The nuclear liquid-gas phase transition at large N_c

Based on 1006.2471

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The plan

Why? Some very difficoult questions...

Why Van Der Waals? to be answered by very simple modles

How? does each parameter scale with N_c?

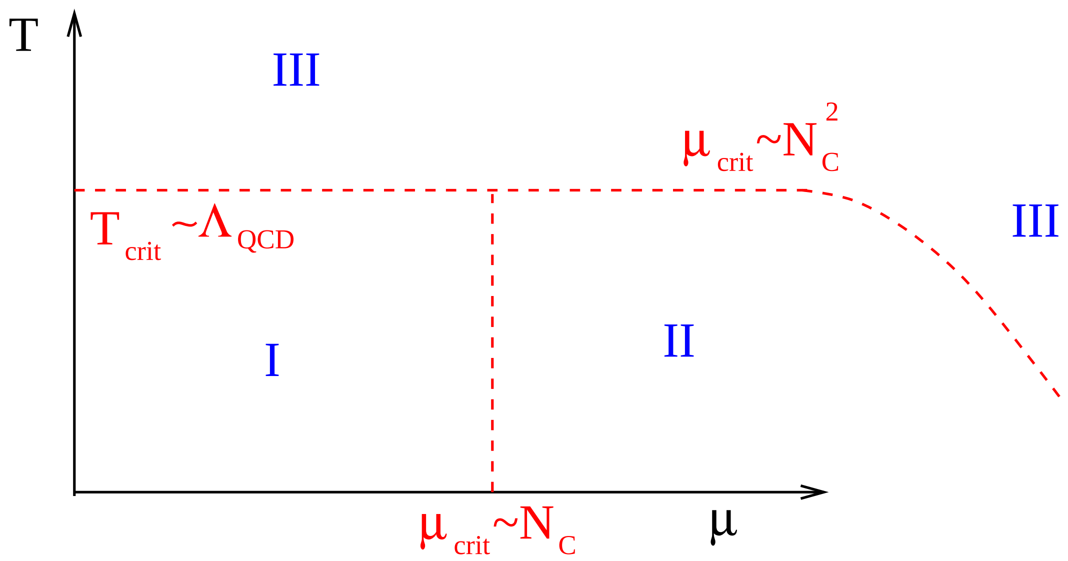
Some results

A conjecture with very boring implications

Experimental signature

conclusions and further work (Chiral symmetry?)

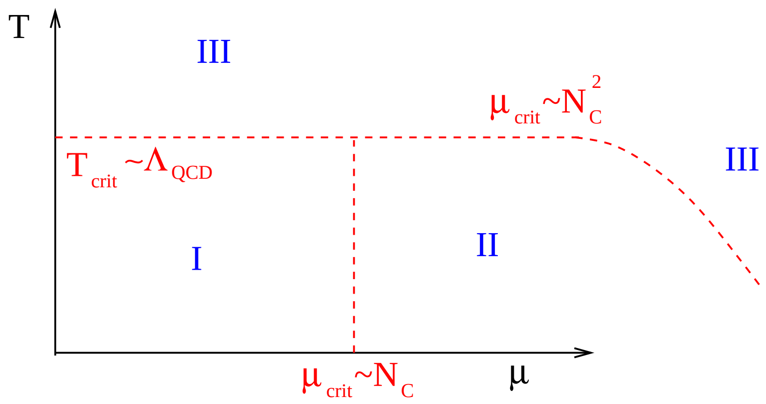
L.McLerran,R.Pisarski NPA 796, 83 (2007) :Phase diagram at large N_c:



I : The "usual" hadron (ie meson) gas, $P \sim N_c^0$

III : The "usual" deconfined gas of quarks and (mostly) gluons, $P \sim N_c^2$

And...



II :A new Quarkyonic phase, with the following characteristics:

Confining , but Chiral symmetry is restored (2007, revised recently)

Baryonic density discontinuity at the phase transition line

$P \sim N_c$!! Confined, but P dominated by quarks inside Fermi Surface

And exciting new phase to be explored! FAIR,supernovae (EoS puzzle),...

How was this phase motivated ?

Baryons at large N_c (E. Witten,Nucl. Phys. B 160, 57 (1979))

Mass $\sim N_c \to \infty$ but size $\sim \Lambda_{QCD}^{-1} \sim N_c^0$

Baryonic density at large N_c abruptly changes from $\exp[-N_c]$ to ~ 1 at $\mu_B/N_c \sim \Lambda_{QCD}$

Interactions strong $\sim N_c$ (E.Witten, Nucl. Phys. B 160, 57 (1979)).

Way of reconciling coupling constant (weak at quark density scale in baryon $\gg \Lambda_{QCD}^3$,strong at baryon size scale $\sim \Lambda_{QCD}^{-1}$ set by confinement)

What about our $N_c = 3$ world?

Well, $N_c \gg 1$ and $m_B \gg \Lambda_{QCD}$! Qualitatively "large N_c" with 30% errors

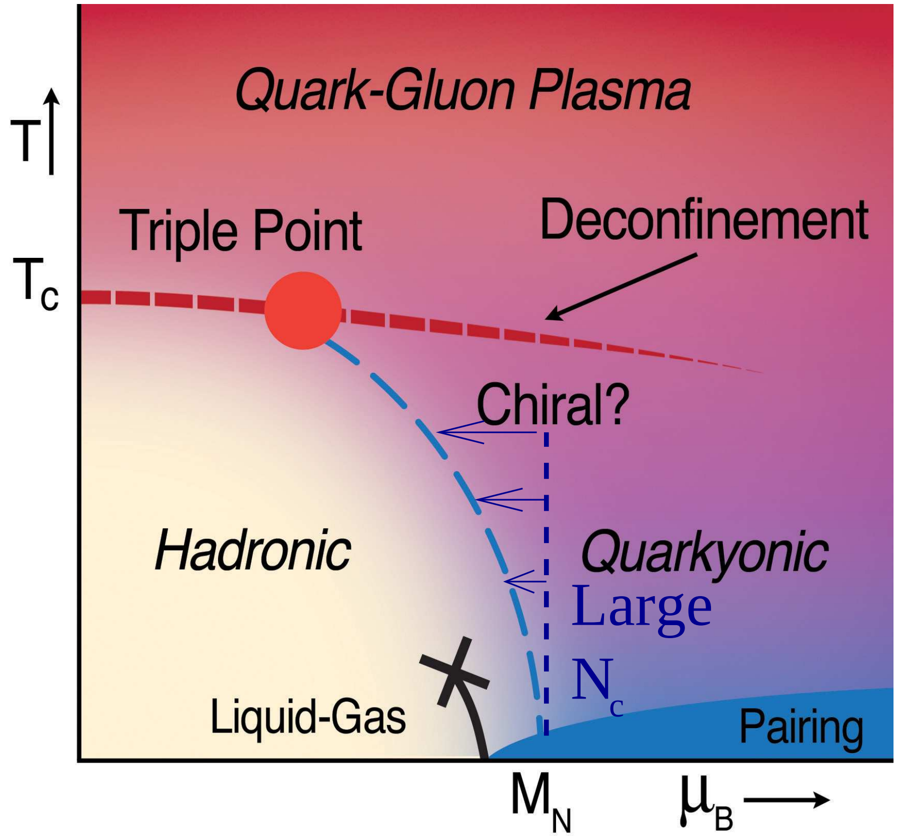
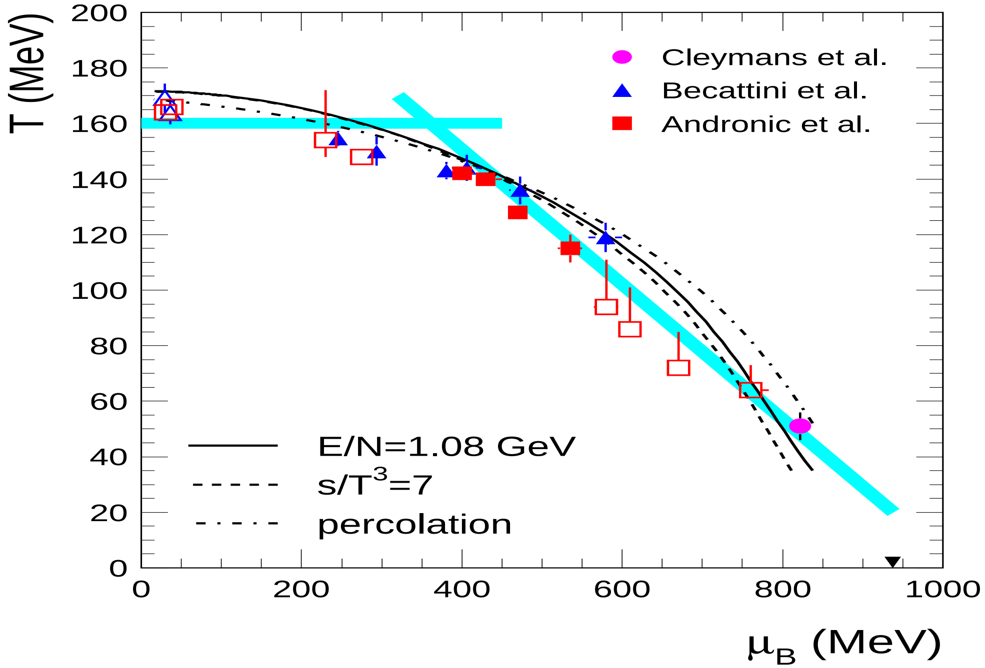
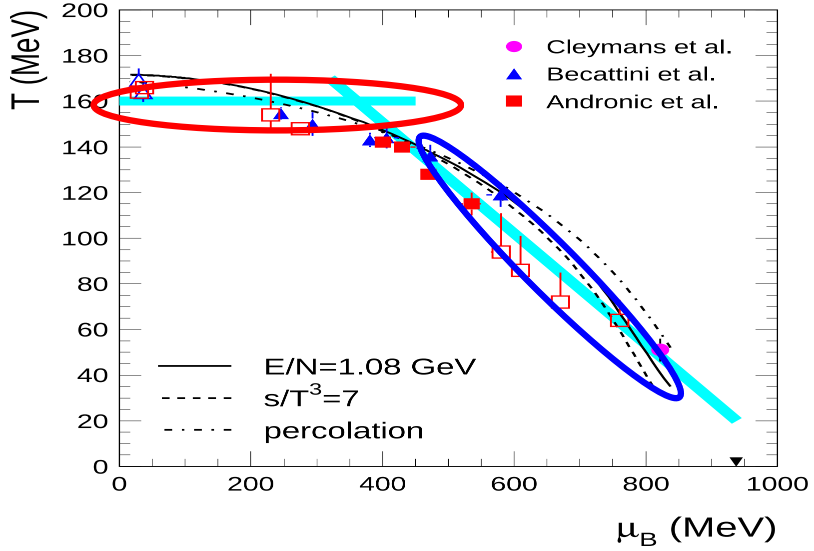


Diagram qualitatively similar but curvature~30% (At high T transition chemical potential decreases)



A.Andronic et al
NPA 837, 65 (2010)

If statistical mechanics in a hadronic quasi-particle system is an indication of a phase transition, we might be seeing the Quarkyonic phase in low energy statistical particle production!!!



Freezeout probes deconfinement

Freezeout probes quarkyonic matter

The quarkyonic phase is therefore accessible at <u>low</u> energy nuclear collisions!
and supernovae (Where it might help with the EoS problem!)

