Quark matter and meson properties in a nonlocal SU(3) chiral model at finite *T*

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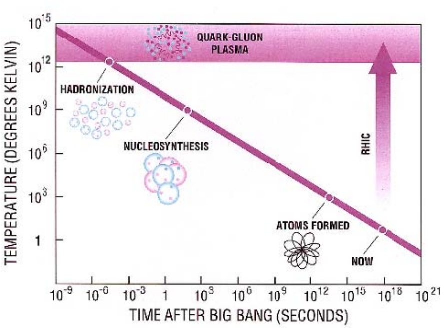
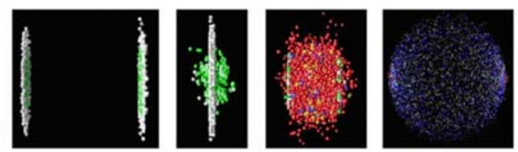
Motivation

The understanding of the behaviour of strongly interacting matter at finite temperature and/or density is a subject of fundamental interest

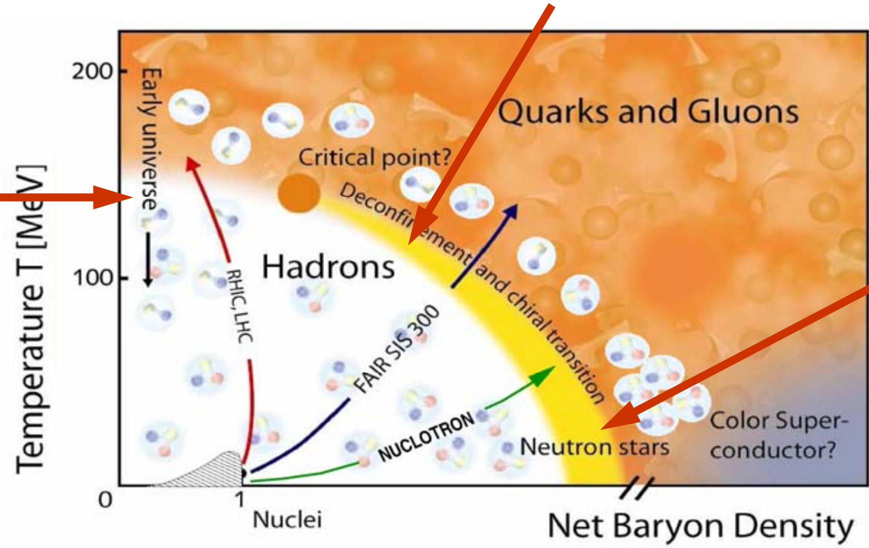
Several important applications :

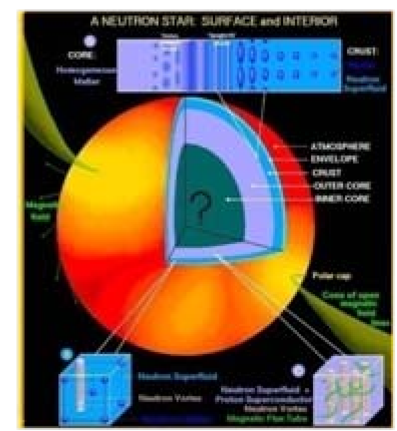
- Cosmology (early Universe)
- Astrophysics (neutron stars)
- RHIC physics





QCD
Phase Diagram





Essential problem: dealing with strong interactions in nonperturbative regimes

Main theoretical approaches:

- Lattice QCD techniques (difficult to implement for nonzero chemical potentials)
- Effective models – Effective quark couplings satisfying QCD symmetry properties

Nambu–Jona-Lasinio (NJL) model: local scalar and pseudoscalar four-fermion couplings + regularization prescription (ultraviolet cutoff)

NJL (Euclidean) action $$S_E = \int d^4x \left\{ \bar\psi\,(-i\partial\!\!\!/ + m_c\,\mathbb{1})\,\psi - \frac{G}{2}\left[(\bar\psi\,\psi)^2 \;+\; (\bar\psi\,i\,\gamma_5\,\vec\tau\,\psi)^2\right]\right\}$$

