Physical Mechanism
of the (Tri)Critical Point Generation

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in collaboration with A.I. Ivanitsky,

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Outline

- What is known about (tri)critical points from exactly solvable statistical models
- The role of surface tension for (tri)critical point existence
- Order parameters: field theoretical vs. stat.mechanical
- A surface tension induced generation of a Triple point
- Conclusions

Well Known Statistical Models with CEP

Model	Reference	Comment
CEP in spin systems - THEORY	K.Wilson, ε expansion...	Is valid at CEP only! Was not systematically extended to other systems
Fisher Droplet Model (FDM)- Condensation of gases	M. Fisher, Physica 3 (1967); J.B. Elliott et al, nucl-ex/0608022 (2006)	Describes the gas only! NO liquid phase!
Statistical Multifragmentation Model (SMM) [without Coulomb interaction]- Liquid-Gas PT in nuclear matter	J. P. Bondorf et al, Phys. Rep. 257(1995); K.A.B., Phys. Part. Nucl. 38 (2007);	Elaborate model, but liquid phase has limiting density! $\Rightarrow$problems at high pressure!

Well Known Statistical Models with CEP

CEP in spin systems -	K.Wilson, ε	Is valid at CEP only! Was not systematically ms

* **PROBLEM:** for liquids ε–expansion requires ODD powers of spin interaction! see Sh. Ma, MThCPh

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Free Energy in Statistical Models

- In FDM and SMM the FREE ENERGY of a V-volume cluster has the Bulk, V**1, Surface, V**(2/3), and Topological, -T ln (V), parts.

- At the phase equilibrium the Bulk part of free energy vanishes (equal pressures).

- At the (tri)critial point the Surface part of free energy vanishes (the energy and entropy gaps between gaseous and liquid phases disappear; recall the critical opalescence).

Free Energy in Statistical Models

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MAIN CONCLUSION:

the surface free energy is vitally necessary for CEP existence!

- At the (tri)critial point the Surface part of free energy vanishes (the energy and entropy gaps between gaseous and liquid phases disappear; recall the critical opalescence).

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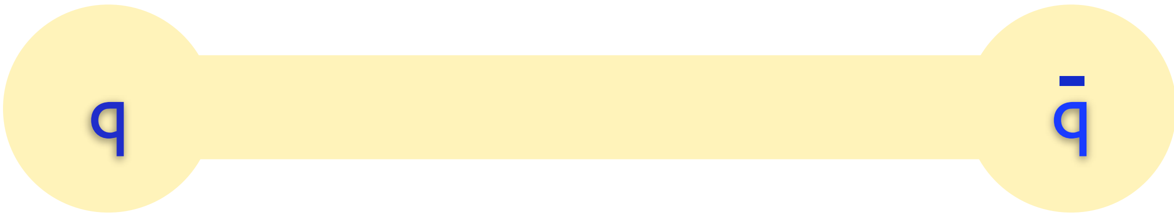
What about the surface tension in QCD ?

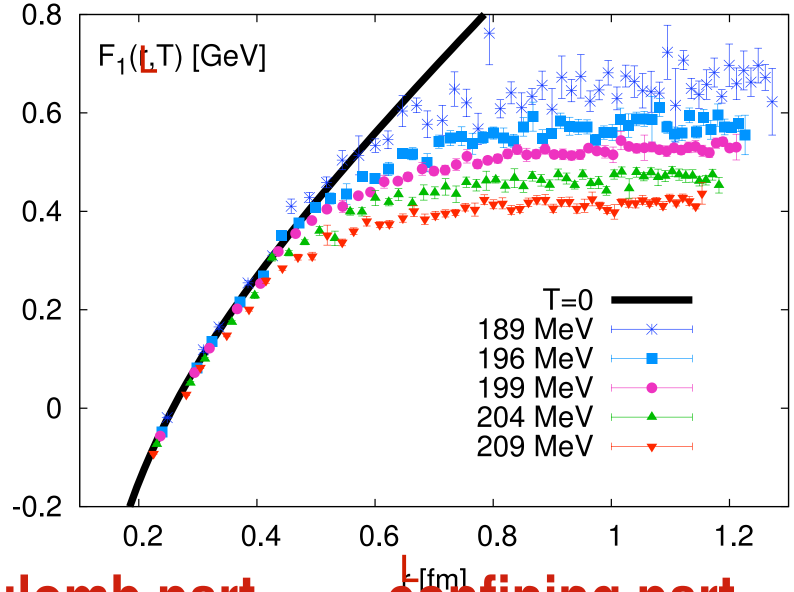
Surface Tension from Confining String

Consider confining (unbroken) string between static q & anti q of length L and radius R<<L

outer pressure Ptot



color anticolor



Coulomb part confining part

Its free energy measured from Polyakov loop correlator is $F_{str} = \sigma_{str} L$

Consider now this tube as the cylindrical bag of length L and radius R<<L

Neglect effects of color sources and get cylinder FREE ENERGY as:

$$F_{cyl}(T, L, R) \equiv -\underbrace{p_v(T)\pi R^2 L}_{bulk} + \underbrace{\sigma_{surf}(T) 2\pi R L}_{surface} + \underbrace{T\tau \ln \frac{V}{V_0}}_{small} \; .$$

String Tension vs Surface Tension

K.A.B., G.M.Zinovjev, arXiv:0907.5518

Equating the cylinder FREE ENERGY to string free energy

$$\boxed{\sigma_{str}(T) = \sigma_{surf}(T)\, 2\pi R \; - \; p_v(T)\pi R^2} \cancel{+ \frac{T\tau}{L} \ln\left[\frac{\pi R^2 L}{V_0}\right]}$$

I. We got a possibility to determine QGP bag surface tension directly from LQCD!

From bag model pressure $p_v(T=0) = -(0.25)^4$ GeV^4, $R = 0.5$ fm and $\sigma_{str}(T=0) = (0.42)^2$ GeV^2 $\Rightarrow$

$$\sigma_{surf}(T=0) = (0.2229\ \mathrm{GeV})^3 + 0.5\, p_v\, R \approx \boxed{(0.183\ \mathrm{GeV})^3} \approx 157.4\ \mathrm{MeV\ fm}^{-2}.$$

II. At T= 0 the bag surface tension is rather large!

III. At the cross-over temperature the bag surface tension must be negative!

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From exactly solvable model of surface deformations K.A.B. et al, PRE 72 (2005) we know that there is NOTHING wrong, if surface F = E - TS < 0 above critical T! This means only that entropy dominates!

