Quarkyonic Chiral Spirals

- Chiral Symmetry in Quarkyonic Matter

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T.K., Y. Hidaka, L. McLerran, R. D. Pisarski ; NPA 843:37-58, 2010.

T.K., R. Pisarski, A.M. Tsvelik ; arXiv:1007.0248 [hep-ph]

Contents

1. Quarkyonic matter: Basics

$1/N_c = 1/3$ expansion

A leading order gives a reasonable portrait of QCD:
(large Nc)

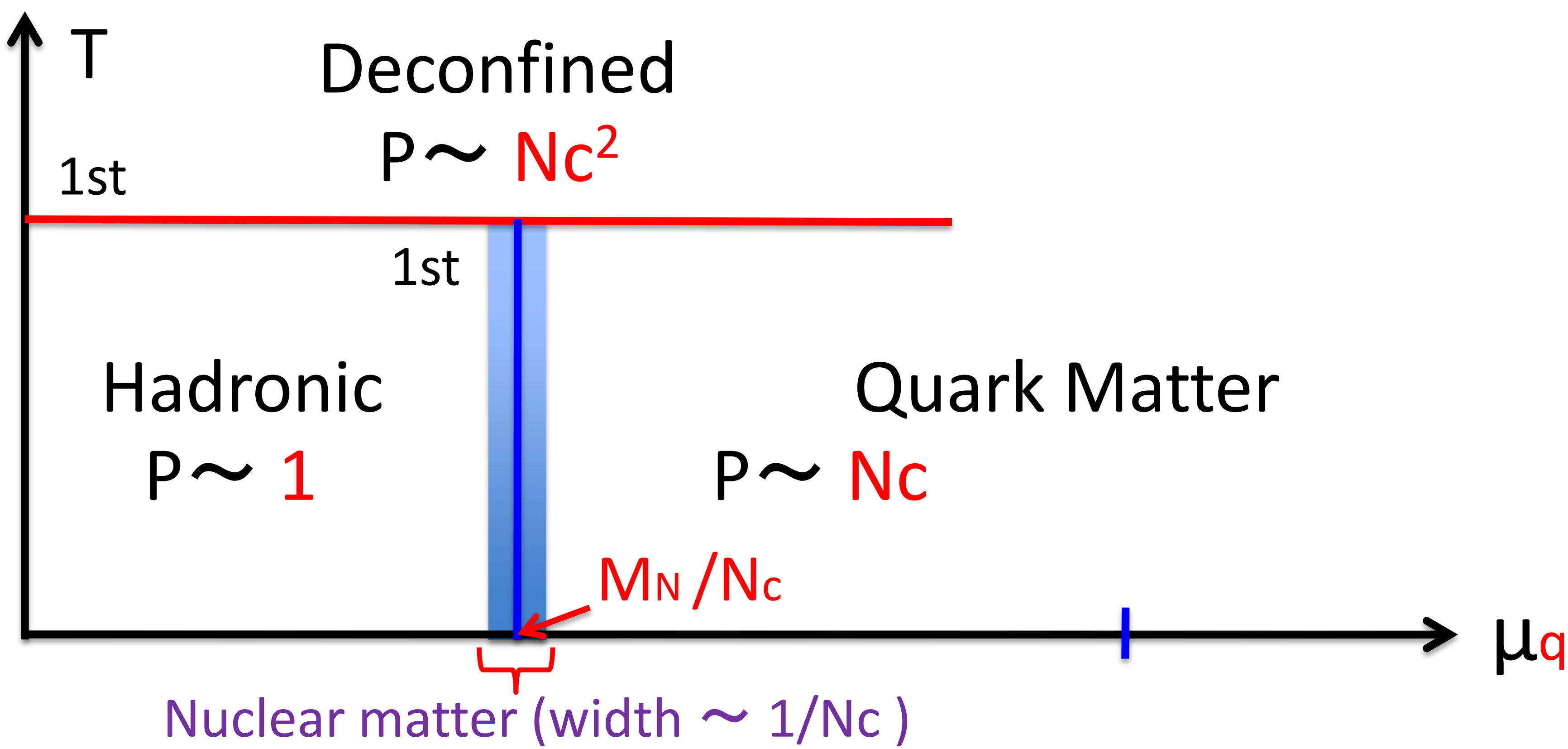
- Exotic hadrons such as multiquarks are not conspicuous.
- OZI suppression (even for low energy)
- Mesons can be described as quasi particles: $\Gamma/M \sim 1/N_c$
- Quenched approximation in lattice QCD

Relevant fact for our arguments is:

Quantum fluctuations $\begin{cases} \text{gluon sector} & \sim O(N_c^2) \\ \text{quark sector} & \sim O(N_c) \end{cases}$

→ Quarks modify gluon sector only in very high density.

Large Nc Phase Diagram : McLerran & Pisarski (2007)



Change from Nuclear matter to Quark matter occurs rapidly.

small change in μ_q → large change in k_F thus n_B

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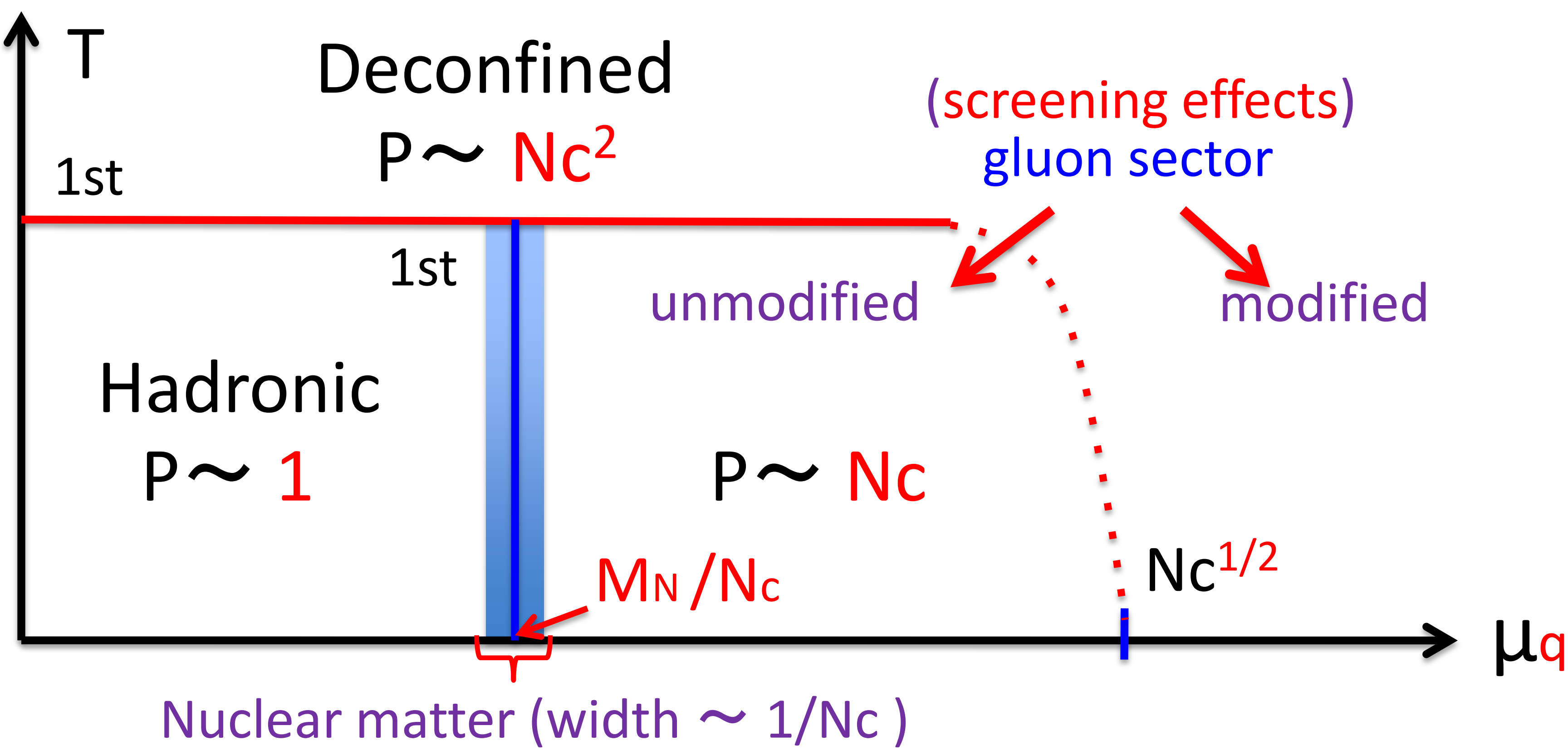

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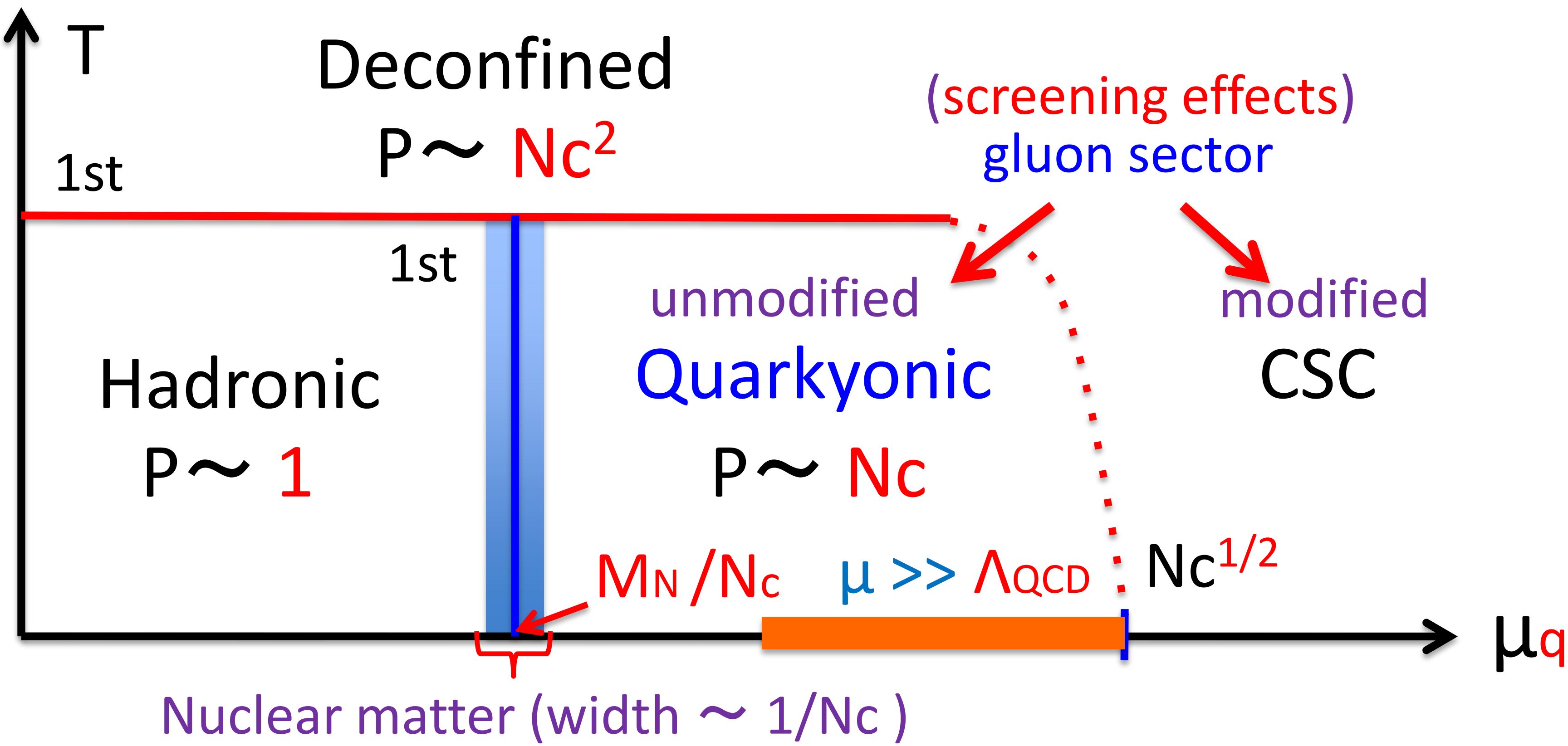


Change from Nuclear matter to Quark matter occurs rapidly.

