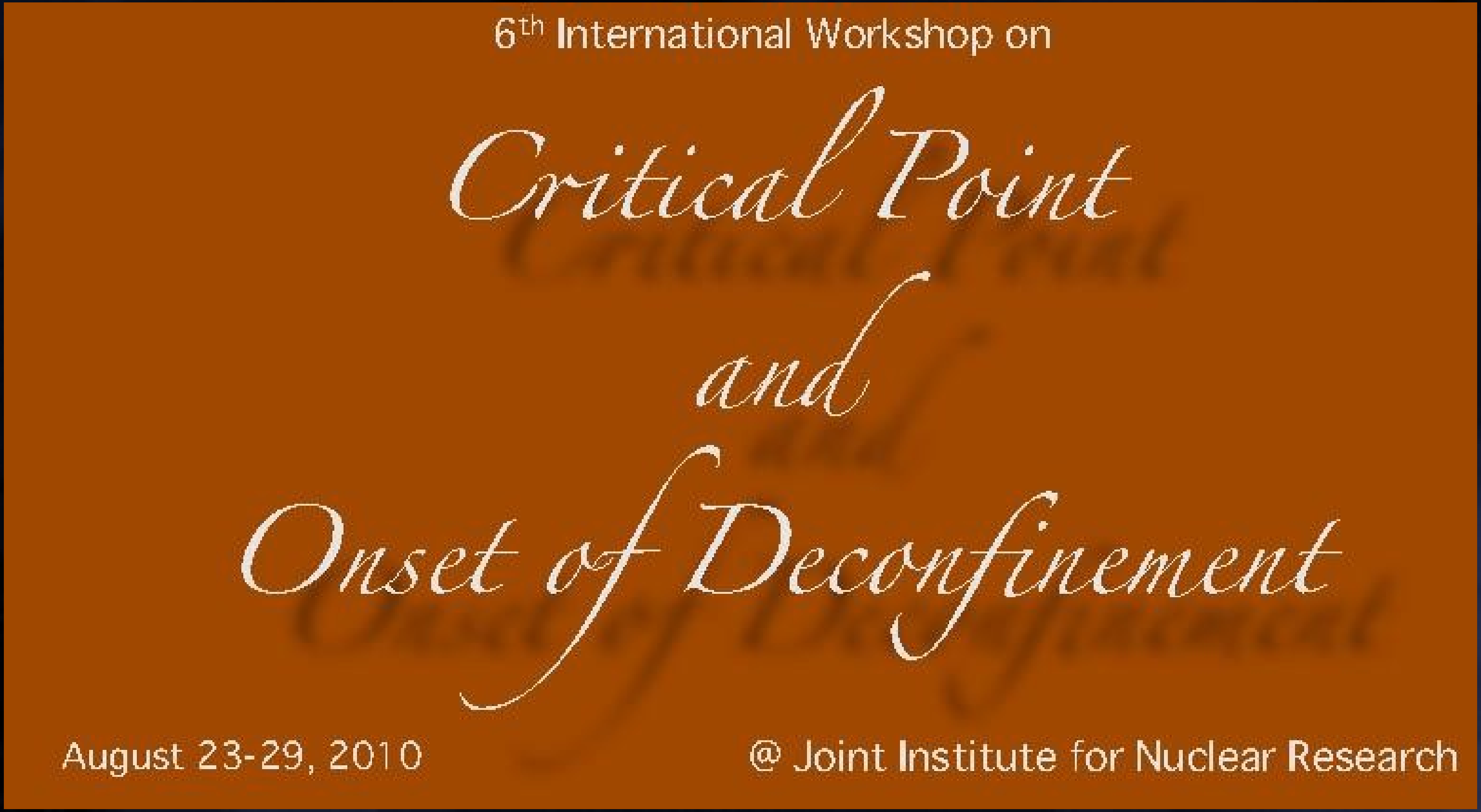


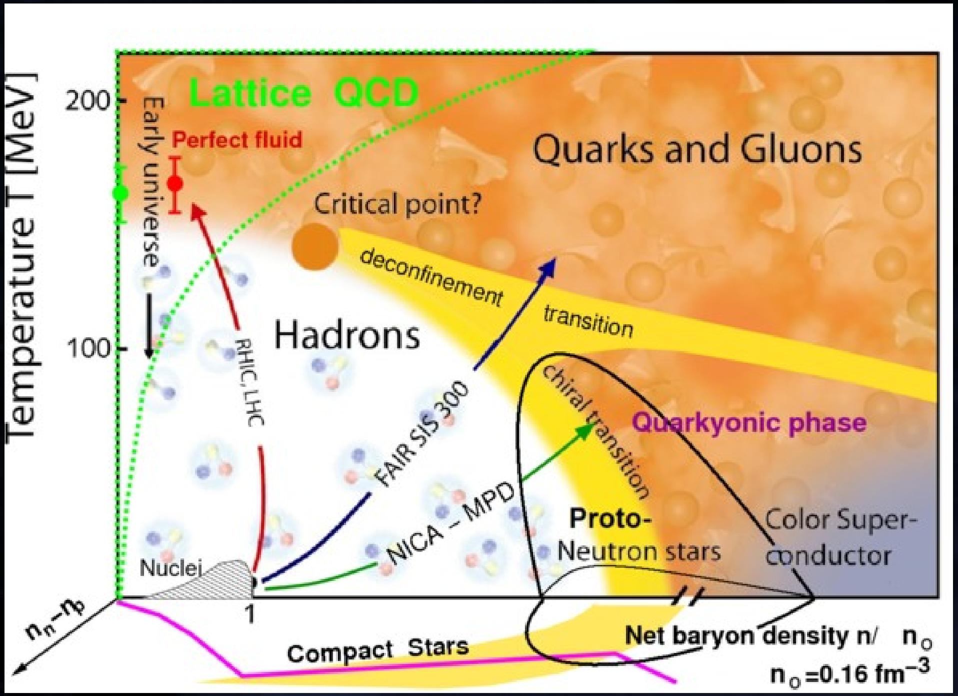

# How to extract physics from $v_{dyn}$

*Peter Christiansen, Lund*
*Work done together with*
*Eva Haslum and Evert Stenlund*

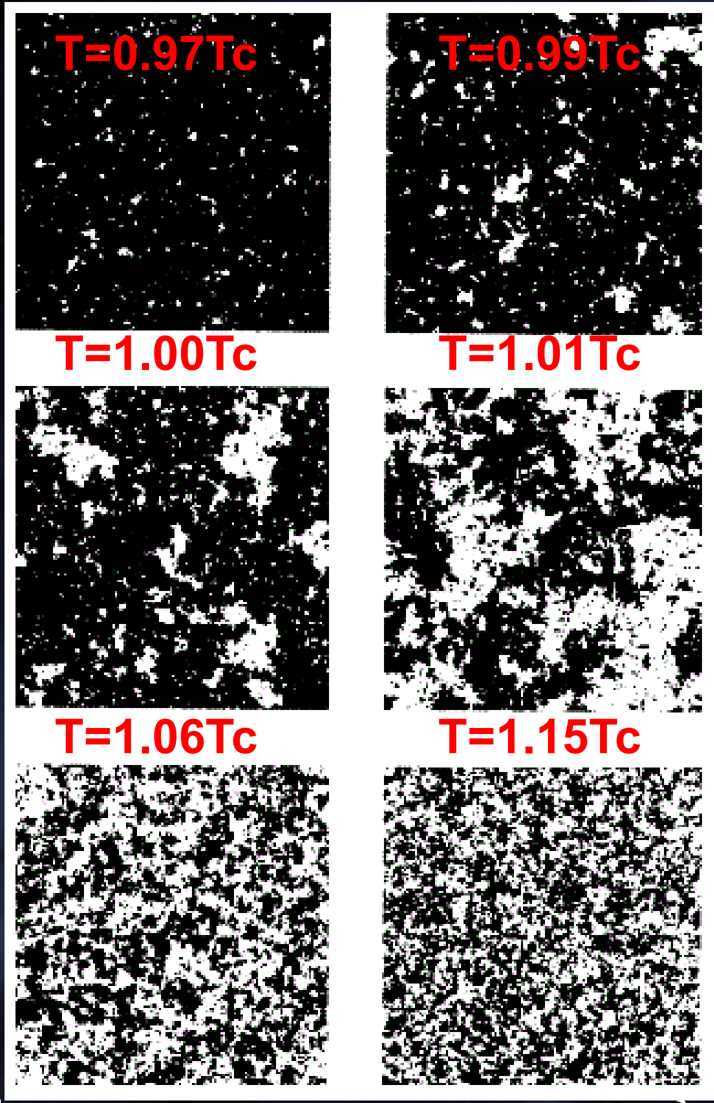
# Outline

- What is $v_{dyn}$ and what does it measure
  - Original idea: S. Voloshin for STAR, INPC2001, arXiv:nucl-ex/0109006v1. C.Pruneau, S.Gavin, and S.Voloshin, (PRC 66, 044904, 2002).
  - Mathematical properties and simple models for $v_{dyn}$ (PRC 80, 034903, 2009)
- Example: Forward-backward fluctuations in pp
- Generalization of $v_{dyn}$ to continuous variables
- Conclusions