III. At the cross-over temperature the bag surface tension must be negative!

Quark Gluon Bags with Surface Tension

Modern paradigm: near deconfinement there is sQGP = strongly interacting liquid

=> QGBST Model exploits the same mechanism for (tri)critical endpoint generation as in usual liquids: surface tension coefficient Σ plays a decisive role in it!

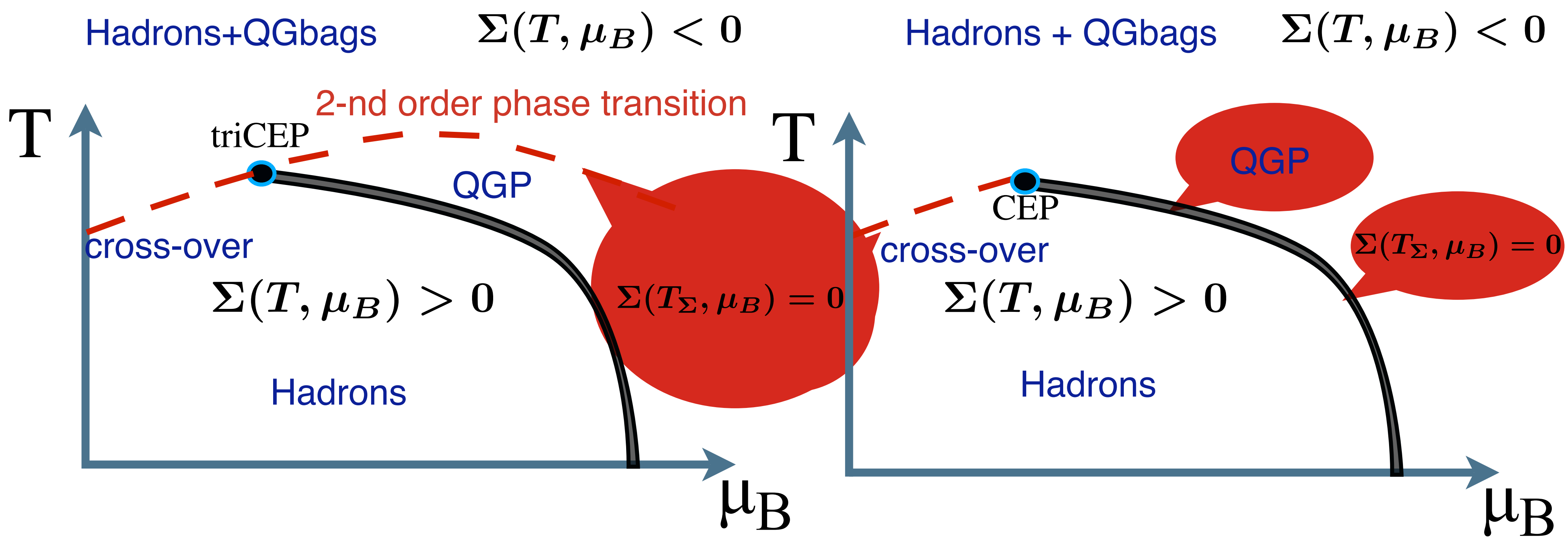
K.A.B., PRC 76 (2007); arXiv:0809.1023;

K.A.B., V.K.Petrov, G.M.Zinovjev, Europhys. Lett. 85 (2009); PRC 79 (2009); arXiv:0904.4420

Hadrons: spherical shape of finite volume

QGP: an infinite cluster of nearly spherical shape

Hadrons+QGbags: highly nonspherical shapes!



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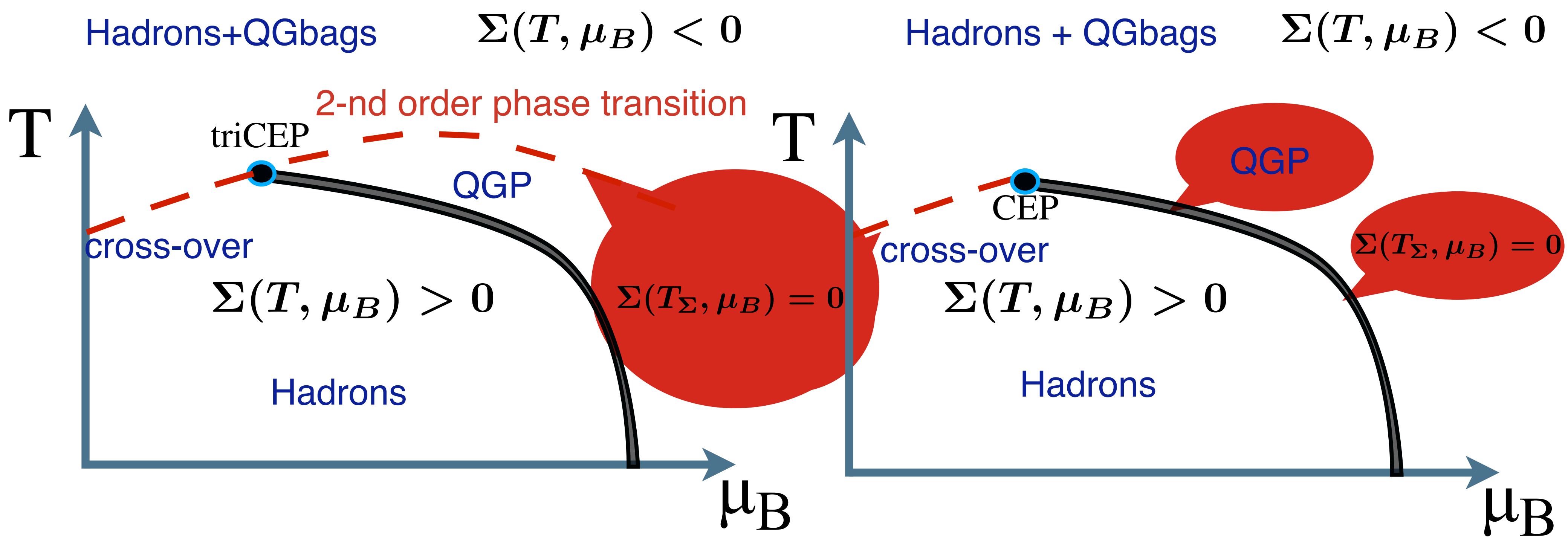
QGBST model: 1-st order deconfinement PT degenerates into cross-over due to $\Sigma < 0$

Hadrons+QGbags: highly nonspherical shapes!



