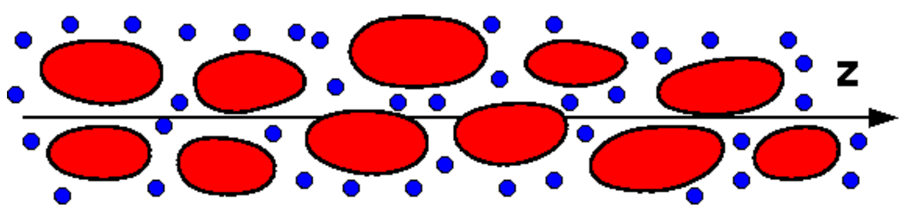
Critical Point and Onset of Deconfinement (CPOD)
23 - 29 August 2010 at Joint Institute for Nuclear Research

Federal Ministry of Education and Research

Spinodal phase decomposition with dissipative fluid dynamics

Jørgen Randrup, LBNL



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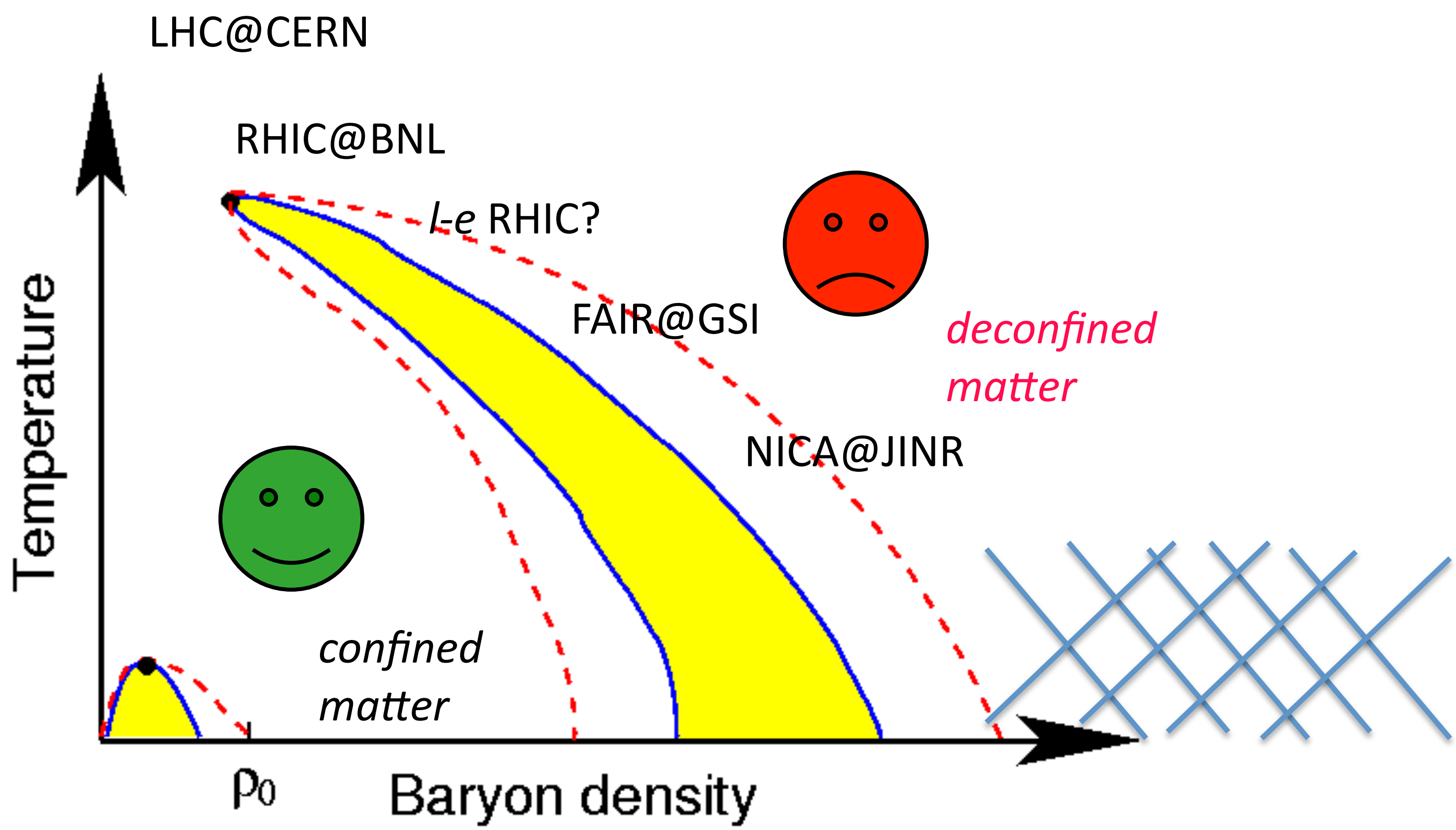
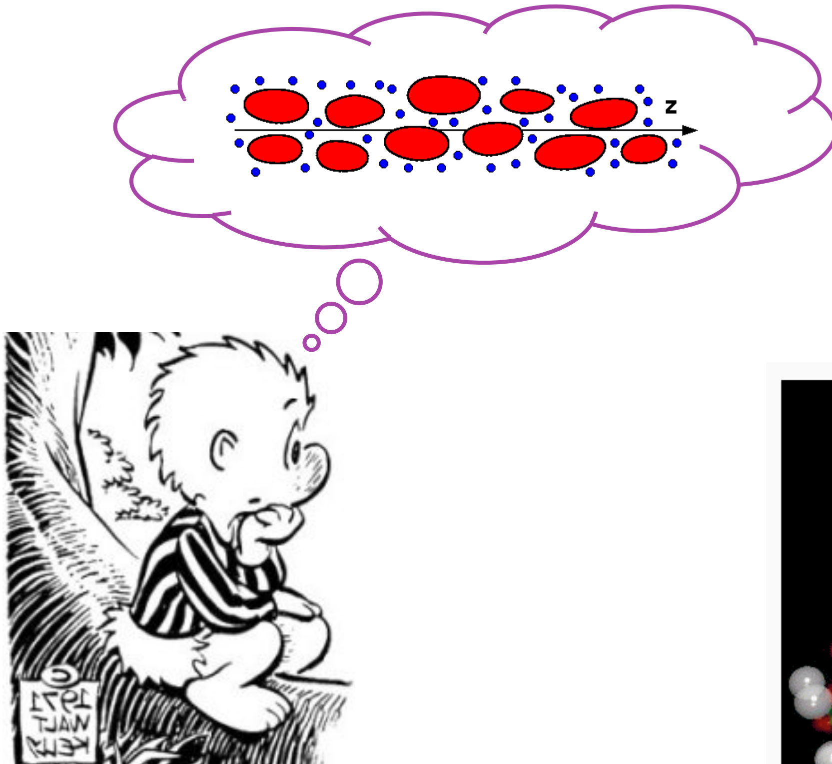
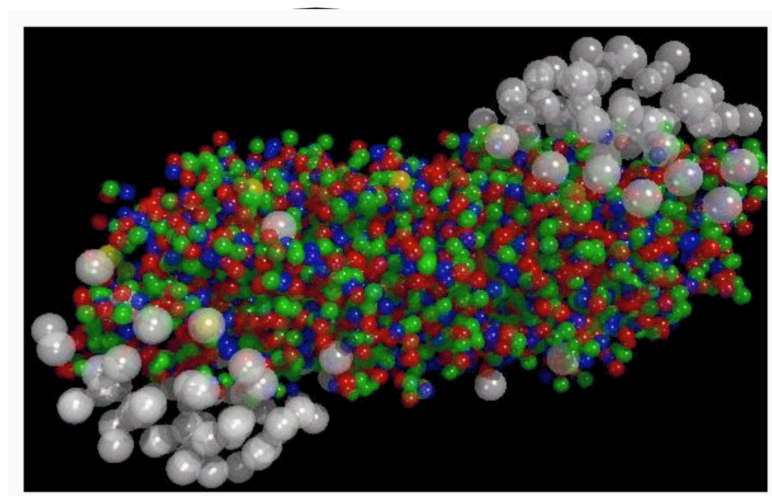
Schematic and simplified

phase diagram of strongly interacting matter



(Under what circumstances) can spinodal phase separation occur in nuclear collisions?