A step towards a more realistic modeling of QCD:

Extension to NJL–like theories that include nonlocal quark interactions

Bowler, Birse, NPA (95); Blaschke et al., NPA (95), Ripka (97); Plant, Birse, NPA (98)

Natural in the context of many approaches to low-energy quark dynamics (instanton liquid model, Schwinger-Dyson resummation)

✓ No sharp momentum cut-offs $\rightarrow$ relatively low dependence on model parameters

✓ Consistency with Lattice QCD

Nonlocal chiral quark model with $SU(3)_f$ symmetry

Euclidean action:

$$S_E = \int d^4x \Big\{ \bar\psi(x)\left(-i\partial\!\!\!/ + \hat m_c\right)\psi(x) - \frac{G}{2}\left[j_a^S(x) j_a^S(x) + j_a^P(x) j_a^P(x)\right] - \frac{H}{4} A_{abc}\left[j_a^S(x)\, j_b^S(x)\, j_c^S(x) - 3\, j_a^S(x)\, j_b^P(x)\, j_c^P(x)\right]\Big\}$$

- Three active flavors, isospin symmetry
- Nonlocal four fermion coupling + six-fermion ‘t Hooft interaction

Here $$A_{abc} = \frac{1}{3!}\,\epsilon_{ijk}\,\epsilon_{mnl}\,(\lambda_a)_{im}\,(\lambda_b)_{jn}\,(\lambda_c)_{kl}$$ $a = 0, 1, \ldots 8$

m_c : u, d, s current quark mass matrix ($m_u = m_d$); G, H : free model parameters

$j_a(x)$: nonlocal quark-antiquark currents (based on OGE interactions)

$$\left\{ \begin{array}{c} j_a^S(x) \\ j_a^P(x) \end{array} \right\} = \int d^4z\; g(z)\; \bar\psi(x+\frac{z}{2})\; \left\{ \begin{array}{c} \mathbb{1} \\ i\gamma_5 \end{array} \right\} \lambda_a\; \psi(x-\frac{z}{2})$$

$g(z)$: nonlocal, well behaved covariant form factor

Further steps:

- Hubbard-Stratonovich transformation: standard bosonization of the fermion theory. Introduction of bosonic fields σ_a and π_a

- Mean field approximation (MFA) : expansion in powers of meson fluctuations

$$\sigma_a(x) = \bar{\sigma}_a + \delta\sigma(x) \ , \quad \bar{\sigma}_a \neq 0 \ \text{ for } \ a = 0, 3, 8$$
$$\pi_a(x) = \delta\pi_a(x)$$

Scalar and pseudoscalar mesons a_0 , κ , σ_8 , σ_0 , π , K , η_8 , η_0

- Minimization of S_E at the mean field level ⟹ coupled gap equations that allow to determine $\bar{\sigma}_a$

Momentum-dependent effective quark masses $\Sigma_q(p) = m_q + g(p^2)\ \bar{\sigma}_q$, $q = u , d , s$

Quark-antiquark condensates $\langle \bar{u}u \rangle = \langle \bar{d}d \rangle \ , \quad \langle \bar{s}s \rangle$

General, DGD, Scoccola, PLB(01); DGD, Scoccola, PRD(02); DGD, Grunfeld, Scoccola, PRD (06)

Beyond the MFA : low energy meson phenomenology

Quadratic Euclidean action (e.g. for the pseudoscalar sector)

$$S_E^{quad}\Big|_P = \frac{1}{2}\int \frac{d^4p}{(2\pi)^4}\Big\{G_\pi(p^2)\left[\pi^0(p)\ \pi^0(-p) + 2\,\pi^+(p)\ \pi^-(-p)\right] + G_K(p^2)\left[2\,K^0(p)\ \bar{K}^0(-p) + 2\,K^+(p)\ K^-(-p)\right] + G_\eta(p^2)\ \eta(p)\ \eta(-p) + G_{\eta'}(p^2)\ \eta'(p)\ \eta'(-p)\Big\}$$

Meson masses from $G_P(-m_P^2) = 0$

Meson decay constants from $\langle\, 0 \,|\, A_\mu^a(0) \,|\, \pi^b(p)\, \rangle \;=\; i\ f^{ab}\ p_\mu$