# Motivation for studying fluctuations

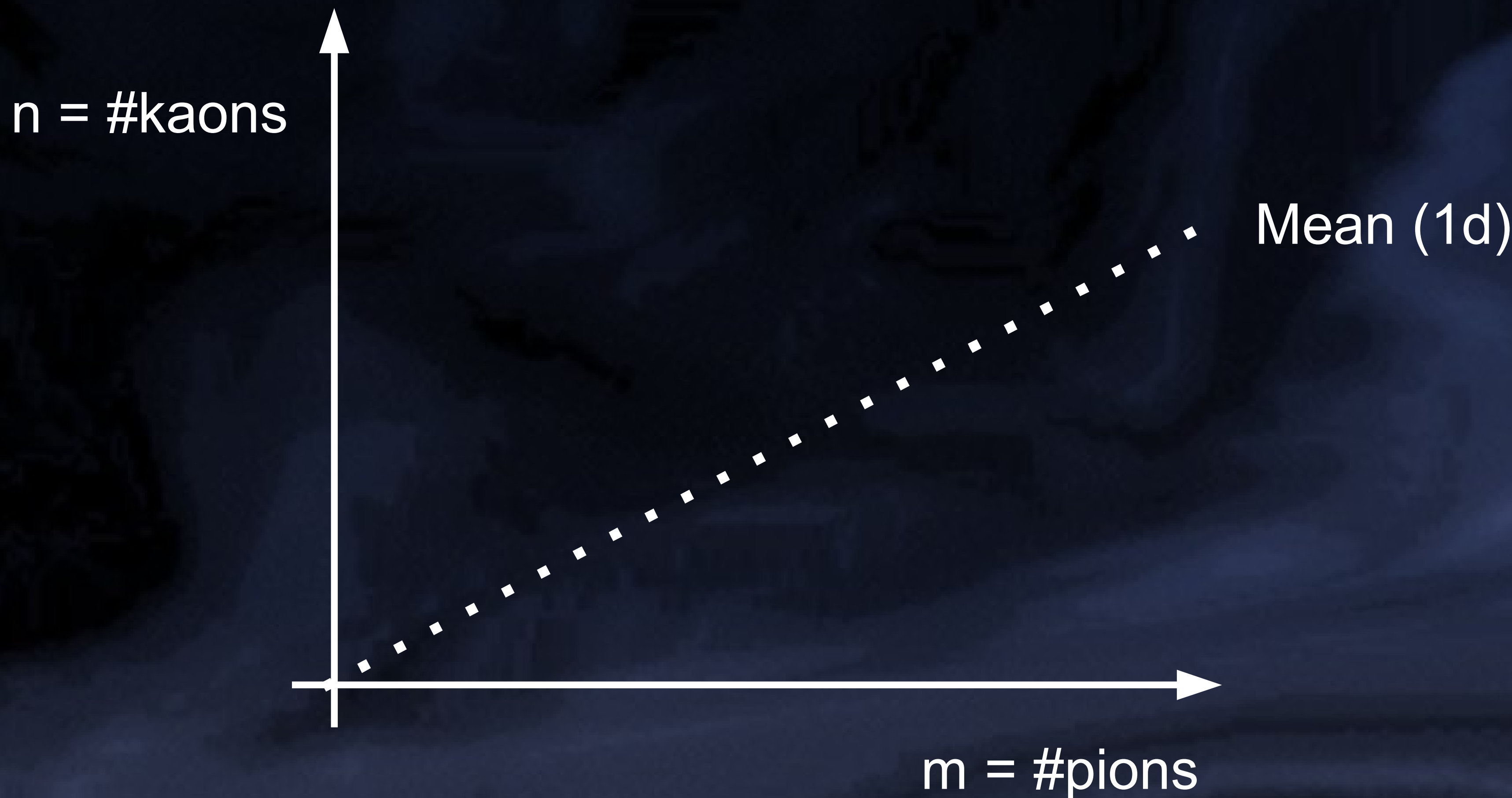



Ex.:2d ising model



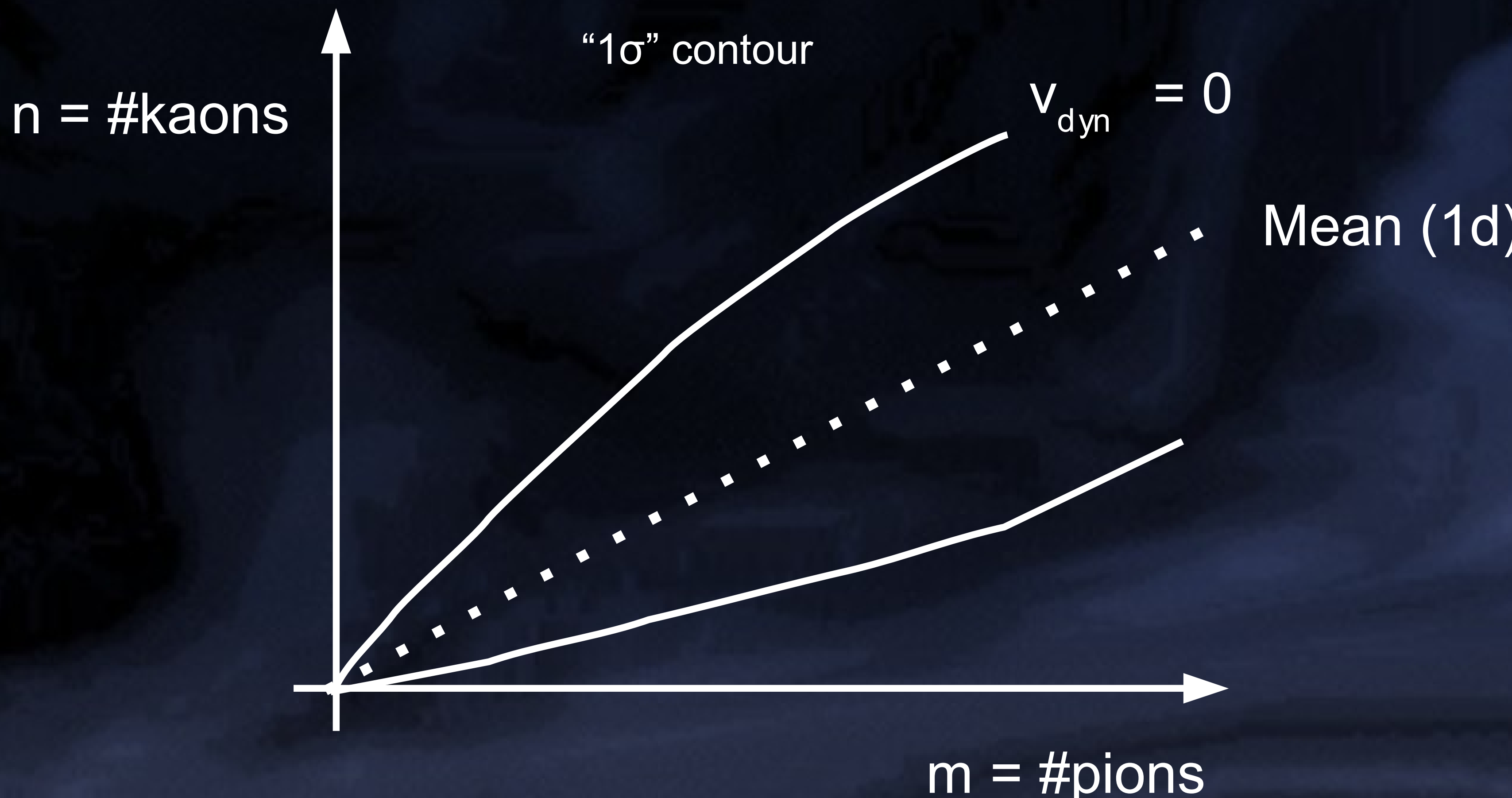

- Large fluctuations expected near QCD tri-critical point
- Provide "L2" discrimination between models (not only mean, but also fluctuations)