Spinodal phase decomposition with dissipative fluid dynamics

Nuclear spinodal fragmentation

Equation of state

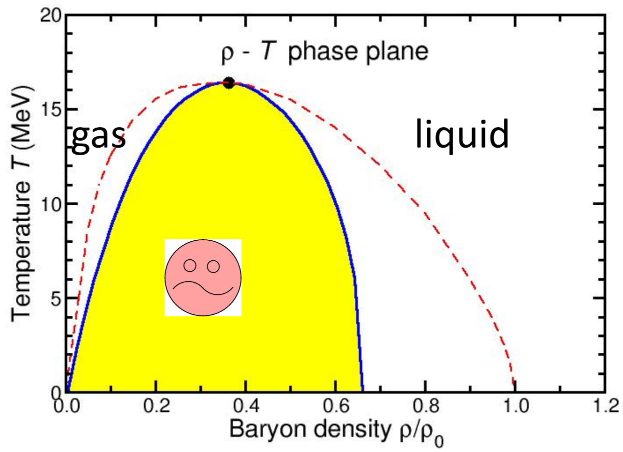
Inclusion of finite range

Dissipative fluid dynamics => dispersion relations

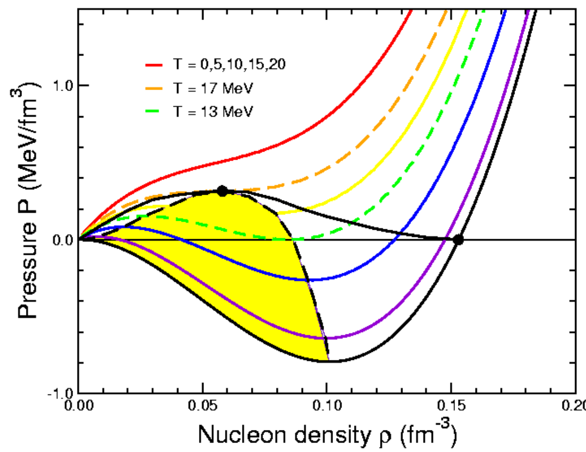
Dynamical phase trajectories => spinodal amplification

Nuclear liquid-gas phase transition

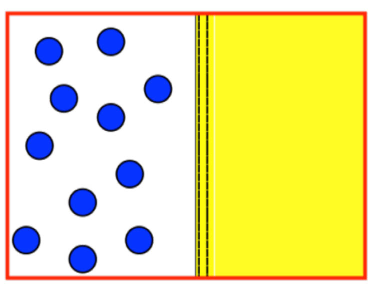
(ρ, T) phase diagram



Equation of state: $p_T(\rho)$



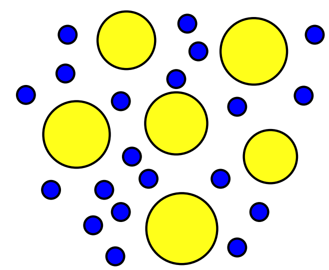
Phase coexistence



gas liquid

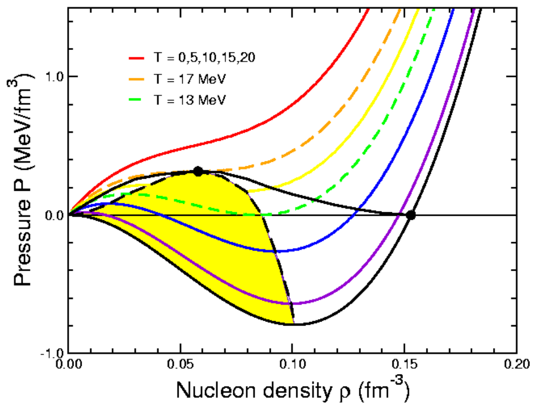
$\neq$

Phase mixture

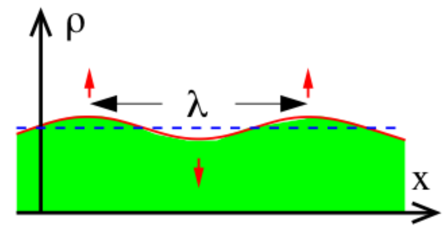


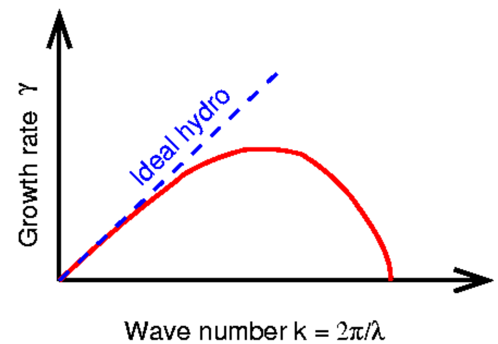
nucleons & fragments

Nuclear spinodal fragmentation



Density undulations
are amplified in
the spinodal region:



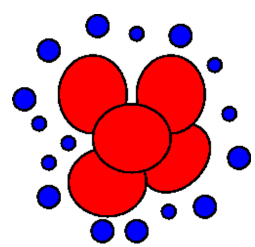


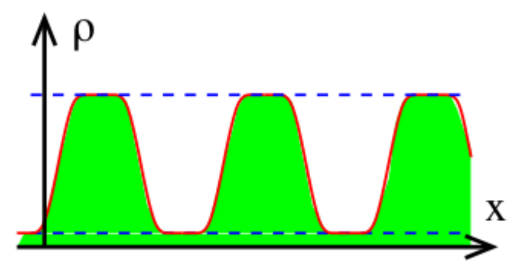
Long-wavelength distortions grow slowly
(it takes time to relocate the matter)

Short-wavelength distortions grow slowly
(they are hardly felt due to finite range)

There is an *optimal length scale*
that grows faster than all others:

Ph Chomaz, M Colonna, J Randrup:
Nuclear Spinodal Fragmentation,
Physics Reports 389 (2004) 263





=> Equal-size fragments!