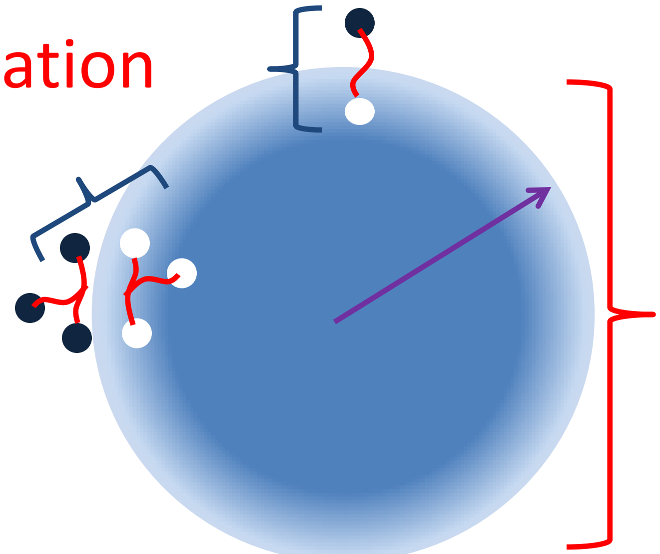
small change in μ_q → large change in k_F thus n_B

Quarkyonic Matter

(For theoretical foundation, see lecture on Thursday)

hadronic excitation

- Transport properties
- Phase structures (condensation)
- etc.



Bulk properties:
(EOS, Pressure, etc.)

perturbative quarks
(forming color singlet B.G.)

Quark Fermi sea + baryonic Fermi surface → Quarkyonic
(hadronic)

- Large Nc: screening by quarks : $M_D \sim N_c^{-1/2} \to 0$
  → vacuum gluon propagator unchanged.

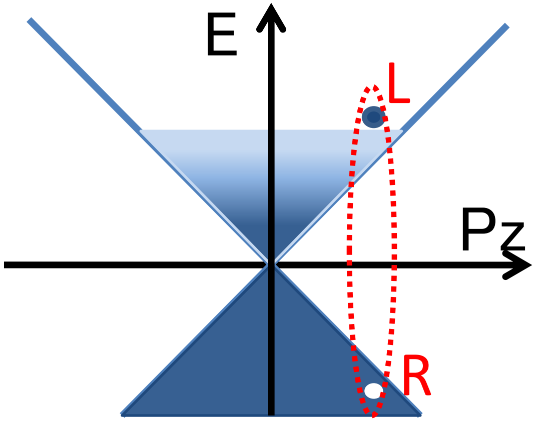
Quarkyonic regime holds for $\mu_q \sim O(1)$.

2. Quarkyonic Chiral Spirals

How is Chiral Sym. realized ?

- Candidates which spontaneously break Chiral Symmetry

Dirac Type

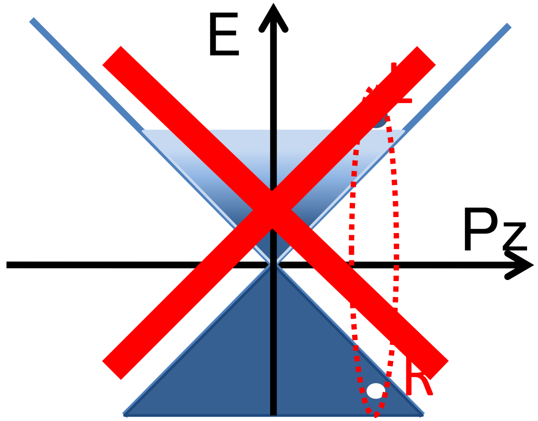


P_{Tot}=0 (uniform)

How is Chiral Sym. realized ?

▪Candidates which spontaneously break Chiral Symmetry

Dirac Type



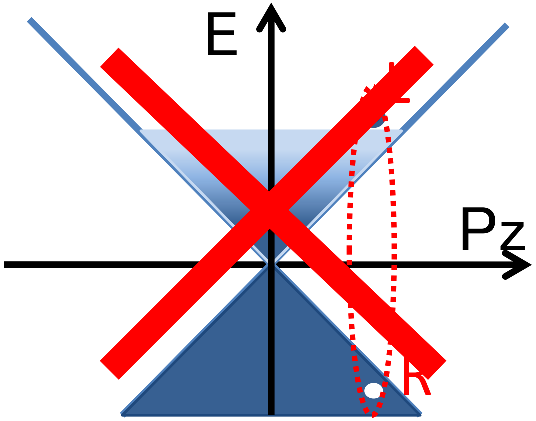
$P_{Tot}=0$ (uniform)

It costs large energy,
so does not occur spontaneously.

How is Chiral Sym. realized ?

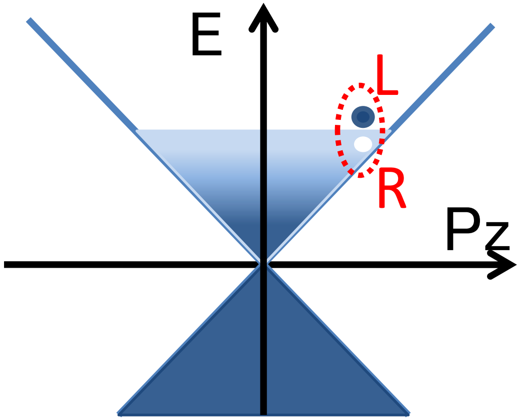
▪ Candidates which spontaneously break Chiral Symmetry

Dirac Type



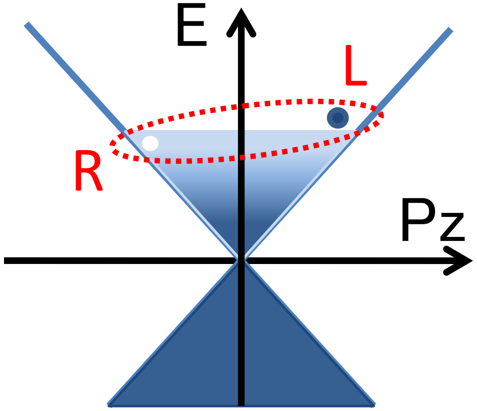
$P_{Tot}=0$ (uniform)

Exciton Type

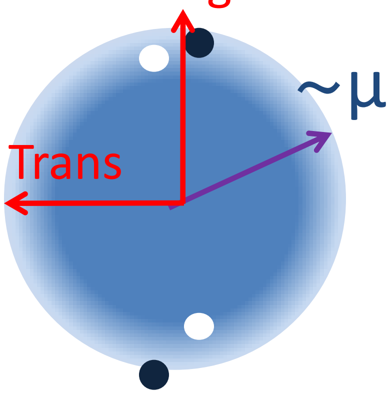


$P_{Tot}=0$ (uniform)

Density wave



$P_{Tot}=2\mu$ (nonuniform)



We will identify the most relevant pairing:
Exciton & Density wave solutions
will be treated and compared simultaneously.

A simple model of linear confinement

- Confining propagator for quark-antiquark (quark-hole):

$$D_{\mu\nu} = C_F \times g_{\mu 0} g_{\nu 0} \times \frac{\sigma}{(\vec{p}^2)^2}$$ (linear rising type)

strong IR enhancement

cf) leading part of Coulomb gauge propagator (ref: Gribov, Zwanziger)

- Absence of $q\bar{q}$ continuum in mesonic channel

$\longrightarrow$ linear confinement

- We will apply nonperturbative treatments:

Schwinger-Dyson & Bethe-Salpeter equations.

- We dimensionally reduce these from (3+1)D to (1+1)D.

(Pert. regime; Deryagin-Grigoriev-Rubakov '92, Shuster-Son 99, etc)

On the IR prescription

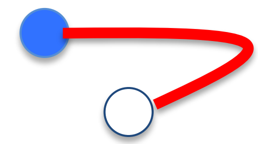
linear potential

$$\frac{\sigma}{(\vec{k}^2)^2} \xrightarrow{\text{IR cut}} \frac{\sigma}{(\vec{k}^2+\Lambda_{\rm IR}^2)^2} \xrightarrow{\text{F.T.}} \underline{-\frac{\sigma}{\Lambda_{\rm IR}}} + \sigma r + O(\Lambda_{\rm IR} r^2)$$

▪Probe colored objects:

IR div.: const. from naïve IR cutoff

▪Color singlet sector:



IR const. → irrelevant.
(Linear conf. without IR const.)

▪As far as color-singlet sector is concerned,
we can get the same results even if we drop off div. const.
(principal value IR regulation; e.g., Coleman, Aspects of Symmetry)

▪S-D eqs. → just sub-diagrams in B-S eqs.

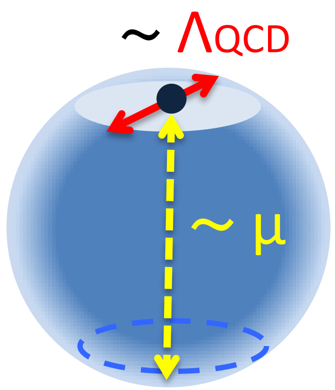
▪Div. of poles will be used as color selection rules at best.

Dim. reduction of integral eqs.

1, Virtual flucts. are limited within small mom. domain.

2, Quark energies are insensitive to small ΔkT.

(due to flatness of Fermi surface in trans. direction)



e.g.) Schwinger-Dyson eq.

insensitive to kT

$$\not{\Sigma}(p) + \Sigma_m(p) = \int \frac{dk_4 dk_z d^2\vec{k}_T}{(2\pi)^4}\gamma_4 S(k)\gamma_4 \frac{\sigma}{|\vec{p}-\vec{k}|^4}$$

$$\xrightarrow{\text{factorization}} \int \frac{dk_4 dk_z}{(2\pi)^2}\gamma_4 S(k_4, k_z, \underline{\vec{0}_T})\gamma_4 \otimes \underbrace{\int \frac{d\vec{k}_T}{(2\pi)^2}\frac{\sigma}{|\vec{p}-\vec{k}|^4}}_{\text{smeared gluon propagator}}$$

- At leading order:

Dimensional reduction of Non-pert. self-consistent eqs :

4D “QCD” in Coulomb gauge ⟷ 2D QCD in A_1=0 gauge

(confining model)

Dictionary: μ = 0 & μ ≠ 0 in (1+1)D

- μ ≠ 0 2D QCD can be mapped onto μ = 0 2D QCD

$$\Phi = \exp\left(-i\mu z\Gamma^5\right)\Phi' \quad \text{: Chiral rotation}$$

(Opposite shift of mom. for (+, -) moving states)

$$\boxed{\overline{\Phi}\left[i\Gamma^\mu\partial_\mu + \mu\Gamma^0\right]\Phi \to \overline{\Phi}' i\Gamma^\mu\partial_\mu\Phi'}$$

(μ ≠ 0) (μ = 0)

(due to special geometric property of 2D Fermi sea)

- Dictionary between μ = 0 & μ ≠ 0 condensates:

μ = 0 μ ≠ 0

$$\langle\overline{\Phi}'\Phi'\rangle \rightarrow \cos(2\mu z)\langle\overline{\Phi}\Phi\rangle - \sin(2\mu z)\langle\overline{\Phi}i\Gamma^5\Phi\rangle$$

$$\underbrace{\langle\overline{\Phi}'\Gamma_0\Phi'\rangle}_{(=0)} \rightarrow \underbrace{\langle\overline{\Phi}\Gamma_0\Phi\rangle}_{(=0)} + \underline{\frac{\mu}{2\pi}}$$

induced by anomaly

"correct baryon number"

Why Chiral Spirals in (1+1)D ?