# Idea explained via 2d contour plot



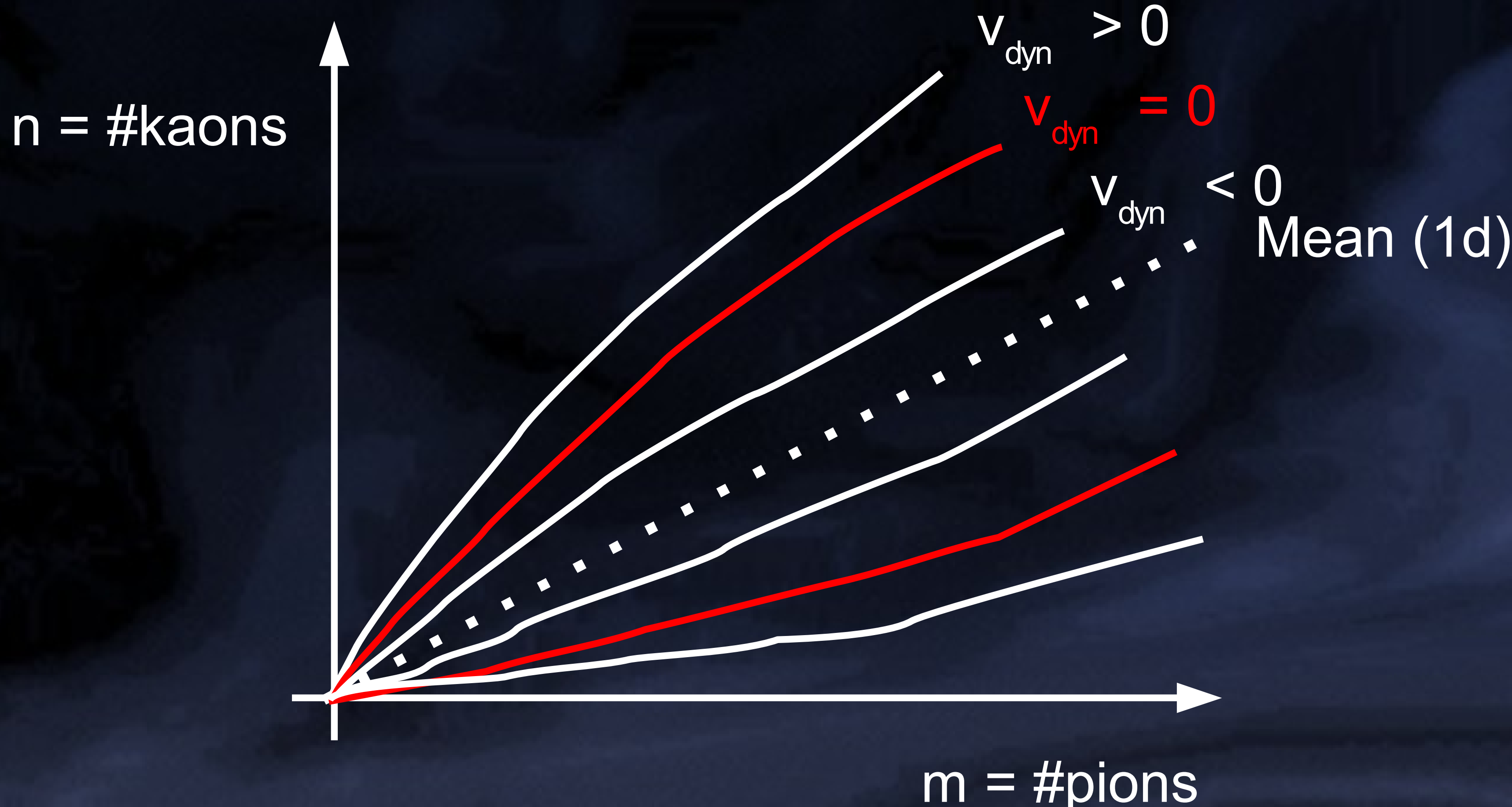

We study systems where the ratios m/N and n/N (N=m+n) are almost constant.

# Idea explained via 2d contour plot




Statistical fluctuations: √m and √n

# Idea explained via 2d contour plot



# Mathematical expression for $\nu_{dyn}$

- Consider $\left(\frac{m}{\langle m\rangle}-\frac{n}{\langle n\rangle}\right)$
  - It has mean 0!
- We define
$$\nu=V\left[\left(\frac{m}{\langle m\rangle}-\frac{n}{\langle n\rangle}\right)\right]=\left\langle\left(\frac{m}{\langle m\rangle}-\frac{n}{\langle n\rangle}\right)^2\right\rangle$$
  - Easy statistical behavior (no event mixing): $\nu_{stat}=\frac{1}{\langle m\rangle}+\frac{1}{\langle n\rangle}$
- Allows the study of dynamical fluctuations:
$$\nu_{dyn}=\nu-\nu_{stat}=\frac{\langle m(m-1)\rangle}{\langle m\rangle^2}-2\frac{\langle m\,n\rangle}{\langle m\rangle\langle n\rangle}+\frac{\langle n(n-1)\rangle}{\langle n\rangle^2}$$
- 3 terms: cancels trivial correlations, e.g., centrality!
- Independent of efficiencies

# What dynamical fluctuations are measured by $\nu_{dyn}$

- Definition: $$\nu_{dyn}=\frac{\langle m(m-1)\rangle}{\langle m\rangle^2}-2\frac{\langle mn\rangle}{\langle m\rangle\langle n\rangle}+\frac{\langle n(n-1)\rangle}{\langle n\rangle^2}$$
- For M=m+n, then the pair probability (mm) is:

$$p=P_m=\frac{\langle m\rangle}{\langle M\rangle} \qquad P_{mm}=\frac{\langle m(m-1)\rangle}{\langle M(M-1)\rangle}$$

- So we can rewrite $\nu_{dyn}$ as:

$$\nu_{dyn}=\frac{\langle M(M-1)\rangle}{\langle M\rangle^2}\left(\frac{P_{mm}}{p^2}-2\frac{P_{mn}}{p(1-p)}+\frac{P_{nn}}{(1-p)^2}\right)$$

- M=2 → pair probabilities can be written uniquely as:

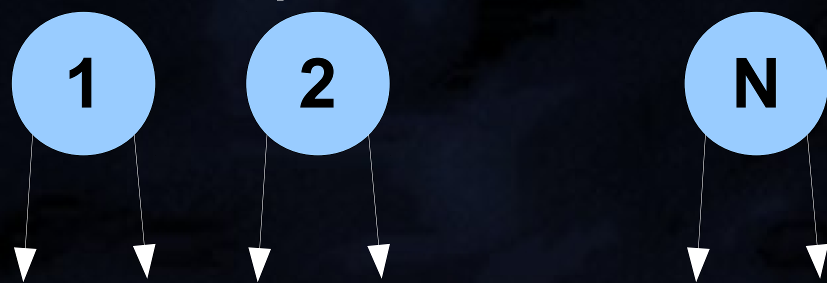
$$P_{mm}=p^2+\varepsilon \quad P_{mn}=2p(1-p)-2\varepsilon \quad P_{nn}=(1-p)^2+\varepsilon \quad \rightarrow \quad \nu_{dyn}=\frac{1}{2}\left(\frac{\varepsilon}{p^2(1-p)^2}\right)$$

- Consider M/2 pairs →

$$\nu_{dyn}=\frac{1}{\langle M\rangle}\left(\frac{\varepsilon}{p^2(1-p)^2}\right)$$

