On the magnetic mass of gluons in Abelian magnetic background field

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Plan
1. Motivation
2. Vacuum energy in a magnetic field
3. Formalism for polarization tensor
4. Magnetic mass for neutral gluon
5. On the existence of the polarization tensor of the charged gluon

Aim

Investigate the one loop polarization tensor of a non Abelian vector field (gluon) in a chromo magnetic background field

$$A^a_\mu(\underline{x}) = \delta_{a,3}\,\frac{B}{2}\begin{pmatrix} 0 \\ -y \\ x \\ 0 \end{pmatrix}$$

homogenous Abelian chromo magnetic field, is solution of the eom

Motivation

A chromo magnetic field is likely to be spontaneously generated by gluons due to their magnetic moment (tachyonic mode)

some definitions

What means 'neutral' and 'charged' gluons?

consider SU(2)-field: $A_\mu^a(x)$ it is a neutral vector field, $a \in SU(2)$

make a transformation:

$$Q_\mu^\pm(x) = \frac{1}{\sqrt{2}}\left(A_\mu^1(x) \pm i A_\mu^2(x)\right)$$

$$Q_\mu(x) = A_\mu^3(x)$$

then $Q_\mu^\pm(x)$ is a charged vector field whereas $Q_\mu(x)$ is a neutral one

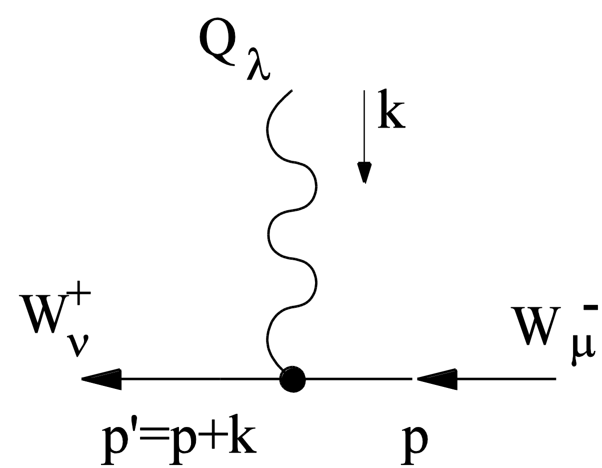
wave equations:

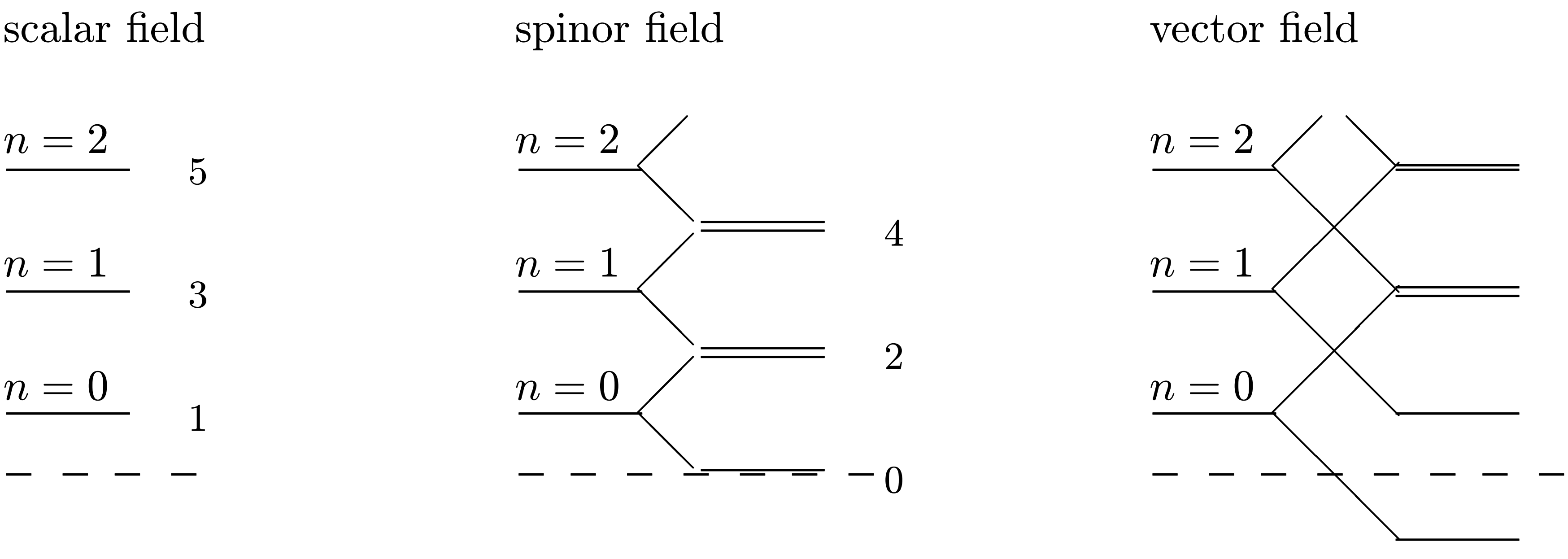
$$\partial_\nu \partial^\nu Q_\mu(x) = 0$$

