Shear viscosity of the Quark-Gluon Plasma from a virial expansion

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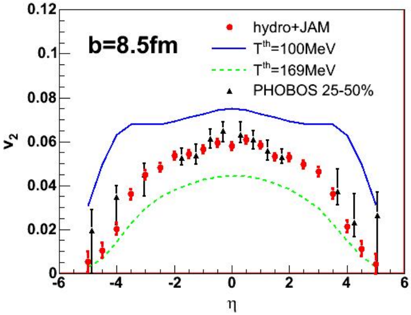
Experiments and shear viscosity

- small viscosity η from elliptic flows at RHIC



- almost perfect fluid / strong coupling QGP
- universal lowest bound $\eta/s \geq 1/4\pi$

- Ratio of viscosity to entropy density

$$\frac{\eta}{s} = \frac{4}{5}\frac{T}{s\sigma_{\mathrm{t}}}$$

- Transport cross section

$$\sigma_{\mathrm{t}}(\hat{s}) = \sigma_0 4z(1+z)\left[(2z+1)\ln(1+1/z) - 2\right], \quad z \equiv \mu_{\mathrm{scr}}^2/\hat{s}$$

- Debye mass

$$\mu_{\mathrm{scr}}^2 = 4\pi\alpha_{\mathrm{s}} T^2$$

- Inputs:
  - Equation of state
  - temperature dependent coupling constant α_{s}

- Our approach:
  - Derivation of a realistic Equation of State starting from a specific interaction (S.M and W. Cassing, J. Phys. G36, 125003, 2009)
  - Derivation of the temperature dependent coupling constant α_{s} from this interaction

Theory

- Partition Function

$$\ln Z = d \int_V d^3\mathbf{r} \int d^3\mathbf{p}\, \eta \log\left(1+\eta \mathrm{e}^{-\beta H(\mathbf{r},\mathbf{p})}\right), \qquad \eta \pm 1$$

- Interaction free: Stefan Boltzmann limit $\ln Z \sim T^3$
- For interacting systems?

- N-body Hamiltonian

$$
\begin{aligned}
H_1(\mathbf{r_1},\mathbf{p_1}) &= E_{\mathrm{k}}(p_1) \\
H_2(\mathbf{r_1},\mathbf{r_2},\mathbf{p_1},\mathbf{p_2}) &= E_{\mathrm{k}}(p_1) + E_{\mathrm{k}}(p_2) + V_{12} \\
H_3(\mathbf{r_1},\mathbf{r_2},\mathbf{r_3},\mathbf{p_1},\mathbf{p_2},\mathbf{p_3}) &= E_{\mathrm{k}}(p_1) + E_{\mathrm{k}}(p_2) \\
&\quad + E_{\mathrm{k}}(p_3) + V_{123} \\
&\dots
\end{aligned}
$$

- Virial expansion

$$Z = \sum_{\nu=0}^{\infty} \frac{b_\nu}{\nu!}\zeta^\nu$$

- Coefficients

$$b_1 = 1 \qquad b_2 = \int_V d^3\mathbf{r_2}\ \left(\mathrm{e}^{-\beta V_{12}} - 1\right)$$

Thermodynamics

- Pressure

$$P = T \ln Z = T \sum_{\nu=1}^{\infty} \frac{b_\nu}{\nu!} \zeta^\nu$$

- Further

$$\begin{aligned} \varepsilon &= -\frac{\partial \ln Z}{\partial \beta} \\ s &= \frac{\partial P}{\partial T} \\ c_{\mathrm{s}}^2 &= \frac{dP}{d\varepsilon} = \varepsilon \frac{dP/\varepsilon}{d\varepsilon} + \frac{P}{\varepsilon} \end{aligned}$$

Confined phase

- Good description by phenomenological resonance-gas (1026 resonance)
- Generalized resonance gas in Boltzmann limit

$$\ln Z(V,T) = \sum_{i=1} \frac{VTm_i^2}{2\pi^2}\rho(m_i)K_2\left(\frac{m_i}{T}\right)$$