# Comment on 1/<M> dependence

- Consider N independent sources



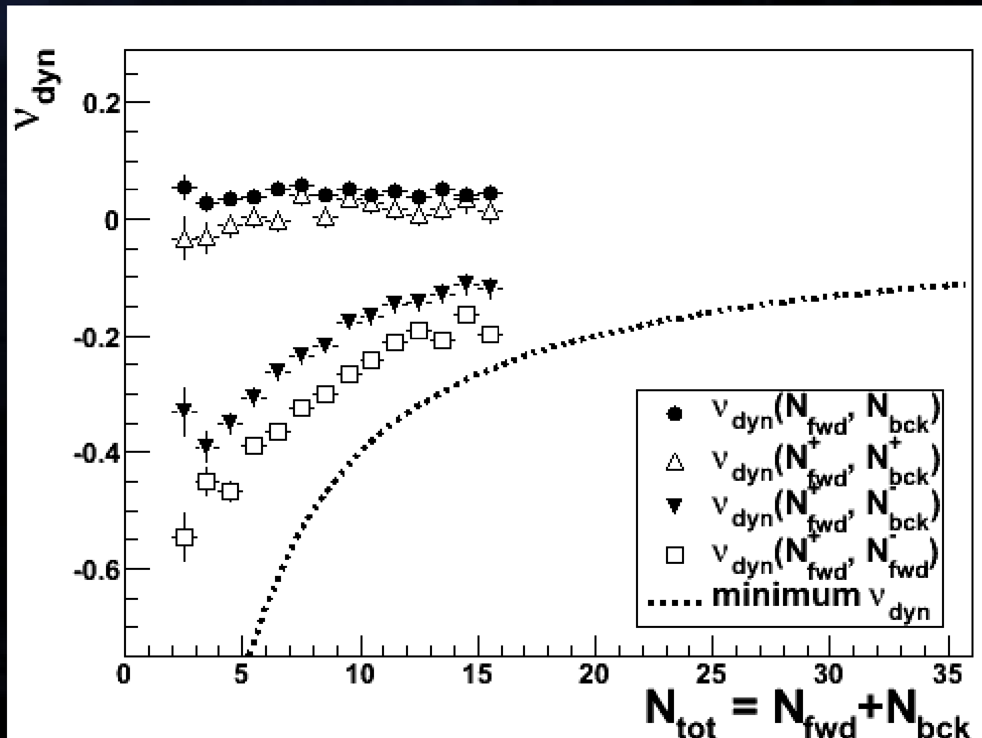

- Number of correlated pairs goes as N
- Number of uncorrelated pairs goes as N*(N-1)
- → 1/N dependence in this case!
- Feature of the model!
  - It measures the pair correlations

- NB! It does not have to go as 1/<M> if you increase or decrease (CGC?) the correlations in central collisions

# An example: Forward-backward correlations


- For symmetric intervals we have p=½, e.g.:
  - Fwd: $1 < \eta < 2$ and Bck: $-2 < \eta < -1$
- Interest from phenomenology
  - Strong correlation between $N_{fwd}$ and $N_{bck}$ (UA5+STAR)
    - Signals collective behavior in pp
  - NB! $\nu_{dyn}$ filters out these correlations

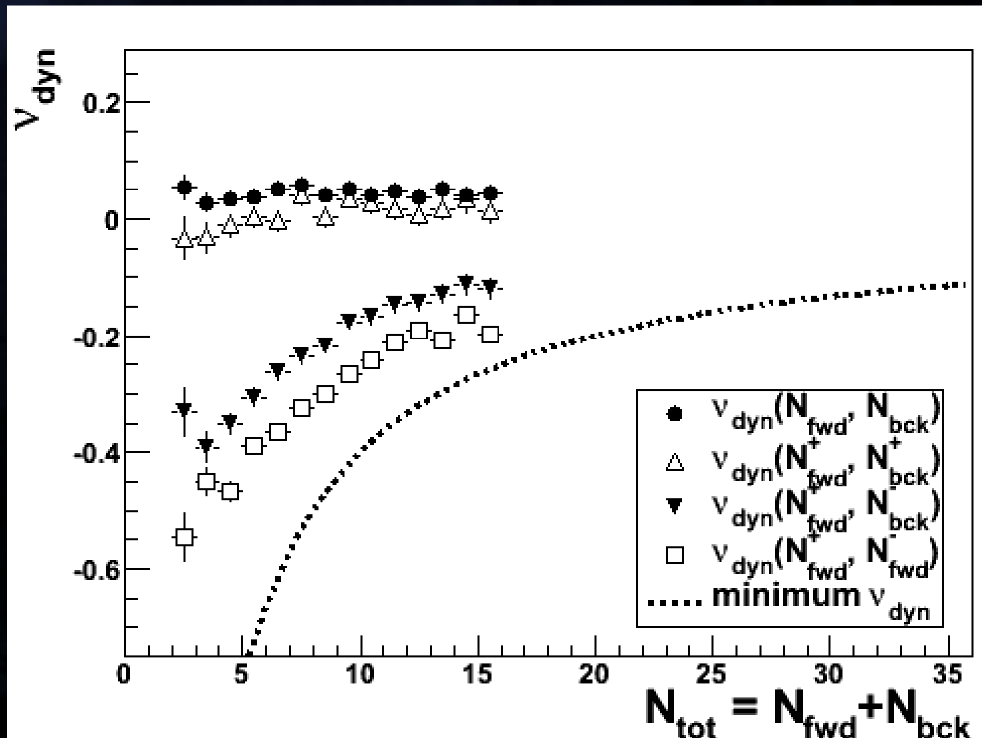
# Forward backward fluctuations




**PYTHIA 8.108**
**Min bias pp**
**$\sqrt{s}$ = 200 GeV**
**No $p_T$ cut!**
**Bck:$-1<\eta<0$**
**Fwd:$0<\eta<1$**

| | | |
|---|---|---|
| $N_{bck}^{ch}$ | $N_{fwd}^{ch}$ | 1 term |
| $N_{bck}^{+} + N_{bck}^{-}$ | $N_{fwd}^{+} + N_{fwd}^{-}$ | 6 terms (3 different) |

# Forward backward fluctuations




**PYTHIA 8.108**
**Min bias pp**
**$\sqrt{s}$ = 200 GeV**
**No $p_T$ cut!**
**Bck:-1<η<0**
**Fwd:0<η<1**

$N_{bck}^{ch}$ $N_{fwd}^{ch}$

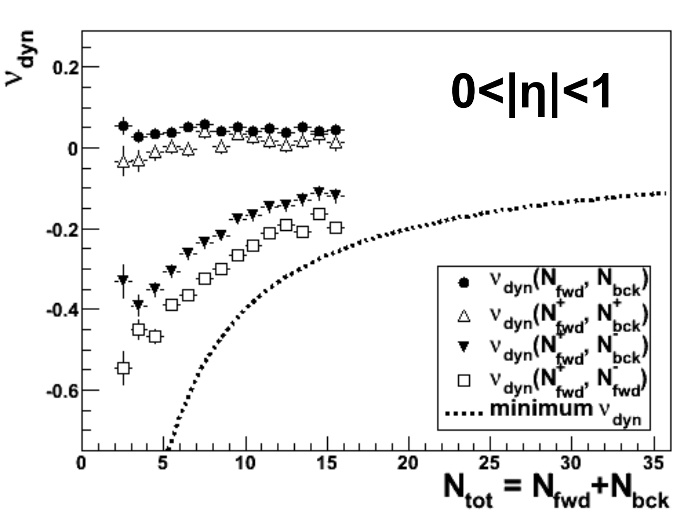
$N_{bck}^{+} + N_{bck}^{-}$ $N_{fwd}^{+} + N_{fwd}^{-}$

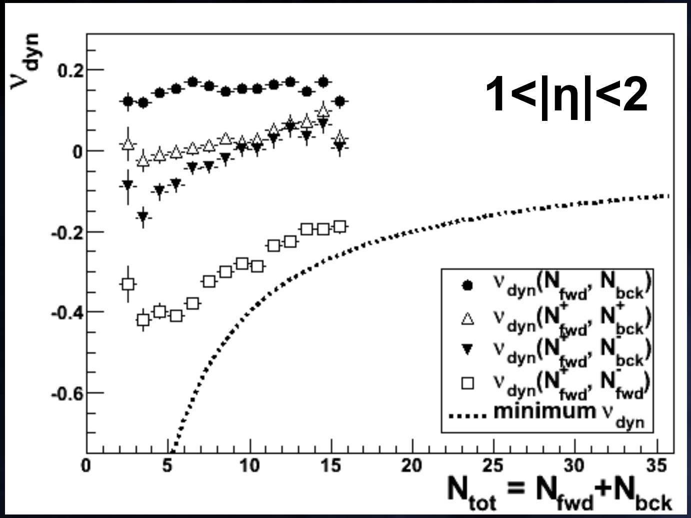
- (Stenlund) sum rule:

$$\nu_{dyn}(N_{fwd}^{ch}, N_{bck}^{ch}) = \frac{1}{2}\left[\nu_{dyn}(N_{fwd}^{plus}, N_{bck}^{plus}) + \nu_{dyn}(N_{fwd}^{plus}, N_{bck}^{minus}) - \nu_{dyn}(N_{fwd}^{plus}, N_{fwd}^{minus})\right]$$