$$\left(D_\nu^\pm D^{\pm\nu} g_{\mu\mu'} - 2igF_{\mu\mu'}\right) Q_{\mu'}^\pm(x) = 0$$

with the covariant derivative $D_\mu^\pm = \partial_\mu \pm i B_\mu$ in an Abelian magnetic background field $B_\mu^a = \delta^{a,3} B_\mu$

coupling between the fields:





$E_n^2 = B(2n+1)$ $\qquad$ $E_n^2 = B(2n+1 \pm \frac{1}{2})$ $\qquad$ $E_n^2 = B(2n+1 \pm 1)$

In a technical sense, it is interesting to extend the formulas known from QED in magnetic field (Schwinger proper time formalism) to non Abelian vector field

while we expected 'only' technical complications, there appeared more serious ones

Vacuum energy in a homogeneous magnetic background field

vacuum energy $\widehat{=}$ effective potential
use zeta functional regularization

$$\text{scalar:} \quad E_0 = \frac{B}{2\pi}\int\frac{dk_3}{2\pi}\sum_{n\geq 0}\left(m^2+k_3^2+B(2n+1)\right)^{\frac{1}{2}-s}$$

$$\text{spinor:} \quad E_0 = -\frac{B}{2\pi}\int\frac{dk_3}{2\pi}\sum_{n\geq 0}\sum_{\sigma=\pm 1}\left(m^2+k_3^2+B(2n+1+\sigma)\right)^{\frac{1}{2}-s}$$

$$\text{vector:} \quad E_0 = \frac{B}{2\pi}\int\frac{dk_3}{2\pi}\sum_{n\geq 0}\sum_{\sigma=\pm 1}\left(k_3^2+B(2n+1+2\sigma)\right)^{\frac{1}{2}-s}$$

use proper time representation (pseudo Euclidean)

$$\left(m^2+k_3^2+B(2n+1)\right)^{\frac{1}{2}-s} = i^{\frac{1}{2}-s}\int_0^\infty \frac{dt}{t}\,\frac{t^{s-\frac{1}{2}}}{\Gamma\left(s-\frac{1}{2}\right)}\,e^{-it\left(m^2+k_3^2+B(2n+1)-i\epsilon\right)}$$

carry out the summation over n and perform Wick rotation $t \to -it$

$$\text{scalar:} \qquad E_0 = -\frac{B}{8\pi^2}\,c_s\int_0^\infty \frac{dt}{t}\,t^{s-1}\,\frac{1}{2\sinh(Bt)}\,e^{-tm^2}$$

$$\text{spinor:} \qquad E_0 = \frac{B}{8\pi^2}\,c_s\int_0^\infty \frac{dt}{t}\,t^{s-1}\,\coth(Bt)\,e^{-tm^2}$$

$$\text{vector:} \qquad E_0 = -\frac{B}{8\pi^2}\,c_s\int_0^\infty \frac{dt}{t}\,t^{s-1}\,\left[e^{Bt}+e^{-Bt}\coth(Bt)\right]$$

$c_s = 1+(\gamma+\ln 4-2)s+\dots$

This is the contribution from the tachyonic mode,
here one needs to make the 'Anti'-Wick rotation

perform the renormalization and calculate the finite parts

$$\begin{aligned}
\text{scalar:} \quad E_0^{\rm ren} &= \frac{\gamma}{12} + \frac{\ln 2}{6} + \frac{1}{6} + \zeta'(-1) \sim 0.16 \\
\text{spinor:} \quad E_0^{\rm ren} &= -\frac{\gamma}{3} - \frac{\ln 2}{2} + \frac{5}{6} - 4\zeta'(-1) \sim 1.07 \\
\text{vector:} \quad E_0^{\rm ren} &= -\frac{5\gamma}{6} + \frac{\ln 2}{3} + \frac{1}{3} + 2\zeta'(-1) \sim -0.25
\end{aligned}$$

the 'Anti'-Wick rotation in the tachyonic mode is so far the only complication beyond pure technical ones propagators