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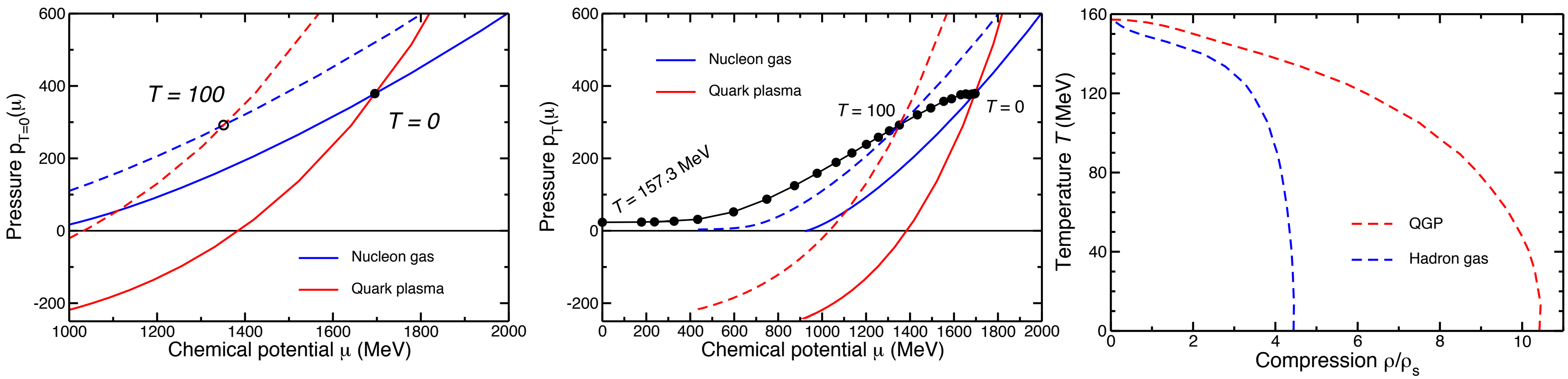
Idealized equations of state: HG -> QGP



Hadron Gas:

$$p^H = p_\pi + p_N + p_{\bar{N}} + p_w$$

$$p_\pi(T) = -g_\pi \int_{m_\pi}^{\infty} \frac{p\epsilon d\epsilon}{2\pi^2} \ln[1 - \mathrm{e}^{-\beta\epsilon}]$$

$$p_N(T,\mu_0) = g_N \int_{m_N}^{\infty} \frac{p\epsilon d\epsilon}{2\pi^2} \ln[1 + \mathrm{e}^{-\beta(\epsilon-\mu_0)}]$$

$$p_{\bar{N}}(T,\mu_0) = g_N \int_{m_N}^{\infty} \frac{p\epsilon d\epsilon}{2\pi^2} \ln[1 + \mathrm{e}^{-\beta(\epsilon+\mu_0)}]$$

$$p_w(\rho) = \rho\partial_\rho w(\rho) - w(\rho)$$

$$w(\rho) = \left[-A\left(\frac{\rho}{\rho_s}\right)^\alpha + B\left(\frac{\rho}{\rho_s}\right)^\beta\right]\rho$$

Quark-Gluon Plasma:

$$p^Q = p_g + p_q + p_{\bar{q}} - B$$

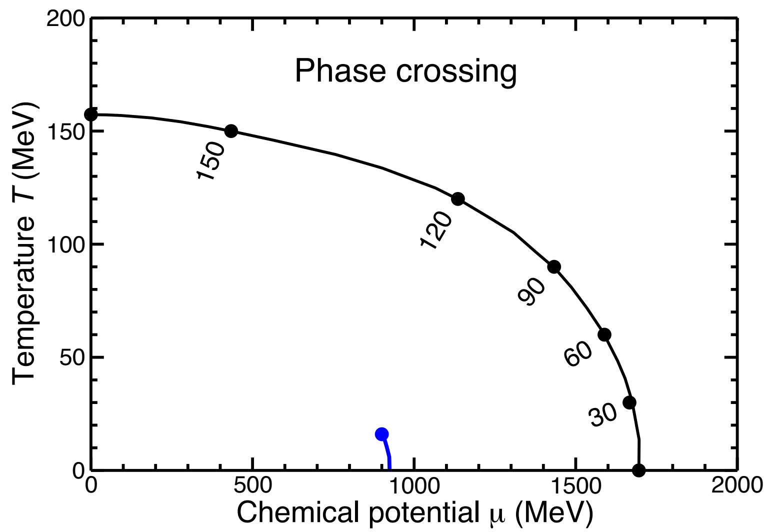
$$p_g = g_g \frac{\pi^2}{90} T^4$$

$$p_q + p_{\bar{q}} = g_q \left[\frac{7\pi^2}{360} T^4 + \frac{1}{12}\mu_q^2 T^2 + \frac{1}{24\pi^2}\mu_q^4\right]$$

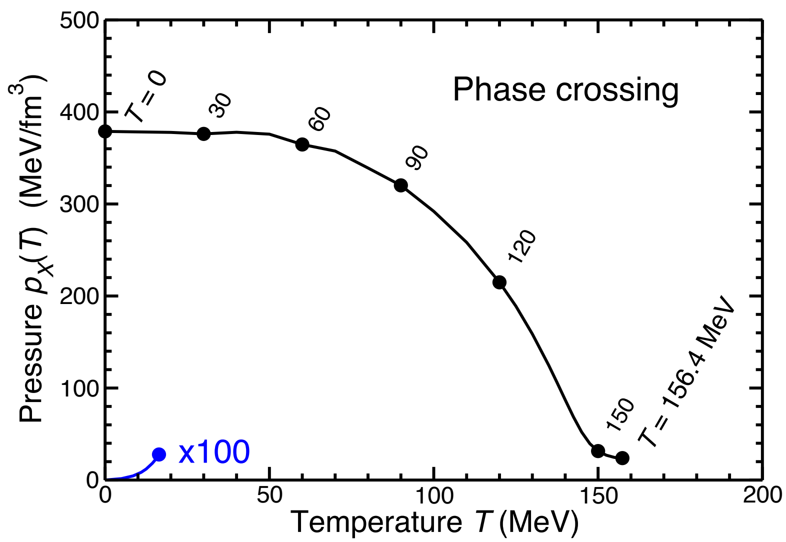
$$\mu = \mu_0 + \partial_\rho w = 3\mu_q$$