- Key observation: Moving direction = (1+1)D Chirality

$$\langle \bar{\varphi}_- \varphi_+ \rangle = \Delta e^{2i\mu z} \qquad \langle \bar{\varphi}_+ \varphi_- \rangle = \Delta e^{-2i\mu z}$$

Opposite phase

$$\boxed{\langle \bar{\varphi} \Gamma_5 \varphi \rangle = \langle \bar{\varphi}_- \varphi_+ \rangle - \langle \bar{\varphi}_+ \varphi_- \rangle = \Delta i \sin 2\mu z \neq 0}$$

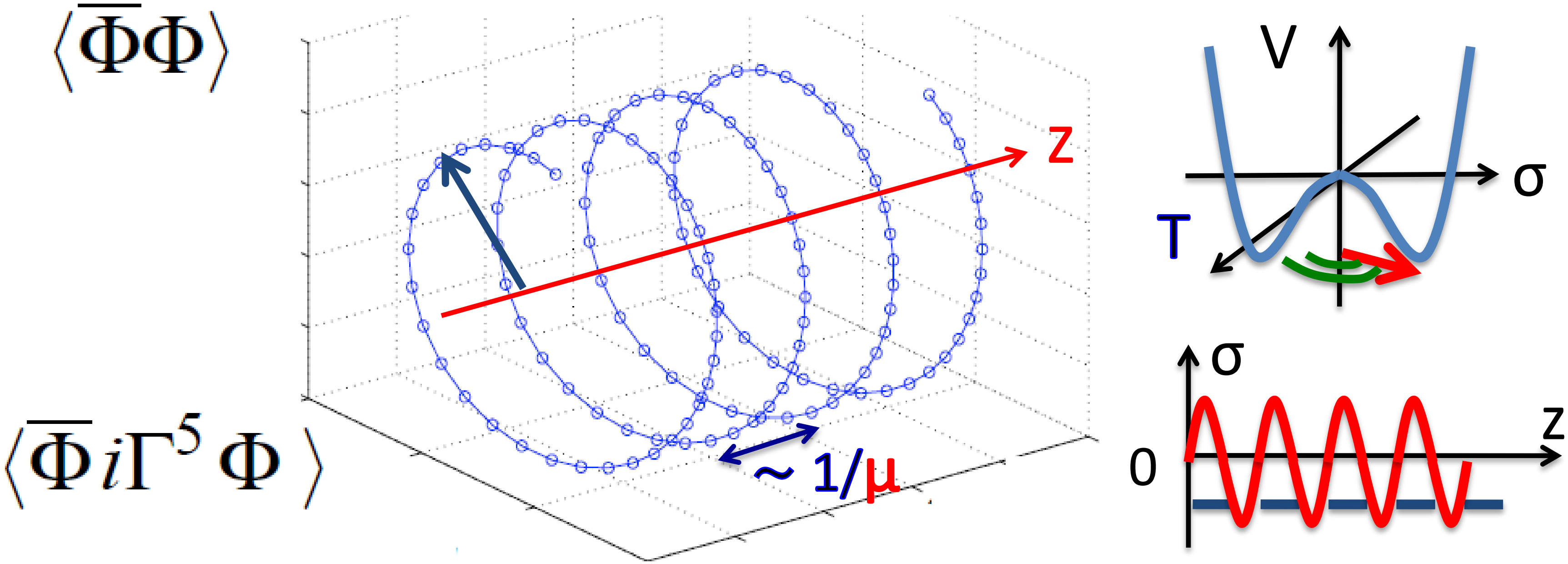
Density wave of $\bar{\Phi}\Phi$ inevitably accompanies $\bar{\Phi} i \Gamma^5 \Phi$

(because of phase mismatch)

(1+1)D: Chiral Density wave → Chiral Spiral

Solutions: Chiral Spirals in (1+1)D

- At μ ≠ 0: periodic structure (crystal) which oscillates in space.



cf) - Chiral Gross Neveu model (with continuous chiral symmetry)

Schon & Thies, hep-ph/0003195; 0008175; Thies, 06010243

Basar & Dunne, 0806.2659; Basar, Dunne & Thies, 0903.1868

- `tHooft model, massive quark (1-flavor)

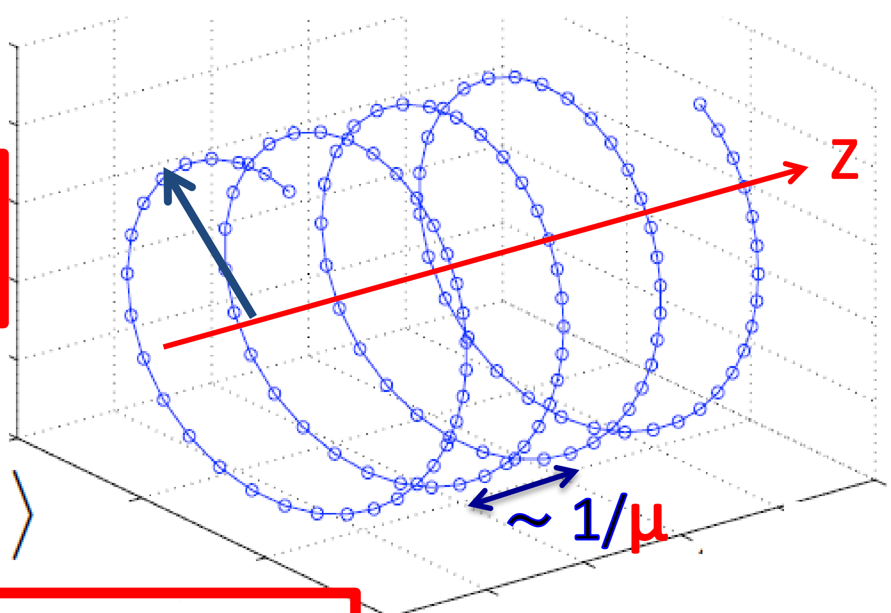
B. Bringoltz, 0901.4035

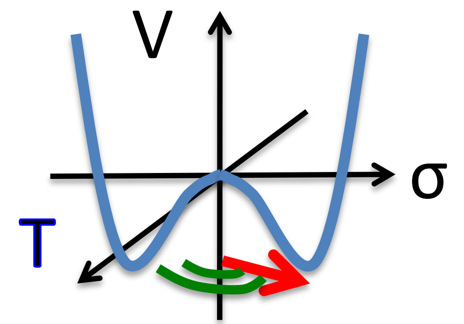
Quarkyonic Chiral Spirals in (3+1)D

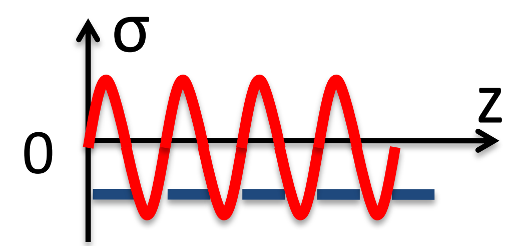
- Chiral rotation evolves in the longitudinal direction:

$\langle \bar{\Phi}\Phi \rangle$

$\boxed{\langle \bar{\psi}\psi \rangle}$







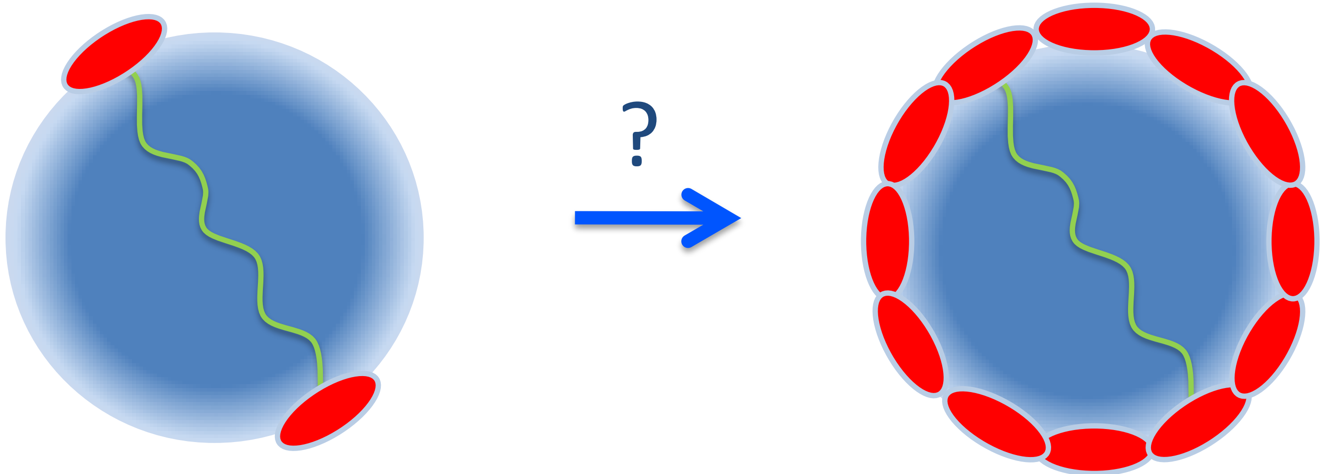
$\langle \bar{\Phi} i\Gamma^5 \Phi \rangle$

$\boxed{\langle \bar{\psi}\gamma^0\gamma^z\psi \rangle}$

(characterized by moving direction)

- Quarkyonic limit:
  - Baryon number is spatially constant.
  - No other condensates.

3. Quarkyonic Chiral Crystals

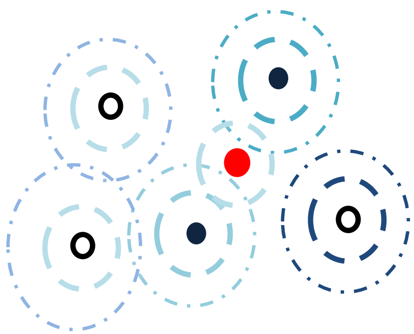


Toward multiple patch construction. 1

One patch results may be good starting point.

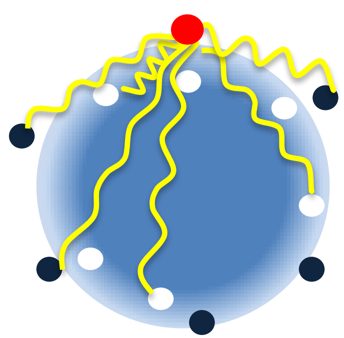
Perturbative gluons

- r - space)



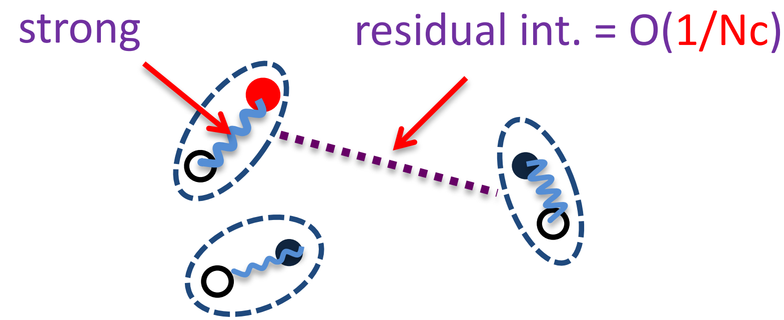
Influence by all other quarks must be treated simultaneously.

- p - space)

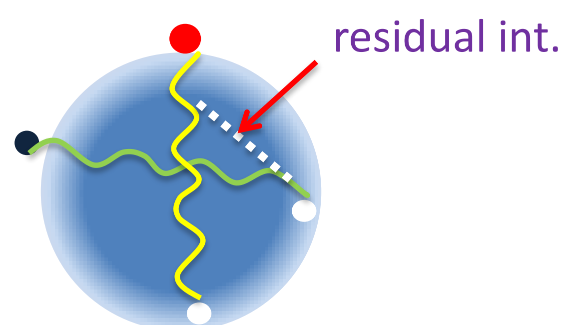


Gap → strongly density dependent.

Confining gluons



After mesonic objects are formed, residual interactions enter.



Gap → weakly density dependent.
(confinement - origin)

Toward multiple patch construction. 2

- e.g.) Quark-Condensate int. in the presence of many QCSs

Sum over all Chiral spirals

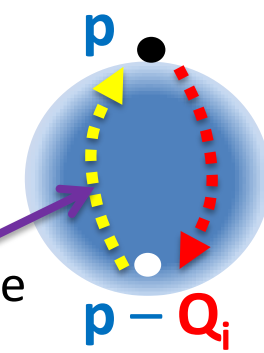
$$\sum_{i=1}^{N_p} \int \frac{d^4 p}{(2\pi)^4} \bar{\psi}(p - \boldsymbol{Q}_i) M(\underline{p;} \boldsymbol{Q}_i) \psi(p)$$

Space-dependent mass self-energy

- Key point: Quarks with high virtuality feel small Chiral Sym. breaking

For both of p^2 and $(p - Q_i)^2$ to be close to Minkovski region:

Angle between **p** and $\mathbf{Q_i}$ ⟶ $|\theta| < \Lambda_{\rm QCD}/p_F$



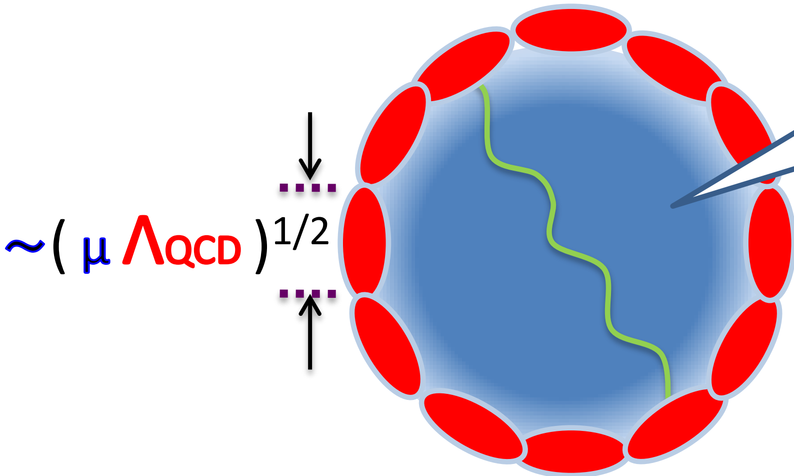
e.g.) θ ~ 0 case

If angles between quark moving direction and QCS are large:

⟶ Chirality changing scatterings are suppressed.