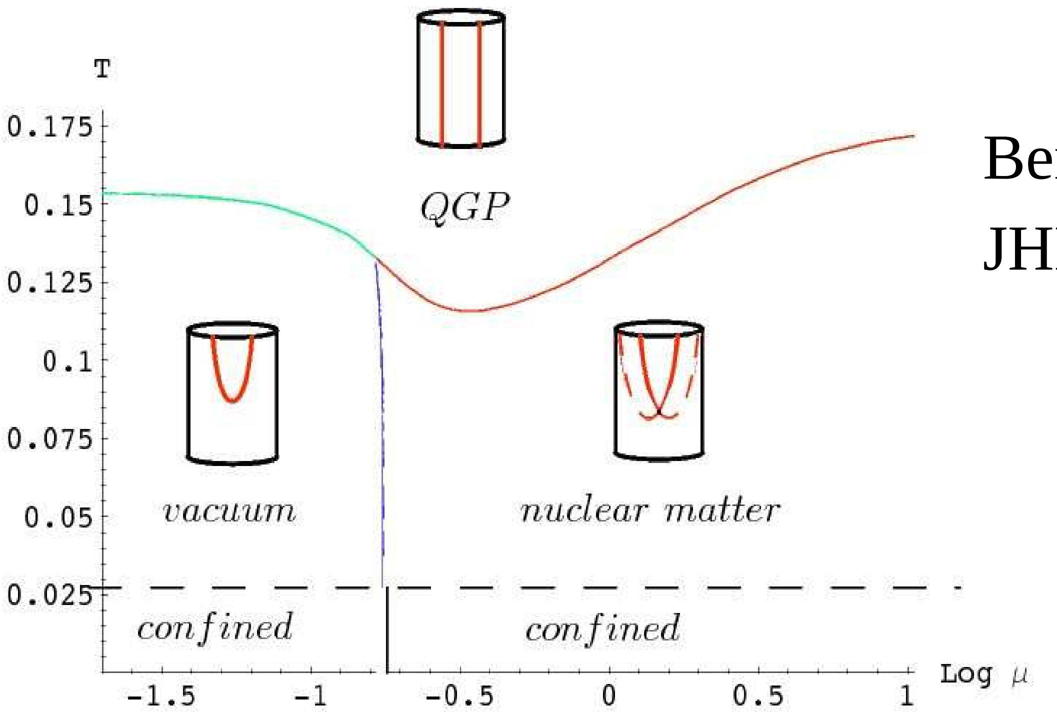
Everything I told you is very speculative

- Applicability of the statistical model not necessarily indicative of a phase transition
  - Other explanations possible
  - System size dependence of thermalization currently controversial

- By everything I also mean theory! There is a good reason for this! We are in the deep strongly coupled semi-confined regime. Difficoult to say anything rigorously.

  And RHIC experience shows making "naive" assumptions dangerous (nearly conformal $\rightarrow$ ideal gas... or perfect liquid?)

  The only approach which claims strong coupling rigorously computed dynamics... links it to 5d black holes and adds a few supersymmetries! does it have anything to say here?



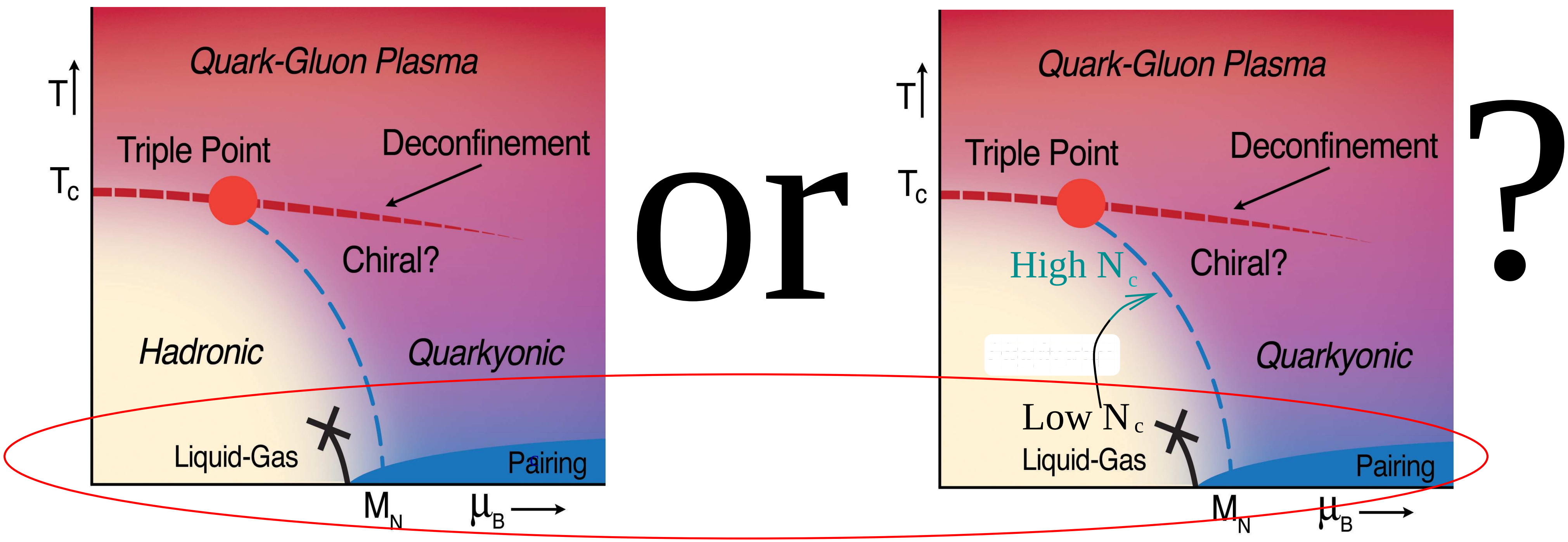
Bergman,Lifschytz,Lippert
JHEP0711:056,2007

AdS/CFT to the (non)-rescue...

Sakai-Sugimoto model (top-down model with a kind of confinement, "a kind of" chiral symmetry breaking shows same shape phase diagram as quarkyonic matter!!! but...

The quarkyonic phase is <u>confined</u> and χ-restored. Its just "nuclear matter".

An important suspicion arises.... Is "quarkyonic matter" simply the large N_c limit of the liquid-gas transition, discovered and studied since the '70s?



An important suspicion arises.... Is "quarkyonic matter" simply the large N_c limit of the liquid-gas transition, discovered and studied since the '70s?

if so , experimentalists can safely leave the auditorium
(But start working on The variable N_c detector!)
(And theorists should explain why $N_c = 3$ is so different!)

if not , experimentalists should look for quarkyonic matter!

The plan :Investigate these questions with a simple universal model

- known to describe the liquid-gas transition
- With few parameters, whose scaling with N_c is intuitively clear

The best physicist in the USSR is Yakov Frenkel, who uses in his papers only quadratic equations.
I am slightly worse, I sometimes use differential equations.

L.D.Landau, quoted in
BULLETIN OF THE American Mathematical Society
Volume 43, Number 4, October 2006, Pages 563−565

Nuclei and their interactions at large N_c

The Van Der Waals equation of state

$$\left(\rho^{-1} - b\right)\left(P + a\rho^2 - g\rho^3\right) = T$$

b Is the excluded volume

a,g are the interaction. For any radial interaction $V(r)$,they came out as terms in the expansion of

$$\prod_{ij} \int dx_{ij} \exp\left[\frac{V\left(x_{ij}\right)}{T}\right]$$

Why Van Der Waals?

- Solvable analytically!
- Has phase transition, similar to nuclear gas-liquid transition
- universal...

The Van Der Waals equation is universal

ANY interaction term can be expanded in a series of

$$2\pi T \int_{\alpha^{1/3}}^{\infty} dr r^2 \left(1 - \exp\left[-\frac{V(r)}{T} \right] \right)$$

with the first two terms giving the Van Der Waals gas

Expansion in (Excluded volume)$\langle \rho^{-1} \rangle$, so higher order corrections $\rightarrow \rho^n$ terms. Corrections due to 3-body $V(x_{12}, x_{23})$ and higher should also show up as ρ^n

Intriguingly for strongly coupled QCD, charged black holes in supergravity part of the VdW universality class (A. Chamblin, R. Emparan, C. V. Johnson and R. C. Myers, PRD 60, 064018 (1999) Phase transition line they derive intriguing...)

BUT universality has limits ...

- No chiral symmetry (Ask me at the end!)

- in VdW, interactions integrated out so carry no entropy.

  inappropriate for measuring the entropy content of,say,electron gas (interaction-dominated).

  So might be inappropriate for understanding the liquid phase if its entropy resonance (or residual quark interaction)dominated as in the quarkyonic conjecture

  ...but can still give phase transition line!

Only scale at of theory large N_c is Λ_{QCD} !

This is the inverse of the confinement scale.

Now we add fermions and chiral symmetry, but empirically we know that $\Lambda_{QCD}^3 \sim \langle \psi \overline{\psi} \rangle$. physically, this is not surprising: A constituent quark mass is, to a good approximation, the Heisenberg uncertainity of the massless particle in the confining potential.

So even with Fermions Only scale at of theory large N_c is Λ_{QCD}

Only scale at of theory large N_c is Λ_{QCD} !
It is therefore natural to decompose VdW equation into dimensionless components (functions of N_c) and the appropriate power of Λ_{QCD}

$$\left(\rho^{-1} - \alpha\right)\left(P + \beta\rho^2 - \gamma\rho^3\right) = T$$

α is in Λ_{QCD}^{-3}

β is in Λ_{QCD}^{2}

γ is in Λ_{QCD}^{5}

Factors of Λ_{QCD} neglected henceforward

How does α depend on N_c?

- α can't go below unity (deconfinement).

- In the large N_c limit, the only scale is Λ_{QCD} . It is therefore natural that

$$\lim_{N_c \to \infty} \alpha = \Lambda_{QCD}^{-3}$$

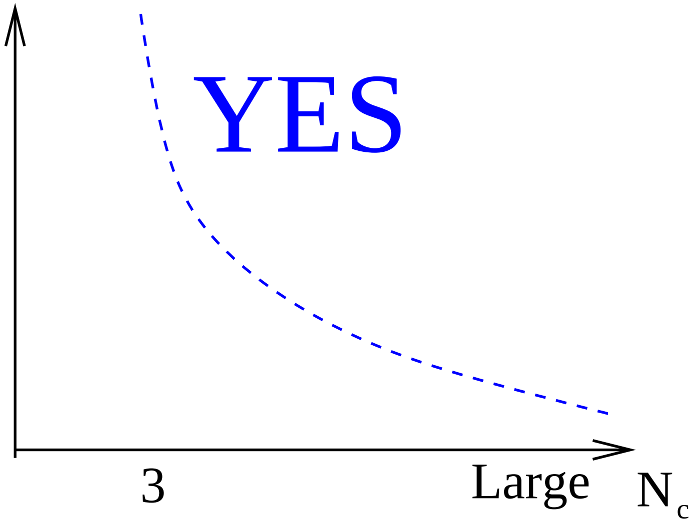
It can not have an $N_c^{a>1}$ leading term, since Baryon size does not diverge. But in our world, $\alpha \gg \Lambda_{QCD}^{3}$

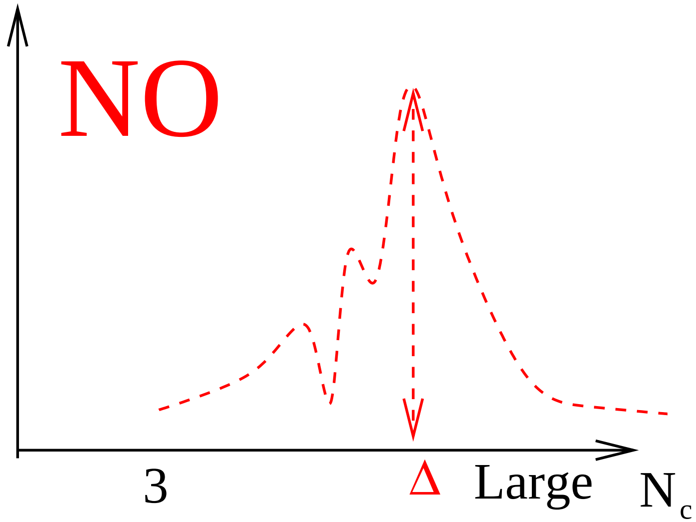
$$\alpha \sim 1 + \frac{A}{N_c}$$

and the A term dominates!