Phase coexistence: Comparison to LG

(μ,T) phase diagram



(T,p) phase diagram

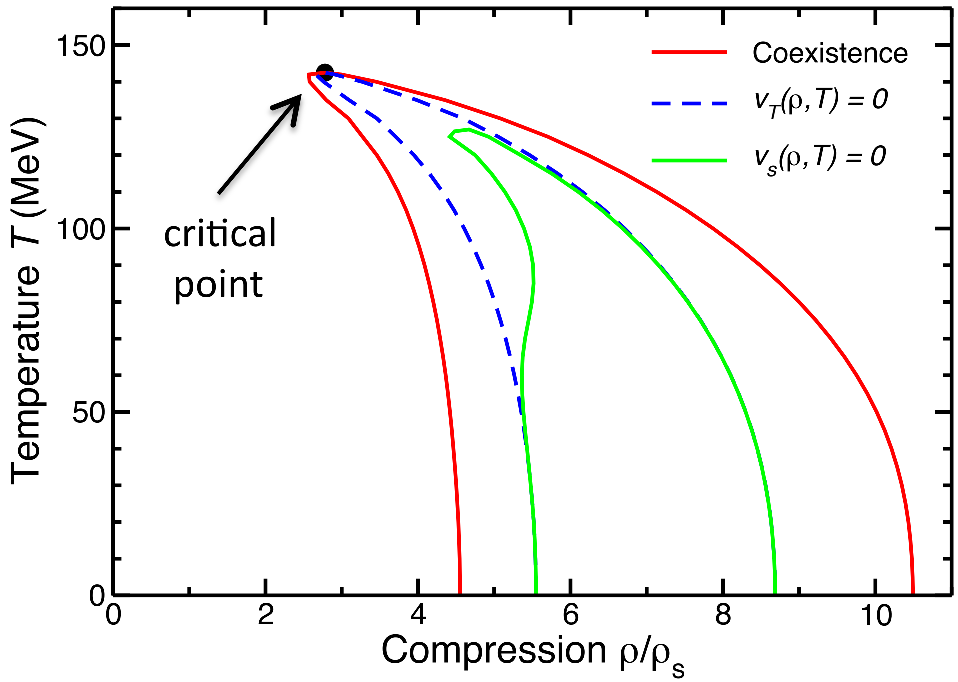


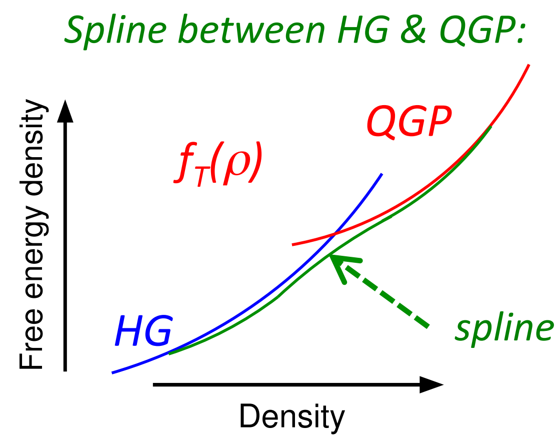
- qualitatively similar to LG

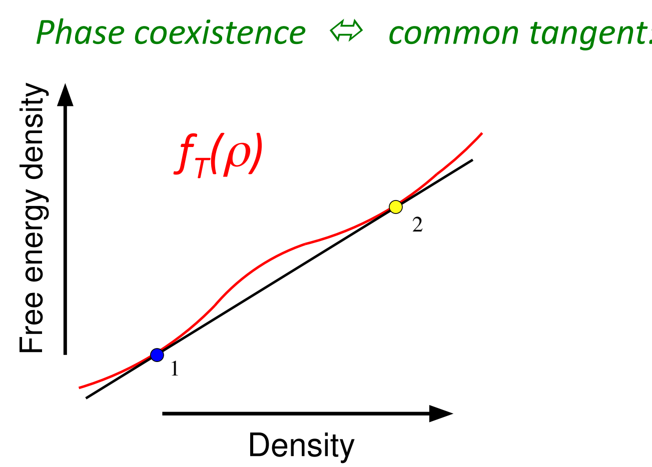
- qualitatively different from LG!

[Igor Iosilevsky]

Phase diagram (for uniform matter)







$v_T = 0$: isothermal spinodal

$v_s = 0$: isentropic spinodal

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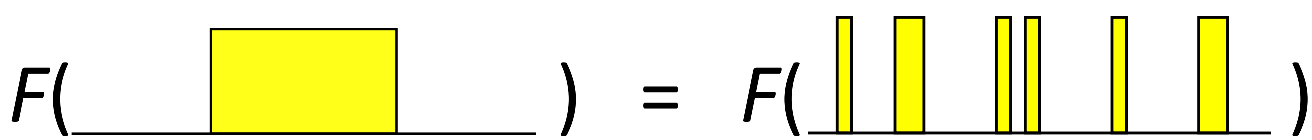
Equation of state: Finite range

Free energy density for uniform matter: $f_0(\rho, T)$

But we need to treat non-uniform systems: $\tilde{\rho}(\boldsymbol{r}),\ \tilde{T}(\boldsymbol{r})$

The local density approximation: $f[\tilde{\rho}(\cdot), \tilde{T}(\cdot)](\boldsymbol{r}) \doteq f_0(\tilde{\rho}(\boldsymbol{r}), \tilde{T}(\boldsymbol{r}))$

… implies:



No good! => Finite range *must* be taken into account

Non-uniform densit$\tilde{\rho}(\boldsymbol{r})$

gradient contribution $\quad \tilde{w}(\boldsymbol{r}) \equiv w_0(\tilde{\rho}(\boldsymbol{r})) + \frac{1}{2}C(\boldsymbol{\nabla}\tilde{\rho}(\boldsymbol{r}))^2$

local entropy density: $\quad \tilde{\sigma}(\boldsymbol{r}) \equiv \sigma(\tilde{\varepsilon}(\boldsymbol{r}), \tilde{\rho}(\boldsymbol{r}))$

$\Rightarrow$ total entropy: $\quad S[\tilde{\varepsilon}(\boldsymbol{r}), \tilde{\rho}(\boldsymbol{r})] \equiv \int \tilde{\sigma}(\boldsymbol{r}) d\boldsymbol{r}$

$$\Rightarrow \begin{cases} \tilde{\beta}(\boldsymbol{r}) \equiv \dfrac{\delta S}{\delta \tilde{\varepsilon}(\boldsymbol{r})} \Rightarrow \tilde{T}(\boldsymbol{r}) \\ \tilde{\alpha}(\boldsymbol{r}) \equiv \dfrac{\delta S}{\delta \tilde{\rho}(\boldsymbol{r})} \Rightarrow \tilde{\mu}(\boldsymbol{r}) \end{cases}$$

$\Rightarrow$ local pressure $\tilde{p}(\boldsymbol{r})$ & local enthalpy density $\tilde{h}(\boldsymbol{r})$ & ...

Free energy density: $\quad \tilde{f}_T(\boldsymbol{r}) = f_T(\tilde{\rho}(\boldsymbol{r})) + \frac{1}{2}C(\boldsymbol{\nabla}\tilde{\rho}(\boldsymbol{r}))^2$

J. Randrup, Phys. Rev. C 79, 054911 (2009)

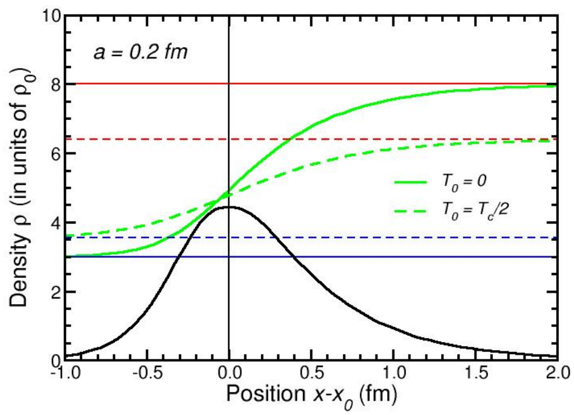
H. Heiselberg *et al.*, Phys. Rev. Lett. 70, 1355 (1993)

Interface equilibrium

Global equilibrium requires constant *T*, μ, *p*:

$$0 \doteq \delta S - \beta_0 \delta E - \alpha_0 \delta N = \int dx \left\{ [\tilde{\beta}(x) - \beta_0]\delta\tilde{\varepsilon}(x) + [\tilde{\alpha}(x) - \alpha_0]\delta\tilde{\rho}(x) \right\}$$

$$\Rightarrow \quad \tilde{\beta}(x) = \beta_0 \;\;\&\;\; \tilde{\alpha}(x) = \alpha_0 \quad \Rightarrow \quad \tilde{p}(x) = p_0$$

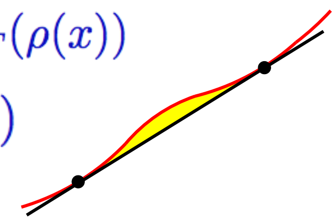


$$\tilde{f}_T(\boldsymbol{r}) \;=\; f_T(\tilde{\rho}(\boldsymbol{r})) + \tfrac{1}{2}C(\boldsymbol{\nabla}\tilde{\rho}(\boldsymbol{r}))^2 \qquad \Rightarrow$$