 $= \frac{1}{k^2} = \int\limits_0^\infty ds\ e^{-sk^2} \qquad k^2 = k_4^2 + k_3^2 + k_2^2 + k_1^2$

 $= \frac{1}{p^2} = \int\limits_0^\infty ds\ e^{-sp^2} \qquad p^2 = p_4^2 + p_3^2 + B(2n+1)$

μ $\mu' \qquad = \frac{g_{\mu\mu'}}{k^2} = \int\limits_0^\infty ds\ g_{\mu\mu'} e^{-sk^2} \qquad k^2 = k_4^2 + k_3^2 + k_2^2 + k_1^2$

μ  $\mu' \qquad = \left(\frac{1}{p^2 + 2iF}\right)_{\mu\mu'} = \int\limits_0^\infty ds\ E_{\mu\mu'} e^{-sp^2} \qquad p^2 = p_4^2 + p_3^2 + B(2n+1)$

with $E_{\mu\mu'} = \delta^{||}_{\mu\mu'} + iF_{\mu\mu'} \sinh(2sB) + \delta^{\perp}_{\mu\mu'} \cosh(2sB)$

one loop polarization tensor (gluon self energy)

basic loop $\Theta = e^{-sp^2} e^{-t(p-k)^2}$

integration over the loop momentum p (Schwinger 1973)

neutral

$$\langle\Theta\rangle = \frac{\exp\left[-\frac{st}{s+t}(l_3^2+l_4^2)+\frac{ST}{S+T}(l_1^2+l_2^2)\right]}{(4\pi)^2(s+t)\sinh(s+t)}$$

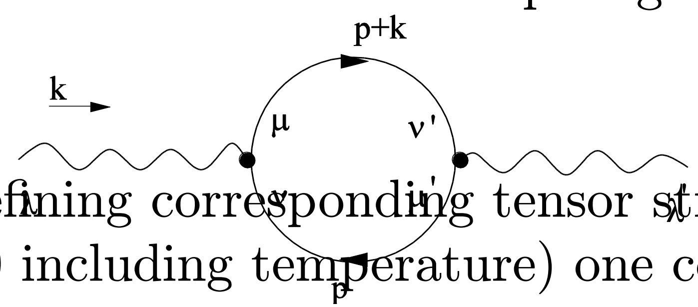
$$S = \tanh(s)$$
$$T = \tanh(t)$$

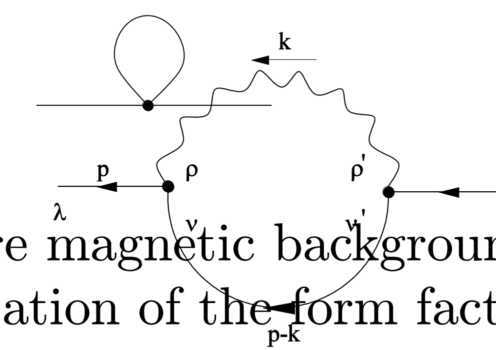
charged

$$\langle\Theta\rangle = \frac{\exp\left[-\frac{st}{s+t}(l_3^2+l_4^2)-m(s,t)(2n+1)\right]}{(4\pi)^2(s+t)\sqrt{\Delta}}$$

$$\Delta = \mu_+\mu_- \ , \quad m = s + \frac{1}{2}\ln\frac{\mu_-}{\mu_+}$$
$$\mu_\pm = t + \sinh(s)\, e^{\pm s}$$

in addition there are tadpole graphs and ghosts





defining corresponding tensor structures (6 in a pure magnetic background, 10 including temperature) one comes to a representation of the form factors general structure

$$\Pi_i = \frac{B}{(4\pi)^2} \int_0^\infty \int_0^\infty ds\, dt\;\; M_i(s,t)\;\; \langle \Theta \rangle$$

