KINETICS OF CHIRAL TRANSITION IN DENSE QUARK MATTER

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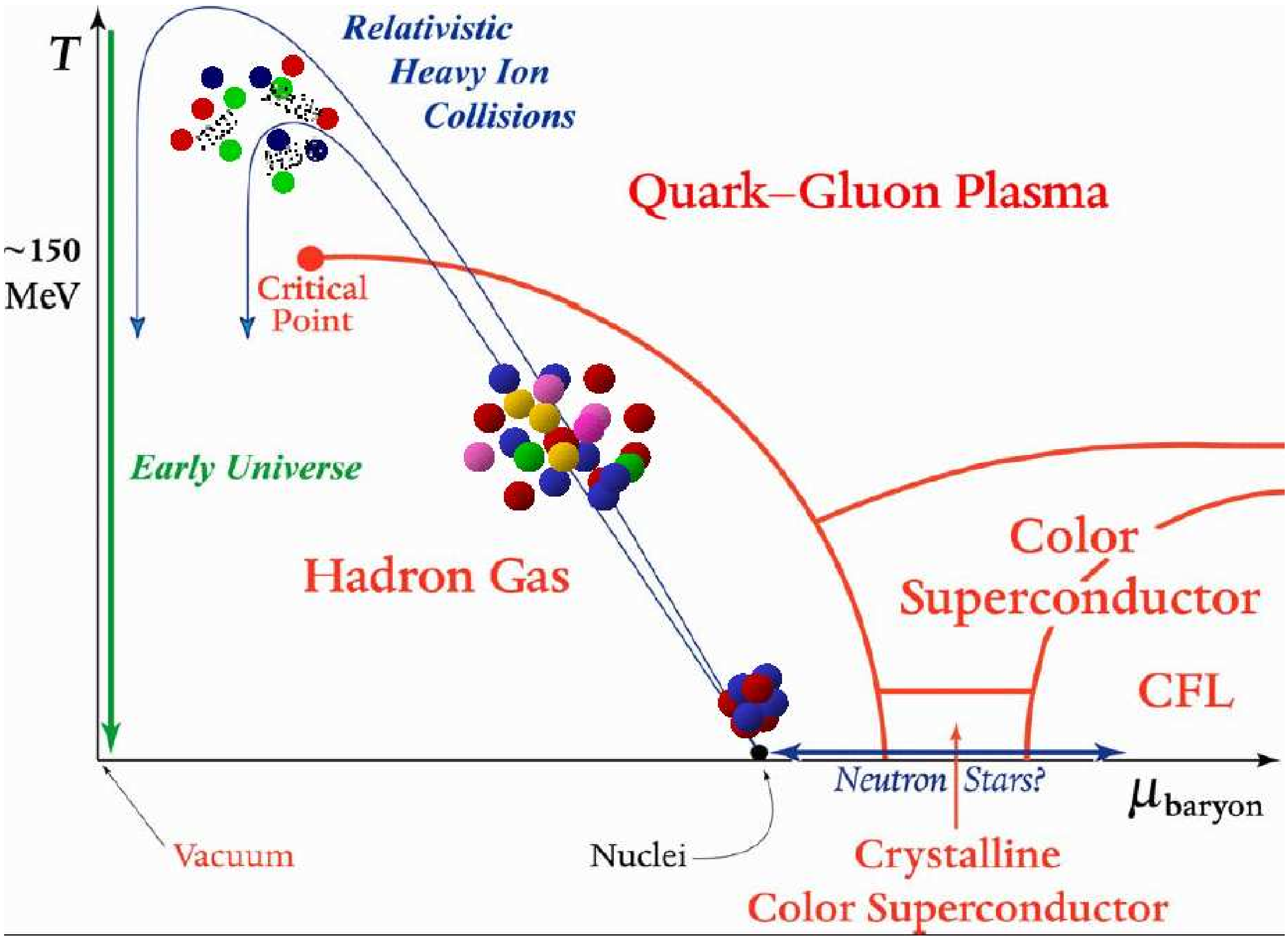
OUTLINE

- **Introduction**
- **QCD phase digram**
- **Vacuum with quark condensates in NJL model and phase diagram**
- **Gingburg-Landau expansion and TDGL equation**
- **Quenching through second order transition and domain growth**
- **Quenching through first order transition (bubble nuclation and spinodal decomposition)**
- **Summary and Outlook**

QCD UNDER EXTEREME CONDITION

- Extreme conditions exist in the universe. (Compact astrophysical objects, Cosmology)
- Exploring QCD phase diagram is important to understand the phase we live in
- Fundamental properties of QCD

QCD PHASE DIAGRAM (SCHEMATIC)



CSB AND VAC. STRUCTURE IN NJL MODEL

$$\mathcal{L}_{NJL} = i\bar{\psi}\partial\!\!\!/\psi + G[(\bar{\psi}\psi)^2 + (\bar{\psi}i\gamma_5\tau\psi)^2]$$