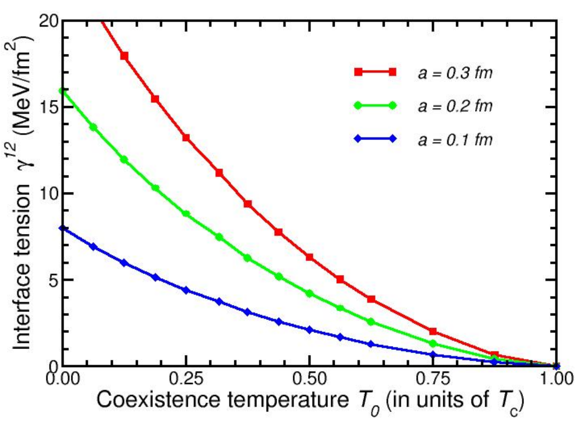
The interface density profile is determined by

$$C\partial_x^2\rho(x) \doteq \mu_T(\rho(x)) - \mu_0 = \partial_\rho \Delta f_T(\rho(x))$$

where $\quad \Delta f_T(\rho) \equiv f_T(\rho) - f_T^M(\rho)$



$$f_T^M(\rho) \;\equiv\; f_T(\rho_i) + \mu_0(\rho - \rho_i) \;\leq\; f_T(\rho)$$



The interface tension is given by

$$\boxed{\gamma_T^{12} = \int_{\rho_1}^{\rho_2} d\rho \, [2C\Delta f_T(\rho)]^{\frac{1}{2}}}$$

[*ρ(x) not needed!*]

J. Randrup, Phys. Rev. C 79, 054911 (2009)

Dynamical effect of the gradient term:
The local pressure is modified

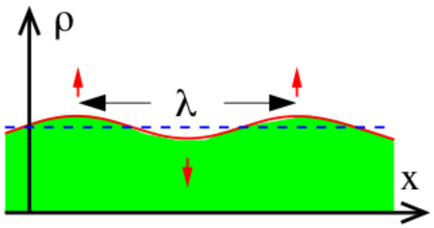
Small deviations from uniformity:

$$\tilde{p}(\boldsymbol{r}) \;\approx\; p_0(\tilde{\varepsilon}(\boldsymbol{r}), \tilde{\rho}(\boldsymbol{r})) - C\rho_0 \nabla^2 \tilde{\rho}(\boldsymbol{r})$$

Small harmonic density undulations:

$$\tilde{\rho}(\boldsymbol{r}, t) = \rho_0 + \delta\rho(x, t) \doteq \; \rho_0 + \rho_k \mathrm{e}^{ikx - i\omega t}$$

$$\boxed{p_k \;\longrightarrow\; p_k + C\rho_0 k^2 \rho_k}$$



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Transport model: Dissipative fluid dynamics

Energy-momentum tensor:

$$T^{00} \approx \varepsilon \quad \& \quad T^{0i} \approx (\varepsilon + p)v^i + q^i \quad \&$$

$$T^{ij} \approx \delta_{ij}p - \eta[\partial_i v^j + \partial_j v^i - \tfrac{2}{3}\delta_{ij}\partial^k v^k] - \zeta\delta_{ij}\partial^k v^k$$

A Muronga, PRC 76, 014909 (2007)

$|\rho_k| \ll \rho_0 \;\Rightarrow\; |v| \ll 1$

$$\boldsymbol{\nabla}\cdot\boldsymbol{T} \;\approx\; \boldsymbol{\nabla}p - \eta\Delta\boldsymbol{v} - [\tfrac{1}{3}\eta + \zeta]\boldsymbol{\nabla}(\boldsymbol{\nabla}\cdot\boldsymbol{v}) \;\asymp\; \partial_x p - [\tfrac{4}{3}\eta + \zeta]\partial_x^2 v$$

Eckart frame

Equations of motion:

$$\begin{cases} C: \;\; \partial_t\rho \;\doteq\; -\rho_0\boldsymbol{\nabla}\cdot\boldsymbol{v} \;\Rightarrow\; \omega\rho_k \doteq \rho_0 k v_k & \text{charge} \\ \boldsymbol{M}: \;\; h_0\partial_t\boldsymbol{v} \;\doteq\; -\boldsymbol{\nabla}[p - \zeta\boldsymbol{\nabla}\cdot\boldsymbol{v}] - \boldsymbol{\nabla}\cdot\boldsymbol{\pi} - \partial_t\boldsymbol{q} & \text{momentum} \\ E: \;\; \partial_t\varepsilon \;\doteq\; -h_0\boldsymbol{\nabla}\cdot\boldsymbol{v} - \boldsymbol{\nabla}\cdot\boldsymbol{q} & \text{energy} \end{cases}$$

Sound equation:

$$\partial_t E - \boldsymbol{\nabla}\cdot\boldsymbol{M}: \;\; h_0\partial_t^2\varepsilon \;\doteq\; \Delta[p - \zeta\boldsymbol{\nabla}\cdot\boldsymbol{v}] + \boldsymbol{\nabla}\cdot(\boldsymbol{\nabla}\cdot\boldsymbol{\pi})$$

$$\omega^2\varepsilon_k \;\doteq\; k^2 p_k - i[\tfrac{4}{3}\eta + \zeta]\frac{\omega}{\rho_0}k^2\rho_k \qquad \xi \equiv \tfrac{4}{3}\eta + \zeta$$

Heat flow:

$$\boldsymbol{q} \approx -\kappa[\boldsymbol{\nabla}T + T_0\partial_t\boldsymbol{v}]: \;\; q_k = -i\kappa[kT_k - \frac{T_0}{\rho_0}\frac{\omega^2}{k}\rho_k] \qquad T_k \approx \frac{1}{1 + i\kappa k^2/\omega c_v}\frac{T_0}{\rho_0}\left(\frac{\partial p}{\partial\varepsilon}\right)_\rho \rho_k$$

Equation of state:

$$p_T(\rho) \;\Rightarrow\; p_k = \left(\frac{\partial p}{\partial\varepsilon}\right)_\rho c_v T_k + \frac{h_0}{\rho_0}v_T^2\rho_k$$