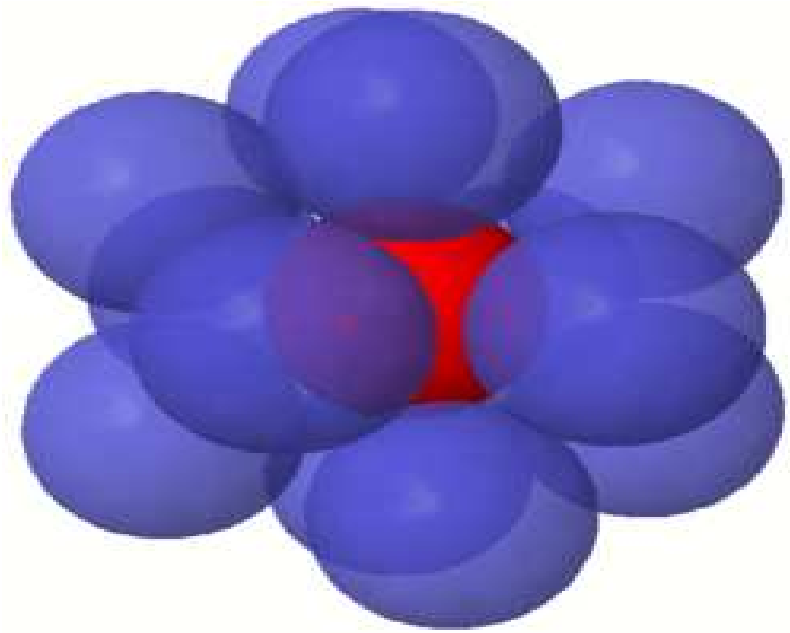
$$\alpha \sim 1 + \frac{A}{N_c}$$

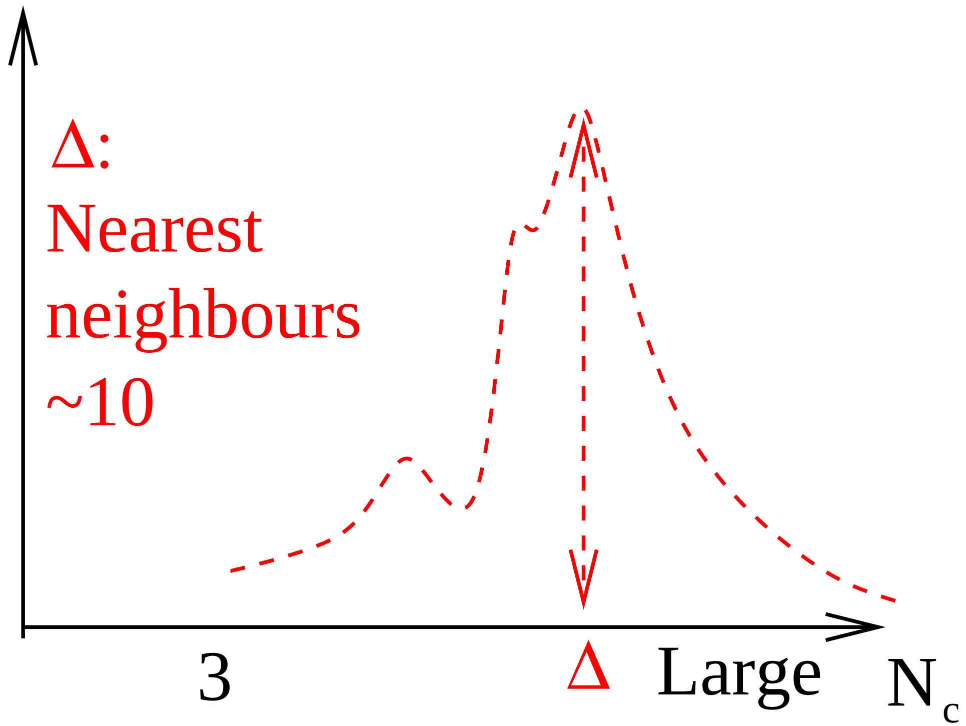
It is an experimental fact that $A \gg N_c$ in our world. Do we really live in a large-N_c world? This is a quantitative question



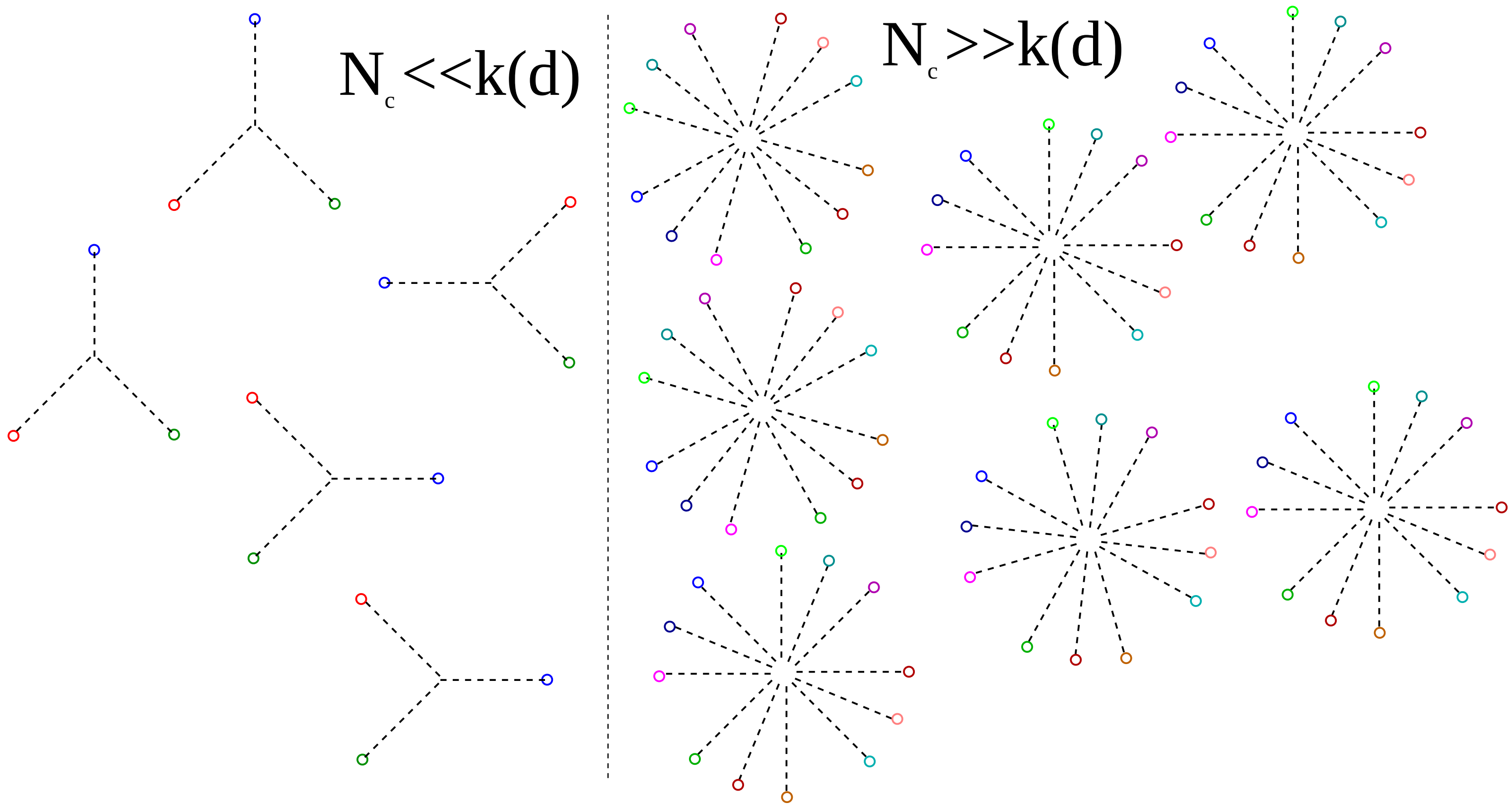


My guess is no!





The other scale of the problem is the the number of neighbours in tightly packed system!
"kissing number", exact dependence on d unknown
$k(d) \sim 2^{\zeta d}, k(1,2,3,4) = 2, 6, 10, 24$, of course $\sim N_c^0$, $k(d=3) \gg 3$



Pauli exclusion principle in valence picture irrelevant for $N_c \gg k(d)$, but not for $N_c = 3$. Keeps nuclei further apart than Λ_{QCD}^{-1}

$$\alpha \sim 1 + \frac{N_N}{N_c} \sim 1 + \frac{k(d)}{N_c} \sim 1 + \frac{10}{N_c}\Big|_{3d}$$

- Fits nuclear VdW at $N_c = 3$
- Compatible with strongly coupled nuclear matter at $N_c \gg 3$
- Understandable by Pauli exclusion principle
  Spin, flavor complicates things. But in our world $\Delta E|_{spin flip} \sim \Lambda_{QCD}$,
  flipping flavor suppressed

$$\alpha \sim 1 + \frac{N_N}{N_c} \sim 1 + \frac{k(d)}{N_c} \sim 1 + \frac{10}{N_c}\bigg|_{3d}$$

What this means:

- confinement scale $\gg$ nuclear separation up to $\sim$ deconfinement potential!

- Expansion in $\rho^n/\Lambda_{QCD}^{3n}$ progressively worse but always converges

Trust diagram, but not factors of $\mathcal{O}(1)$

- β,γ Have to scale the same way, since same interaction

- Witten 's solitonic picture of the nucleon: $\beta,\gamma \sim N_c$
  Weak ($\ll$ even m_π) nuclear force an accidental cancellation.
  Witten says that all $(2,3,n)$ body forces scale as N_c . Weinberg 's hyerarchy, $n-body$ nuclear forces $\sim (k/\Lambda_{QCD})^n \sim \left(\rho^{1/3}/\Lambda_{QCD}\right)^n$ complementary:N body forces all $\sim N_c$ but $2>3>...n$ Same as VdW expansion! .