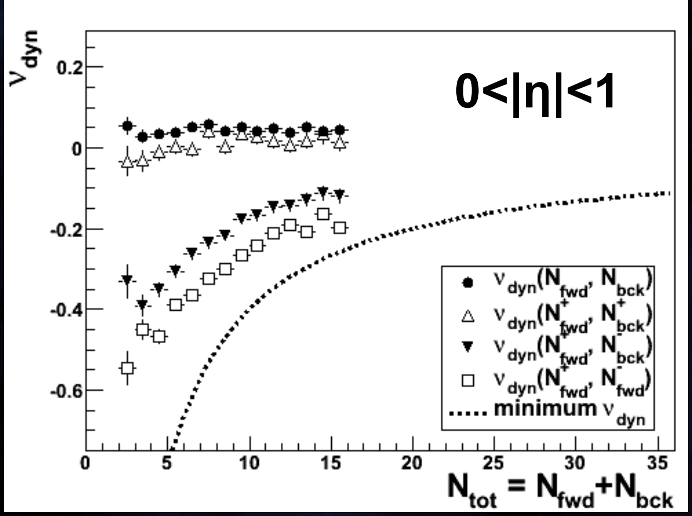
- NB! Note the minus sign.
- Primarily probe charge conservation and transport

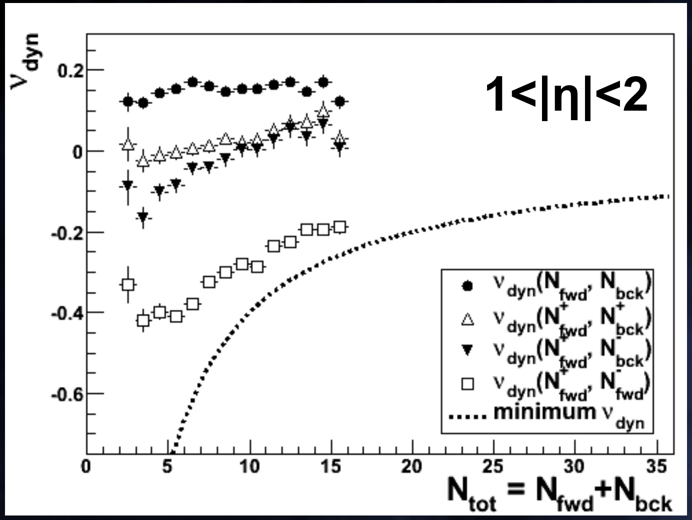
# Pseudo rapidity dependence



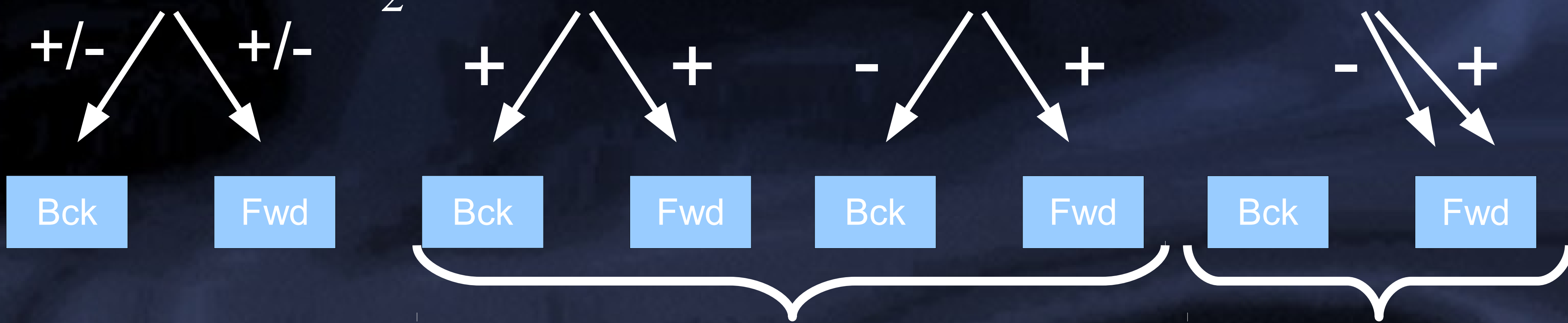



# Pseudo rapidity dependence



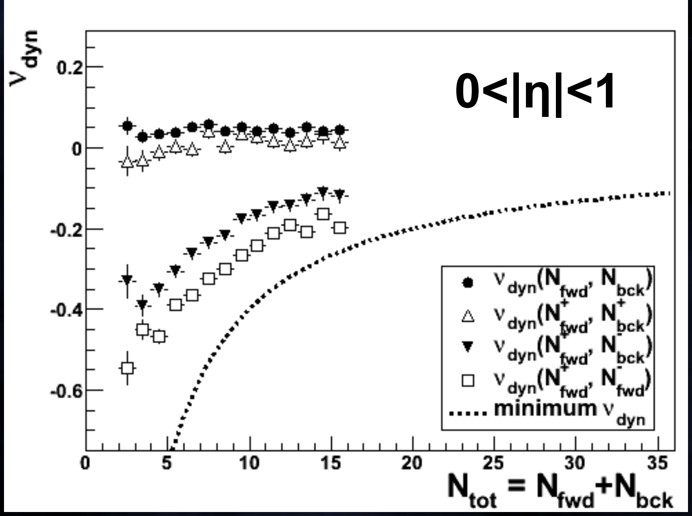

$$\nu_{dyn}(N^{ch}_{fwd}, N^{ch}_{bck}) = \frac{1}{2}\left[\nu_{dyn}(N^{plus}_{fwd}, N^{plus}_{bck}) + \nu_{dyn}(N^{plus}_{fwd}, N^{minus}_{bck}) - \nu_{dyn}(N^{plus}_{fwd}, N^{minus}_{fwd})\right]$$