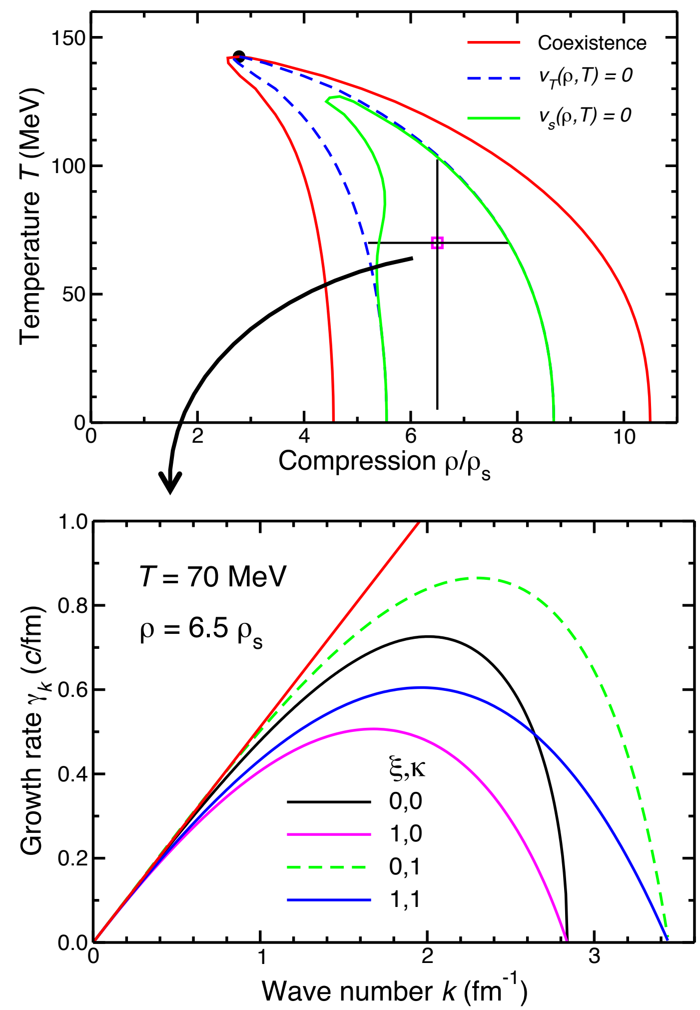
Dispersion equation:

$$\boxed{\omega^2 \;\doteq\; v_T^2 k^2 + C\frac{\rho_0^2}{h_0}k^4 - i\xi\frac{\omega}{h_0}k^2 + \frac{v_s^2 - v_T^2}{1 + i\kappa k^2/\omega c_v}k^2}$$

Heiselberg, Pethick, Ravenhall, Ann. Phys. 233, 37 (1993)

Dispersion relations

$$\boxed{\omega_k \;=\; \epsilon_k + i\gamma_k}$$



Ideal fluid dynamics: $\omega^2 \doteq v_s^2 k^2$

+ gradient term: $\omega^2 \doteq v_s^2 k^2 + C\dfrac{\rho_0^2}{h_0}k^4$

+ shear & bulk viscosity: $\omega^2 \doteq v_s^2 k^2 + C\dfrac{\rho_0^2}{h_0}k^4 - i\xi\dfrac{\omega}{h_0}k^2$

$$\xi \equiv \tfrac{4}{3}\eta + \zeta$$

+ heat conduction: $\omega^2 \doteq v_T^2 k^2 + C\dfrac{\rho_0^2}{h_0}k^4 - i\xi\dfrac{\omega}{h_0}k^2 + \dfrac{v_s^2 - v_T^2}{1 + i\kappa k^2/\omega c_v}k^2$

Transport coefficients

$$\boxed{\begin{array}{l}\eta_0 \geq 1 \\ \kappa_0 \geq 1\end{array}}$$

1) Bulk viscosity ζ: Ignore $\quad \zeta \ll \eta \Rightarrow \xi \equiv \frac{4}{3}\eta + \zeta \approx \frac{4}{3}\eta$

2) Shear viscosity η:

$$\rho = 0: \quad h \equiv p + \varepsilon = T\sigma$$

$$\rho = 0: \quad \eta \geq \frac{\hbar}{4\pi}\sigma = \frac{\hbar}{4\pi}\frac{h}{T}$$

$$\boxed{\eta(\rho,T) = \eta_0 \frac{c_0}{c} d(\rho,T)\, h(\rho,T)}$$

$$\rho > 0,\ T \ll mc^2: \quad h \asymp mc^2 n \gg T\sigma$$

$$\rho = 0: \quad n \sim T^3 \quad \Rightarrow \quad \frac{\hbar c}{T} = 4\pi c_0\, d \qquad d \equiv n^{1/3}$$

$$\lambda_{\rm visc} \equiv \frac{1}{c}\frac{\xi(\rho,T)}{h(\rho,T)/c^2} \approx \tfrac{4}{3}\eta_0\, c_0\, d(\rho,T)$$

3) Heat conductivity κ:

$$\eta \approx \tfrac{1}{3} n\bar{p}\ell \qquad \kappa \approx \tfrac{1}{3}\bar{v}\ell c_v$$

$$\frac{\kappa}{\eta} \approx \frac{c_v}{h/c^2}$$

$$\bar{p} = m\bar{v} \qquad h \asymp mc^2 n$$

$$c_v \equiv \partial_T \varepsilon_T(\rho) \qquad c_v \asymp \tfrac{3}{2} n$$

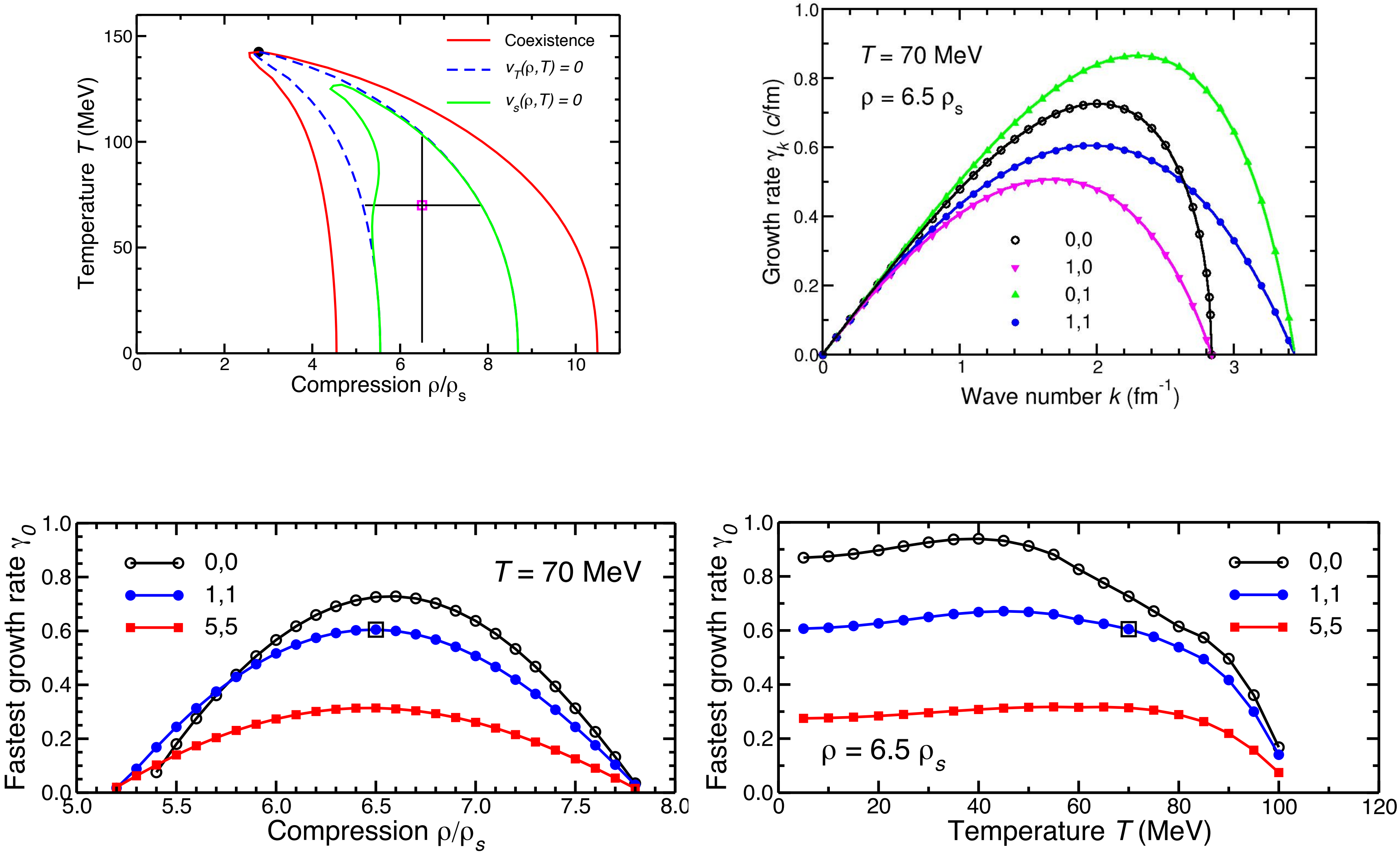
$$\boxed{\kappa(\rho,T) = \kappa_0\, c_0\, c\, d(\rho,T)\, c_v(\rho,T)}$$