QGBST Model

Volume spectrum of bags in isobaric ensemble

discrete = hadrons **continuous = QG bags**

$$F(s,T,\mu_B) \equiv F_H(s,T,\mu_B) + F_Q(s,T,\mu_B) =$$

$$\sum_{j=1}^{n} g_j e^{\frac{\mu_j}{T} - v_j s} \phi(T,m_j) + u(T) \int_{V_0}^{\infty} dv \, \frac{\exp\left[(s_Q(T,\mu_B) - s)\, v - \Sigma(T,\mu_B)\, v^{\varkappa}\right]}{v^{\tau}}$$

thermal particle density of bags of mass m_k and eigen volume v_k and degeneracy g_k

$$g_k \, \phi(T,m_k) \equiv \frac{g_k}{2\pi^2} \int_0^{\infty} p^2 dp \; e^{-\frac{(p^2 + m_k^2)^{1/2}}{T}}$$

Main parameters: $s_Q(T,\mu_B)$ defines pressure of QG bags $p_Q = T s_Q(T,\mu_B)$

$\Sigma(T,\mu_B)$ is reduced surface tension coefficient

$\varkappa = \frac{2}{3}$, Fisher exponent $\tau > 1$

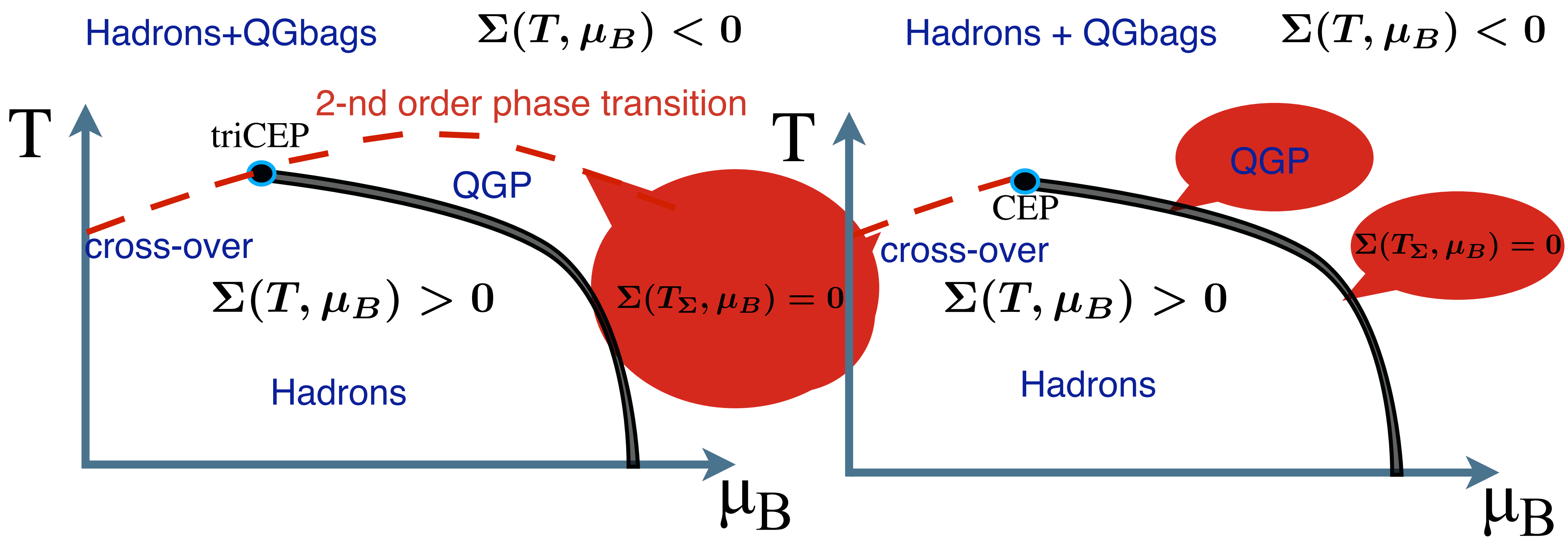
QGBST Model incorporates the best features of Hadron Gas Model, Bag Model and Fisher droplet model

Surface Tension Parameterization

$$\Sigma(T,\mu_B) = \begin{cases} \Sigma^- > 0, & \text{for} \quad T \to T_\Sigma(\mu_B) - 0 \\ 0, & \text{for} \quad T = T_\Sigma(\mu_B) \\ \Sigma^+ < 0, & \text{for} \quad T \to T_\Sigma(\mu_B) + 0 \end{cases}$$

TriCEP: $\partial\Sigma$ is continuous function at $T_\Sigma(\mu_B)$ =>
surface tension induced 2-nd order PT

CEP: $\partial\Sigma$ has discontinuity at $T_\Sigma(\mu_B)$ => deconfinement PT is
a surface tension induced 1-st order PT



Order Parameter in LQCD

Should unambiguously distinguish all phases and all transitions: hadrons, QGP, QG bags + hadrons above cross-over; deconfinement, cross-over & surface tension induced PT

LQCD: SU(3)c x SU(3)f with physical quark masses = Im(Polyakov loop)

Borisenko, Petrov, Zinovjev, PLB 221 (1989)

20 years later it is confirmed in SU(3)c gluodynamics!
Gattringer et al arXiv1004.2200

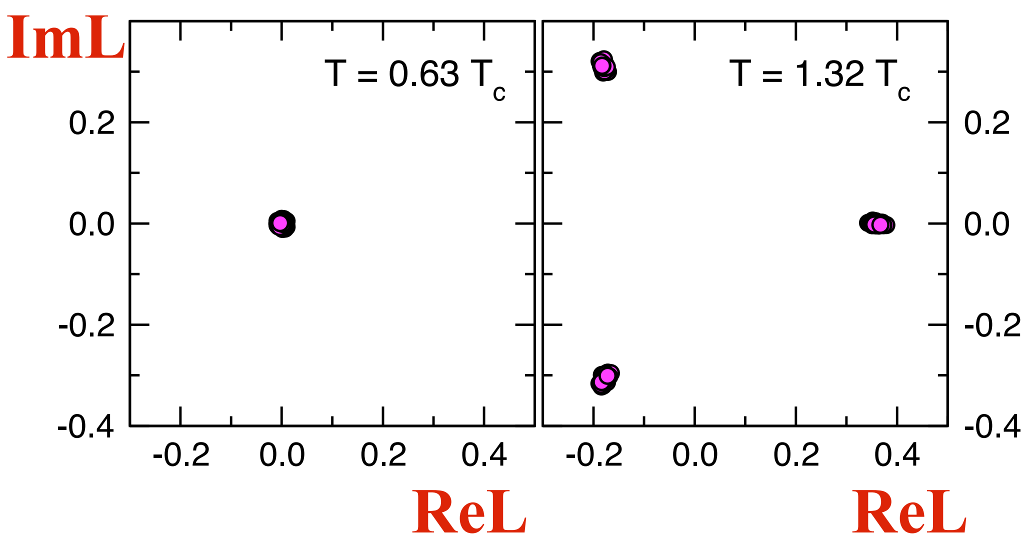


Figure 1: Scatter plots of the spatially averaged Polyakov loop P in complex plane for configurations below (lhs. panel) and above T_c (rhs.).