- Hagedorn hypothesis

$$\rho(m) = f(m)\exp\{m/T_{\mathrm{H}}\} \qquad f(m) = am^{-\alpha}, \quad \alpha = 3/2$$

Deconfined phase

- Potential including non perturbative effects of dimension two condensate

$$V(r,T) = \left(\frac{\pi}{12}\frac{1}{r} + \frac{\mathcal{C}_2}{2N_\mathrm{c}T}\right)\mathrm{e}^{-M(T)r}$$

- Debye mass

$$M(T) = \sqrt{N_\mathrm{c}/3 + N_\mathrm{f}/6}\, gT = \tilde{g}\,T$$

- Fit in gluondynamics

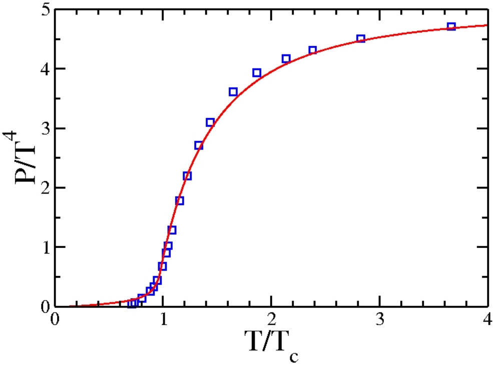
$$\mathcal{C}_2 = (0.9\mathrm{GeV})^2 \qquad \tilde{g}_{\mathrm{PL}} = 1.26$$

- Confined phase: resonace-gas

$$\ln Z(V,T) = aV\left(\frac{T}{2\pi}\right)^{3/2}\frac{\exp\{-m_0 b\}}{b}, \quad b = \frac{1}{T} - \frac{1}{T_{\mathrm{H}}}$$

- Deconfined phase: virial expansion with $\tilde{g}$ free parameter
- Calculation of P, c_s and $W = \varepsilon - 3P$

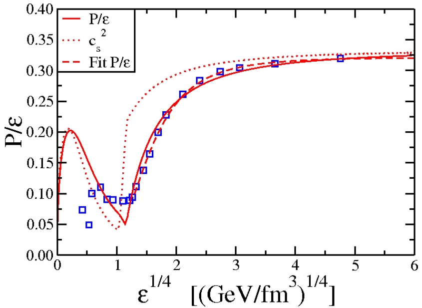
Pressure



Good agreement for $\tilde{g} = 1.30$

Lattice data: M. Cheng *et al.*, Phys. Rev. D77, 014511, 2008

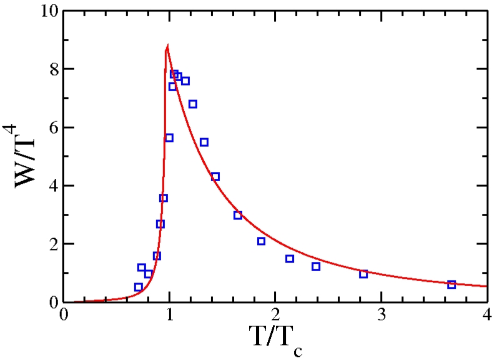
Sound velocity



- Comparison with the *Ansatz*:

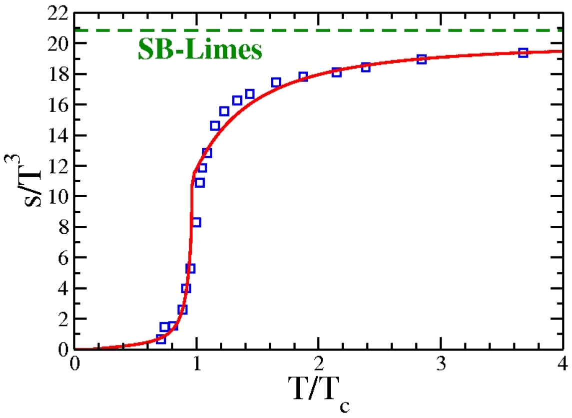
$$\frac{P}{\varepsilon} = \frac{1}{3}\left(C - \frac{A}{1 + B\varepsilon\ \mathrm{fm}^3/\mathrm{GeV}}\right)$$