Two flavor, massless.

$$|vac\rangle = \exp(q^0(\mathbf{k})^\dagger \sigma \cdot \hat{\mathbf{k}} h(\mathbf{k}) \tilde{q}^0(-\mathbf{k}) d\mathbf{k} - h.c.)|0\rangle$$

$q^0|0\rangle = 0$

Determine the condensate function h(**k**) by minimising energy (T=0,μ=0),/free energy ($T \neq 0$,$\mu = 0$)/, thermodynamic potential ($T \neq 0$, $\mu \neq 0$).

$$\tan 2h(\mathbf{k}) = \frac{M}{|\mathbf{k}|} = \frac{-2g\langle\bar{\psi}\psi\rangle}{|\mathbf{k}|}$$

$g = G(1 + \frac{1}{4N_c})$

NJL MODEL CONTD. $\cdots$

Thermodynamic potential

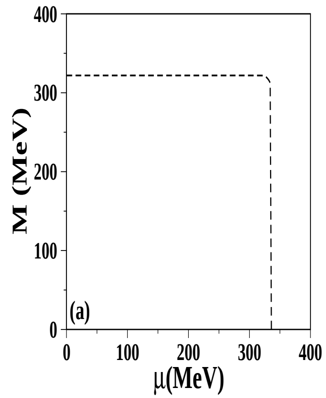
$$
\begin{aligned}
\Omega &= -\frac{12}{(2\pi)^3}\int(\sqrt{\mathbf{k}^2+M^2}-|\mathbf{k}|)d\mathbf{k} \\
&- \frac{12}{(2\pi)^3}\int[\log(1+\exp(-\beta\omega_-)+\log(1+\exp(-\beta\omega_+)]d\mathbf{k} \\
&+ \frac{M^2}{4g}
\end{aligned} \tag{1}
$$

$\omega_\mp = \sqrt{\mathbf{k}^2+M^2} \mp \nu$, $\nu = \mu - G\rho_v/N_c$.

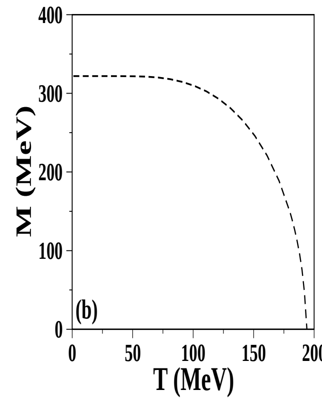
Mass gap equation

$$M = 2g\frac{2N_cN_f}{(2\pi)^3}\int\frac{M}{\sqrt{\mathbf{k}^2+M^2}}[1-n_-(\mathbf{k},\beta,\mu)-n_+(\mathbf{k},\beta,\mu)]d\mathbf{k}$$