⟶ Each QCS behaves incoherently (except matching point of patches)

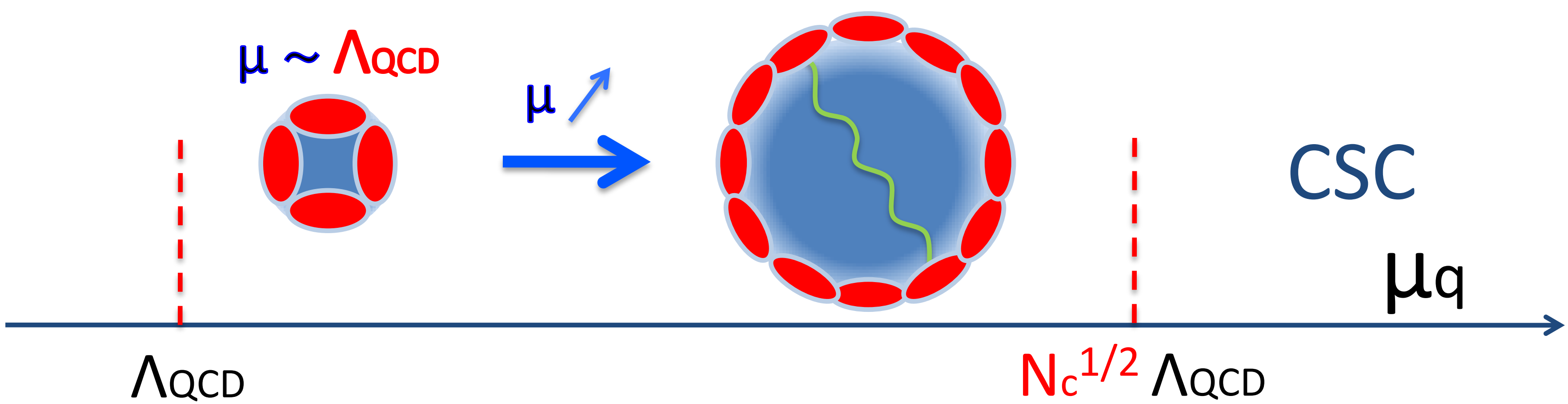
Summary



- **Deep inside:**
  Very likely Chiral sym. restored

- **Near Fermi surface:**
  Quarkyonic Chiral Spirals
  → Local violation of P & Chiral sym.

- **Number of Patches** $\sim \mu^2 / \mu \Lambda_{QCD} \sim \mu / \Lambda_{QCD}$
  total surface area (μ^2), 1-patch area ($\mu \Lambda_{QCD}$)



Appendix

Quarkyonic Chiral Spirals vs …..

1, Perturbative gluon propagator : Deryagin, Grigoriev, & Rubakov '92

- Scalar CDW (not spirals) was studied in large Nc, high density regime.
- Gaps are small, and reach $\sim \Lambda_{QCD}$ when **$\mu \sim$ 100 GeV.**

2, + Screening effects : Shuster & Son 99 Park-Rho-Wirzba-Zahed 99

- Spirals (same structure as QCS) are found in large Nc.
- Screening mass develops faster than pert. gap, so no spirals in Nc=3.

3, Effective models : Nakano-Tatsumi 04, Nickel08, Carignano-Nickel-Buballa10 Ralf-Shuryak-Zahed01

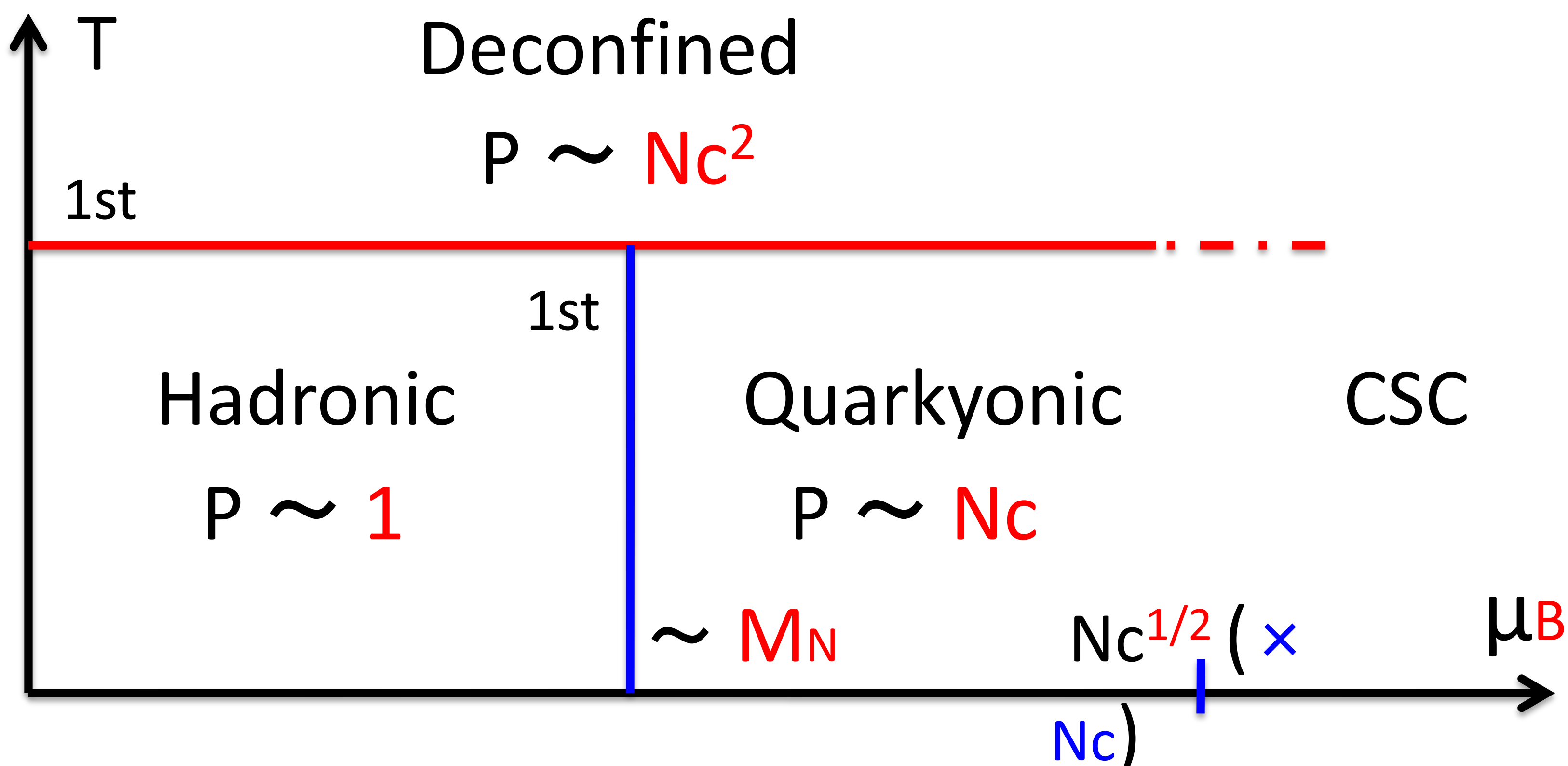
- Relatively low density regime.
- CDW or CS or solitons in σ-π (not σ-Tensor) channels are studied.

4, Non-Perturbative gluon propagator : This work

- Spirals are studied in large Nc, relatively high density regime.
- gap is confinement origin $\sim \Lambda_{QCD}$ (>> perp. gap) ,

  it may be possible to have QCS before screening mass fully develops.

Large Nc Phase Diagram : McLerran & Pisarski (2007)



1, Nuclear matter regime: $\mu_B \sim M_N + k_F^2/M_N$

small change in μ_B → large change in k_F or n_B

2, ChSB is not used to define Quarkyonic phase.

On the IR prescription

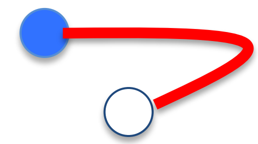
linear potential

$$\frac{\sigma}{(\vec{k}^2)^2} \xrightarrow{\text{IR cut}} \frac{\sigma}{(\vec{k}^2+\Lambda_{\mathrm{IR}}^2)^2} \xrightarrow{\text{F.T.}} \underline{-\frac{\sigma}{\Lambda_{\mathrm{IR}}}} + \sigma r + O(\Lambda_{\mathrm{IR}} r^2)$$

▪ Probe colored objects:

IR div.: const. from naïve IR cutoff

▪ Color singlet sector:



IR const. → irrelevant.
(Linear conf. without IR const.)

▪ As far as color-singlet sector is concerned,
we can get the same results even if we drop off div. const.
(principal value IR regulation; e.g., Coleman, Aspects of Symmetry)

▪ S-D eqs. → just sub-diagrams in B-S eqs.

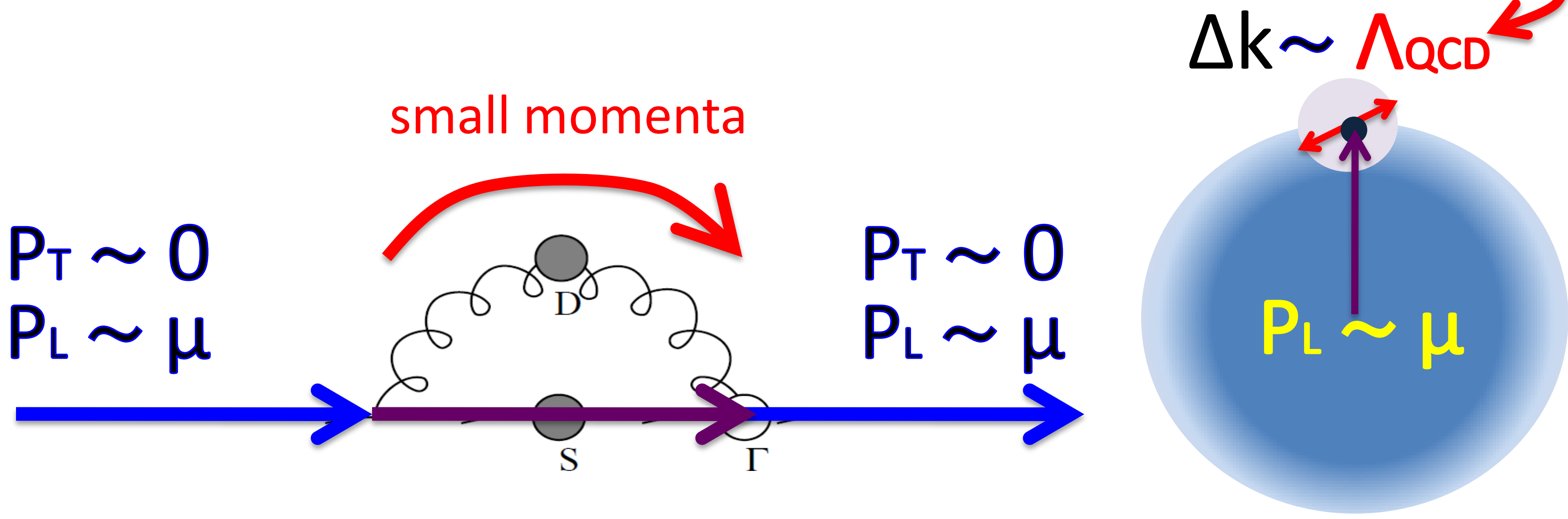
▪ Div. of poles will be used as color selection rules at best.

e.g.) Dim. reduction of Schwinger-Dyson eq. 1

quark self-energy   including Σ

$$\not{\Sigma}(p) + \Sigma_m(p) = \int \frac{dk_4 dk_z d^2\vec{k}_T}{(2\pi)^4} \gamma_4 S(\vec{k}) \gamma_4 \frac{\sigma}{|\vec{p}-\vec{k}|^4}$$

- **Note1**: Mom. restriction from **confining** interaction.



e.g.) Dim. reduction of Schwinger-Dyson eq. 2

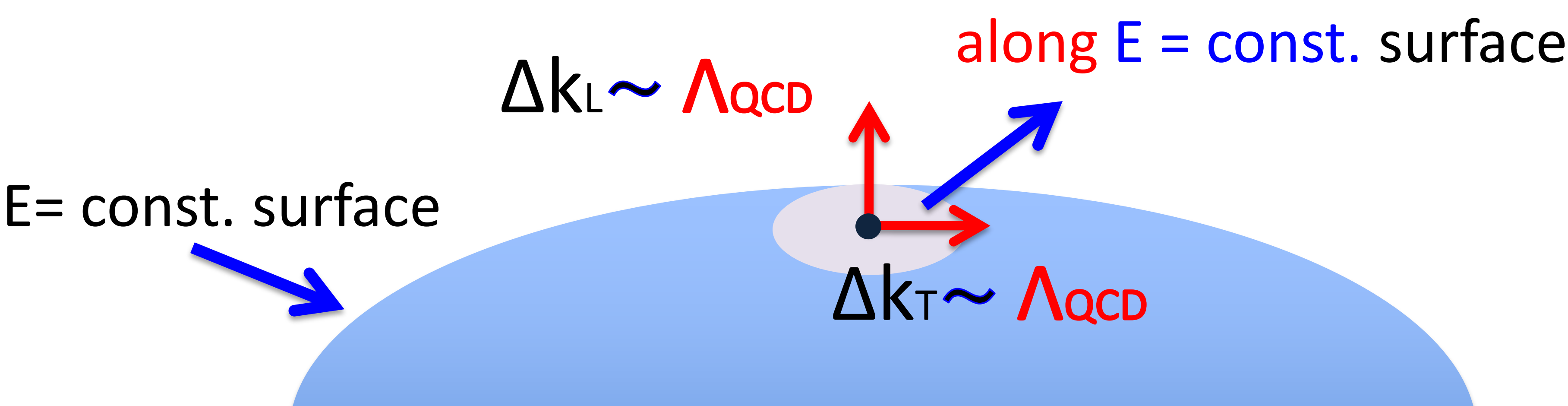
quark self-energy

$$\not\Sigma(p) + \Sigma_m(p) = \int \frac{dk_4 dk_z d^2\vec{k}_T}{(2\pi)^4}\, \gamma_4 \underline{S(k)} \gamma_4\, \frac{\sigma}{|\vec{p}-\vec{k}|^4}$$

