}{R} = \sqrt{m_{\mathcal{L}}(m_p + \frac{1}{4}m_{\mathcal{L}})} = 41.481\text{MeV},$$

it corresponds

$$\boxed{R = 24.1\,GeV^{-1} = 4.75\,10^{-13}\text{cm}},$$

$$\left[g_{eff} = g_{\pi NN}\exp(-m_\pi R) \sim 0.5, \quad (g_{\pi NN}^2/4\pi = 14.6)\right],$$

$$M \sim R^{-1}\left(M_{Pl}/R^{-1}\right)^{2/(d+2)}|_{d=6} \sim 5\,\text{TeV}.$$

Let us build the Kaluza-Klein tower of KK excitations by equation

$$M_n = 2\sqrt{m_p^2 + \frac{n^2}{R^2}}, \quad (n = 1, 2, 3, \dots)$$

and compare it with the observed irregularities in the spectrum of mass of the diproton system.

Kaluza-Klein tower of KK excitations of diprotons

n	$M_n(MeV)$	$M^{pp}_{exp}(MeV)$	Refs.
1	1878.38	1877.5 ± 0.5	[3]
2	1883.87	1886 ± 1	[1]
3	1892.98	1898 ± 1	[1]
4	1905.66	1904 ± 2	[4]
5	1921.84	1916 ± 2	[1]
		1926 ± 2	[4]
		1937 ± 2	[1]
		1942 ± 2	[4]
6	1941.44	1945 ± 2.5	[2]
		1955 ± 2	[2]
		1956 ± 3	[10]
7	1964.35	1965 ± 2	[1]
		1969 ± 2	[5]
8	1990.46	1980 ± 2	[1]
		1999 ± 2	[1]
9	2019.63	2017 ± 3	[1]
		2035 ± 8	[10]
10	2051.75	2046 ± 3	[1]
		2050 ± 3.2	[6]
11	2086.68	2087 ± 3	[1]
		2120 ± 3.2	[6]
12	2124.27	2121 ± 3	[7]
		2129 ± 5	[1]
		2140 ± 9	[10]
13	2164.39	2150 ± 12.6	[6]
		2172 ± 5	[1]
		2192 ± 3	[7]
14	2206.91	2217	[11]
		2220	[9]
15	2251.67	2238 ± 3	[1]
		2240 ± 5	[7]
16	2298.57	2282 ± 4	[1,8]
17	2347.45	2350	[9]

CONCLUSION

Investigations of three-body forces open many new pages in the study of fundamental dynamics of particles and nuclei:

- Three-body forces define the dynamics of one-particle inclusive reactions.
- We have to take into accout a contribution of three-body forces in scattering from deuteron.
- New scaling characteristics in shadow dynamics in scattering from deuteron have been established by account of three-body forces.
- Introduction of three-body forces resulted the discovery of global structure of (anti)proton-proton total cross sections.
- Investigating the three-body forces allowed us to predict a new particle ($\mathcal{L}$-particle), describing a new scale of internucleon distances, where strong Yukawa forces compared with electromagnetic ones.

Multidimensional space is a natural space to describe the properties of three-body forces. Geniusly simple formula provided by Kaluza-Klein approach so accurately described the mass spectrum of diproton system, and certainly it was not an accidental coincidence. This means that the existence of the extra dimensions was experimentally proved in the experiments at very low energies where the nucleon-nucleon dynamics had been studied, but we did not understand it. However, now it seems we understand it.

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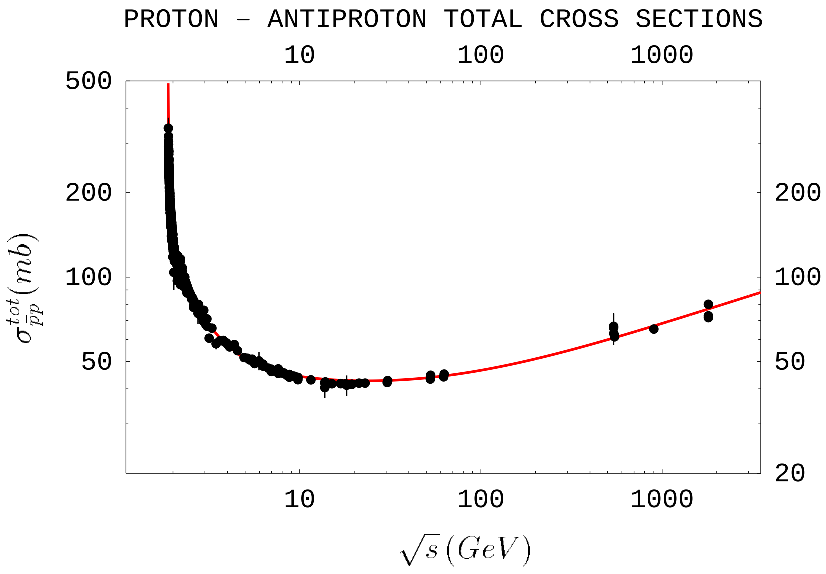

Figure 1: The proton-antiproton total cross sections versus $\sqrt{s}$ compared with the theory. Solid line represents our fit to the data. Statistical and systematic errors added in quadrature.