Interaction measure



Gluondynamics fit: agreement for $1.26 \leq \tilde{g} \leq 1.45$

Entropy density



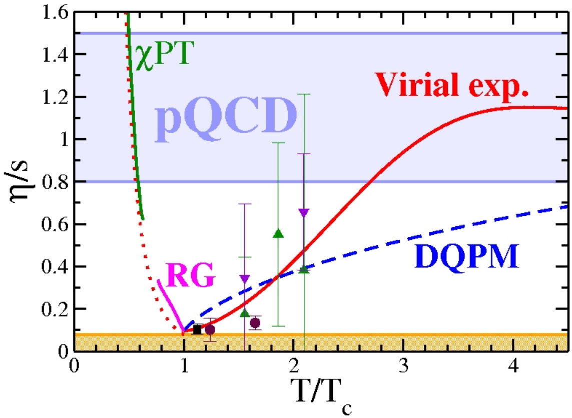
- Sizeble deviation at the critical temperature

- Derivation of the coupling

$$\alpha_{\mathrm{V}}(T) \equiv \alpha_{\mathrm{qq}}(r_{\mathrm{max}}, T), \qquad \alpha_{\mathrm{qq}}(r, T) \equiv -\frac{12}{\pi} r^2 \frac{\mathrm{d}V_{12}(r, T)}{\mathrm{d}r}$$

- Ratio viscosity to entropy density

$$\frac{\eta}{s_{\mathrm{q}}} = \frac{4}{5} \frac{T}{s_{\mathrm{q}} \sigma_{\mathrm{t}}}, \qquad s_{\mathrm{q}} = s - s_{\mathrm{g}}$$



- Minimum of η/s near to the critical temperature
- Very good agreement with the experimental and theoretical expectations

- Summary
  - Kinetic theory to calculate the viscosity of QGP
  - Virial expansion:
    - Systemtic derivation of a realistic EoS
    - Consistent dynamical calculation of η/s
- Outlook
  - Calculation of the bulk viscosity ζ
  - Hydrodynamical evolution
  - Molecular dynamics and transport calculation

Further...

Finite density

Deconfined phase

- Dependence on the chemical potential in the Debye mass

$$M(T,\mu_q) = M(T,\mu_q = 0) F\left(\frac{\mu_q}{T}\right)$$

- Modification from perturbation theory

$$F_{\mathrm{PT}}\left(\frac{\mu_q}{T}\right) = \sqrt{1 + \frac{3N_{\mathrm{f}}}{(2N_{\mathrm{c}} + N_{\mathrm{f}})\pi^2}\left(\frac{\mu_q}{T}\right)^2}$$

- Generalization

$$F\left(\frac{\mu_q}{T}\right) = A\left(\frac{\mu_q}{T}\right) F_{\mathrm{PT}}\left(\frac{\mu_q}{T}\right)$$

$$\text{with} \qquad A\left(\frac{\mu_q}{T}\right) = 1 + \frac{1}{\pi^2}\left(\frac{\mu_q}{T}\right)^2$$