– nontrivial gauge transformation due to nonlocality –

Consistency with low energy ChPT results :

- ✓ GT relation
- ✓ GOR relation
- ✓ FH theorem
- ✓ $\pi^0\gamma\gamma$ coupling

Numerics

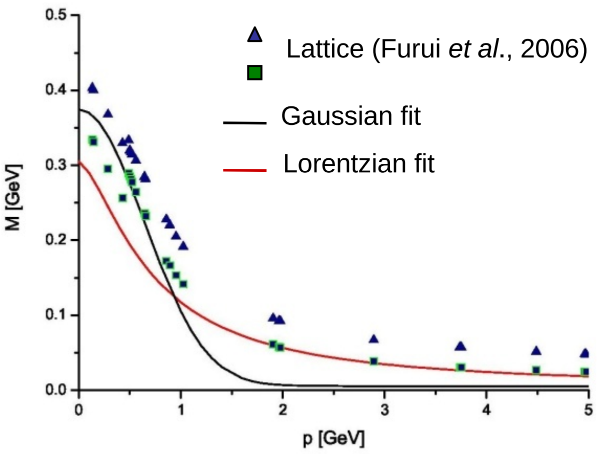
Model inputs :

- Parameters : G , H , m_u , m_s
- Form factor & scale

Gaussian $g(p^2) = \exp(-p^2/\Lambda^2)$

n-Lorentzian $g(p^2) = \dfrac{1}{1+(p^2/\Lambda^2)^n}$

Lattice QCD :



(also presence of a "wave function" form factor)

$$\frac{Z(p)}{\not p + \Sigma_q(p)}$$

Fit of model parameters m_u , m_s , G , H and Λ so as to reproduce empirical values of meson masses and decay constants

Numerical results : meson masses, decay constants and mixing angles

Our input parameters: m_u , m_π , m_K , $m_{\eta'}$, f_π

(+ Gaussian form factor)

$\eta_8 - \eta_0$ sector: two mixing angles θ_η , $\theta_{\eta'}$

Four decay constants f_η^a , $f_{\eta'}^a$, $a = 0, 8$

$$\begin{pmatrix} f_\eta^8 & f_\eta^0 \\ f_{\eta'}^8 & f_{\eta'}^0 \end{pmatrix} = \begin{pmatrix} f_8 \cos\theta_8 & -f_0 \sin\theta_0 \\ f_8 \sin\theta_8 & f_0 \cos\theta_0 \end{pmatrix}$$

NLO ChPT + 1/Nc :

$\theta_8 = -20.5°\,,\ \theta_0 = -4°$ ✓

Current algebra:

$m_s/m = (2m_K^2 - m_\pi^2)/m_\pi^2 \simeq 25$ ✓

	Our Model	Empirical & Phenomenological
$\bar{m}$ [MeV]	5*	(3.4 - 7.4)
m_s [MeV]	119	(108 - 209)
m_π [MeV]	139*	139
m_K [MeV]	495*	495
m_η [MeV]	523	547
$m_{\eta'}$ [MeV]	958*	958
m_{a_0} [MeV]	900	980
m_κ [MeV]	1380	1425
m_σ [MeV]	566	400-1200
m_{f_0} [MeV]	1280	980
θ_η	$-2.3°$	
$\theta_{\eta'}$	$-40.3°$	
θ_8	$-24°$	$-(22°$ - $19°)$
θ_0	$-7.7°$	$-(10°$ - $0°)$
f_π [MeV]	92.4*	92.4
f_K/f_π	1.17	1.22
f_η^8/f_π	1.14	(1.17-1.22)
f_η^0/f_π	0.16	(0.11-0.37)
$f_{\eta'}^8/f_\pi$	-0.49	-(0.42-0.46)
$f_{\eta'}^0/f_\pi$	1.16	(0.98-1.16)

(*) Input values

⟹ Adequate overall description of meson phenomenology

Scarpettini, DGD, Scoccola, PRD (04); Contrera, DGD, Scoccola, PRD (10)