- Note2: Suppression of transverse part:

$$S(k) = \gamma_0 S_0 - \underset{\sim \mu}{\gamma_z S_z} - \underset{\sim \Lambda_{QCD}}{\cancel{\vec{\gamma}_T \vec{S}_T}} + S_m$$

- Note3: Quark energy is insensitive to small change of k_T:



e.g.) Dim. reduction of Schwinger-Dyson eq. 3

insensitive to kT

$$\not{\Sigma}(p) + \Sigma_m(p) = \int \frac{dk_4 dk_z d^2\vec{k}_T}{(2\pi)^4} \gamma_4 S(k) \gamma_4 \frac{\sigma}{|\vec{p}-\vec{k}|^4}$$

factorization

$$\gamma_4 \Sigma_4 + \gamma_z \Sigma_z = \int \frac{dk_4 dk_z}{(2\pi)^2} \gamma_4 S(k_4, k_z, \vec{0}_T) \gamma_4 \otimes \int \frac{d\vec{k}_T}{(2\pi)^2} \frac{\sigma}{|\vec{p}-\vec{k}|^4}$$

smearing

confining propagator in (1+1)D: $\boxed{\frac{\sigma}{2\pi} \frac{1}{|p_z - q_z|^2}}$

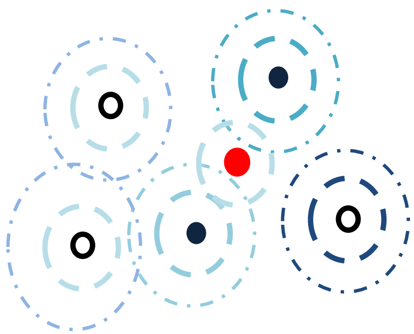
Schwinger-Dyson eq. in (1+1) D QCD in A1=0 gauge

Bethe-Salpeter eq. can be also converted to (1+1)D

Toward multiple patch construction. 1

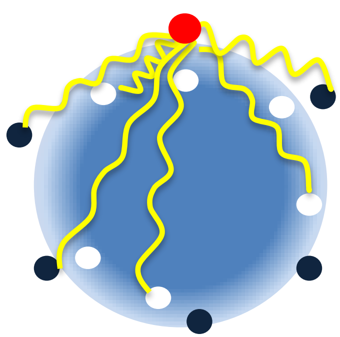
Perturbative gluons

▪ r - space)



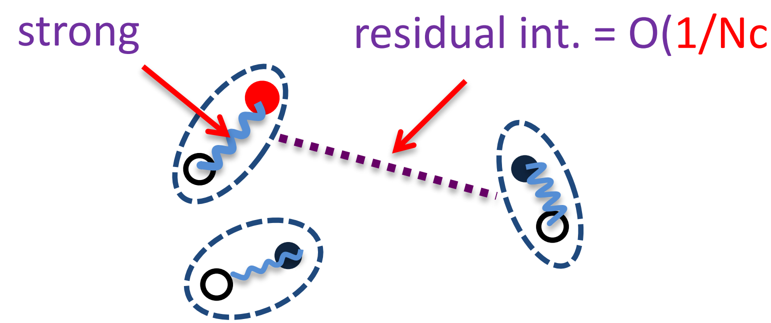
Influence by all other quarks must be treated simultaneously.

▪ p - space)

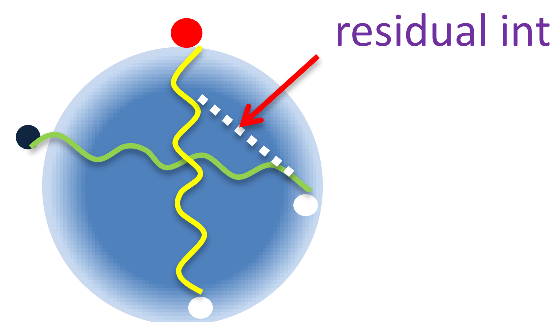


Gap → strongly density dependent.

Confining gluons



After mesonic objects are formed, residual interactions enter.



Gap → weakly density dependent.
(confinement - origin)

One patch results may be good starting point for confining models.

(not for perturbative gluons or contact int. models: Rapp-Shuryak-Zahed 01)

Toward multiple patch construction. 2

- e.g.) Quark propagator in the presence of many QCSs

Sum over all Chiral spirals

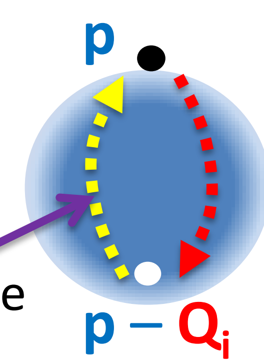
$$\sum_{i=1}^{N_p} \int \frac{d^4 p}{(2\pi)^4} \bar{\psi}(p - \boldsymbol{Q}_i) M(\underline{p}; \boldsymbol{Q}_i) \psi(p)$$

Space-dependent mass self-energy

- Hypothesis: quarks with high virtuality feel small Chiral Sym. breaking

For both of p^2 and $(p - Q_i)^2$ to be close to Minkovski region:

Angle between **p** and **Q_i** $\longrightarrow$ $|\theta| < \Lambda_{\mathrm{QCD}}/p_F$



e.g.) θ ~ 0 case

If angles between quark moving direction and QCS are large:

$\longrightarrow$ Chirality changing scatterings are suppressed.

$\longrightarrow$ Each QCS behaves incoherently (except matching point of patches)

Modified Fermi surface

1/Nc & virtuality arguments

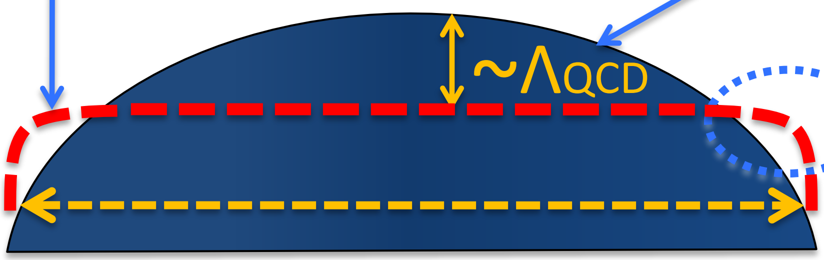
Multiple QCSs ~ Incoherent sum of single QCSs

Modified Fermi surface
(for nesting)

Gap ~ 1-patch result → weakly density dep.

(Luttinger theorem: volume conservation)

compensate squeezed volume
in long. direction



Trans. size ~ $(\mu \Lambda_{QCD})^{1/2}$

Near edge: $(\Delta P_T)^2/2\mu \sim \Lambda_{QCD}$

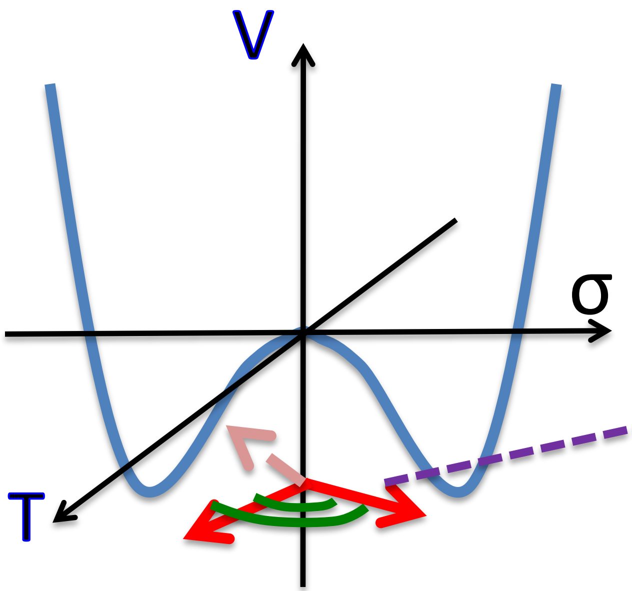
- **Number of Patches** ~ $4\mu^2 / \mu \Lambda_{QCD} \sim \mu / \Lambda_{QCD}$

total surface area   1-patch area

As density increases, a number of QCSs also increases.

Phase fluctuation effects

Fluctuation effects are stronger in lower dimension:



e.g.) Coleman's theorem in 1+1 D (T=0)

$$\langle \rho \rangle \neq 0$$

But $\langle e^{i\theta} \rangle = 0$ (No SSB)

IR divergence in phase dynamics

(power-law correlations : Quasi-long range order)

Strong IR behaviors $\longleftrightarrow$ Restricted dissociation

Including trans. kinetic perturbations to 1+1 D QCD,

we wish to generate (3+1) D eff. Lagrangian for phase fields.

Pert. of transverse curvature

- 1-D chains with transverse coupling:

$$H = \sum_{\boldsymbol{r}} H_{1D}(\boldsymbol{r}) + T_{tunn}$$

$\propto 1/\mu$

continuum limit:

$$\frac{1}{2p_F}(\nabla_\perp R^\dagger \nabla_\perp R + \nabla_\perp L^\dagger \nabla_\perp L)$$

- H_{1D} → Bosonization → "Charge-Flavor-Color separation"

U(1) free bosons (g, h: matrix field for flavored & colored bosons)

$$S = \underbrace{S_{U(1)}[\phi] + S^{flavor}_{k=N_c}[g]}_{\text{conformal}} + \underbrace{S^{color}_{k=N_f}[h] \quad + \text{gauge int.}}_{\text{dimensionful}}$$

- Trans. terms → expanded perturbatively and then resumed:

Massive sector (colored) → integrated out.

→ leaving only color singlet sectors.

Collective modes (near the center of patches)

- U(1): $\mathcal{L}^{U(1)}_{k=N_c N'_f} = \frac{N'_f N_c p_F M}{8} \left[(\partial_L \Phi)^2 + \frac{\eta M}{p_F} (\partial_\perp \Phi)^2 \right]$
- SU(2Nf): $\mathcal{L}^{SU(N'_f)}_{k=N_c} = \frac{N_c p_F M}{4} \left[\mathcal{L}_{WZW}[g] + \frac{\eta' M}{p_F} \mathrm{tr}[\partial_\perp g \partial_\perp g^\dagger] \right]$

(momentum measured from Fermi surface) $M \sim \Lambda_{\rm QCD} \quad \eta,\ \eta' \sim 1$

- A number of Goldstone modes: $(4Nf^2 - 1) + \underline{1}$

(anomaly is not included yet)

- Decay constant $\sim (Nc\, \underline{\mu\, \Lambda_{QCD}})^{1/2}$

degeneracy in trans. direction

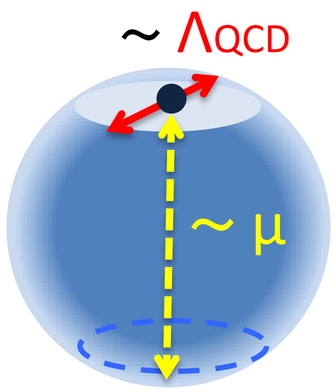
Interactions between Goldstone modes → O(1/Nc)

- Transverse dispersion is suppressed by Λ_{QCD} / μ .

System gets closer to quasi-long range order as μ increases.

(fluctuations become more and more important in higher density)

Dim. reduction of integral eqs.



1, Virtual flucts. are limited within small mom. region.

2, Quark energies are insensitive to small ΔkT.