$$\lambda_{\rm heat} \equiv \frac{1}{c}\frac{\kappa(\rho,T)}{c_v(\rho,T)} = \kappa_0\, c_0\, d(\rho,T)$$

$$c_0 = \frac{1}{4\pi}\left[(g_g + \tfrac{3}{3}g_q)\frac{\zeta(3)}{\pi^2}\right]^{\frac{1}{3}} \approx 0.12779$$

V Koch & J Liao, PRC81, 014902 (2010)

Spinodal growth rates



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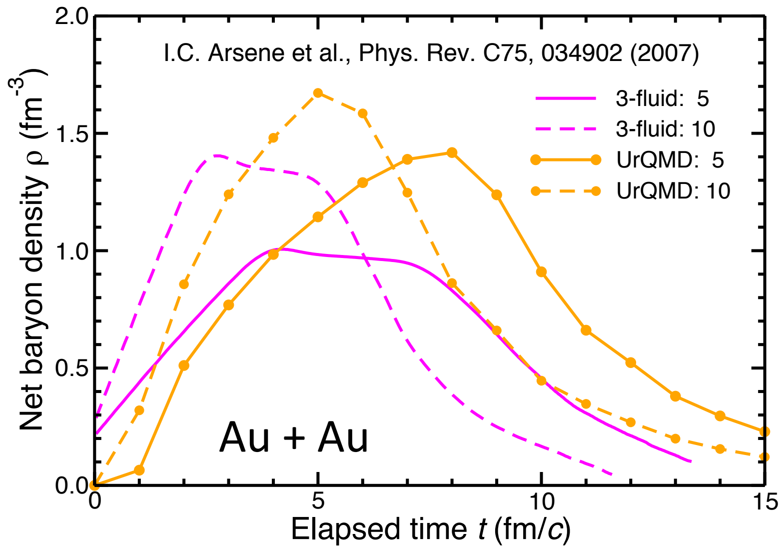
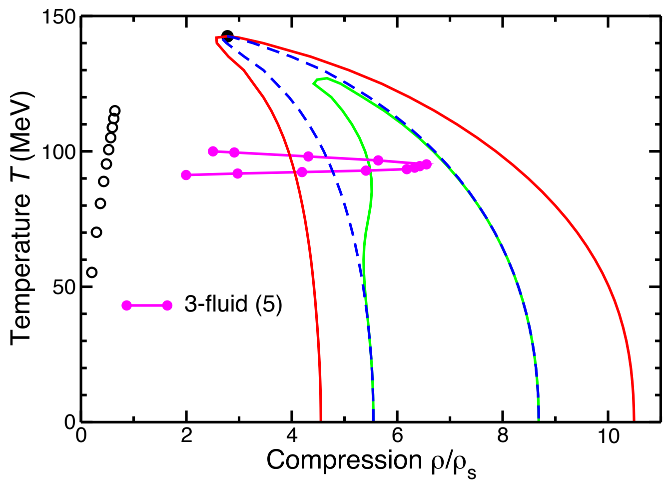
Dynamical phase trajectories

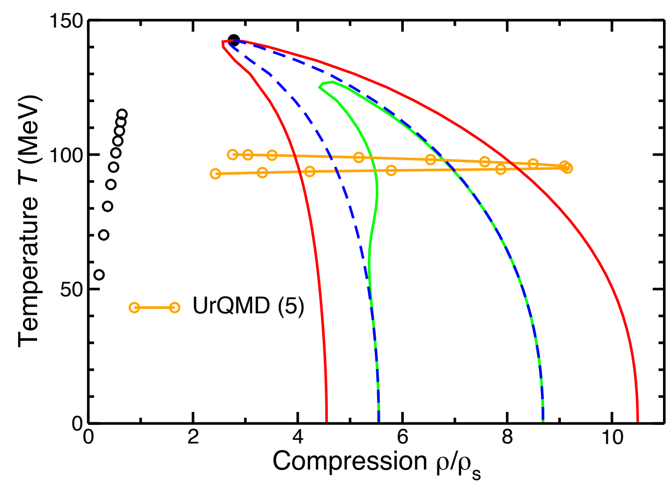


UrQMD
3-fluid
PHSD
QGSM
GiBUU

3-fluid



UrQMD



Dynamics of collective modes in many-body systems

Amplitude evolution: $\omega_\nu = \epsilon_\nu + i\gamma_\nu$

$$\frac{d}{dt}A_\nu(t) \;=\; -i\omega_\nu A_\nu(t) \;+\; B_\nu(t)$$

Correlation function:

$$\sigma_{\nu\mu}(t_1,t_2) \;\equiv\; \prec A_\nu(t_1)\, A_\mu(t_2)^* \succ$$

Markovian noise:

$$\prec B_\nu(t)\, B_\mu(t')^* \succ \;=\; 2\mathcal{D}_{\nu\mu}\,\delta(t-t')$$

Evolution:

$$\frac{d}{dt}\sigma_{\nu\mu}(t) \;=\; \underbrace{2\mathcal{D}_{\nu\mu}}_{\textit{seed}} \;-\; \underbrace{i(\omega_\nu - \omega^*_\mu)\sigma_{\nu\mu}}_{\textit{feedback}}$$

Variance of a single mode:

$$\boxed{\frac{d}{dt}\sigma^2_\nu \;=\; 2\mathcal{D}_\nu \;+\; 2\gamma_\nu\sigma^2_\nu}$$

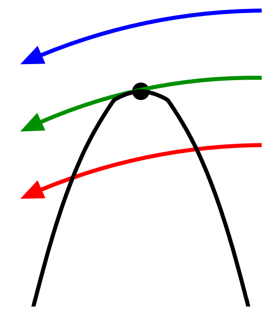
Lalime Equation

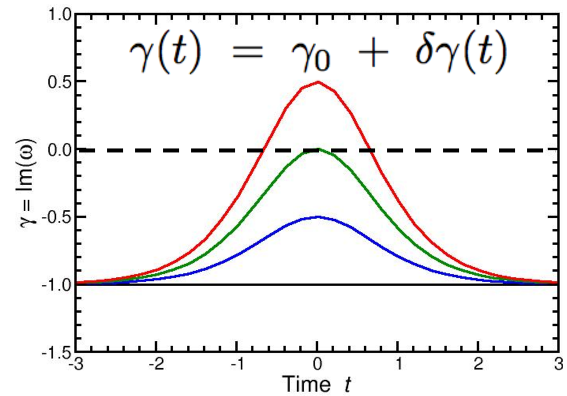
Amplification coefficient:

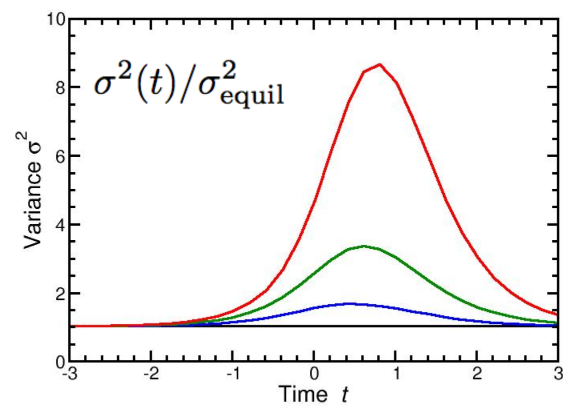
$$\Gamma_\nu(t) \equiv \int_0^t \gamma_\nu(t')dt'$$

=> $$\sigma^2_\nu(t) \;=\; \left[2\mathcal{D}_\nu \int_0^t \mathrm{e}^{-2\Gamma_\nu(t')}dt' + \sigma^2_0\right]\mathrm{e}^{2\Gamma_\nu(t)}$$

$$\gamma_\nu < 0: \quad \sigma^2_\nu(t) \;\rightarrow\; -\mathcal{D}_\nu/\gamma_\nu$$

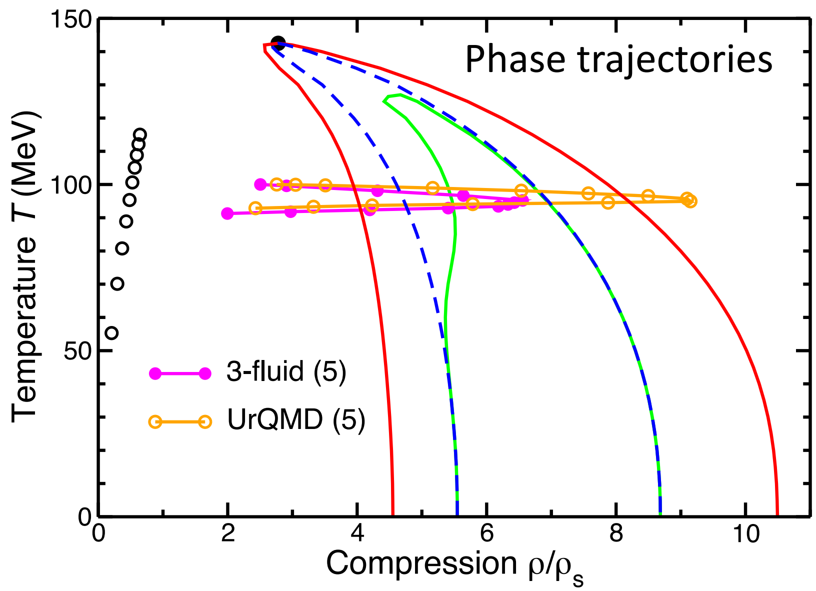






Colonna, Chomaz, Randrup, NPA 567 (1994) 637

Spinodal amplification

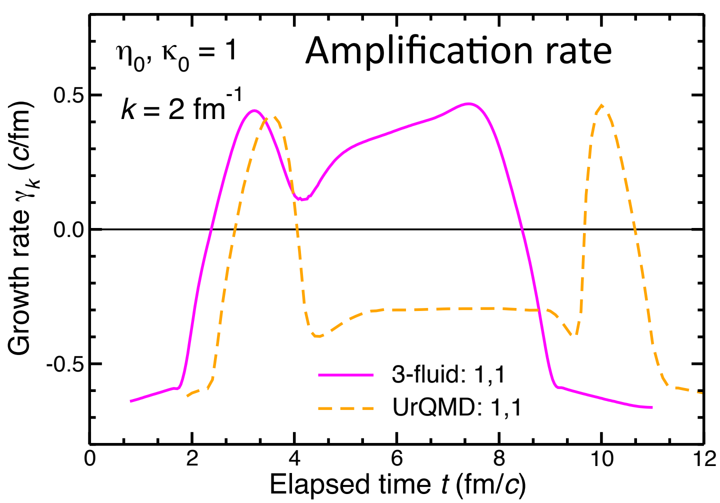


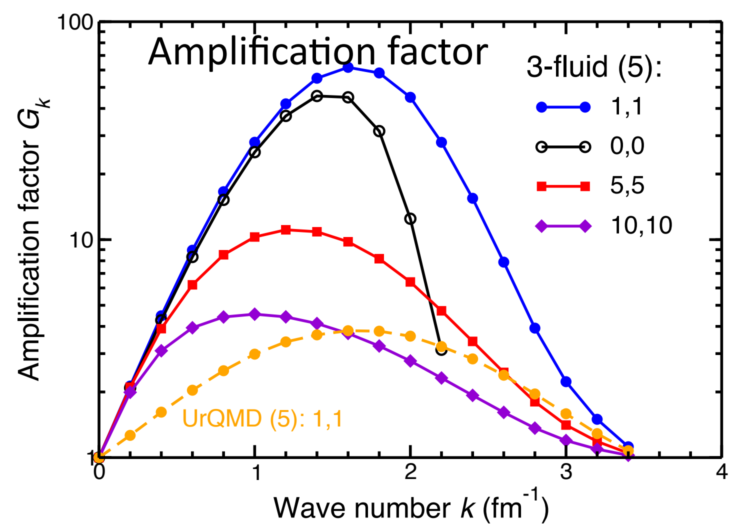
Amplification coefficient:

$$\Gamma_\nu(t) \equiv \int_0^t \gamma_\nu(t')dt'$$

Amplification factor:

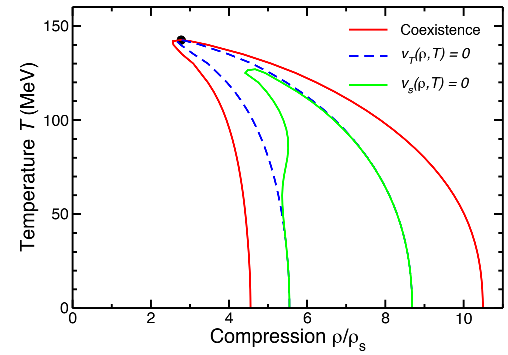
$$G_k = \mathrm{e}^{2\Gamma_k}$$





Conclusion

(Under what circumstances) can spinodal phase separation occur in nuclear collisions?

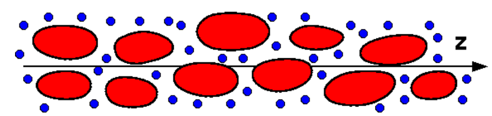


Spinodal decomposition *may* occur at $E_{\mathrm{lab}} \approx$ 5-10 GeV/A (FAIR & NICA):

1) Exists an optimal E range

2) Significant amplification

- but:



Full dynamical calculations are needed!

MODEL REQUIREMENTS:

EoS $p(\rho,T)$ with phase transition

Confined & deconfined phases

Phase coexistence, incl interface*

Spinodal modes*

Dynamics with instabilities*

* *Requires finite range*

Suggestion: *Use fluid dynamics with finite range*

Need: *Transport coefficients* $\eta(\rho,T)$, $\zeta(\rho,T)$, $\kappa(\rho,T)$

Important