- Y. Hidaka, T. Kojo, L. McLerran and R. D. Pisarski, 1004.2261 :
  This picture is wrong (skyrmion unstable, stabilized by large quantum corrections which put N_c-1 quarks into diquarks).
  Nuclear force carried by remaining quark,so $\beta,\gamma \sim N_c^0$ Weak nuclear force natural

Room for phenomenological playing: Try $\beta,\gamma \sim N_c^\nu, \nu=0,1$

Some results... Critical point can be found by solving for

$$\frac{dP}{d\rho} = \frac{d^2P}{d\rho^2} = 0$$

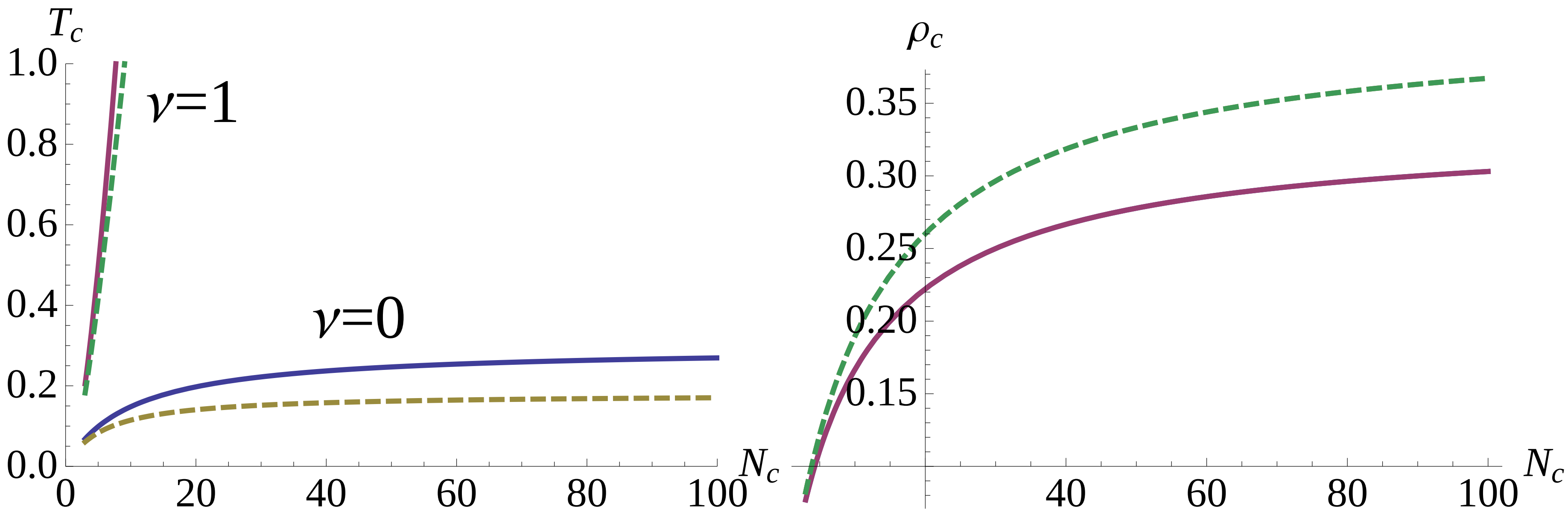
with the equation of state being an additional constraint. We obtain

$$T_c \sim \frac{24N_c^4 + 4N_c^2 N_N F_1 + 2\sqrt{3}N_c N_N^2 D - 3N_N^3 F_1 + 8N_c^3 F_2}{288(N_c + N_N)^2} N_c^{\nu-2} \sim N_c^{\nu} g_1(N_c)$$

$$\rho_c \sim \frac{\sqrt{3}\sqrt{8N_c^2 + 8N_c N_N + 3N_N^2} - 3N_N}{12(N_c + N_N)} \sim N_c^0 g_2(N_c)$$

If $\gamma = 0$, these reduce to the textbook

$$T_c \sim \frac{8}{27}\frac{\beta}{\alpha} \sim \left(\frac{N_c^{1+\nu}}{N_N + N_c}\right) \quad , \quad \rho_c = \frac{N_c}{N_c + N_N}$$



If $\beta \sim N_c^0$, T_c and ρ_c tends to an asymptotic value Λ_{QCD} to the appropriate power. Similar to Quarkyonic!
(Converges to $\sim \Lambda_{QCD}/3$ but Dont trust $\mathcal{O}(1)\,@N_c \to \infty$)
If $\beta \sim N_c$, T_c, ρ_c diverge, quickly $(N_c \simeq 10)$ overtaking the deconfinement temperature. Applicability of VdW equation is an accident of low N_c

The phase diagram for $\gamma = 0$: J. Lekner, AJP, **50**, 2, 161-163 (1982)

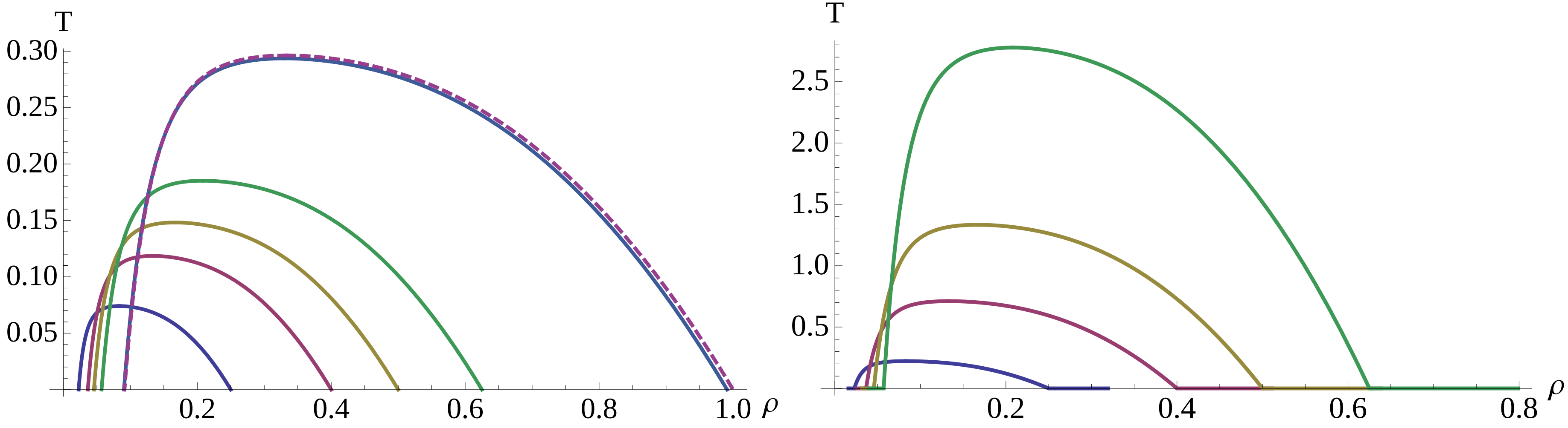
$$\alpha\beta\rho^3 - \beta\rho^2 + \rho\left(T + \alpha P\right) - P = 0$$

Given that the entropy difference between the liquid and gas phases is Δs ,

$$x_{+,-} = \frac{\alpha}{\rho_{g,l}^{-1} - \alpha} = e^{\pm\Delta s/2} f\left(\frac{\Delta s}{2}\right) \quad , \quad f\left(y\right) = \frac{y\cosh y - \sinh y}{\sinh y \cosh y - y}$$

where, $\rho_{g,l}$ is the density in the gas and liquid phases respectively and The temperature can be found from the same parameters and the requirement that pressure in the liquid and gas phase transitions has to be the same. solving for pressure, equalising and doing some algebra gets us

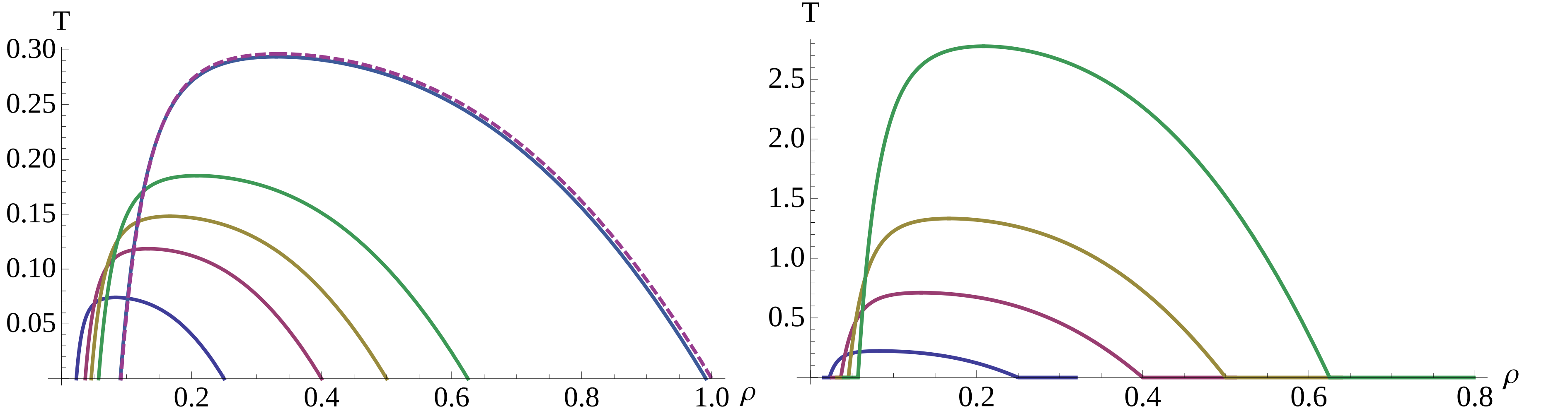
$$T = \left[\beta\left(\rho_l - \alpha\right)\left(\rho_l^{-2} - \rho_g^{-2}\right)\right]\left[1 - \frac{\rho_l^{-1} - \alpha}{\rho_g^{-1} - \alpha}\right]^{-1}$$



We confirm the previous results (Remember $\mathcal{O}(1)$ warning!)

$\nu = 0$ means the transition looks quarkyonic

$\nu = 1$ means no large N_c limit



At large N_c , all ρ from ~ 0 up to $\rho \sim \Lambda_{QCD}^3$ in mixed phase . reasonable, since Baryon density e^{-N_c} but "fractional baryons" impossible

μ_B: In relativistic systems mass makes an appearance, and the baryon mass is $m_B = N_c m_q^{constituent} \sim N_c \Lambda_{QCD}$. Dont forget quantum corrections

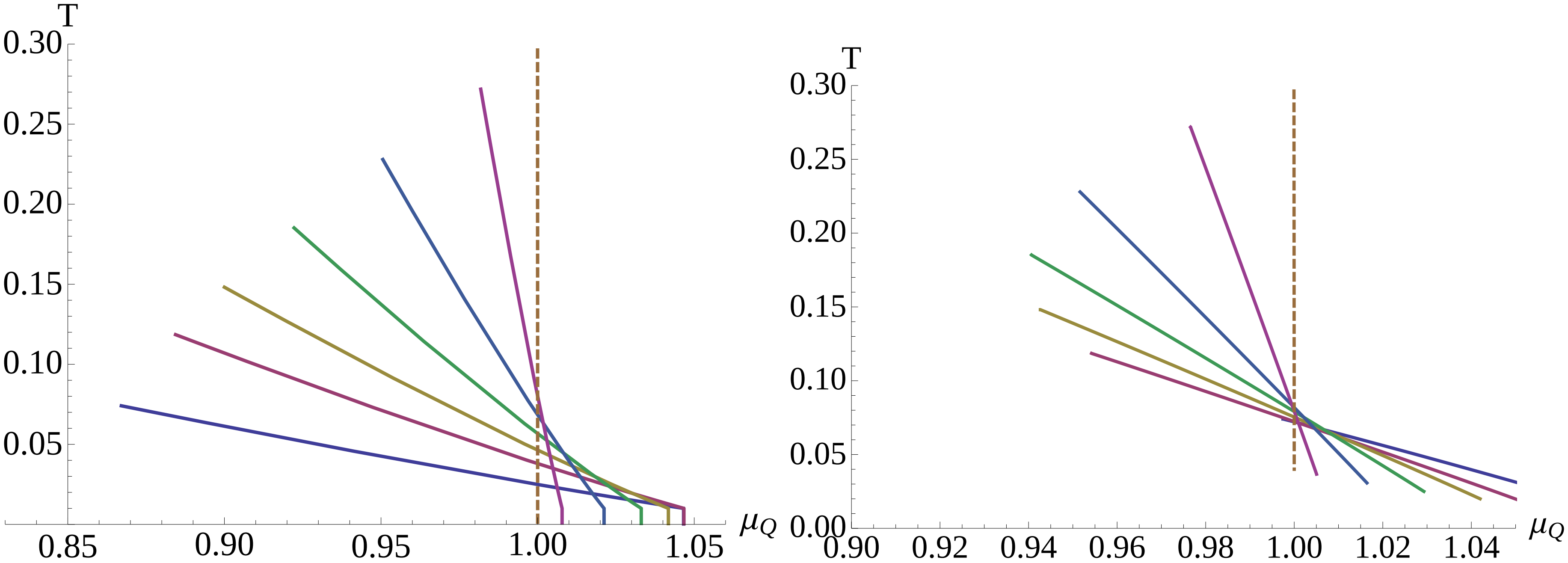
$$\mu_q = \frac{\mu_B}{N_c} = 1 + \frac{1}{N_c}\left[\int_0^\rho f\left(\rho', T\right) d\rho' + F\left(T\right) + \Delta\mu_{FD}\left(\rho, T\right)\right]$$

$$f(\rho, T) \quad = \left(\frac{dP}{d\rho}\right)_T \frac{1}{\rho}$$

$F(T) = -f\left(\rho \to 0, T\right)$ ensures $\mu_Q(\rho = 0, T) \to 0$, $\sim T^{-3/2}$ @ideal gas