Confined phase

- Dependence on the chemical potential in the constant

$$a \longrightarrow a^* \left(\frac{\mu_B}{T}\right)$$

- Using the modification from perturbation theory

$$a^* \left(\frac{\mu_B}{T}\right) = a \, F_{\mathrm{PT}} \left(\frac{\mu_B}{T}\right)$$

$$\text{with} \qquad \mu_B = 3\mu_q$$

Virial expansion vs. Lattice

- Scaled pressure difference

$$\frac{\Delta P(T,\mu_q)}{T^4} = \frac{P(T,\mu_q) - P(T,\mu_q = 0)}{T^4}$$

- Scaled quark number density

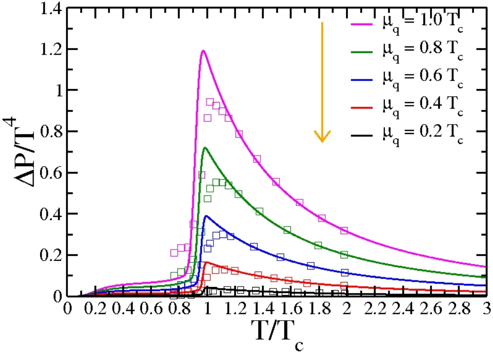
$$\frac{n_q(T,\mu_q)}{T^3} = \frac{1}{T^3}\frac{\partial P(T,\mu_q)}{\partial \mu_q} = \frac{1}{T^4}\frac{\partial P(T,\mu_q)}{\partial(\mu_q/T)}$$

- Lattice results via Taylor expansion

$$\frac{P(T,\mu_q)}{T^4} = \frac{\ln Z(T,\mu_q)}{T^3} = \sum_{n=0}^{\infty} c_n(T)\left(\frac{\mu_q}{T}\right)^n$$

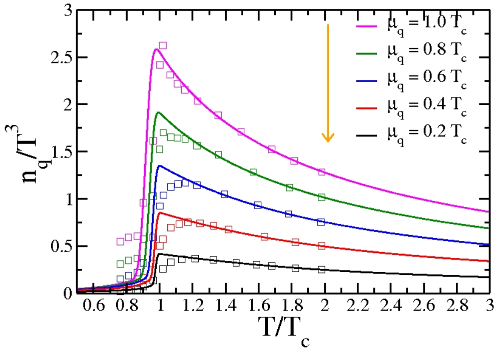
$$\text{with} \qquad c_n(T) = \frac{1}{n!\,T^3}\frac{\partial^n \ln Z}{\partial(\mu_q/T)^n}$$

Pressure difference



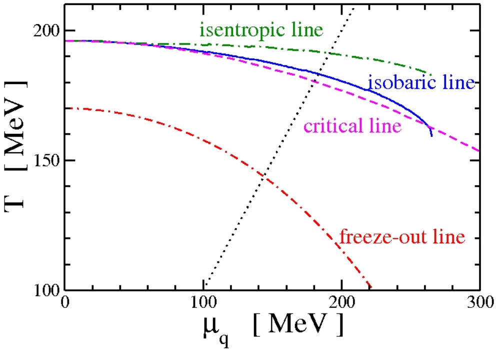
Overestimation at T_c

Quark number density



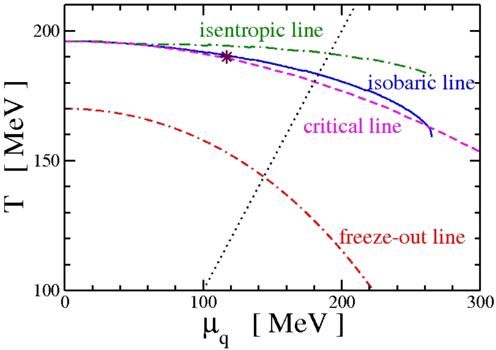
Overestimation at T_c becomes smaller

Phase diagram



isobaric line agrees with the PT from the lattice

Critical Endpoint



- Crossover parametrized by:

$$T/T_c = 1 - \tilde{C}\mu_q^2/T_c^2$$

Equation of state

- Pressure difference vs. quark density

$$\Delta P \qquad \text{vs.} \qquad n_{\mathrm{q}}$$

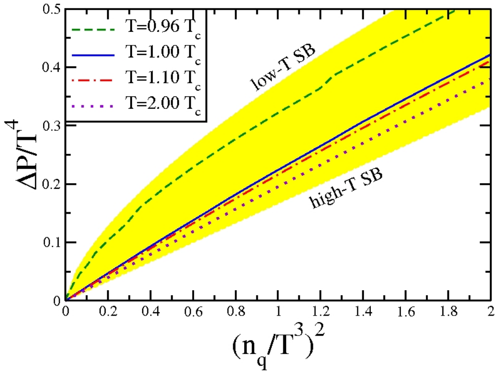
- Low-T SB limit

$$\Delta P = \frac{1}{4}(\pi^2/N_{\mathrm{f}})^{1/3} n_{\mathrm{q}}^4$$

- High-T SB limit

$$\Delta P = \frac{1}{2N_{\mathrm{f}}} n_{\mathrm{q}}^2$$

EoS



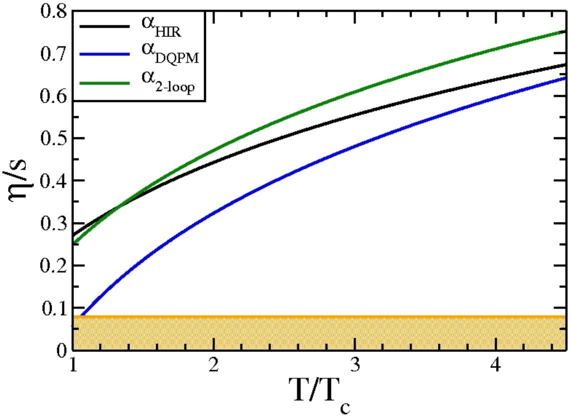
Change of the functional dependence on the temperature

weak QGP

- Stefan-Boltzmann equation of state

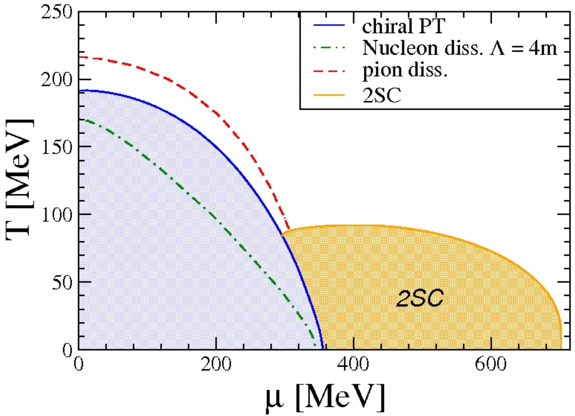
$$\left(\frac{\eta}{s}\right)_{\mathrm{SB}} = \frac{3}{5}\frac{T}{\sigma_{\mathrm{t}}}\frac{1}{K_{\mathrm{SB}}\,T^3}, \qquad K_{\mathrm{SB}} = \frac{\pi^2}{30}\left[2(N_{\mathrm{c}}^2-1)+\frac{7}{8}12N_{\mathrm{f}}\right]$$

- Parametrization for $\alpha_{\mathrm{s}}(T)$:
  - Hirano
    $\alpha_{\mathrm{HIR}}(T) = 4\pi/[18\ln(4T/T_{\mathrm{c}})]$
  - Quasiparticle model (DQPM)
    $\alpha_{\mathrm{DQPM}}(T) = \frac{12\pi}{11N_{\mathrm{c}}\ln(\lambda^2(T/T_{\mathrm{c}}-T_{\mathrm{s}}/T_{\mathrm{c}}))}$
  - Two-loop coupling constant
    $\alpha_{2-\mathrm{loop}}(T) = \frac{1}{4\pi}\frac{1}{2\beta_0\ln\left(\frac{\mu T}{\Lambda_{\overline{\mathrm{MS}}}}\right)+\frac{\beta_1}{\beta_0}\ln\left(2\ln\left(\frac{\mu T}{\Lambda_{\overline{\mathrm{MS}}}}\right)\right)}$



- Bad description of η/s
- HERE: EoS and interaction fully uncorrelated!

to David's question ...



- (Dynamically calculated) correlations are important!!