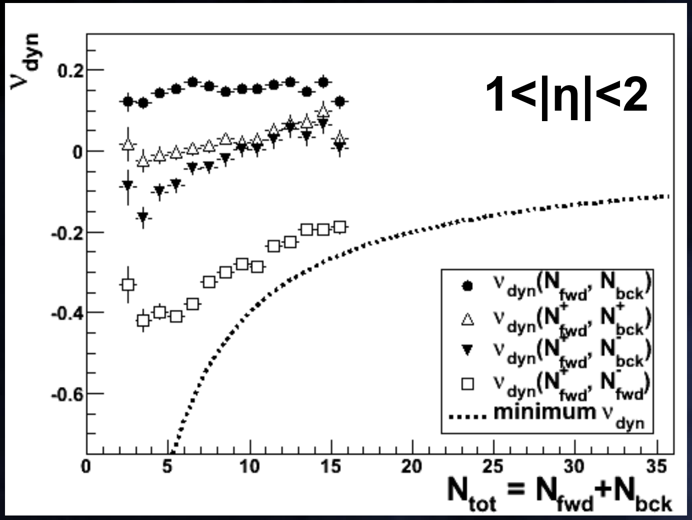



→ 0 for Δη → ∞

~const for all Δη

# Pseudo rapidity dependence

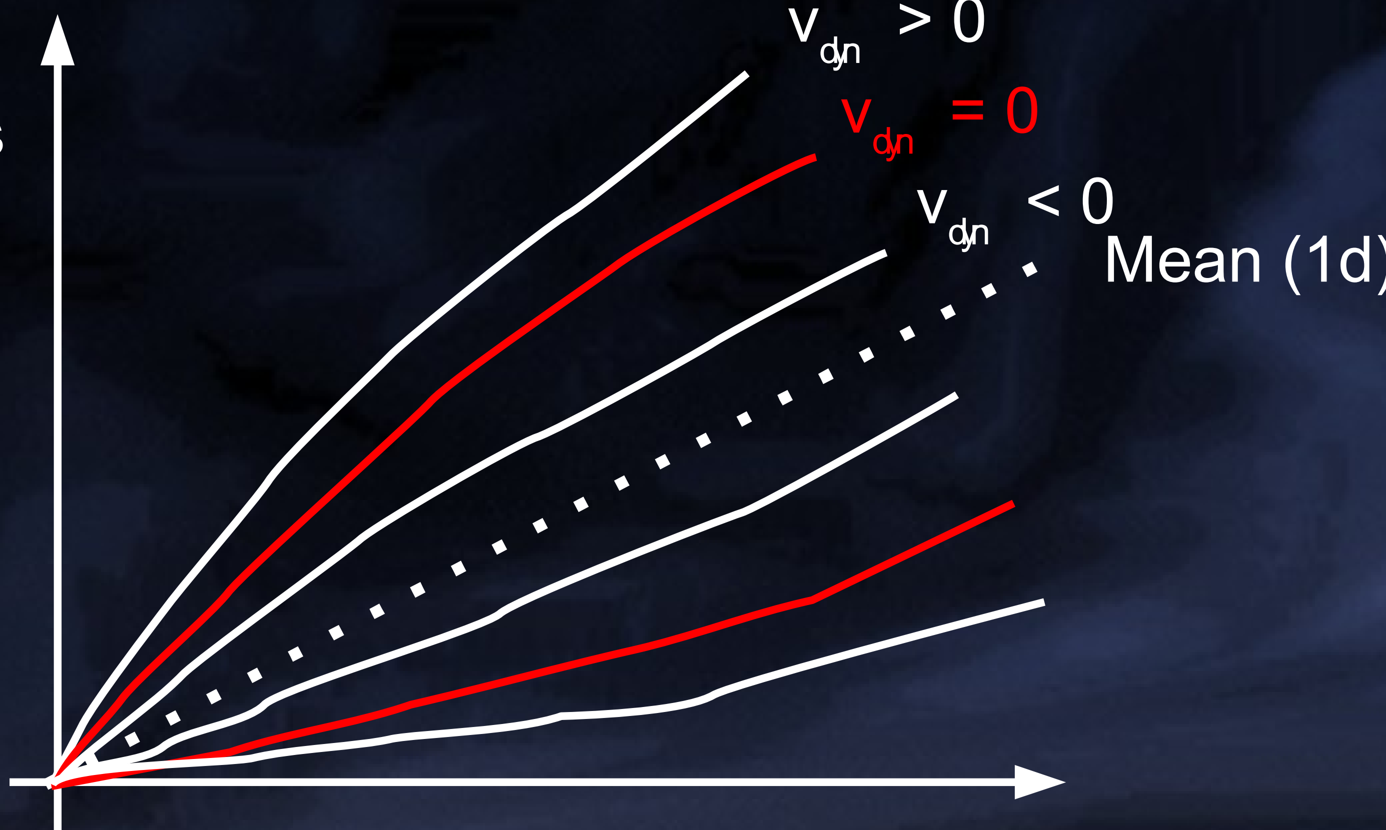



- Fluctuations measured by $v_{dyn}$ for PYTHIA are dominated by charge conservation
  - general limit of method: strangeness ($K^-+K^+$), baryon number (p+pbar) → negative dynamical fluctuations
    - NB! Results in positive $v_{dyn}$ for sums, e.g., $m=K^-+K^+$
- To search for long range correlations: $v_{dyn}$ (+,+) vs (+ & +)
- Alternative: Study e.g., $\pi^0$ vs $\pi^+$

# Generalization to continuous variables

n = #kaons → y= $\Sigma p_T$ stock j $CO_2$




m = #pions → x = N, stock i, T

# Generalized expression and interpretation

- The general expression becomes:

$$\nu_{dyn} = \frac{\langle x^2 \rangle - \langle \sigma_x^2(x) \rangle}{\langle x \rangle^2} - 2\frac{\langle x\, y \rangle}{\langle x \rangle \langle y \rangle} + \frac{\langle y^2 \rangle - \langle \sigma_y^2(y) \rangle}{\langle y \rangle^2} \qquad \sigma_{\nu_{dyn}} \sim \frac{\nu}{\sqrt{N_{events}}}$$

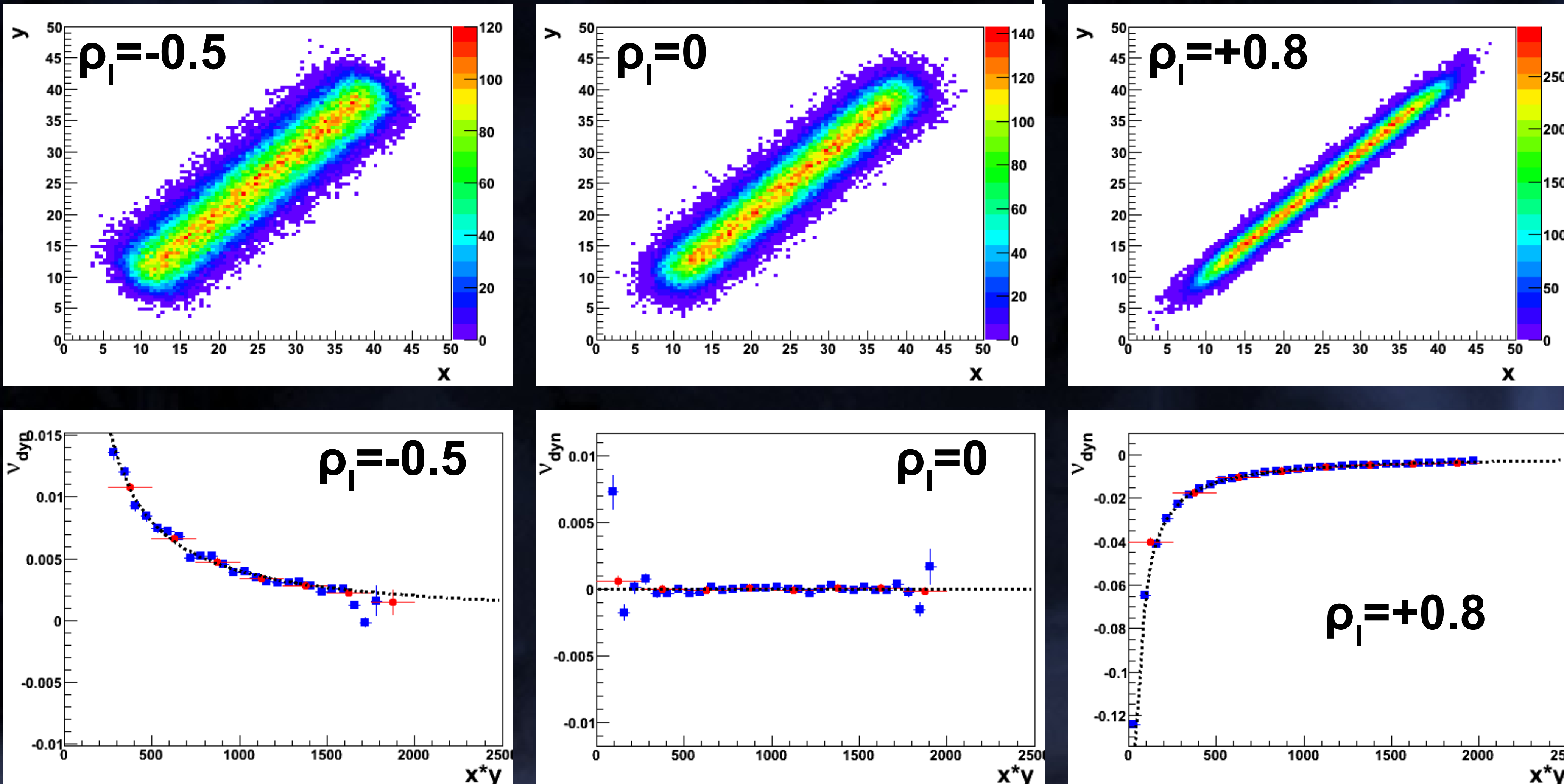
- And is consistent with the old definition because for numbers: $\sigma_x^2 = x$
- Because we know that $\nu_{dyn}$ cancels trivial correlations we can consider a local Gaussian model:

$$P(x, y) = \frac{1}{2\pi\sigma_x\sigma_y\sqrt{1-\rho_l^2}} \exp\left[-\frac{1}{2}\frac{1}{1-\rho_l^2}\left(\frac{(x-\mu_x)^2}{\sigma_x^2} + \frac{(y-\mu_y)^2}{\sigma_y^2} - 2\rho_l \frac{(x-\mu_x)(y-\mu_y)}{\sigma_x\sigma_y}\right)\right]$$

- Then we find that

$$\nu_{dyn} = -2\rho_{local}\left(\frac{\sigma_x\sigma_y}{\langle x \rangle \langle y \rangle}\right)$$

# Simulated examples




- Results are as predicted on previous slide (curves)
- Statistical error increases when $\rho_l$ is decreased

# Problem: How to determine σ?

- For numbers there is a natural statistics:
  - Poisson
- There is no general continuous statistic
  - We have to determine σ example by example
    - And it is not clear how! And if it is at all possible....
- If we compare to a model we can always use ν:

$$\nu = \frac{\langle x^2 \rangle}{\langle x \rangle^2} - 2\frac{\langle x\, y \rangle}{\langle x \rangle \langle y \rangle} + \frac{\langle y^2 \rangle}{\langle y \rangle^2}$$

- But then the result we get is model dependent. In one model the fluctuations can be dynamical and in another not (or have different sign!).