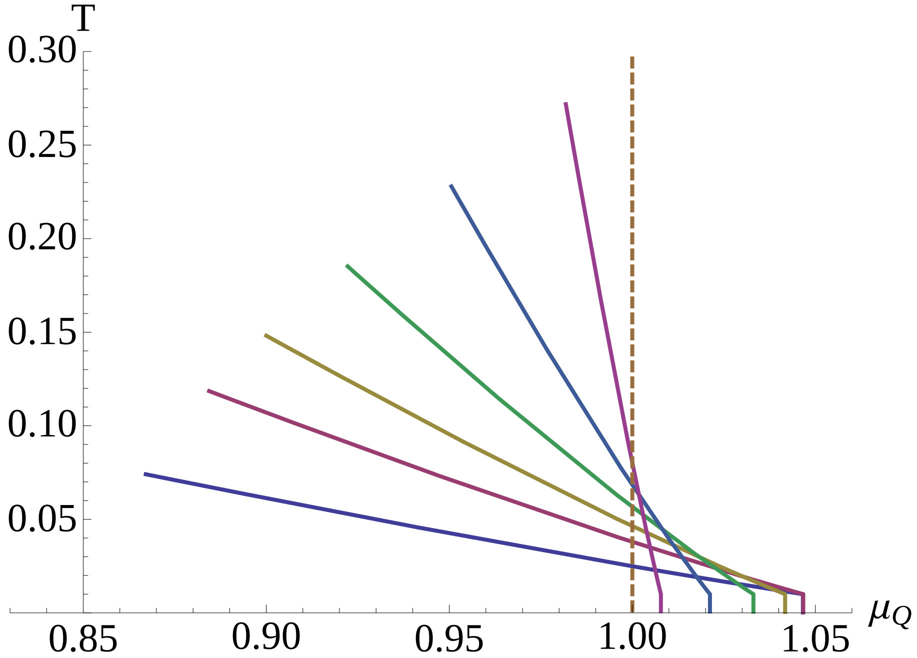
$\Delta\mu_{FD}(\rho, T)$ solves

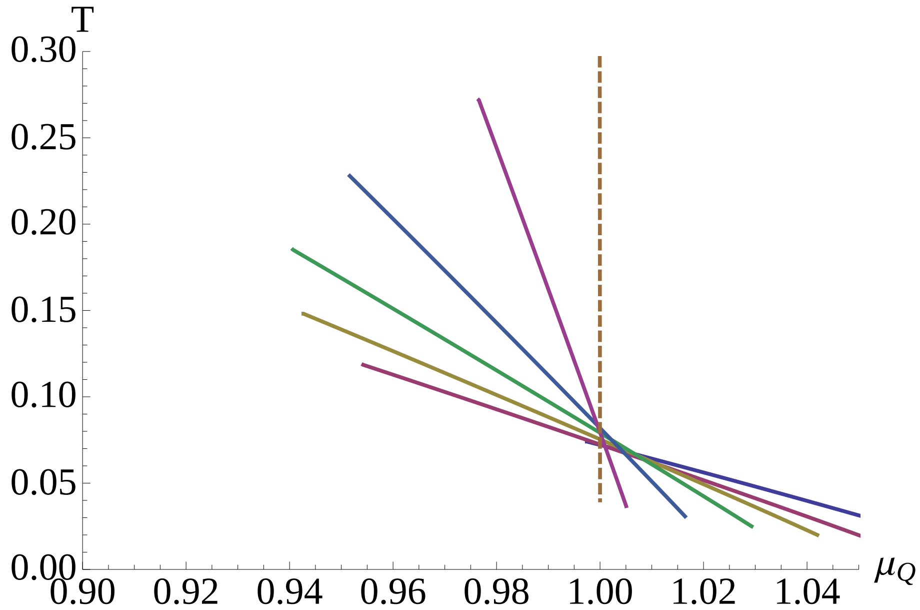
$$\Delta\mu_{FD} = \frac{T}{N_c}\left[\log(z) - \log\left(\rho\lambda^3\right)\right], \lambda^3\rho = \frac{4}{\sqrt{\pi}}\int_0^\infty \frac{x^2 dx}{\frac{\exp(x^2)}{z} + 1}$$



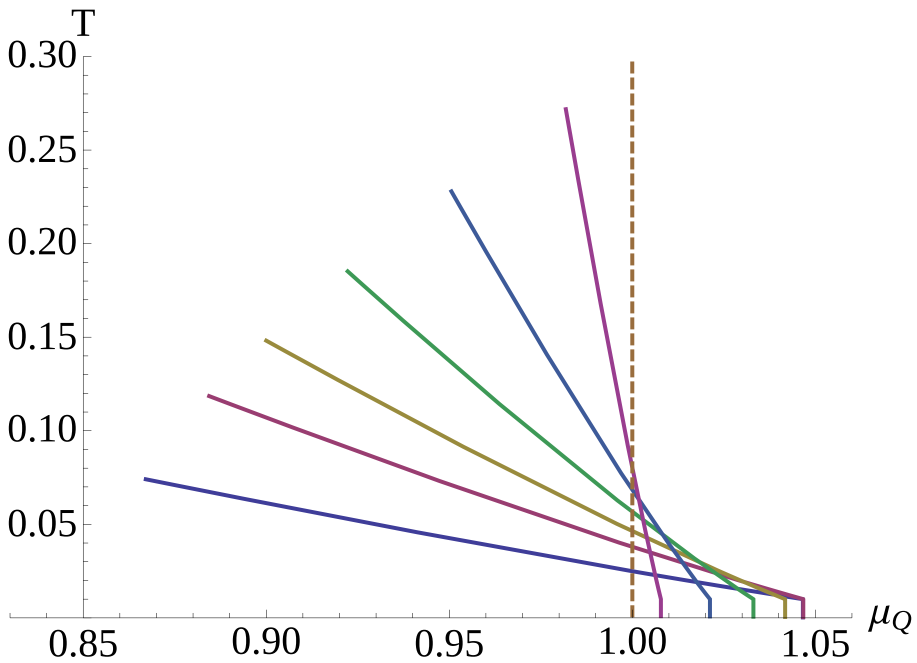
Phase diagram for $\gamma = 0$ looks quarkyonic .

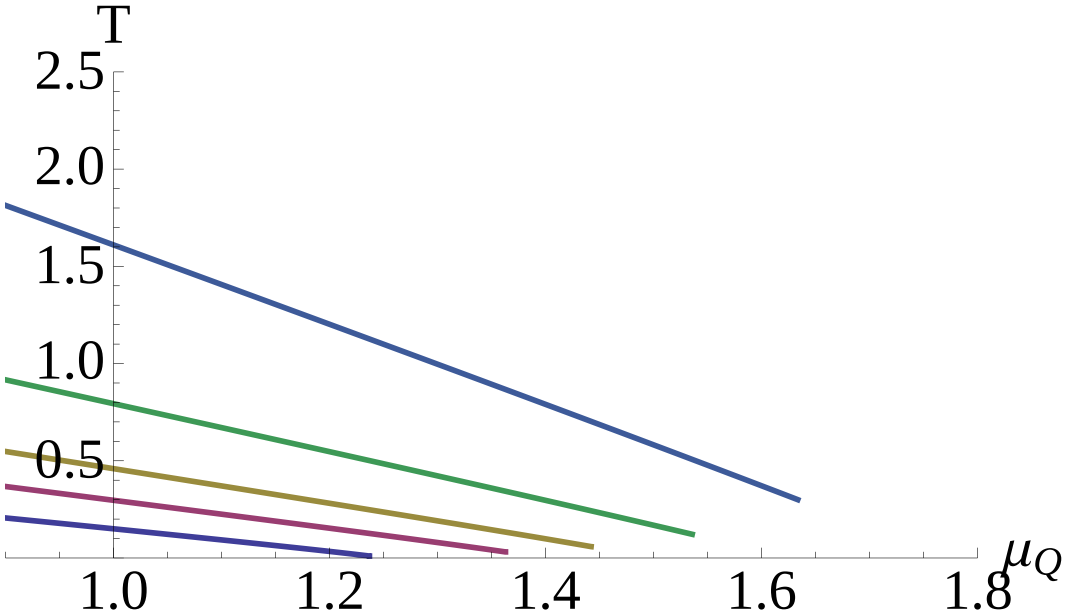
Easy to see why: All T -dependence of phase diagram $\sim N_c^{-1}$





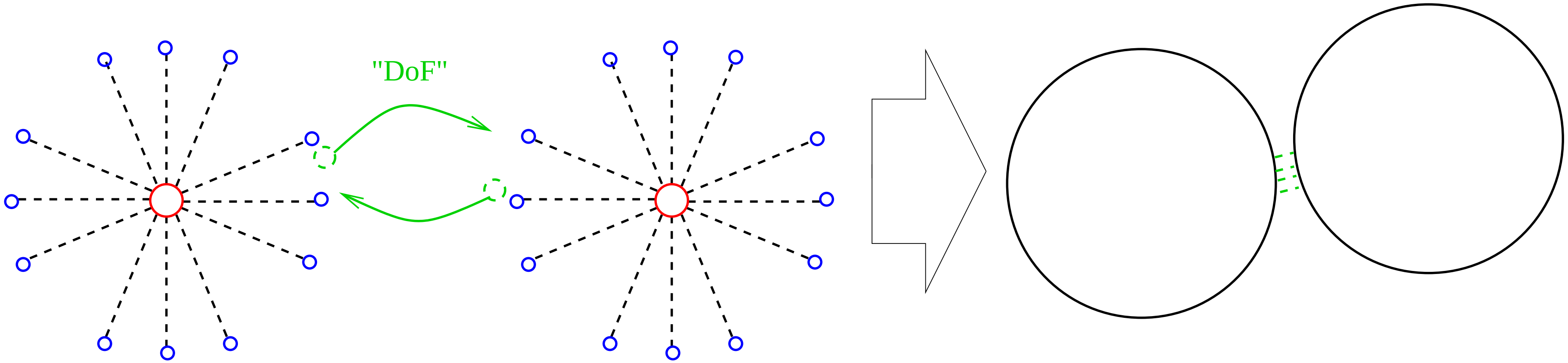
Quantum corrections also vanish. Also, note that widely used $\Delta\mu_{FD} \simeq \frac{e_f}{N_c}\left(1 - \frac{\pi^2}{8}\left[\frac{T}{e_f}\right]^2\right)$ inappropriate as its an expansion in T/e_f and $\lim_{N_c\to\infty} e_f = 0$. Large N_c nuclear matter classical VdW!