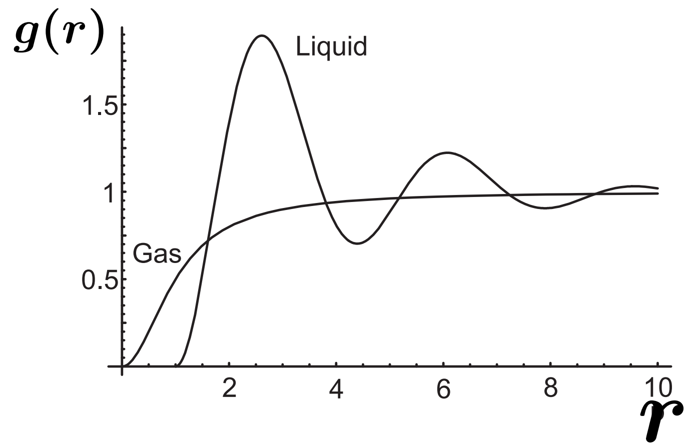
However:

1. **It has to be confirmed in full QCD**

2. **It is unclear how to distinguish phases on both sides of cross-over**

Order Parameter in Liquids

At QM-2005 M. Thoma suggested to use the usual pair radial distribution function g(r) both for liquid QGP (=sQGP) and for gaseous QGP(=?)



$$g(r) = \frac{n_\Delta(r)}{4\,\pi r^2 \Delta r\,\rho}$$

Signals near order

$n_\Delta(r)$ is number of particles in a layer of width Δr

located at distance r, ρ – mean particle density

Clearly, the same should work for hadron gas to sQGP phase transition!

However:

1. **It is unclear how can g(r) distinguish phases on both sides of cross-over**

2. **It is unclear how does g(r) behaves at (tri)CEP**

Condensation in Liquids

CONDENSATION in liquids is another physical process signaling 1-st PT:

liquid phase is a single infinite fragment;
gaseous phase consists of small fragments;
mixed phase is their mixture

Gas to liquid PT is a formation of an infinite fragment =>
Formation of infinite fragment is a natural order parameter for deconfining sQGP

However, QGBST model shows that just mean volume is not suited for this role

It is necessary to account for the surface tension effect!

For a pressure $p = T\, F(\frac{p}{T}, T, \mu_B)$ find an average of

$\langle\, v^q \exp\left[+\Sigma(T,\mu_B)\, v^{\varkappa}\right] \rangle$ over v-spectrum for $q \geq \tau - 1$

$$\langle\, v^q \exp\left[+\Sigma(T,\mu_B)\, v^{\varkappa}\right] \rangle =$$

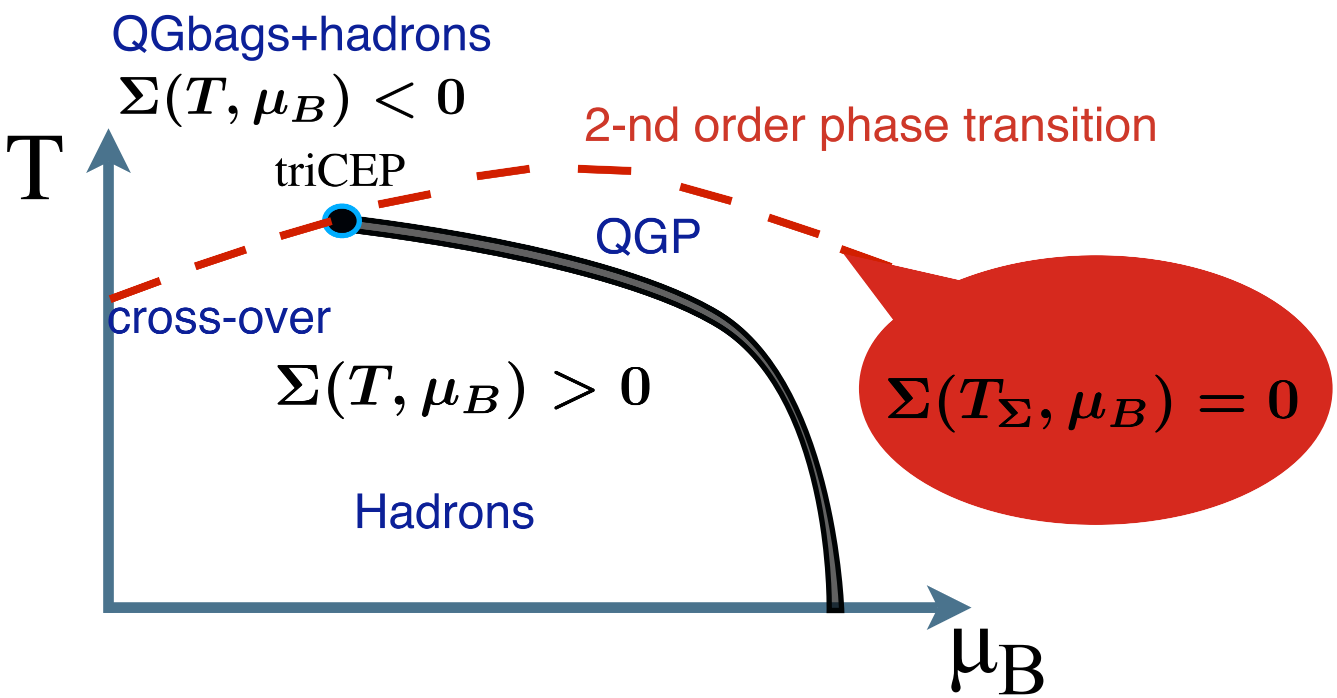
$$\frac{T}{p}\left[\sum_{j=1}^{n} g_j e^{\frac{\mu_j}{T} - v_j \frac{p}{T}} \phi(T,m_j)\, v_j^q\, e^{\Sigma(T,\mu_B)\, v_j^{\varkappa}} \quad + \quad u(T) \int\limits_{V_0}^{\infty} dv\, \frac{\exp\left[(s_Q(T,\mu_B) - p/T)\, v\right]}{v^{\tau - q}}\right]$$