# Conclusions

- Dynamical fluctuations of number variables
  - Measures non-trivial part of pair probabilities
  - Limited by conservation e.g. charge and strangeness
  - Sum rule is needed to understand $\nu_{dyn}$ for sums
- Dynamical fluctuations of continuous variables
  - Mathematically easy to define and interpret
  - Determine of the statistical σ is not straight forward and could introduce model dependence
- Could provide 2nd order discrimination for models that describe the global correlations

# Backup slides

# $\Sigma p_T$ vs N dynamical fluctuations

- Interest to understand the rise of $\langle p_T \rangle$ with multiplicity in pp → try to use $v_{dyn}$

$$\nu = \frac{\langle m^2 \rangle}{\langle m \rangle^2} - 2\frac{\langle m \sum p_T \rangle}{\langle m \rangle \langle \sum p_T \rangle} + \frac{\langle (\sum p_T)^2 \rangle}{\langle \sum p_T \rangle^2}$$

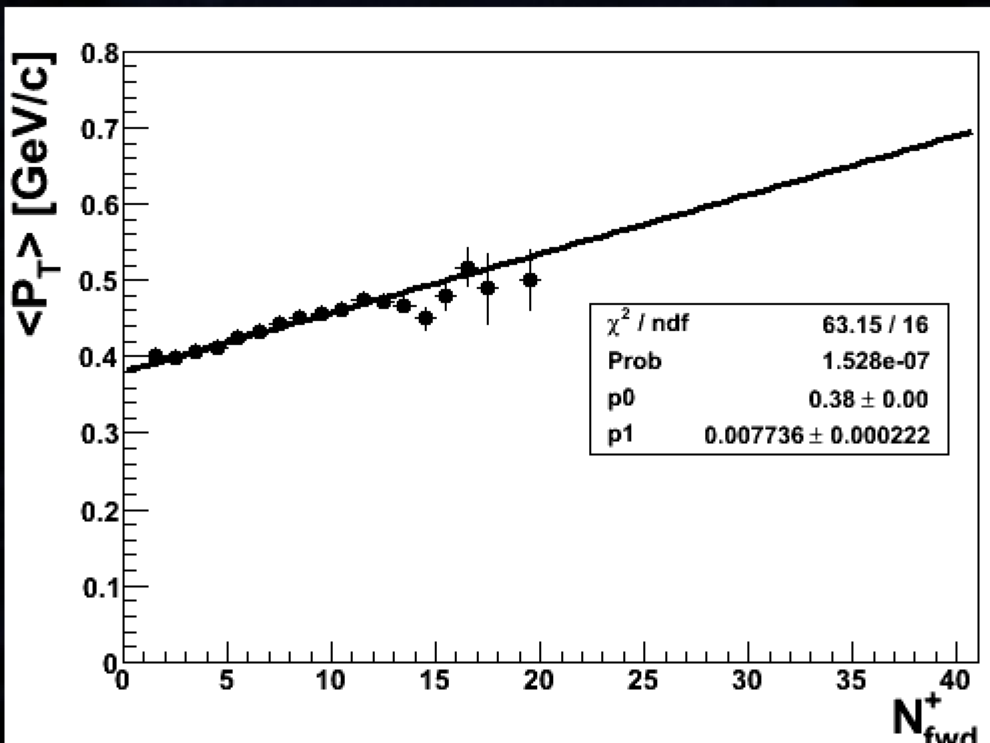
- We need $\Sigma p_T$ vs N and not $\langle p_T \rangle$ vs N for v to be able to remove the trivial correlations
- The statistical fluctuations of m and pT are:

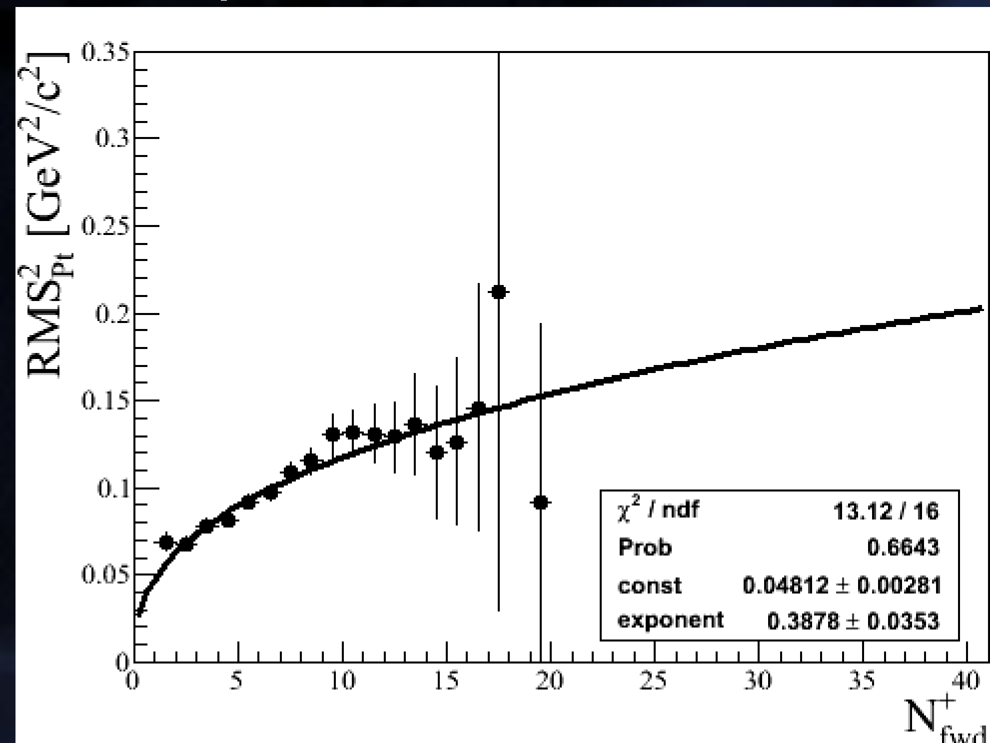
$$\sigma_m^2 = m \qquad \sigma_{\sum p_T}^2 = \langle p_T \rangle^2 n + \sigma_{p_T}^2 n$$

- So that $v_{dyn}$ becomes:

$$\nu = \frac{\langle m(m-1) \rangle}{\langle m \rangle^2} - 2\frac{\langle m \sum p_T \rangle}{\langle m \rangle \langle \sum p_T \rangle} + \frac{\langle (\sum p_T)^2 \rangle - \langle \sigma_{\sum p_T}^2 \rangle}{\langle \sum p_T \rangle^2}$$

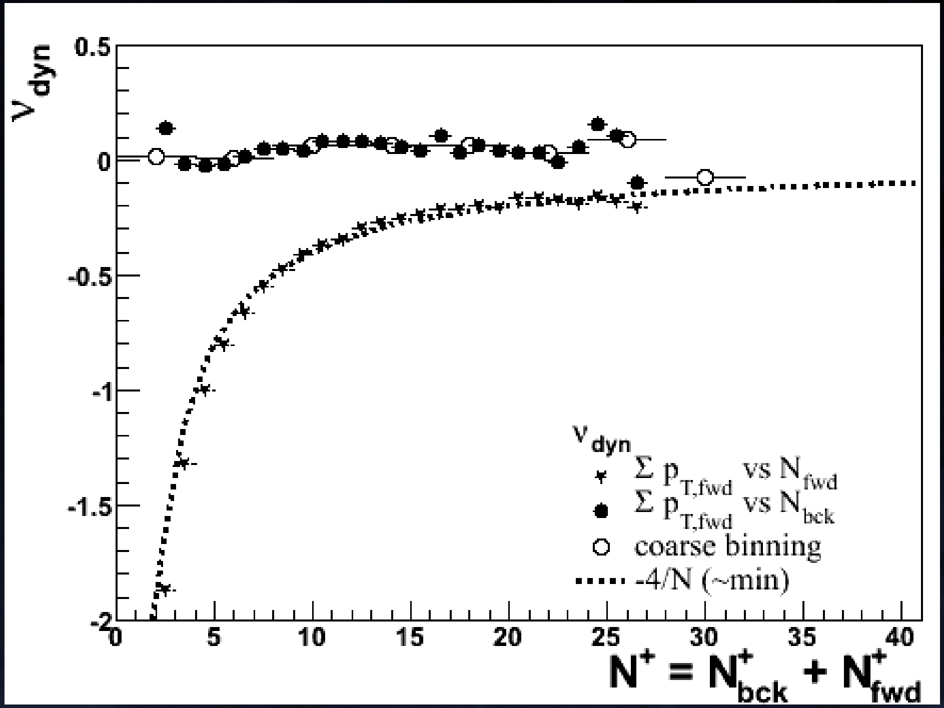
# Extracting $\sigma^2_{\Sigma pt}(N^+_{fwd})$




- **PYTHIA √s = 200 GeV,** fwd: $0 < \eta < 2$, bck: $-2 < \eta < 0$
- No $p_T$ cut and only positive particles to avoid charge conservation effects

# $v_{dyn}$ ($N^+_{fwd}$, Σpt(+,fwd)) and $v_{dyn}$ ($N^+_{bck}$, Σpt(+,fwd))




**PYTHIA √s = 200 GeV,**

fwd: 0 < η < 2,
bck: -2 < η < 0

- As expected we find almost minimal negative fluctuations for $v_{dyn}$ ($N^+_{fwd}$, Σpt(+,fwd)) (*stars*) – this is due to auto correlations!
- For $v_{dyn}$ ($N^+_{bck}$, Σpt(+,fwd)) (*circles*) we find no dynamic fluctuations!
- This observable could perhaps 2nd order discriminate between models: string recombination, dipoles, flow...