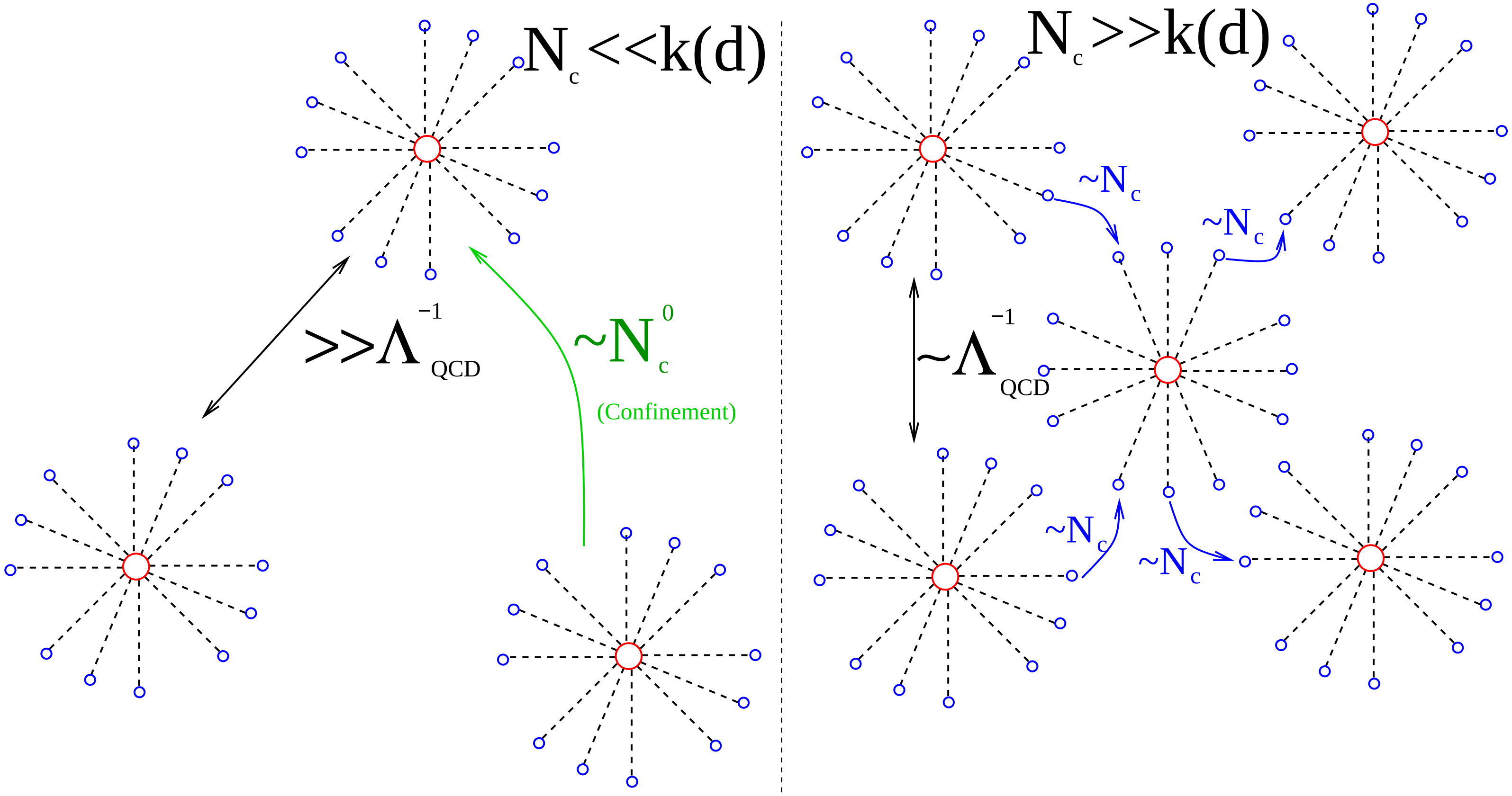


If $\nu = 1, \beta \sim N_c$ curvature does <u>not</u> vanish at $N_c \to \infty$,but then in this case large N_c limit for VdW unphysical

A speculation with boring implications

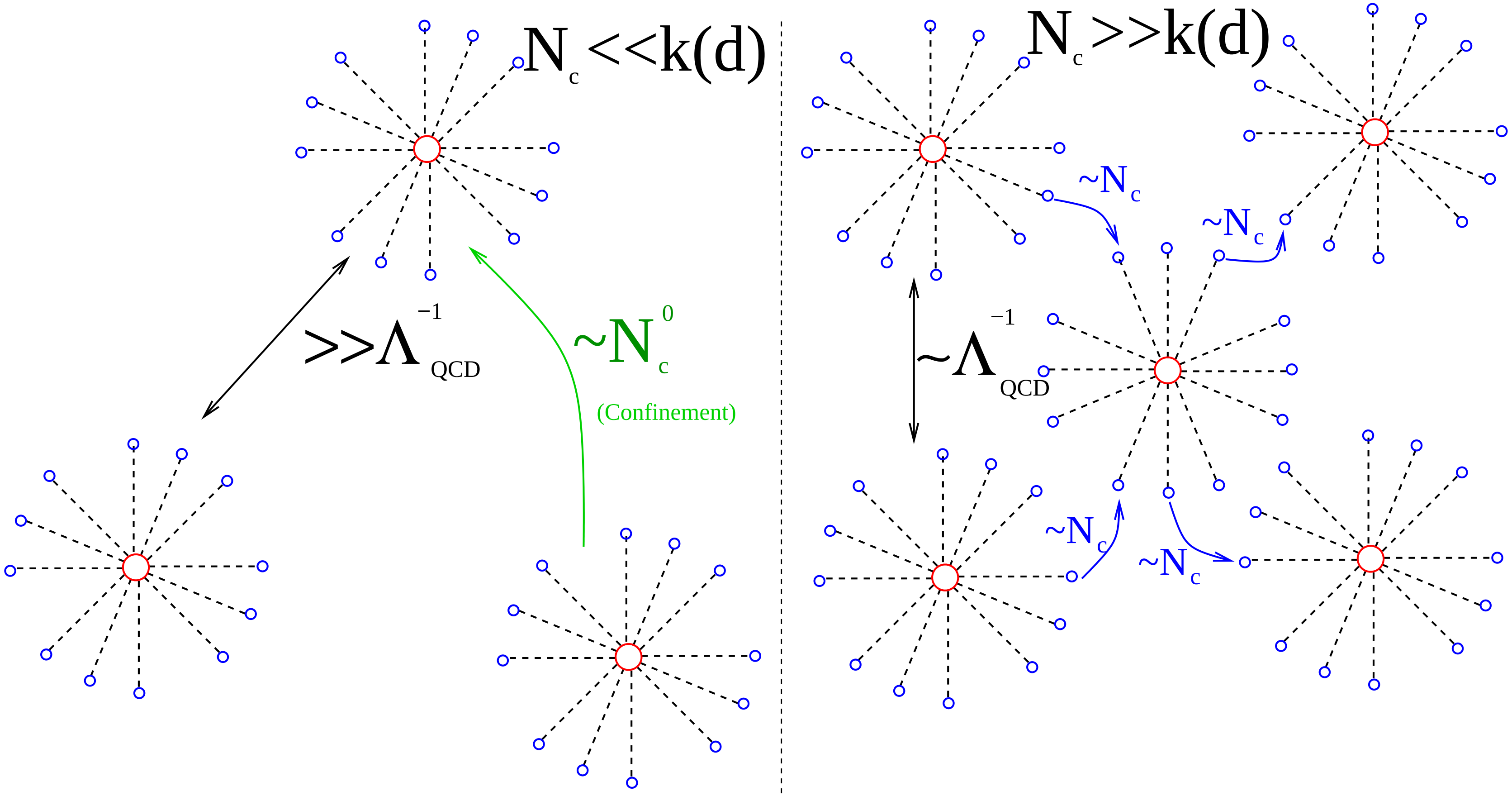


The VdW model "integrates out" the interacting fields into a contact term. Its beauty is that this contact term is universal. But integrating out undercounts the entropy carried by this interaction (Note that if interacting DoFs $\sim N_c$ but interaction N_c-suppressed, $\tau_{eq} \sim N_c$ but so is s_{eq}!) Van Der Waals would undercount entropy in a metal with valence electors, but would be OK for water!

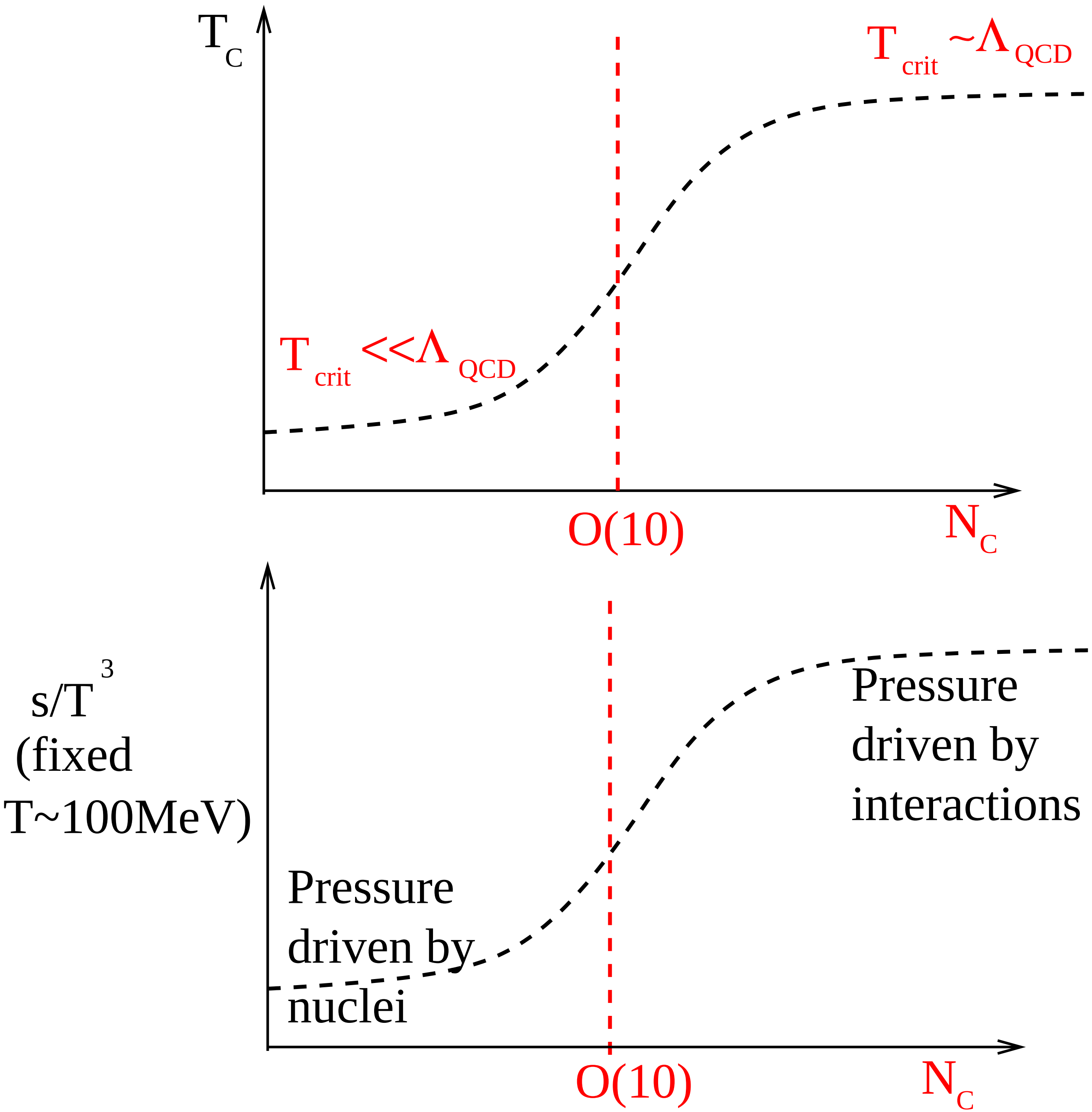


$N_c \ll N_N$ Liquid interbaryon distance $\gg \Lambda_{QCD}^{-1}$ Color exchange suppressed