Natural Order Parameters for TriCEP

Second OP is surface tension coefficient:

1. without Σ one cannot determine a condensation line

2. sign of Σ distinguishes two physically different states at a cross-over



$1 < \tau \leq 2 \Rightarrow q \geq 1$

For this case OP can be mean volume

Natural Order Parameters for TriCEP

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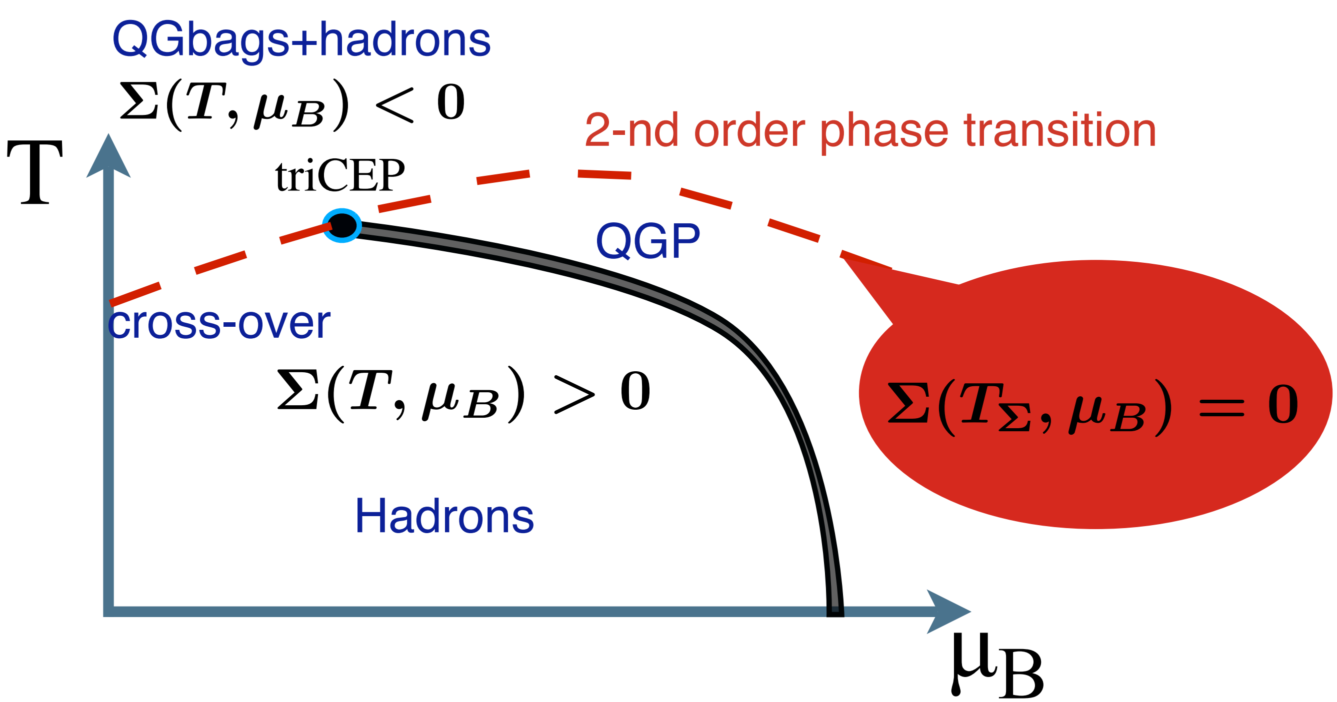
$$\Sigma > 0, \quad \langle\, v^q \,\exp\left[+\Sigma\, v^{\varkappa}\right]\rangle < \infty \quad \equiv \quad \text{Hadronic phase}$$

$$\Sigma > 0, \quad \langle\, v^q \,\exp\left[+\Sigma\, v^{\varkappa}\right]\rangle = \infty \quad \equiv \quad \text{Mixed phase, QGP}$$

$$\Sigma = 0, \quad \langle\, v^q \,\exp\left[+\Sigma\, v^{\varkappa}\right]\rangle = \infty \quad \equiv \quad \text{at triCEP}$$

$$\Sigma < 0, \quad \langle\, v^q \,\exp\left[+\Sigma\, v^{\varkappa}\right]\rangle < \infty \quad \equiv \quad \text{Nonspherical QGBags + Hadrons}$$

$$\Sigma = 0 \text{ line} \quad \equiv \quad \text{phase boundary}$$



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Natural Order Parameters for CEP

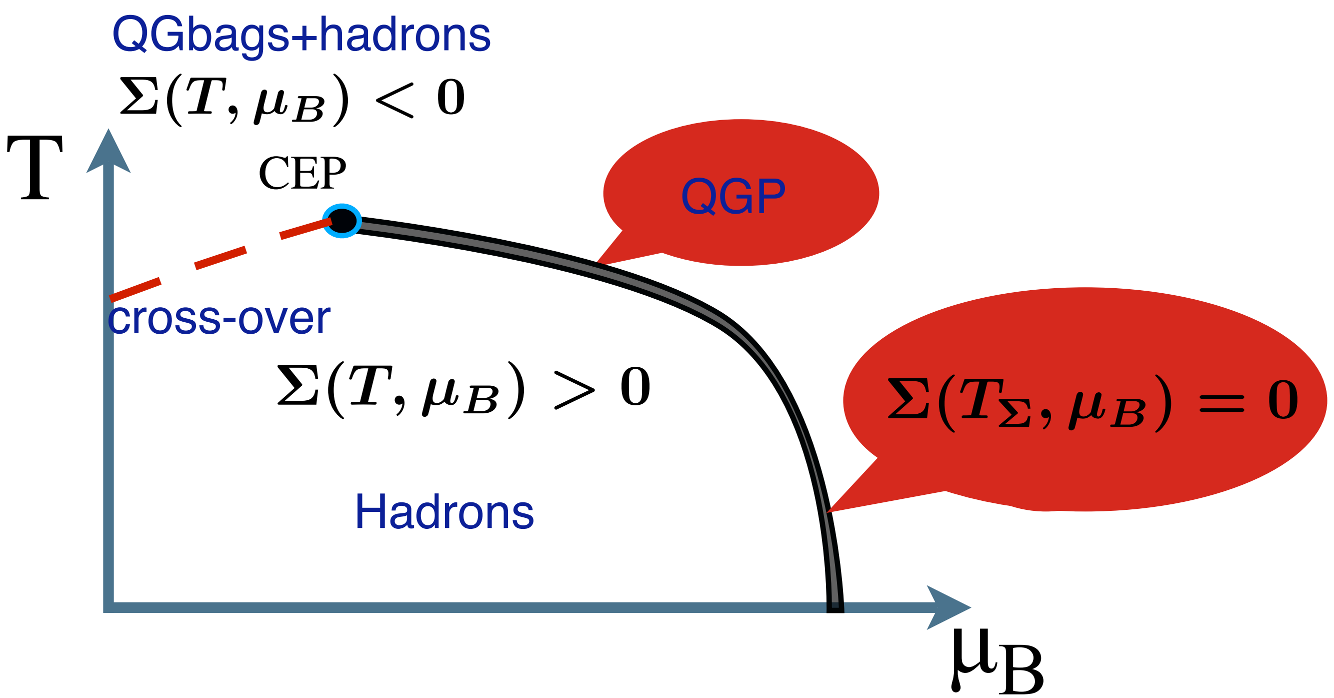
For CEP the situation is similar:

$$\Sigma > 0, \quad \langle\, v^q \,\exp\,[+\Sigma\, v^{\varkappa}]\,\rangle < \infty \quad \equiv \quad \text{Hadronic phase}$$

$$\Sigma = 0, \quad \langle\, v^q \,\exp\,[+\Sigma\, v^{\varkappa}]\,\rangle = \infty \quad \equiv \quad \text{CEP, Mixed phase (QGP)}$$

$$\Sigma = 0, \quad \langle\, v^q \,\exp\,[+\Sigma\, v^{\varkappa}]\,\rangle < \infty \quad \equiv \quad \text{cross-over line}$$

$$\Sigma < 0, \quad \langle\, v^q \,\exp\,[+\Sigma\, v^{\varkappa}]\,\rangle < \infty \quad \equiv \quad \text{Nonspherical QGBags + Hadrons}$$



$\tau > 2 \;\Rightarrow\; q = \tau - 1 > 1$

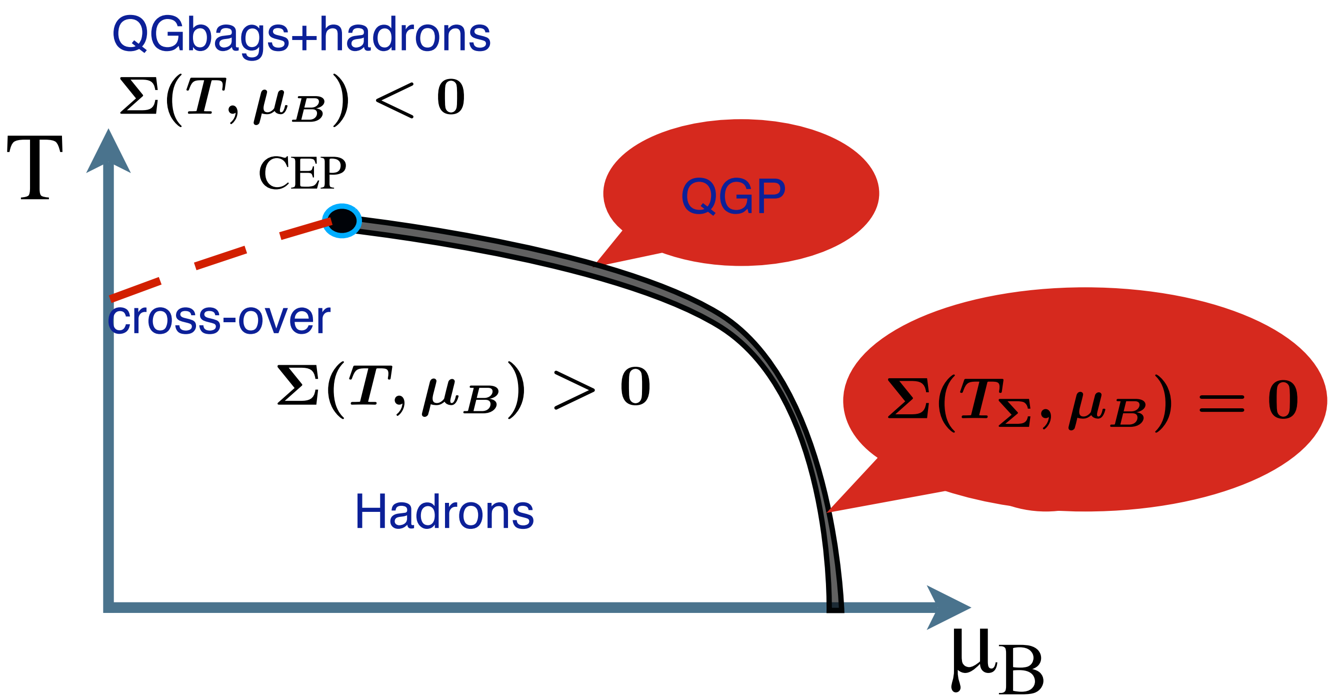
i.e. OP is not just a volume, but its power > 1

Natural Order Parameters for CEP

For CEP the situation is similar:

1. This scheme is valid for simple liquids as well

2. In fact, such a scheme gives an alternative way to find τ and Σ in microscopic models from ν-distribution, e.g. in LQCD