neutral

$$M^{(1)}(s,t) = 4 - 2\left(\frac{s-t}{s+t}\right)^2 \cosh(2(s+t))$$

$$M^{(2,3,4)}(s,t) = \dots$$

$$M^{(5)}(s,t) = -2 - \cosh(2s) + \cosh(2t)$$

charged

$$M^{(1)}(s,t) \;=\; 4 - 2\left(\frac{s-t}{s+t}\right)^2 \cosh(2s)$$

$$M^{(2)}(s,t) \;=\; \frac{\cosh(2s)t(-2t^3 + \dots)}{2(s+t)\Delta} + \dots$$

$$M^{(3,4,5)}(s,t) \;=\; \dots$$

$$M^{(6)}(s,t) \;=\; \frac{\sinh(2s)(2t^2 - \cosh(2s) + 1)}{2\Delta}$$

$$\Delta = \mu_+ \mu_-, \text{ where } \mu_\pm = t + \sinh(s)\, e^{\pm s}$$

convergence for $s \to \infty$

note: $M_i(s,t) \to e^{2s} m_i(s,t)$

therefore divide $M_i = \underbrace{e^{2s} m_i(s,t)} + \underbrace{M_i - e^{2s} m_i(s,t)}$

and $\Pi_i = \Pi_i^{\mathrm{A}} + \Pi_i^{\mathrm{B}}$

perform 'Anti'-Wick rotation in part A: $(s,t) \to (s,t)\, e^{i\pi}$

after which the integrations do converge (IR) result of calculation for neutral polarization tensor on-shell $(l^2 = 0)$

$$\text{Pi}(2) = (4+4\,i)\sqrt{\pi} + \frac{\left(3 - 3\sqrt{2} - 8\,\pi + 2\sqrt{2}\,\pi\right)\,\text{Zeta}\left[\frac{3}{2}\right]}{2\sqrt{\pi}} \qquad = -5.79894 + 7.08982\,i$$

$$\text{Pi}(3) = (5-5\,i)\sqrt{\pi} + \frac{\left(6 - 6\sqrt{2} - 4\,\pi + \sqrt{2}\,\pi\right)\,\text{Zeta}\left[\frac{3}{2}\right]}{2\sqrt{\pi}} \qquad = 1.04427 - 8.86227\,i$$

$$\text{Pi}(5) = i\sqrt{\pi} + \frac{1}{2}\sqrt{\pi}\left(2 + \left(-4+\sqrt{2}\right)\,\text{Zeta}\left[\frac{3}{2}\right]\right) \qquad = -4.21405 + 1.77245\,i$$

for the charged polarisation tensor we need to go into more dateil

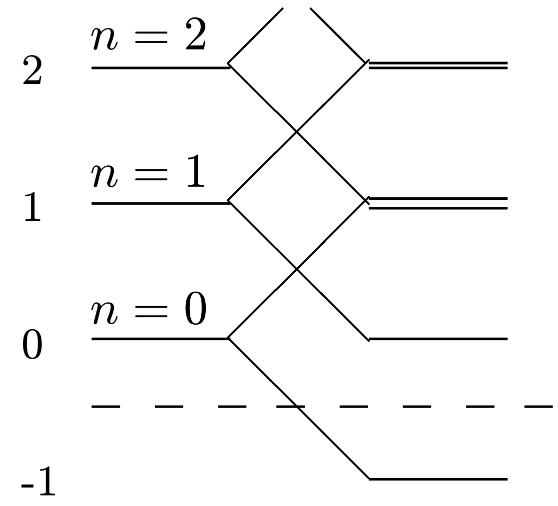
vector field

use variables $s = qu$, $t = q(1-u)$

and notations $\tilde{H} = qu(1-u)\delta + qu^2(2n+1)$

with $\delta = l_4^2 + l_3^2 + 2n + 1$ (on-shell is $\delta = 0$)

$$\Pi_i^{\mathrm{B}} = \int_0^\infty dq \int_0^1 du\, \tilde{M}_i \; \frac{\exp(-\tilde{H})}{\mu_+^{-n} \mu_-^{n+1}}$$