$N_c \geq N_N$ Liquid interbaryon distance $\sim \Lambda_{QCD}^{-1}$ Color exchange dominate

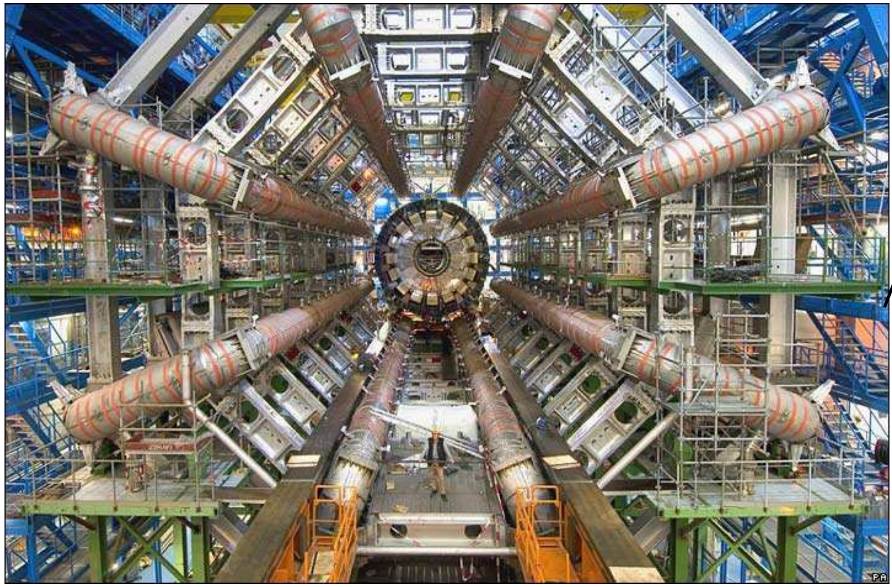


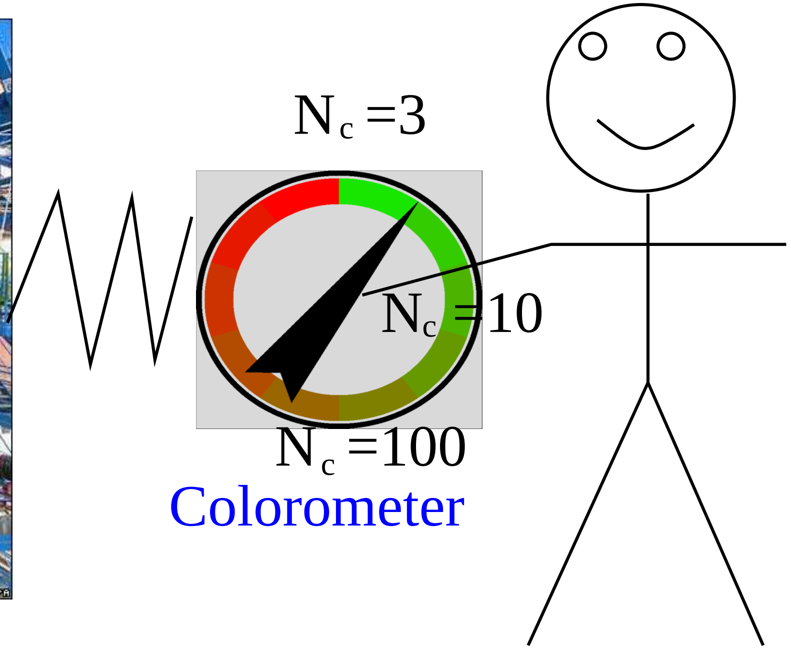
Integrated-out entropy $\sim N_c^0$ in our world but $\sim N_c$ in the large N_c world.
Quarkyonic matter is indeed liquid-gas at large N_c!



The real transition is in $T-N_c$ space

Experimental consequences of this conjecture

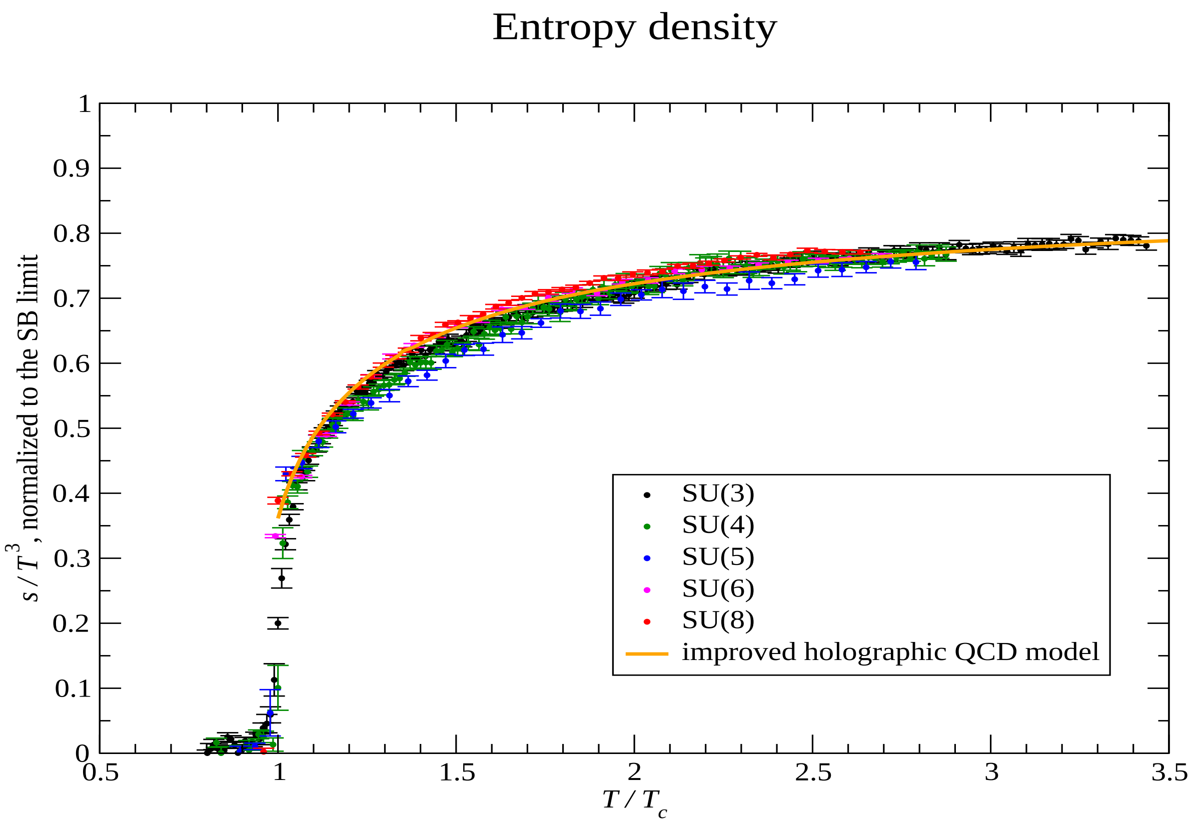




The kind of detector we need!!!! Indeed we have one, its called...

The lattice!

N_c convergence good even at $N_c = 3$ @$\mu_B = 0$



M.Panero

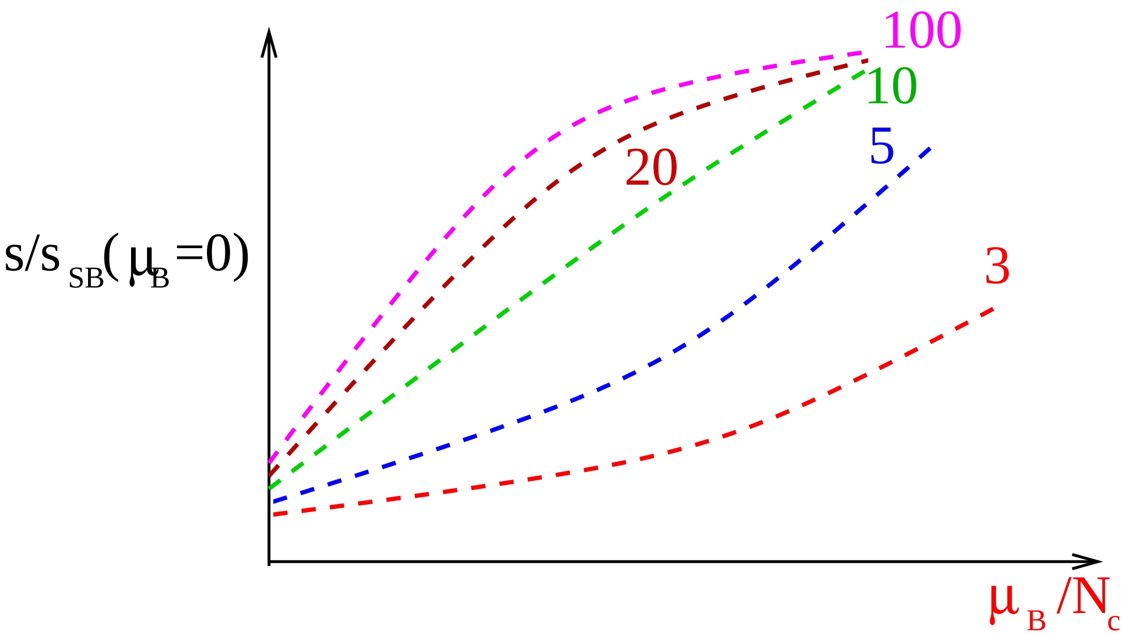
PRL 103, 232001 (2009)

Also

Bringoltz,Teper

PLB 628, 113 (2005)

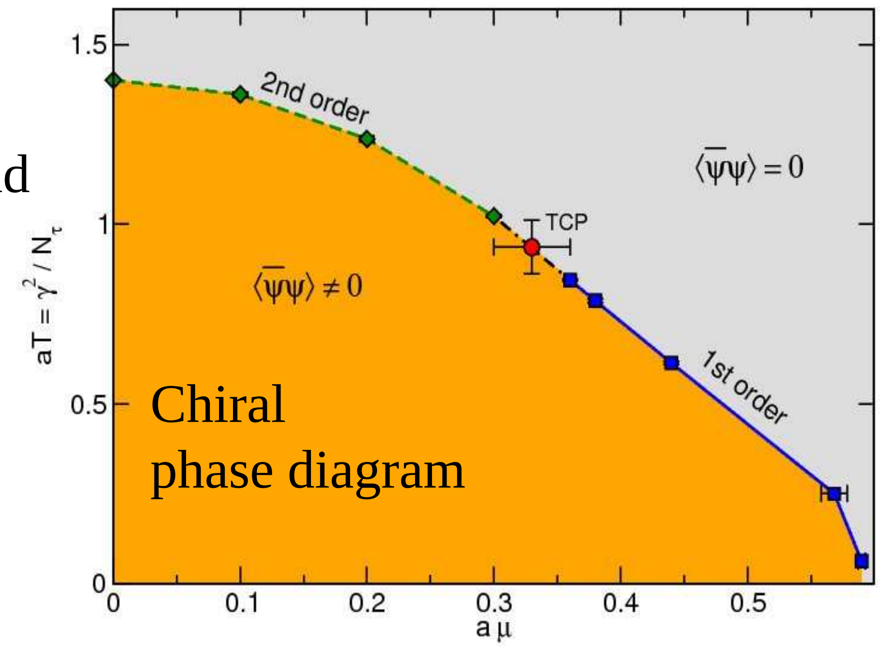
Prediction: At finite μ_B this convergence will break down around $N_c = 3$ but pick up around $N_c \sim k(d)$ Since mixed phase at $\mu \ll \Lambda_{QCD}$, this will be true even if $\mu_B/T < 1$ so one can Taylor-expand

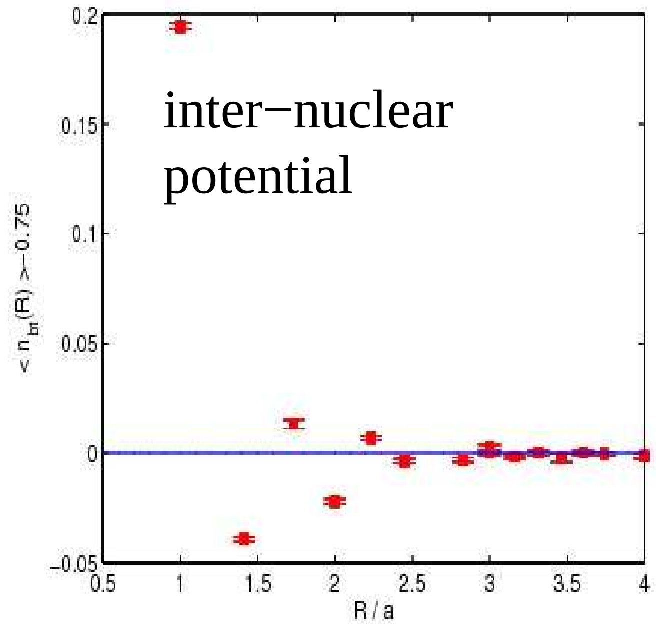


@$d = 3$ $(N_c^{crit} \sim 10)$ verifying this is a few years away. 1d QCD $(N_c^{crit} \sim 1$,so $N_c = 3$ large) and 2d QCD $(N_c^{crit} \sim 4)$, might be feasible now

Maybe results will come sooner than you think...