Extension to finite T + description of deconfinement transition

Finite T partition function obtained through the standard Matsubara formalism

Confinement: quarks coupled to a background color field $\phi = i A_0 = i \frac{g}{2} \delta_{\mu 0} G_a^\mu \lambda^a$

$SU(3)_C$ gauge fields

Traced Polyakov loop $\Phi = \frac{1}{3} \mathrm{Tr} \exp(i\beta\phi)$

Taken as order parameter of deconfinement transition, related with Z(3) center symmetry of color SU(3):

Confinement $\Phi = 0$, deconfinement $\Phi = 1$

Polyakov gauge : ϕ diagonal , $\phi = \phi^3 \lambda^3 + \phi^8 \lambda^8$

Gauge field potential $\mathcal{U}(\Phi, T) = \left[-\frac{1}{2} a(T) \Phi^2 + b(T) \ln(1 - 6\Phi^2 + 8\Phi^3 - 3\Phi^4) \right] T^4$

Group theory constraints satisfied – $a(T)$, $b(T)$ fitted from lattice QCD results

From QCD symmetry properties $\phi^8 = 0$, $\Phi = [2 \cos(\phi^3/T) + 1] / 3$

$T = 0$: $\Phi = 0$, color field decouples

Fukushima, PLB(04); Megias, Ruiz Arriola, Salcedo, PRD(06); Roessner, Ratti, Weise, PRD(07)

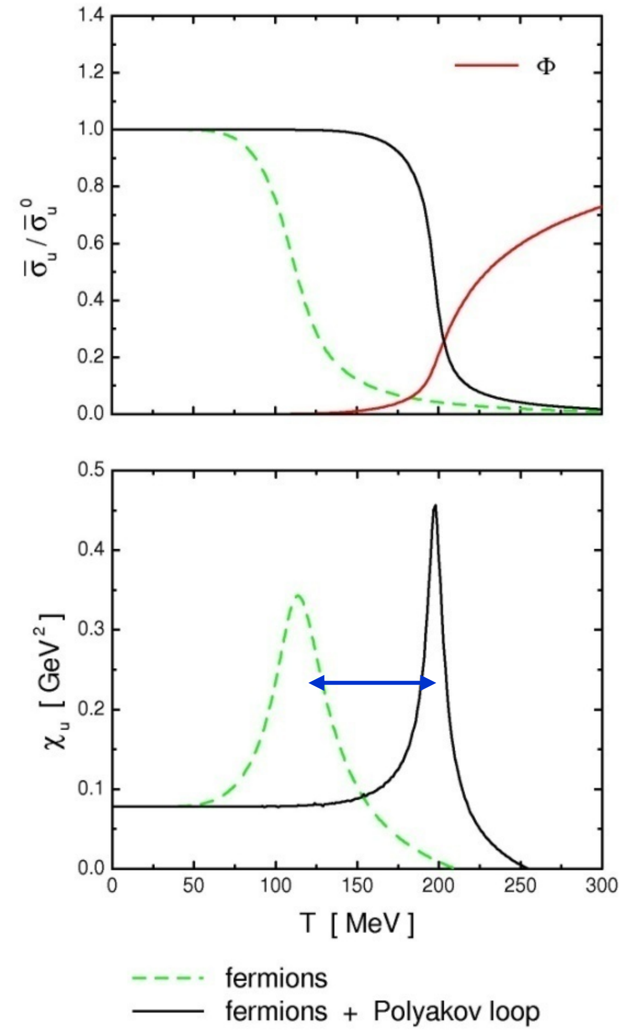
MFA : Grand canonical thermodynamical potential given by

$$\Omega \;=\; \Omega^{\mathrm{MFA}} + \mathcal{U}(\Phi, T) \;\longrightarrow\; p_4 \longrightarrow p_4 - \phi \quad \text{(coupling to fermions)}$$

Now $\Sigma_f(p_{nc}) \;=\; m_f + \bar{\sigma}_f\, g(p_{nc}) \qquad p_{nc} = (\vec{p}\;, \omega_n - \phi_c)$