$\tau > 2$ => $q = \tau - 1 > 1$

i.e. OP is not just a volume, but its power > 1

Triple Point Generation for triCEP

Since sQGP is a strongly interacting liquid =>

1. It is naturally to expect that besides (tri)CEP it has a Triple point

2. in simple liquids triple point temperature < (tri)CEP temperature =>

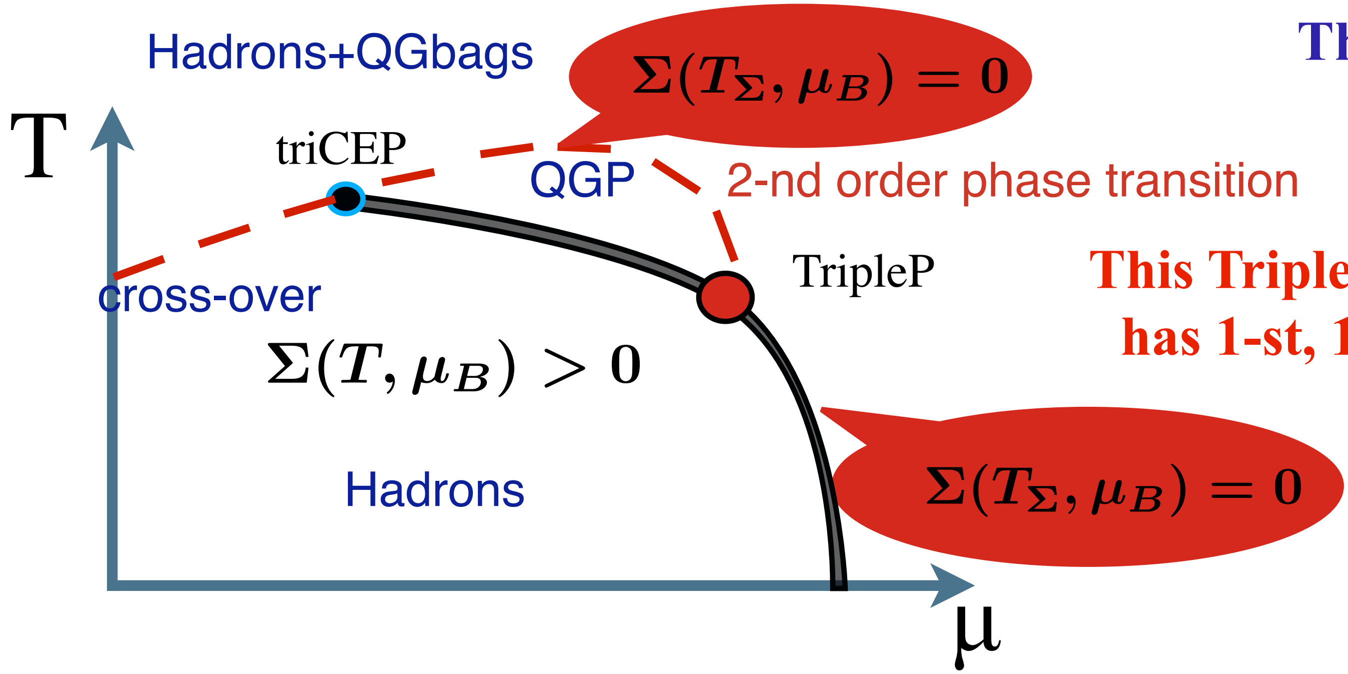
One can expect a Triple point existence

NOT the Triple point of Pisarski & McLerran!

Vanishing surface tension can induce a Triple point!

Motivation: different constituents have different interaction patterns =>

They should have different surface tension!



This Triple point is unusual since it has 1-st, 1-st and 2-nd order PTs

Triple Point Generation for CEP

In case of CEP there are 2 possibilities:

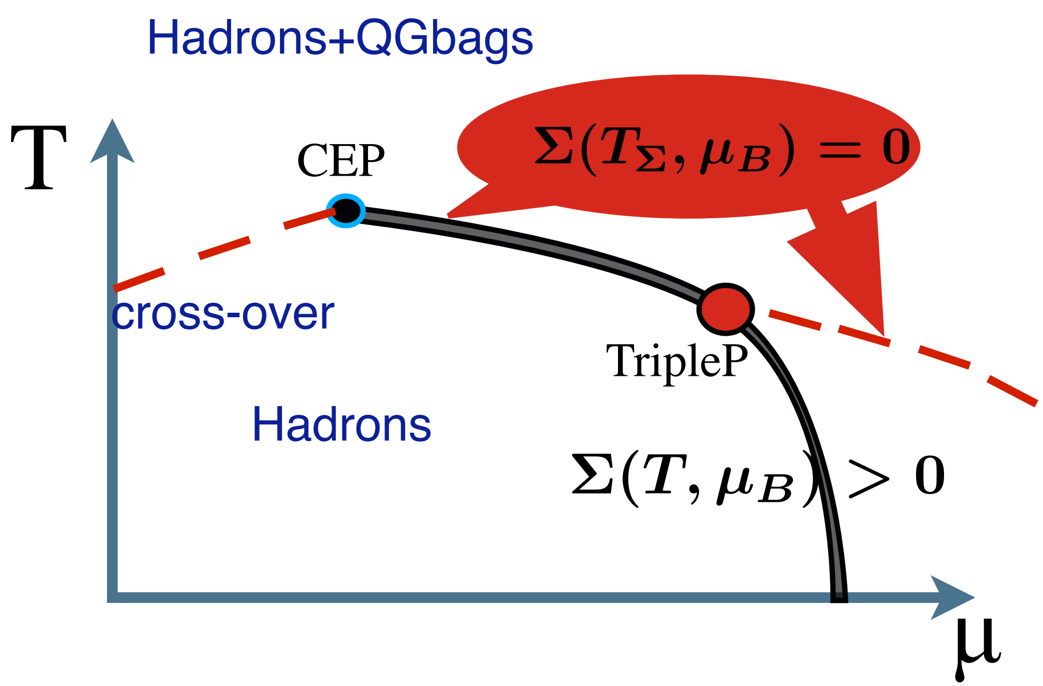
1. If $\partial\Sigma$ has a discontinuity at Σ=0 line on r.h.s. of Triple point =>

Usual Triple point: 1-st, 1-st & 1-st order PTs

2. If $\partial\Sigma$ has no discontinuity at Σ=0 line on r.h.s. of Triple point =>

1-st, 1-st & 2-nd order PTs

Such a possibility is expected for color superconducting PT



In either case this PT is a surface tension induced PT

Conclusions

Exactly solvable models for (tri)critical endpoint show us that the surface tension of QG bags plays a crucial role in the (tri)CEP existence.

QGBST Model allows us to introduce 2 order parameters to distinguish all phases located around (tri)CEP. Both of order parameters are related to surface tension coefficient!

QGBST Model with (tri)CEP can be extended to generate a Triple point. A zero surface tension line plays a KEY ROLE in a Triple point generation.