(due to flatness of Fermi surface in trans. direction)

e.g.) Schwinger-Dyson eq.

insensitive to kT

$$\cancel{\Sigma}(p) + \Sigma_m(p) = \int \frac{dk_4 dk_z d^2\vec{k}_T}{(2\pi)^4}\gamma_4 S(k)\gamma_4 \frac{\sigma}{|\vec{p}-\vec{k}|^4}$$

$$\xrightarrow{\text{factorization}} \int \frac{dk_4 dk_z}{(2\pi)^2}\gamma_4 S(k_4, k_z, \underline{\vec{0}_T})\gamma_4 \otimes \underbrace{\int \frac{d\vec{k}_T}{(2\pi)^2}\frac{\sigma}{|\vec{p}-\vec{k}|^4}}_{\text{smeared gluon propagator}}$$

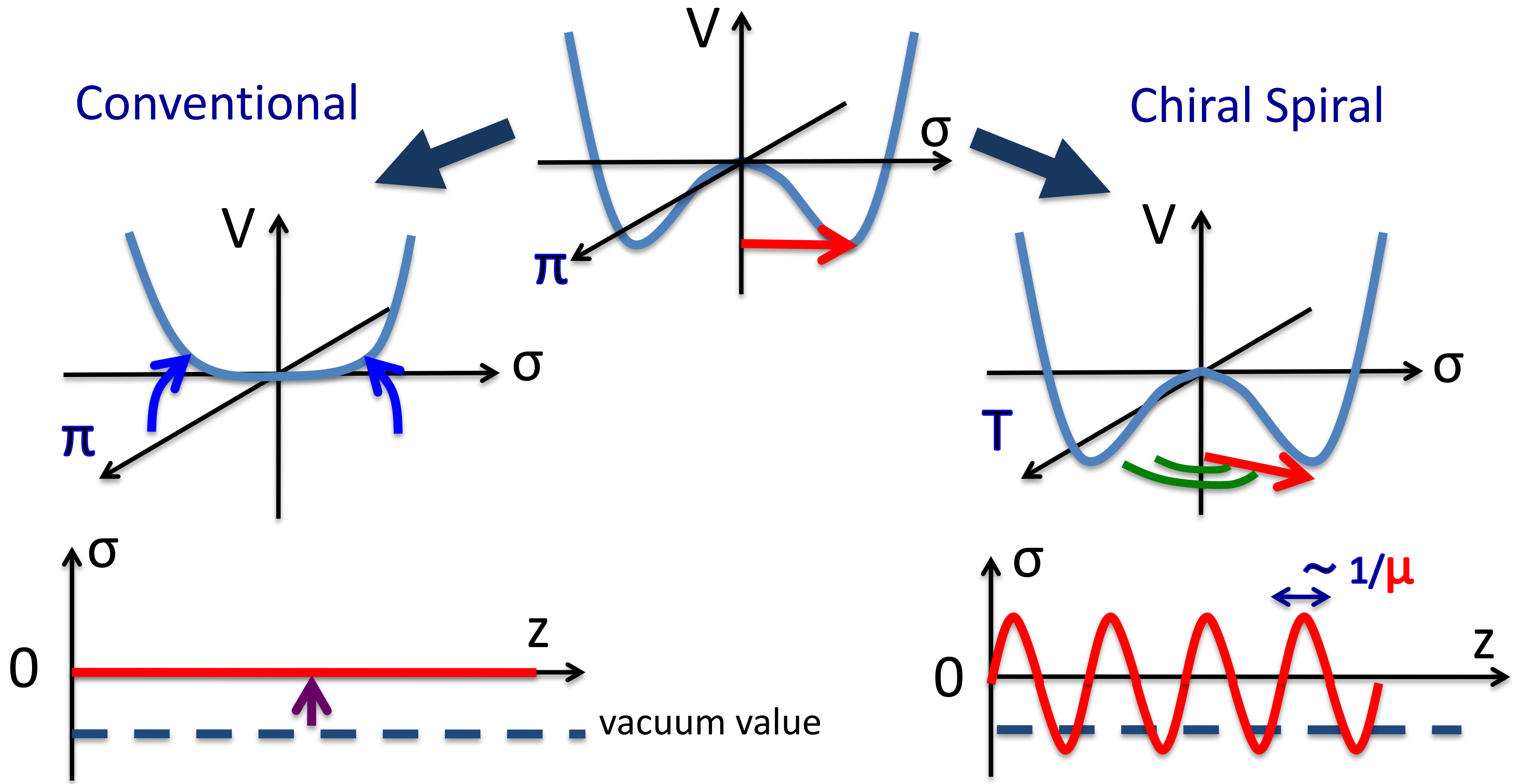
- At leading order:

Dimensional reduction of Non-pert. self-consistent eqs :

4D “QCD” in Coulomb gauge $\longleftrightarrow$ 2D QCD in A_1=0 gauge

(confining model)

Local violation of chiral sym. in dense quark matter



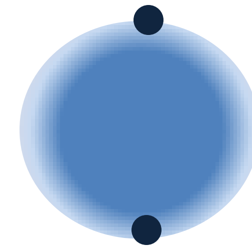
zero everywhere.

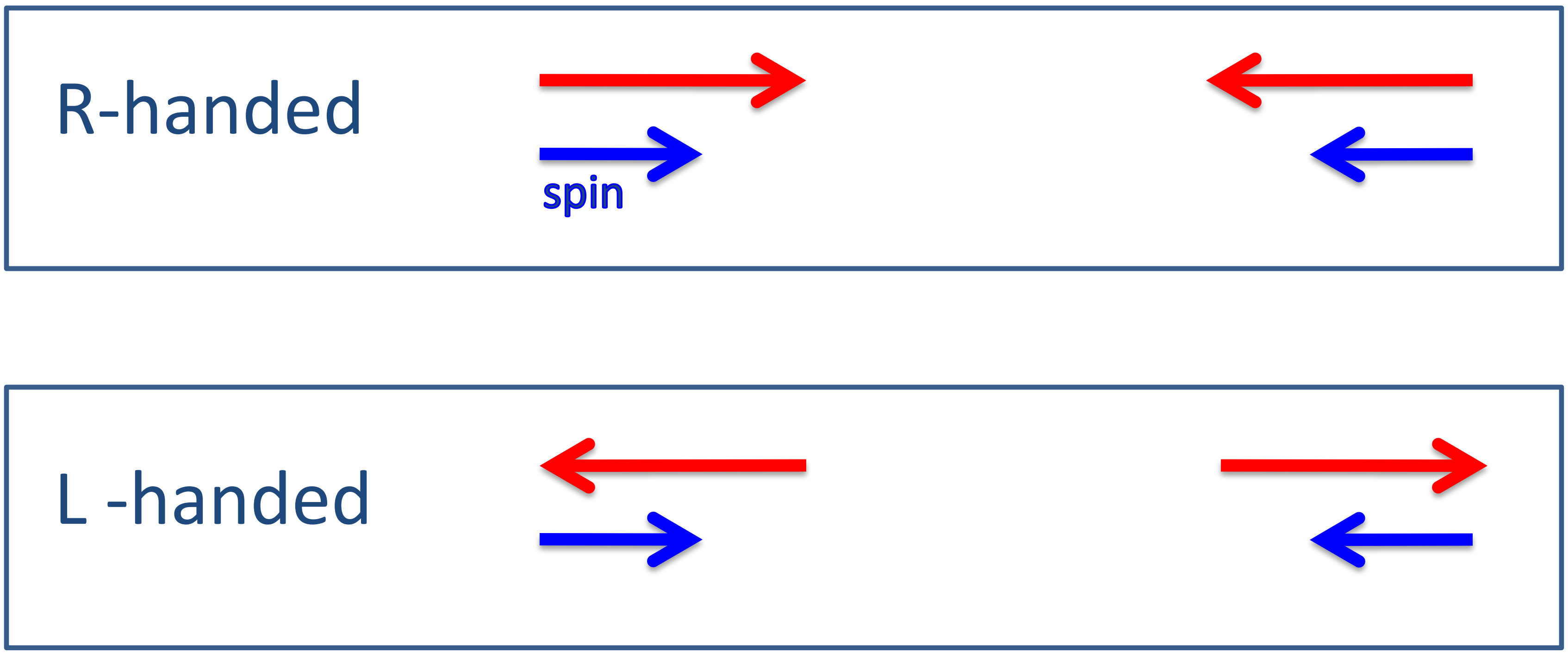
nonzero locally, but globally zero

- Remark: Baryon number is spatially uniform.
  (Only chiral density is spatially modulated.)

Flavor Multiplet

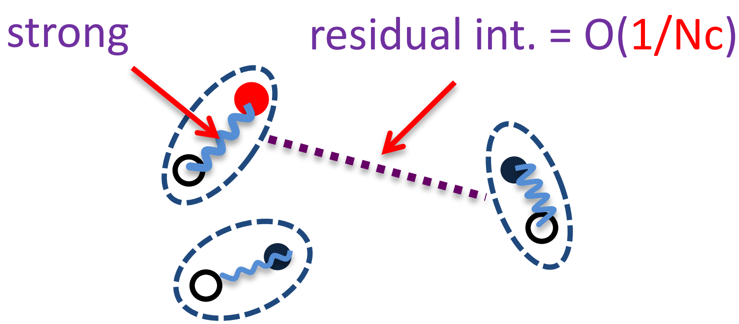
particle near north & south pole

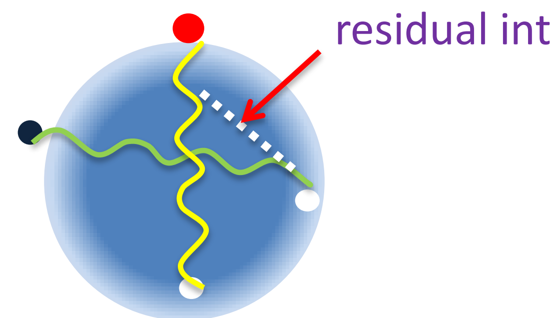




Multiple patch: Chiral Crystals

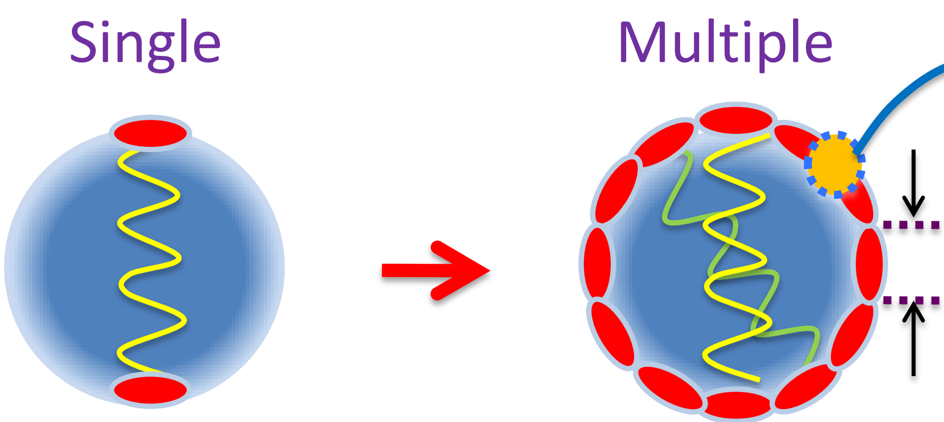
- Special properties of confining models:





Gap → weakly density dep. ~ Λ_{QCD}

- Multiple QCSs ~ Incoherent sum of single QCSs (+ residual interactions b.t.w. patches)



Near edge: gaps start to close

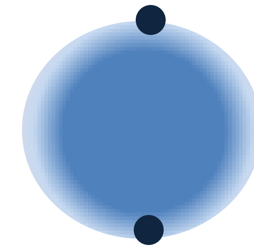
$(\Delta P_T)^2/2\mu \sim \Lambda_{QCD}$