MF values from $\dfrac{\partial \Omega^{\mathrm{MFA}}}{\partial \bar{\sigma}_u\;,\; \partial \bar{\sigma}_s\;,\; \partial \phi^3} \;=\; 0$

Chiral restoration & deconfinement : $\bar{\sigma}_u$ and Φ as functions of the temperature



Main qualitative features :

- SU(2) chiral transition temperature increased due to the presence of the background color field
- Deconfinement transition (smoother)
- Both chiral and deconfinement transition occurring at approximately same temperature

Blaschke, Buballa, Radzhabov, Volkov, YF (08)
Contrera, DGD, Scoccola, PLB (08)

Beyond mean field: quadratic fluctuations at finite T (pseudoscalar sector)

$$S_E^{quad}\Big|_P = \frac{1}{2}\int_{q,m} \Big\{ G_\pi(\vec{q}^{\,2},\nu_m^2)\left[\pi^0(q_m)\,\pi^0(-q_m) + 2\,\pi^+(q_m)\,\pi^-(-q_m)\right]$$
$$+ G_K(\vec{q}^{\,2},\nu_m^2)\left[2\,K^0(q_m)\,\bar{K}^0(-q_m) + 2\,K^+(q_m)\,K^-(-q_m)\right]$$
$$+ G_\eta(\vec{q}^{\,2},\nu_m^2)\,\eta(q_m)\,\eta(-q_m) + G_{\eta'}(\vec{q}^{\,2},\nu_m^2)\,\eta'(q_m)\,\eta'(-q_m)\Big\}$$

Here $q_m = (\vec{q}\,,\nu_m)$, while $\nu_m = 2m\pi T$ are bosonic Matsubara frequencies

Functions G given by loop integrals, e.g.

$$G_\pi(\vec{q}^{\,2},\nu_m^2) = \left[(G + \frac{H}{2}\bar{S}_s)^{-1} + C_{uu}^-(\vec{q}^{\,2},\nu_m^2)\right]$$

where

$$C_{ij}^\pm(\vec{q}^{\,2},\nu_m^2) = -8\sum_c \int_{p,n} g(p_{nc} + q_m/2)\,\frac{p_{nc}^2 + p_{nc}\cdot q_m \mp \Sigma_i(p_{nc}+q_m)\Sigma_j(p_{nc})}{D_i(p_{nc}+q_m)D_j(p_{nc})}$$

$$D_j(s) = s^2 + \Sigma_j^2(s)$$

Meson masses and decay constants given by

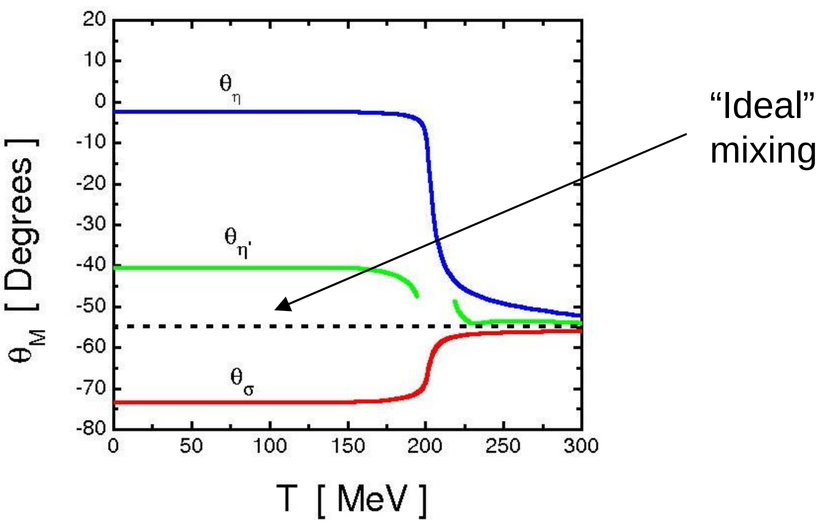
$$G_M(-m_M^2,0) = 0 \qquad\qquad \langle\, 0|A_\mu^a(0)|M_b(p)\,\rangle = i\; f_{ab}\; p_\mu$$