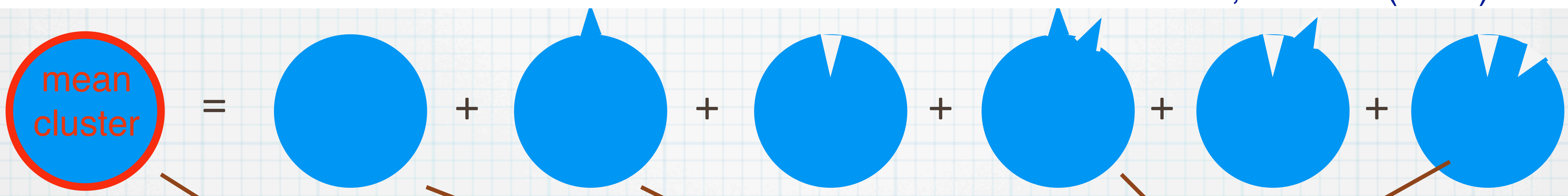
Back up slides

Surface Free Energy: F = E -TS

To find surface F one has to count for ALL surface deformations together with energy costs

Can be exactly done within Hills and Dales Model for v-volume cluster:

K.A.B. et al, PRE 72 (2005)



$$\underbrace{\exp\left[-\frac{\sigma_o v^{2/3}}{T}\right]}_{Energy\ \ part}\ \underbrace{\exp(S)}_{Entropy\ \ part} = \underbrace{\exp\left[-\frac{\sigma_o v^{2/3}}{T}\right]}_{Sphere's\ \ Energy}\times\left\{1 + \left(\underbrace{w_H N_H}_{1\ \ Hill} + \underbrace{w_D N_D}_{1\ \ Dale}\right)\exp\left[-\frac{\sigma_o \Delta S_1}{T}\right] + 2,\ 3,\ \text{etc deformations}\right\}$$

$$= \underbrace{\exp\left[-\frac{\sigma_o v^{2/3}}{T}\right]}_{Energy\ \ part}\ \underbrace{\exp\left[+\frac{\sigma_o v^{2/3}}{T_c}\right]}_{Entropy\ \ part}$$

Simplest case (M. Fisher)

Also one can find supremum and infimum for surface F and surface partition

$$\sigma_0(1-\lambda_L T)\, v^{\frac{2}{3}} \;\geq\; F \;\geq\; \sigma_0(1-\lambda_U T)\, v^{\frac{2}{3}}\,, \quad \lambda_L \approx 0.28\, T_c^{-1}, \quad \lambda_U \approx 1.06\, T_c^{-1}$$

K.A.B. & Elliott, UJP 52 (2007)

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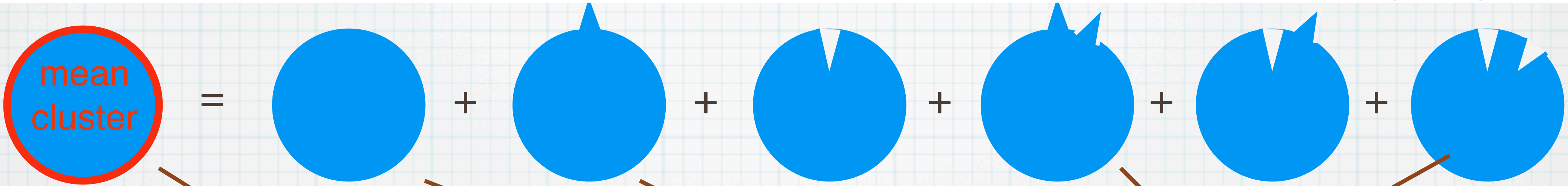

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Thus, there is NOTHING wrong, if surface F < 0 above critical T!
This means only that entropy dominates!

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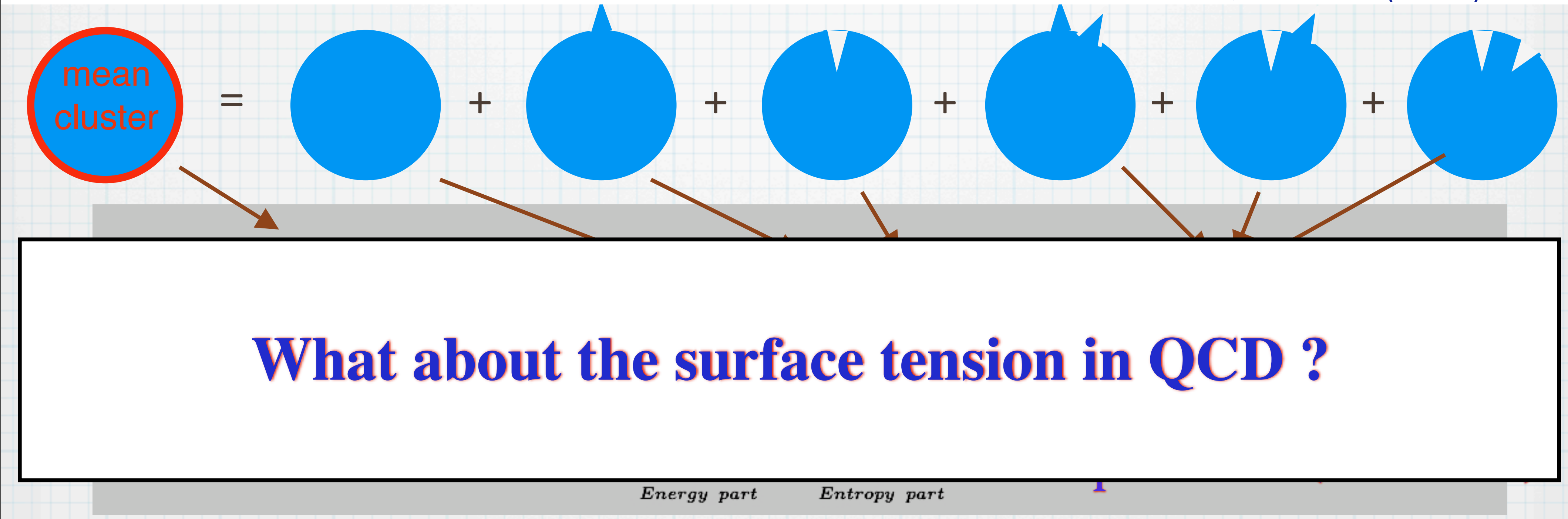

What about the surface tension in QCD ?

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Belief of Many: CEP is Chiral CEP

Chiral Symmetry in QCD is NOT EXACT in our world due to nonzero quark masses

=> there is first principle arguments to believe that CS Restoration happens via 1-st order phase transition

=> (tri)CEP in QCD has another origin than CSR

Such a belief is based on MEAN-FIELD approximation

However: mean-field models are NOT TRULY statistical ones since they do not account for full phase volume

MEAN-FIELD models are good when the surface effect is negligible:

I. small constituents at low densities (e.g. VdWaals EOS)

II. interior of large constituent (e.g. dense liquid EOS)

MEAN-FIELD models do not work, if there is phase separating boundary