# Interpretation of dynamical fluctuations measured by $\nu_{dyn}$

- The expression:

$$\nu_{dyn} = \frac{1}{\langle M \rangle} \left( \frac{\varepsilon}{p^2 (1-p)^2} \right)$$

- Is dynamical in the sense that it measures the part of the pair-probabilities that deviates from that expected from the single particle probabilities
- $\nu_{dyn} < 0$: m and n shows smaller than statistical fluctuations, e.g., conservation.
- $\nu_{dyn} > 0$: m and n shows larger than statistical fluctuations, e.g., phase transition.

# Why study the 3 terms together

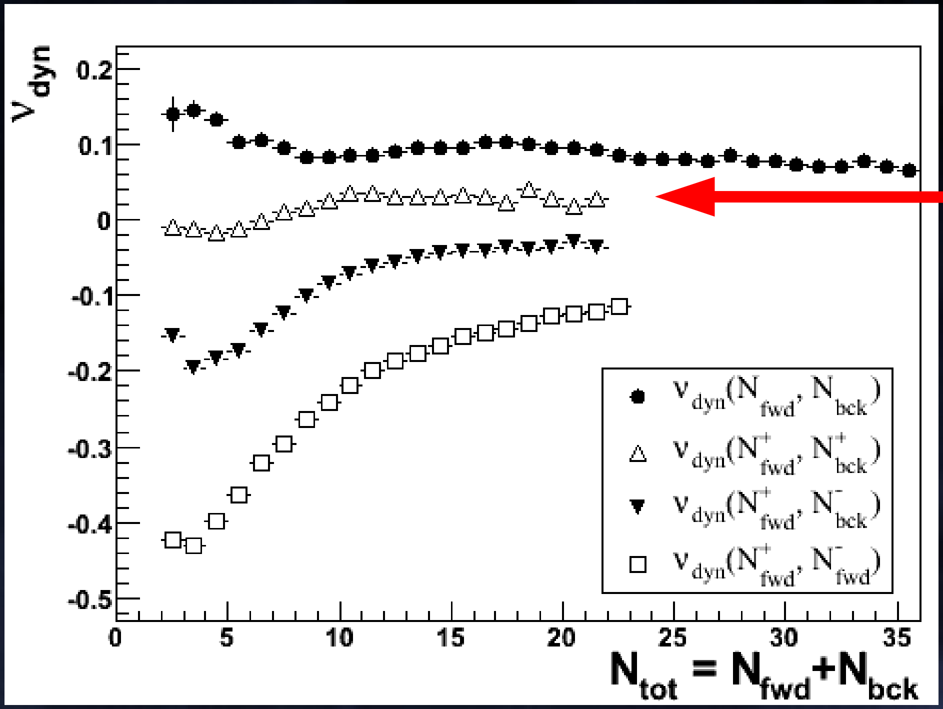
$$\nu_{dyn} = \frac{\langle m(m-1)\rangle}{\langle m\rangle^2} - 2\frac{\langle m n\rangle}{\langle m\rangle\langle n\rangle} + \frac{\langle n(n-1)\rangle}{\langle n\rangle^2}$$

- Each of the terms are sensitive to "centrality variations", i.e., for a sum of Poisson distributions we are sensitive to the global variation:

$$\frac{\langle m(m-1)\rangle}{\langle m\rangle^2} = \frac{\langle G(G-1)\rangle}{\langle G\rangle^2}$$

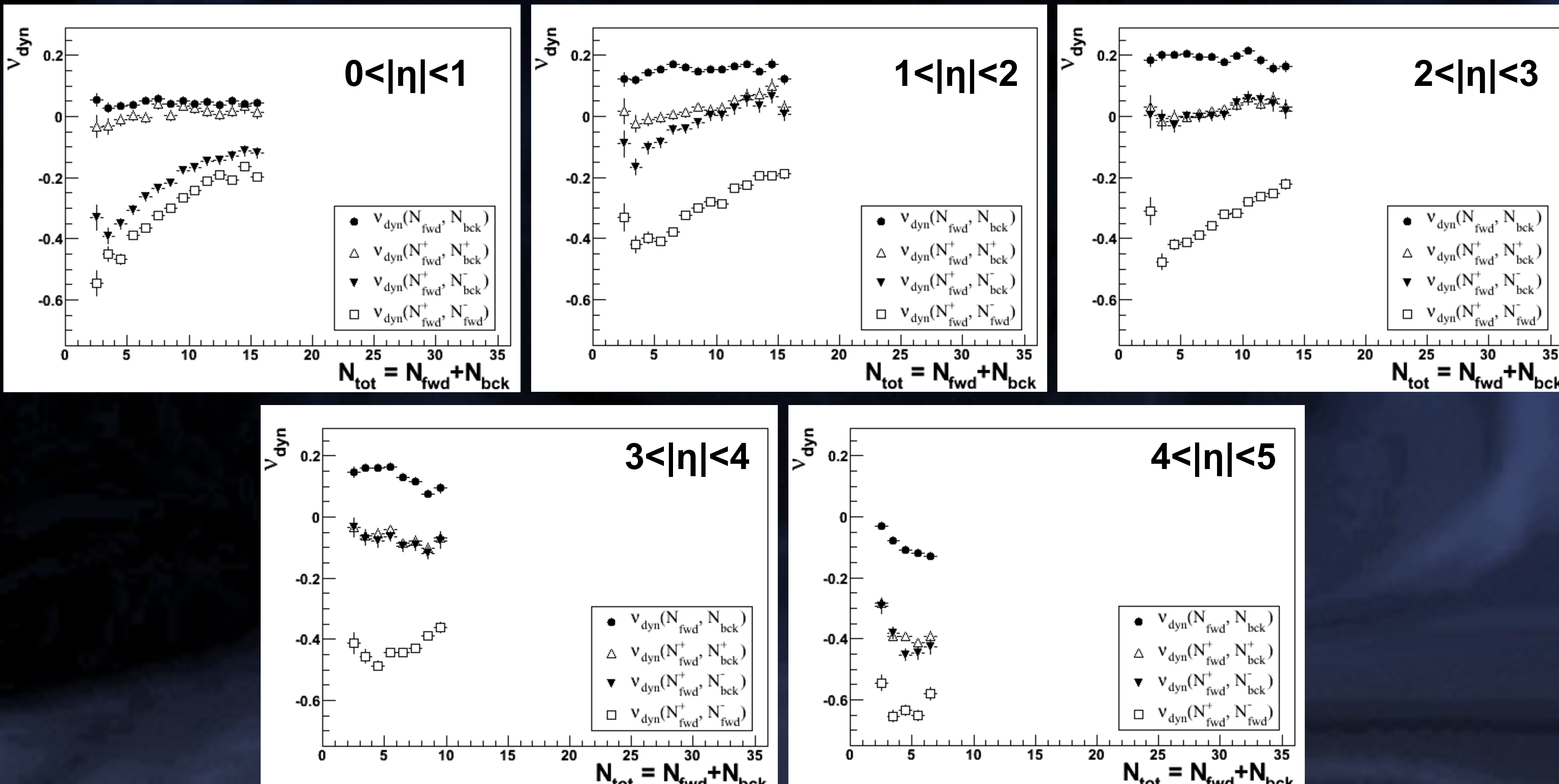
- This global variation is the same for all 3 terms
  - → Adding the 3 terms this depence falls out!

# $v_{dyn}$ ($N_{fwd}$, $N_{bck}$) vs $N_{fwd}$ +$N_{bck}$




- I want to use that $v_{dyn}$ ($N^+_{fwd}$ ,$N^+_{bck}$ )~0 to make sure that when I construct $v_{dyn}$ ($N^+_{bck,}$ Σpt(+,fwd)) I do not have any dynamical fluctuations at the particle level!
  - Otherwise the fluctuations could originate from particle number fluctuations

# Pseudo rapidity dependence




- I believe that the fluctuations in bin 4 <|η|<5 are dominated by the net-protons