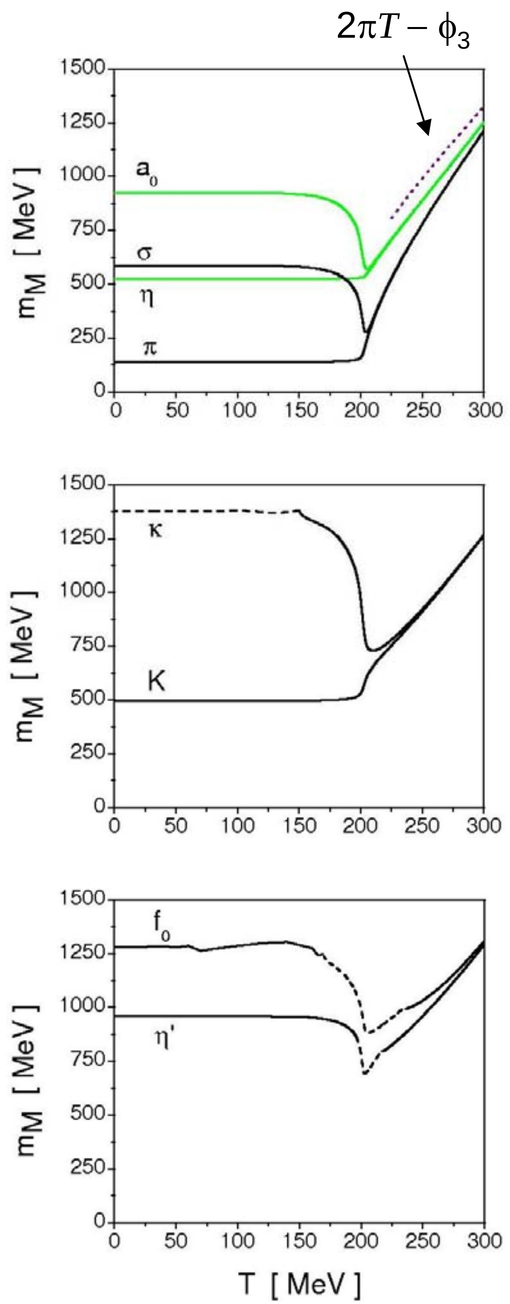
(“screening” masses , m = 0 mode)

Finite T : meson masses and mixing angles

Main qualitative features :

- Masses dominated by thermal energy at large T
- Mass degeneracy of scalar – pseudoscalar partners
- Matching at $T = T_c$ for nonstrange mesons
- “Ideal” mixing at large T





Contrera, DGD, Scoccola, PRD (10)

Summary

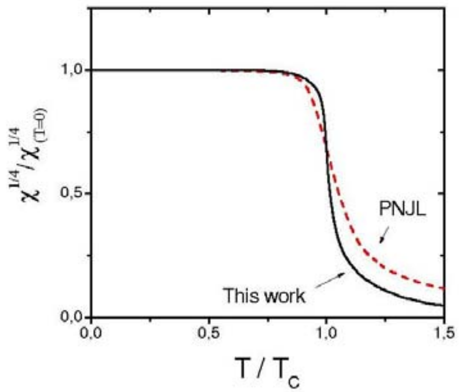
We have studied meson properties at finite temperature within quark models that include effective covariant nonlocal interactions. These models can be viewed as an improvement of the NJL model towards a more realistic description of QCD

- Chiral relations at T = 0 properly satisfied
- Good description of low energy scalar and pseudoscalar meson phenomenology
- Coupling with the Polyakov loop increases T_c up to 200 MeV. Chiral restoration and deconfinement transitions occurr in the same temperature range.
- Behavior of meson masses with temperature: scalar and pseudoscalar chiral partners become degenerate right after the chiral restoration. Ideal mixing and vanishing chiral susceptibility at large T.

To be done

- Form factors taken from Lattice QCD – effective mass & wave function momentum dependence (already done for $SU(2)_f$ model)
- Effect of mesonic correlations on T_c
- Extension of Polyakov loop nonlocal model for finite chemical potential

Finite T behaviour of chiral susceptibility



Finite T behaviour of pseudoscalar meson decay constants