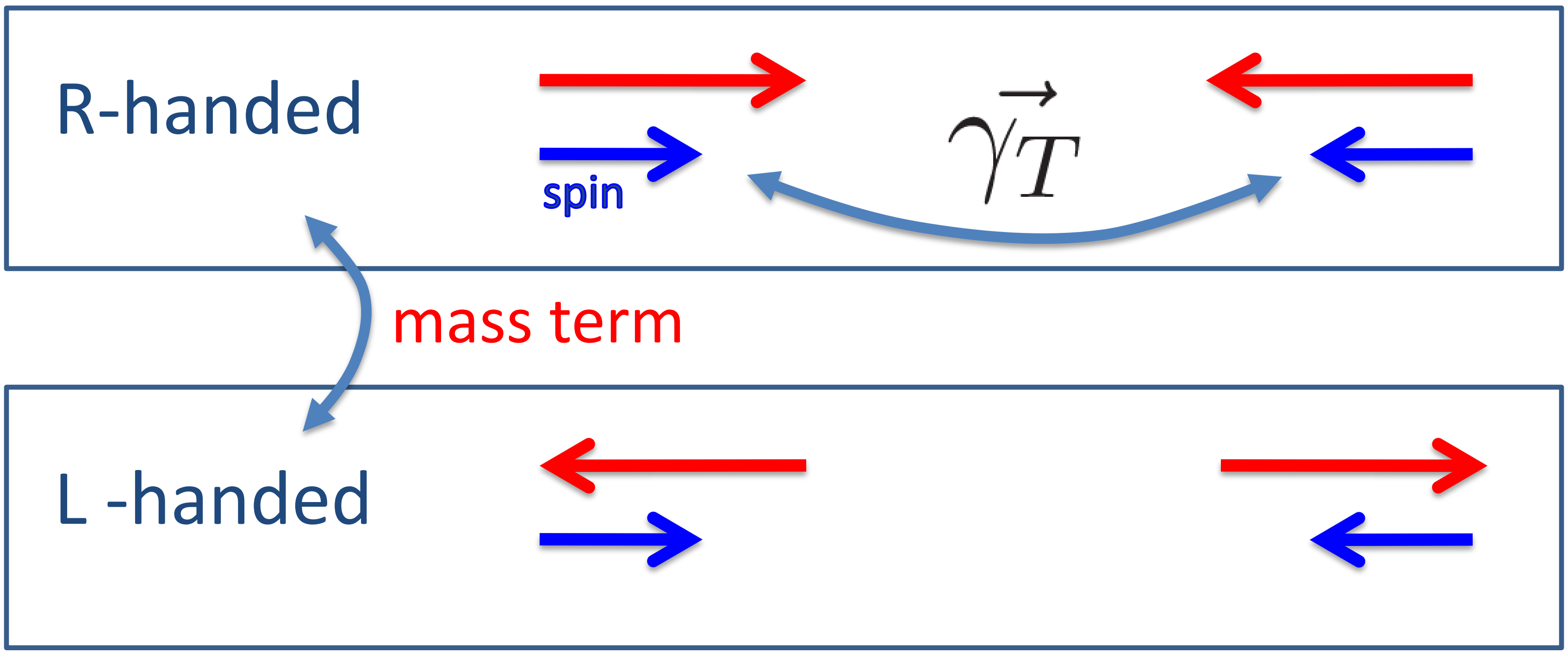
$\sim (\mu \Lambda_{QCD})^{1/2}$

transverse size of single patch

Flavor Multiplet

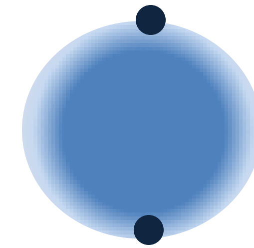
particle near north & south pole

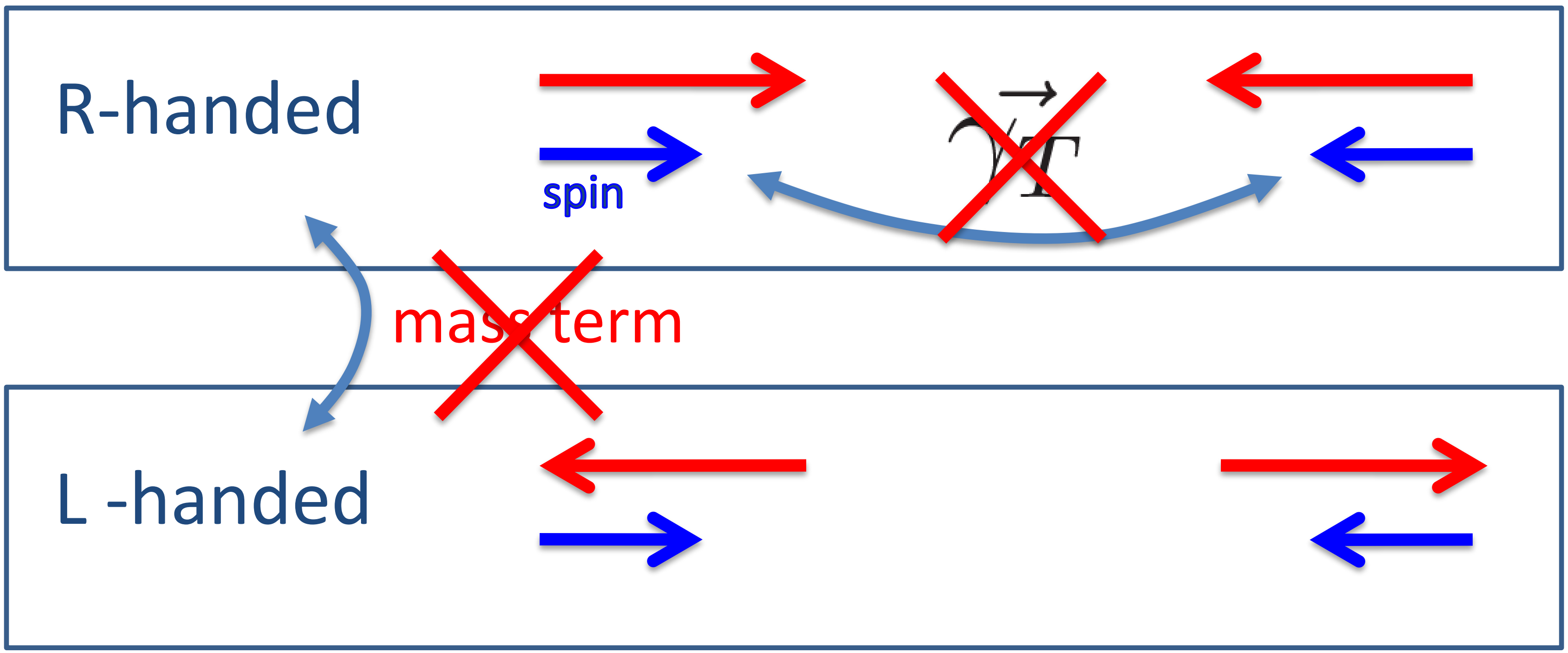




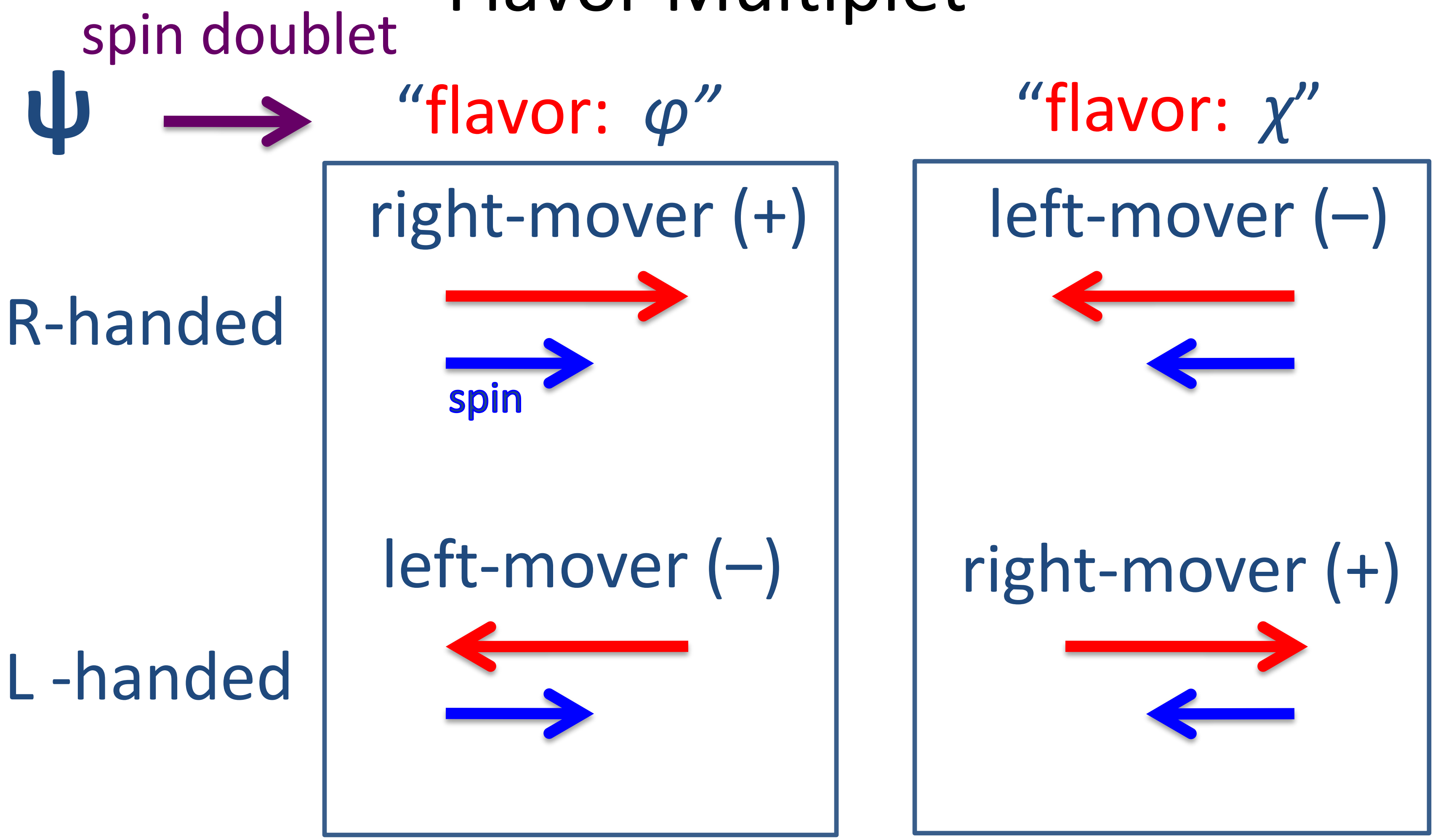
Flavor Multiplet

particle near north & south pole





Flavor Multiplet



Moving direction: (1+1)D "chirality"

(3+1)D – CPT sym. directly convert to (1+1)D ones

Relations between composite operators

- 1-flavor (3+1)D operators without spin mixing:

$$\bar{\psi}\psi \downarrow \bar{\Psi}\Psi \qquad \bar{\psi}\gamma^0\psi \downarrow \bar{\Psi}\Gamma^0\Psi \qquad \bar{\psi}\gamma^z\psi \downarrow \bar{\Psi}\Gamma^z\Psi \qquad \bar{\psi}\gamma^0\gamma^z\psi \downarrow \bar{\Psi}\Gamma^5\Psi$$

Flavor singlet in (1+1)D

- All others have spin mixing:

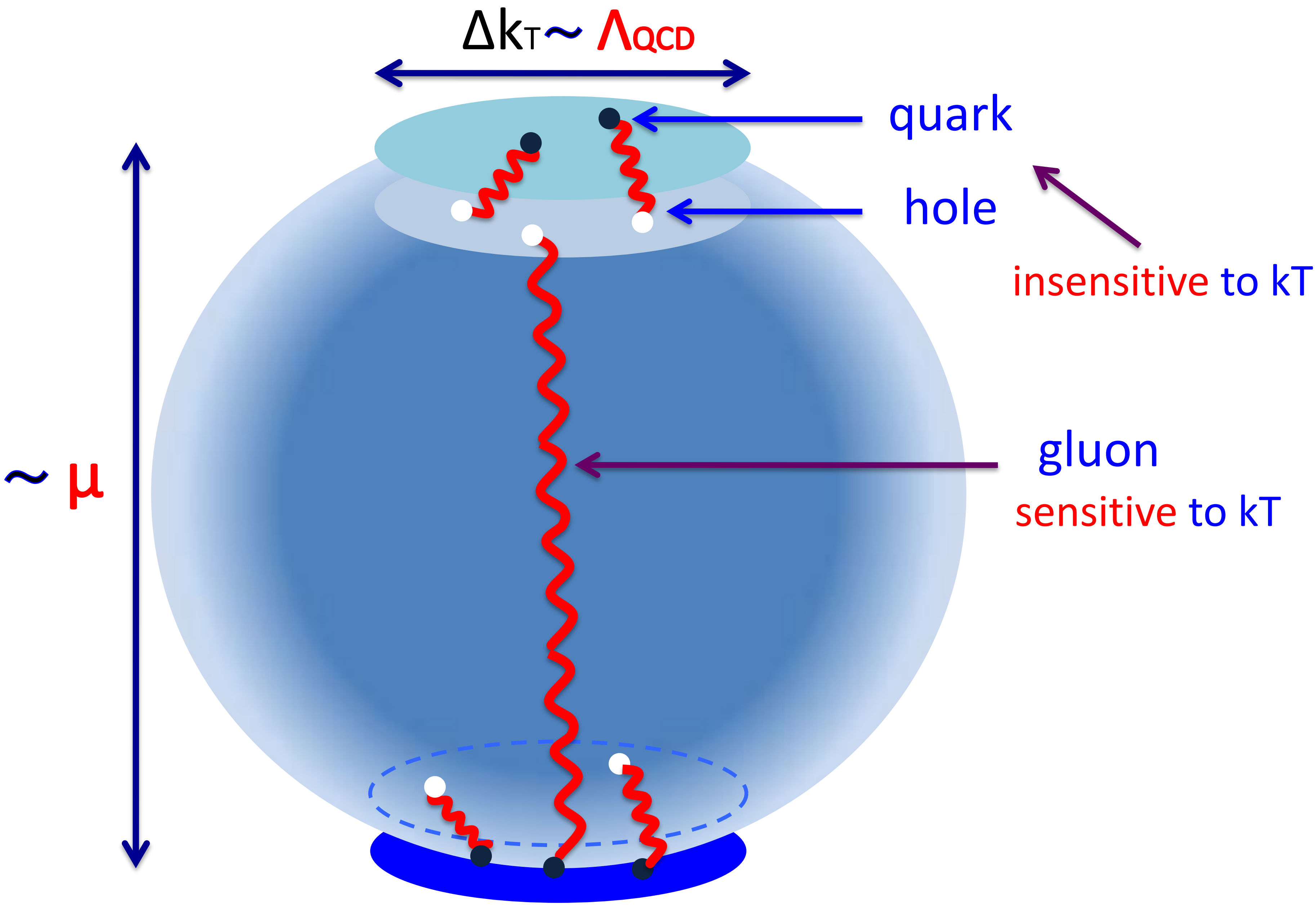
ex) $\bar{\psi}\gamma^5\psi \longrightarrow \bar{\Psi}\tau_3\Psi$