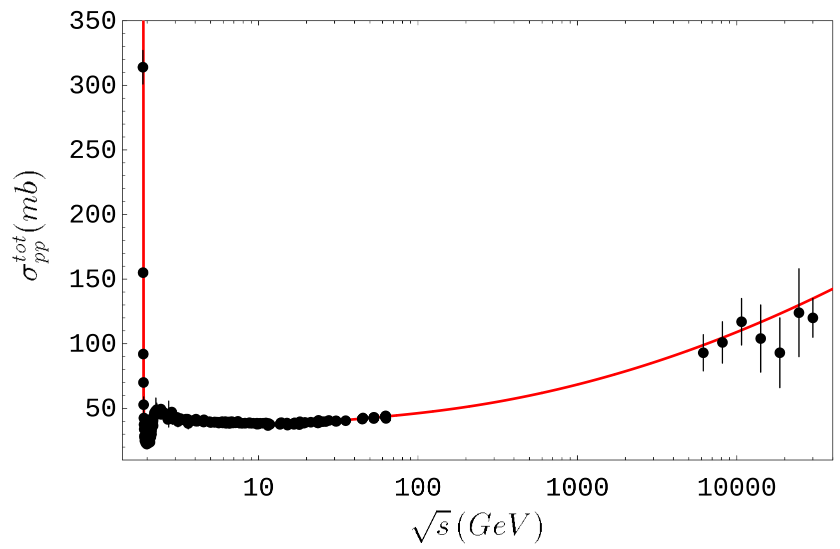

Figure 2: The proton-proton total cross-section versus $\sqrt{s}$ with the cosmic rays data points from Akeno Observatory and Fly's Eye Collaboration. Solid line corresponds to our theory predictions.

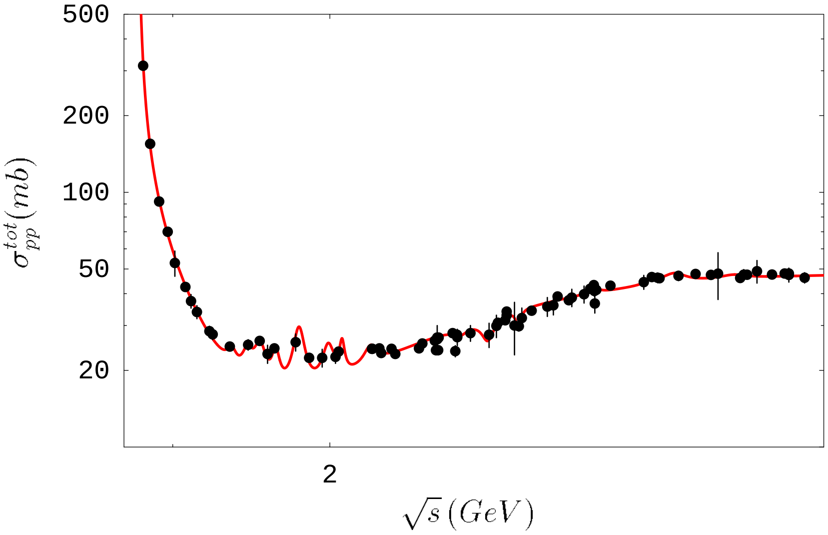

Figure 3: The proton-proton total cross-section versus $\sqrt{s}$ at low energies. Solid line corresponds to our theory predictions.

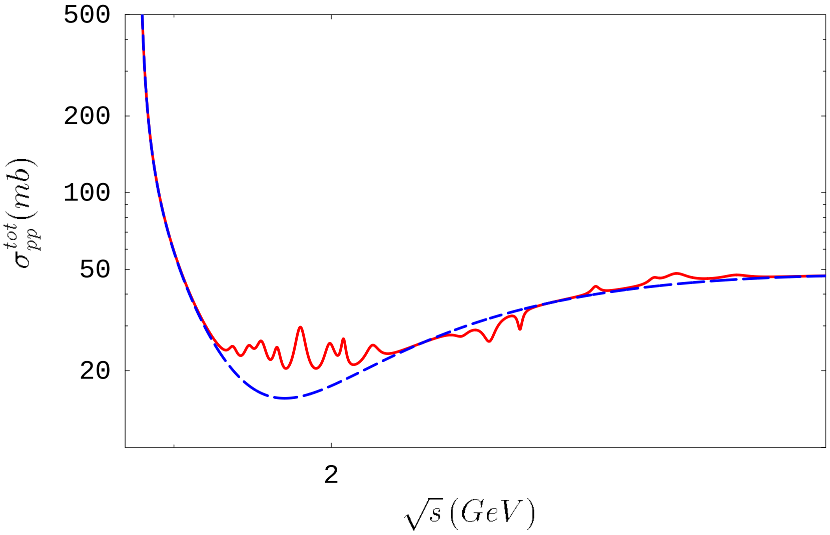

Figure 4: The resonance structure for the proton-proton total cross-section versus $\sqrt{s}$ at low energies. Solid line is our theory predictions. Dashed line corresponds to the "background" where all resonances are switched off.