M.Fromm
P.DeForcrand
0912.2524
[PRL]





Strong coupling expansion has no sign problem. Strictly speaking, no continuum limit, but confinement, nuclear attraction and hard-core repulsion there!

Conclusions: the large N_c question

Is quarkyonic matter

A new phase to be looked for?

The large N_c limit of the old liquid gas phase?

the VdW universal model seems to suggest the second possibility. But further investigations needed:

Lattice Large N_c ,finite μ_B :Difficoult but not impossible

Experiment? Liquid-gas has been extensively studied, so Find that new phase?

A few words on chiral symmetry (Work in progress)

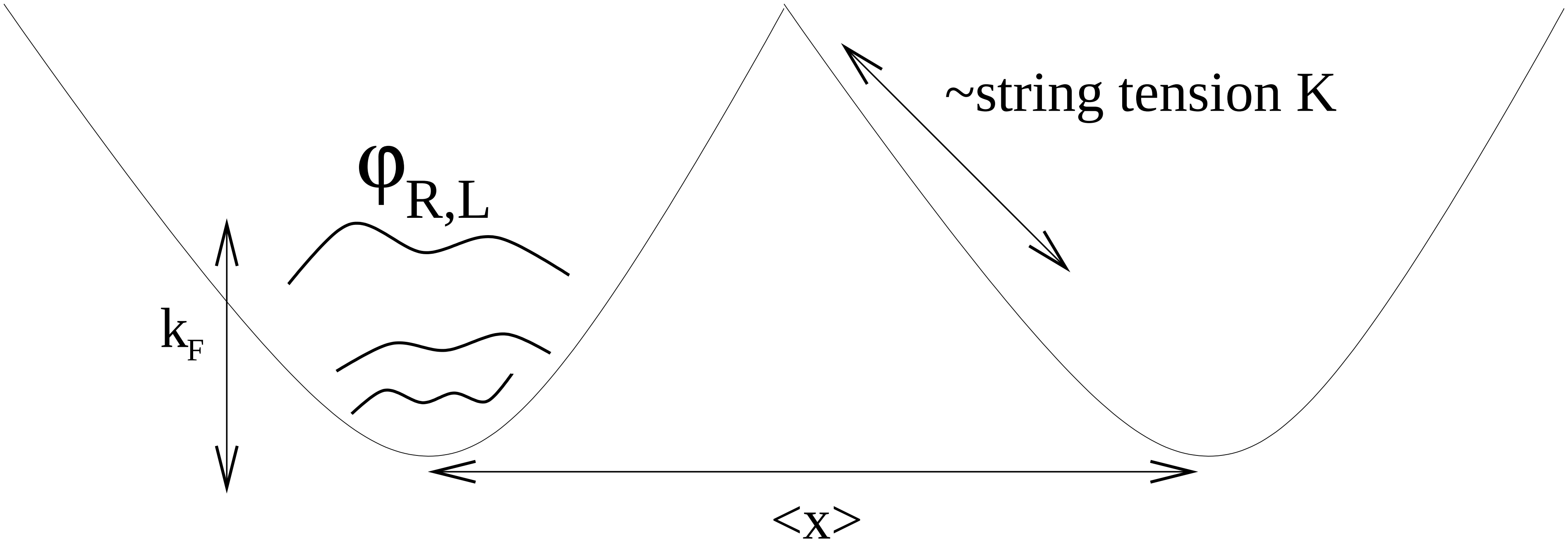
Casher's argument : (A.Casher,PLB **83**, 395 (1979)) A confining theory has to break chiral symmetry.

Implemented through quite sophisticated models (EFTs,Bethe-Salpeter), but origin very simple:

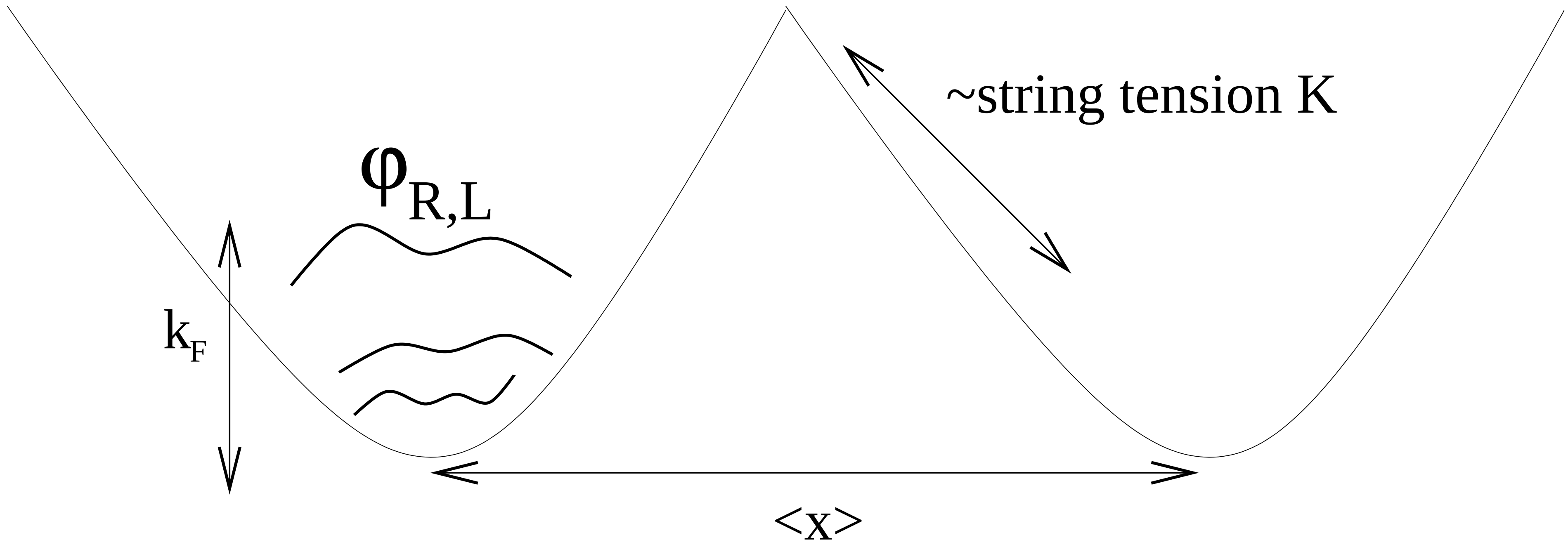
The solution of the Dirac equation in a confining potential has

$$\langle \psi_L \psi_R \rangle > 0$$

An exactly solvable model:



A periodic potential ("Mean field") confinement with N_c fermions ($\sim$ free quarks) per lattice site, in a "confining" periodic potential, $V(x) = K\,|x|$
Solve Dirac equation (or free Dirac field) in this potential



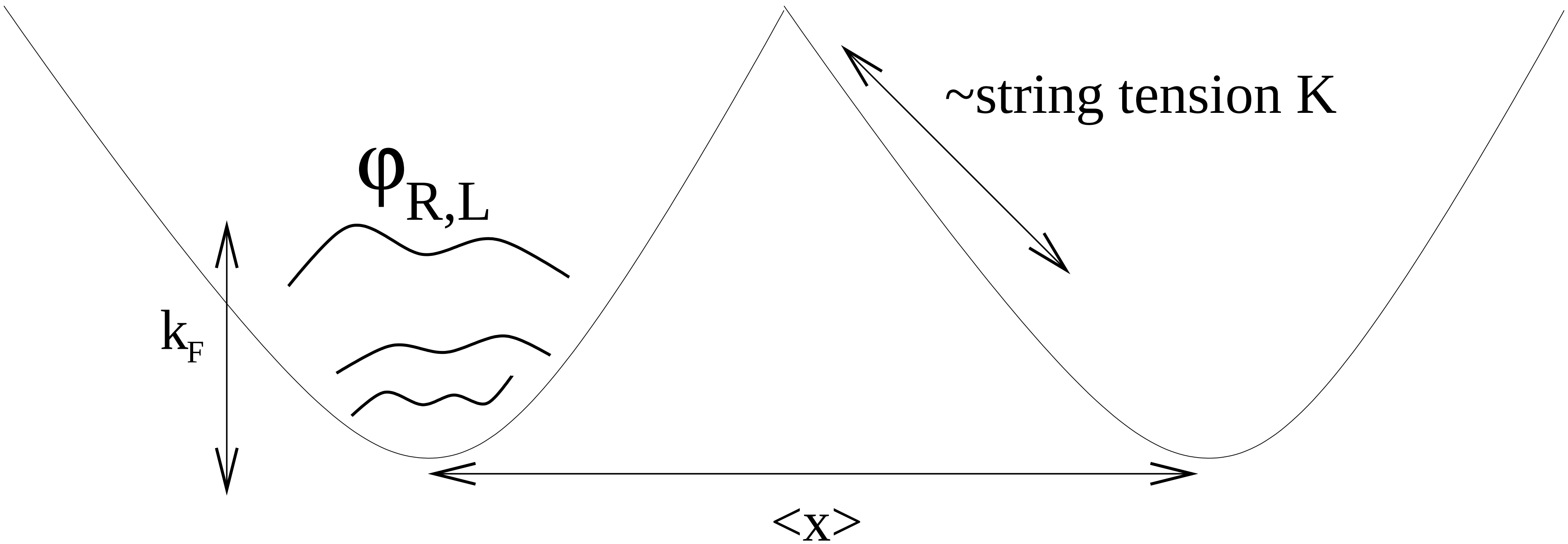
This model is of course very rough, as it misses the correlation of "free" quarks due to confinement (the requirement that color-charged currents vanish at scales above the hadron size).
BUT such corrections can only increase $\langle \psi_L \psi_R \rangle$ So the model provides a lower limit!

Qualitative physics is an interplay of d, K and k_F

$k_F \ll K \left|\langle x \rangle\right|$ Standing waves in lattice sites, quarks confined, $\langle \psi_L \psi_R \rangle \gg k_F$ chiral symmetry broken for "all quarks"

$k_F \gg K \left|\langle x \rangle\right|$ "conduction band" of quasi-massless quarks, for which chiral symmetry restored

Of course $\langle x \rangle, K, k_F$ determined by N_c as per our previous discussion



if $\langle x \rangle$ given by VdW formulae, suspect chiral symmetry broken at large N_c only. Work in progress!!