$E_n^2 = B(2n + 1 \pm 1)$

rewrite levels in the form $E_n^2 = 2n + 1$ with $n = -1, 0, 1, \ldots$

structure of the denominator

remember,

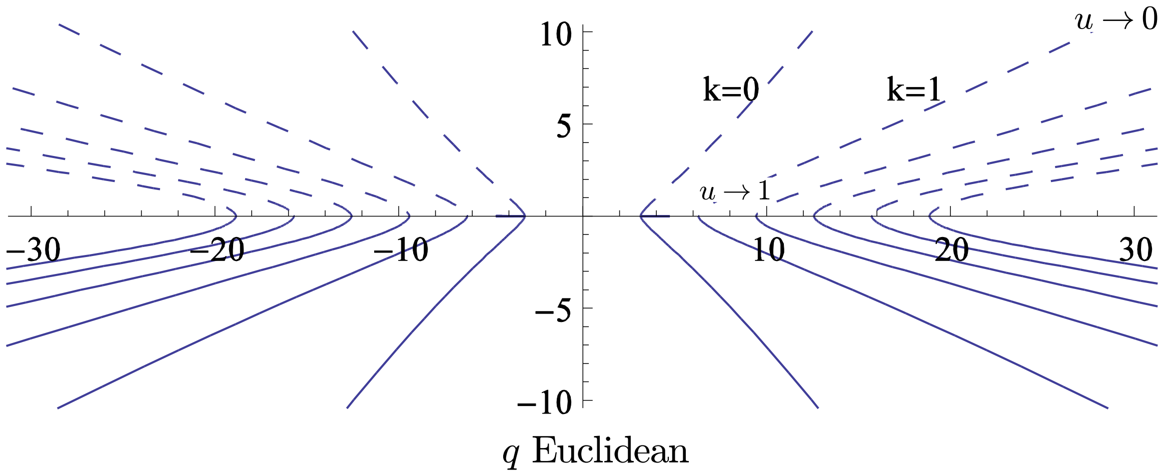
M_1 - no Δ in denominator

$$M_2 = \frac{\cdots}{\Delta^2} = \frac{\cdots}{(\mu_+)^2(\mu_-)^2}$$

$$M_{3,\ldots,6} = \frac{\cdots}{\Delta} = \frac{\cdots}{\mu_+\mu_-}$$

	$n=-1$	$n=0$	$n=1$	$\ldots$
$i=1$	$\frac{1}{1}$	$\frac{1}{\mu_-}$	$\frac{1}{(\mu_-)^2}$	
$i=2$	$\frac{1}{(\mu_+)^3(\mu_-)^2}$	$\frac{1}{(\mu_+)^2(\mu_-)^3}$	$\frac{1}{(\mu_+)^1(\mu_-)^4}$	
$i=3\ldots6$	$\frac{1}{(\mu_+)^2(\mu_-)^1}$	$\frac{1}{(\mu_+)^1(\mu_-)^2}$	$\frac{1}{(\mu_-)^3}$	

the functions μ_+ and μ_- have zeros!



the original integration goes over pseudo Euclidean q's:
in part A we make an 'Anti'-Wick rotation, $q \to iq$, but part A does not have zeros in the denominator
in part B we make a Wick rotation, $q \to -iq$, and we cross the zeros of μ_+

one could think to calculate the pole contributions separately, however, for $u \to 1$ the poles merge on the Euclidean q-axis

in this way, the integrals are not converging from the very beginning!!

	$n=-1$	$n=0$	$n=1$	$\dots$
$i=1$	$\frac{1}{1}$	$\frac{1}{\mu_-}$	$\frac{1}{(\mu_-)^2}$	
$i=2$	$\frac{1}{(\mu_+)^3(\mu_-)^2}$	$\frac{1}{(\mu_+)^2(\mu_-)^3}$	$\frac{1}{(\mu_+)^1(\mu_-)^4}$	
$i=3\dots6$	$\frac{1}{(\mu_+)^2(\mu_-)^1}$	$\frac{1}{(\mu_+)^1(\mu_-)^2}$	$\frac{1}{(\mu_-)^3}$	

this is what we have in scalar case, no factors M_i and no $n=-1$

hence no problems with Wick rotation

similar picture in spinor case

In this way this is a new problem occuring only for a charged vector field

Conclusions

the attempt to calculate the polarization tensor for charged gluons in a magnetic background field revealed unexpected difficulties

standard technical methods, Schwingers proper time representation, could be generalized for the vector case, however, these did not allow to come to a well-defined expression.

For some values of the external momentum, $n = -1, 0, 1$ in $B(2n+1)$, the integration path in pseudo Euclidean region appears quenched between pairs of poles and the integrations in the initial expression do not converge.

It is to be mentioned that these are NOT ultraviolet divergences, these would appear for $q \to 0$

Also, these are NOT infrared divergences which would appear for $q \to \infty$

The problem is puzzling since it appears also for $T \to \infty$ where one would expect perturbation theory to work (to some extent, at least)

Thank you for attention