$\bar{\psi}\gamma^1\psi \longrightarrow \bar{\Psi}\tau_2\Psi$

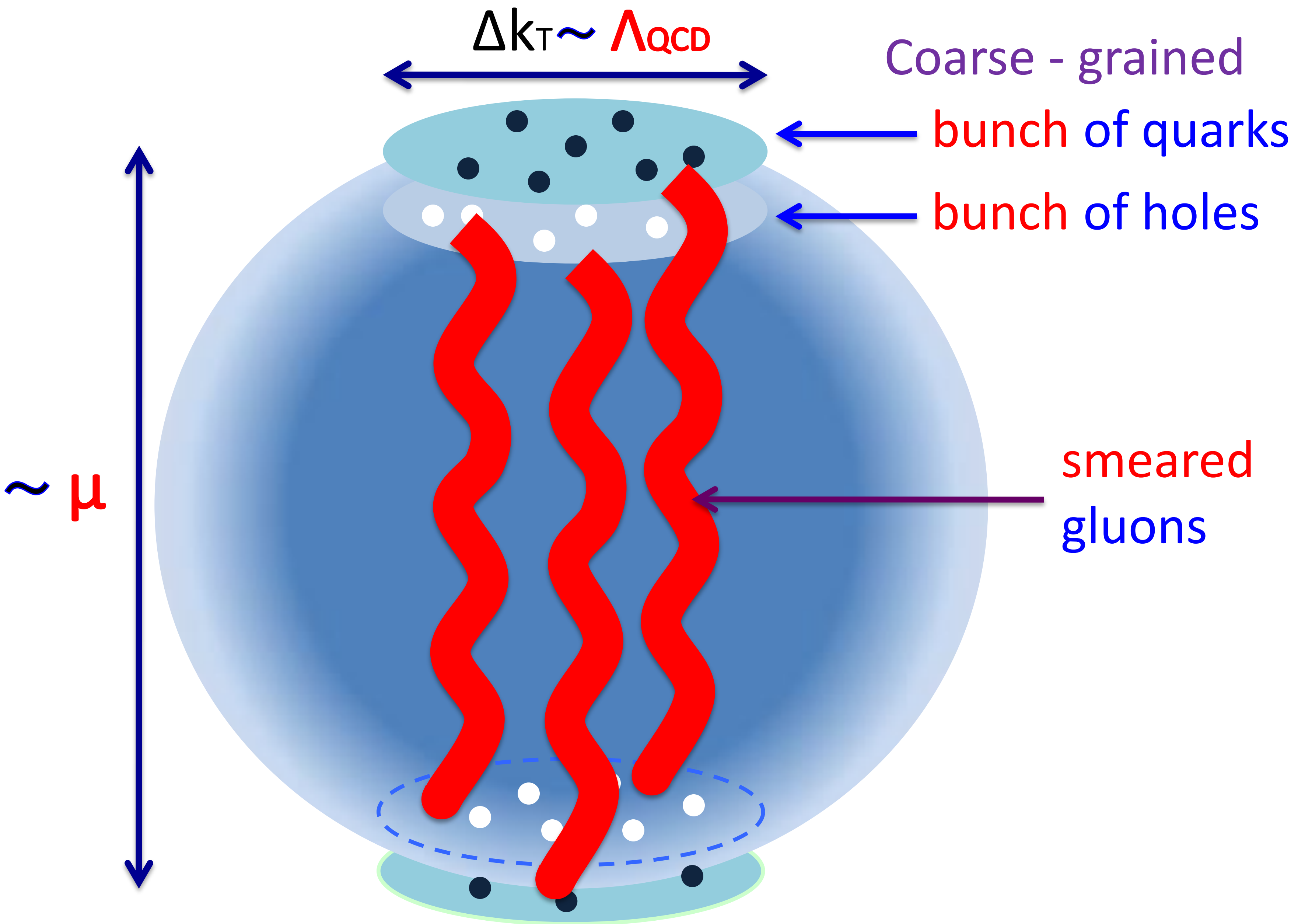
(They will show no flavored condensation)

Flavor non-singlet in (1+1)D

Catoon for Pairing dynamics before reduction



1+1 D dynamics of patches after reduction



Excitation modes in quarkyonic limit

- Fermionic action for (1+1)D QCD:

$$S = \int d^2x [\Psi_+ i\partial_- \Psi_+ + \Psi_- i\partial_+ \Psi_-] \quad + \text{gauge int.}$$

- Bosonized version:

U(1) free bosons & Wess-Zumino-Novikov-Witten action :

(Non-linear σ model + Wess-Zumino term)

"Charge – Flavor – Color Separation"

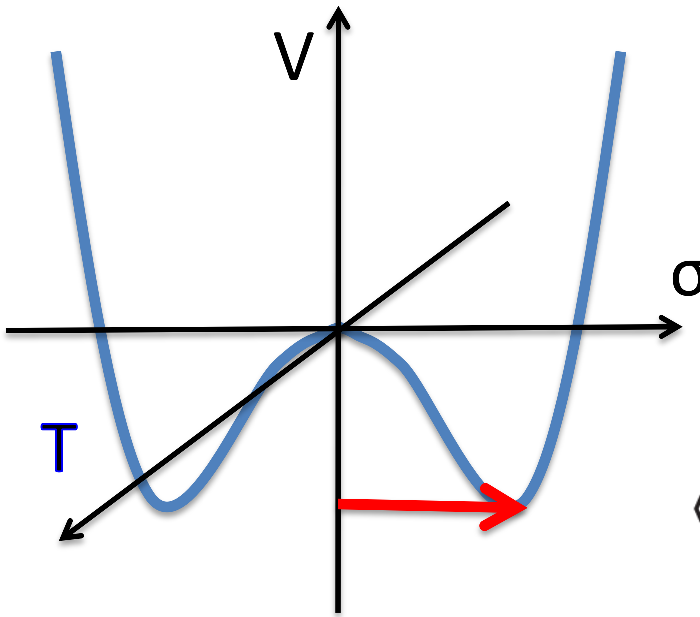
$$S = \underbrace{S_{U(1)}[\phi] + S^{flavor}_{k=N_c}[g]}_{\text{conformal inv.}} + \underbrace{S^{color}_{k=N_f}[h]}_{\text{dimensionful}} \quad + \text{gauge int.}$$

$4N_f^2$ gapless phase modes

gapped phase modes

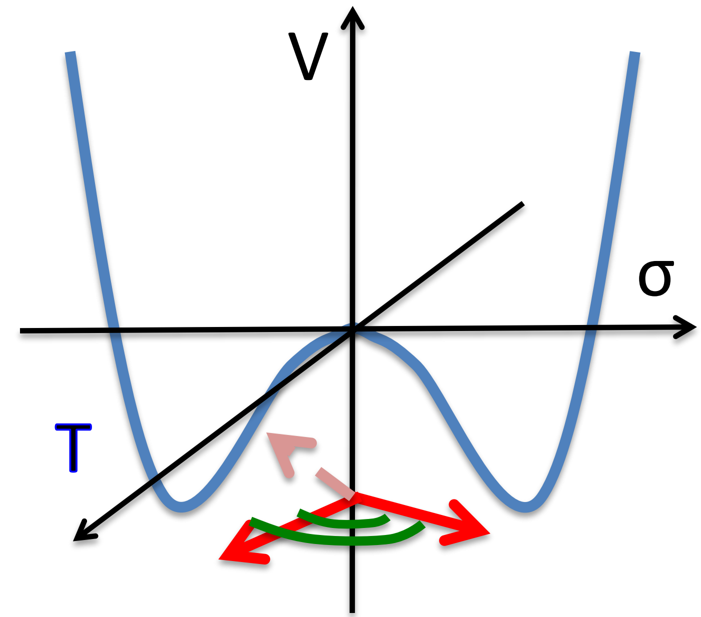
Coleman's theorem ?

- Coleman's theorem: No Spontaneous sym. breaking in 2D



$\langle e^{i\theta} \rangle \neq 0$ (SSB)

$\langle \rho \rangle \neq 0$



$\langle e^{i\theta} \rangle = 0$ (No SSB)

IR divergence in (1+1)D phase dynamics

- Phase fluctuations belong to:

Excitations (physical pion spectra)	ground state properties (No pion spectra)

Quasi-long range order & large Nc

- Local order parameters:

gapless modes gapped modes

$$\langle \bar{\Psi}_+ \Psi_- \rangle \sim \langle e^{i\sqrt{4\pi/N_c N_f}\phi} \rangle \otimes \langle \mathrm{tr} g \rangle \otimes \langle \mathrm{tr} h \rangle$$

due to IR divergent phase dynamics → 0 0 finite

But this does not mean the system is in the usual symmetric phase!

- Non-Local order parameters:

$$\langle \bar{\Psi}_+ \Psi_-(x) \bar{\Psi}_- \Psi_+(0) \rangle \sim \begin{cases} \cancel{e^{-m|x|}} & \text{: symmetric phase} \\ \cancel{\langle \bar{\Psi}_+ \Psi_- \rangle^2} & \text{: long range order} \\ |x|^{\underline{-C/N_c}} & \text{: quasi-long range order} \end{cases}$$

(including disconnected pieces)

(power law)

Quasi-long range order & large Nc

- Local order parameters:

gapless modes gapped modes

$$\langle \bar{\Psi}_+ \Psi_- \rangle \sim \langle e^{i\sqrt{4\pi/N_c N_f}\phi} \rangle \otimes \langle \mathrm{tr} g \rangle \otimes \langle \mathrm{tr} h \rangle$$

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(including disconnected pieces)

large Nc limit (Witten `78)

(power law)

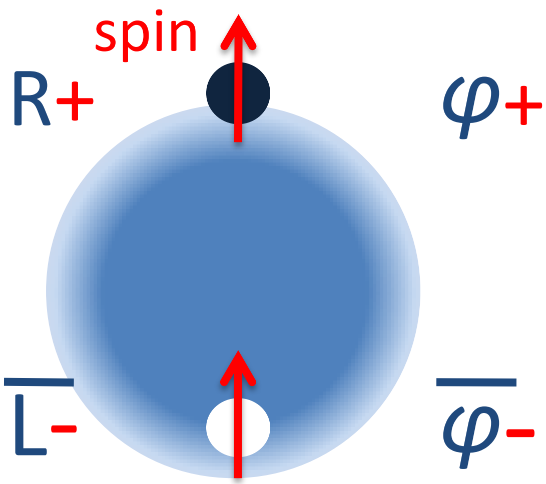
Chiral Density Wave VS Chiral Exciton

IF dimensionally reduced models respect "flavor" symmetry

→ there is no chiral exciton condensates:

Chiral Density Wave

→ R+ $\overline{\text{L-}}$ pairing



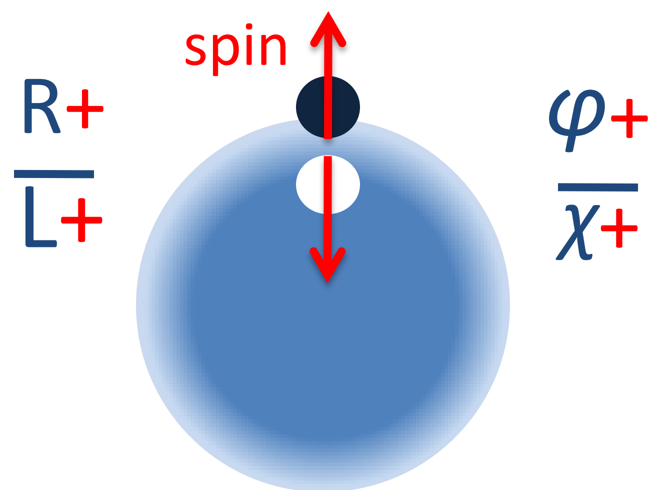
same spin
in 3+1 D
↕
same "flavor"
in 1+1 D

Possible (1+1)D flavor singlet

$$\bar{\Psi}\Psi \qquad \bar{\Psi}\Gamma^5\Psi$$

Chiral Exciton

→ R+ $\overline{\text{L+}}$ pairing



No flavor singlet pairing

→ No chiral exciton!