PHASE DIAGRAM; NJL MODEL



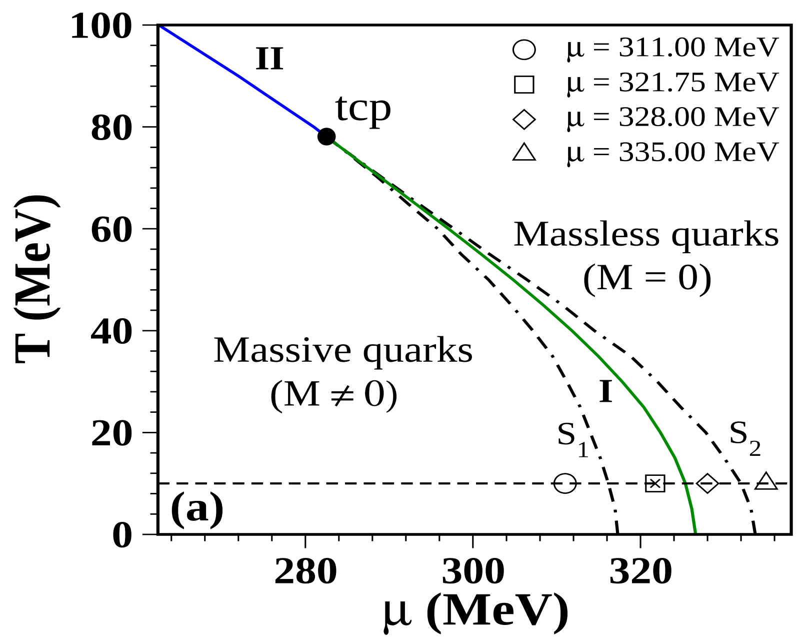
a



b

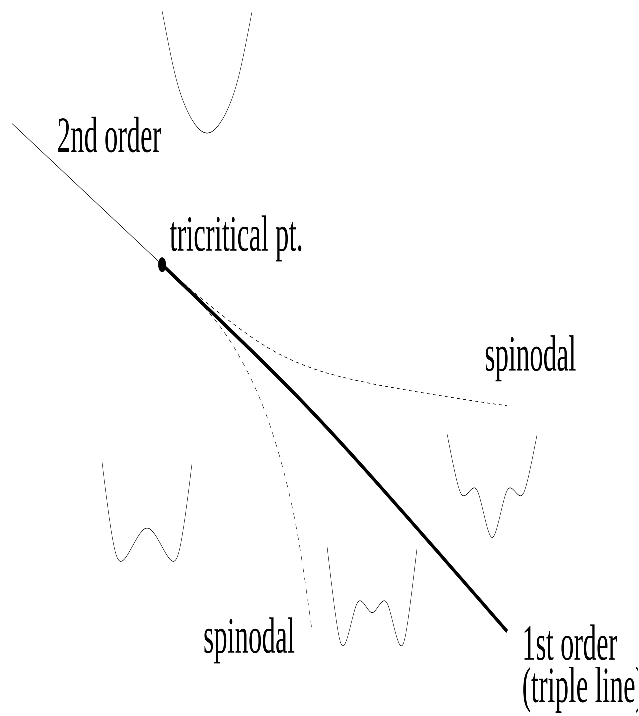
Mass$\sim G\langle\bar{\psi}\psi\rangle$ as a function of μ for T=0 (Fig a) and as a function of T for $\mu = 0$ (Fig b)

PHASE DIAGRAM; NJL MODEL CONTD. · · ·



Phase diagram of the Nambu-Jona-Lasinio (NJL) model in the (μ, T)-plane for zero current quark mass. A line of first-order transitions (I, green) meets a line of second-order transitions (II, blue) at the tricritical point (tcp). We have $(\mu_{\text{tcp}}, T_{\text{tcp}}) \simeq (282.58, 78)$ MeV. The dot-dashed lines S_1 and S_2 denote the spinodals or metastability limits for the first-order transitions.

PHASE DIAGRAM; NJL MODEL CONTD. · · ·



M.Stephanov,arXiv:hep-ph/0402115

$T_1 > T > T_c$, M>0 is a metastable state (superheated liq.)
$T_2 < T < T_c$, M=0 is metastable state (supercooled gas)

GINZBURG LANDAU EXPANSION OF FREE ENERGY

In the mean field approx. close to the phase boundary, the thermodynamic potential may be expanded in power series of the order parameter M:

Sasaki,Friman,Redlich,PRD77, 034024 (2008); Iwasaki,PRD 70, 114031(2004) $\cdots$

$$\tilde{\Omega}(M) = \tilde{\Omega}(0) + \frac{a}{2}M^2 + \frac{b}{4}M^4 + \frac{d}{6}M^6 + \cdots \equiv f(M).$$

a, b, d —functions of (μ, T)
Gap equation:

$$f'(M) = aM + bM^3 + dM^5 = 0.$$

Soln.s

$$\begin{cases} M_0 = 0, \\ M_\pm^2 = \dfrac{-b \pm \sqrt{b^2 - 4ad}}{2d}. \end{cases}$$

G L FREE ENERGY–CONTD$\cdots$

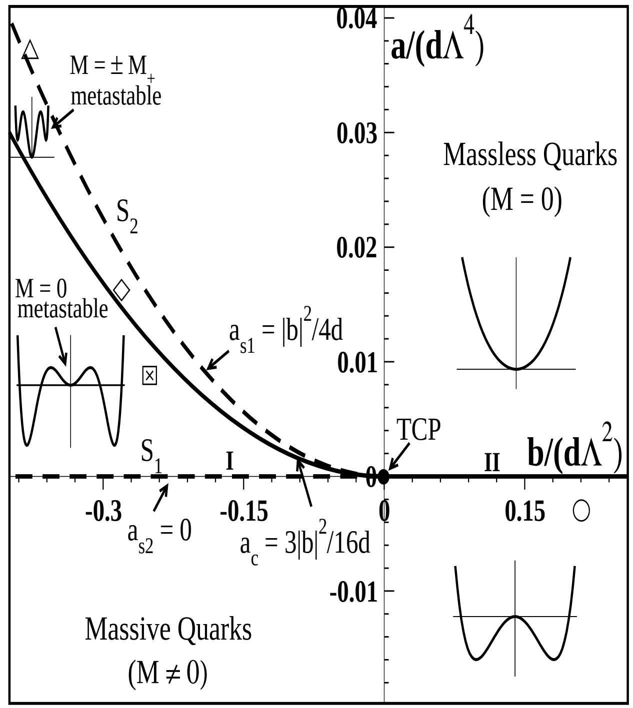
For $b > 0$ transition is second order.
Stationary pt.s are $M = 0$(for $a > 0$) OR M=0,$\pm M_+$(for $a < 0$)
For $b < 0$ phase transition is first order with the soln.s of gap eq.s

$$\begin{aligned} M &= \quad 0, \quad a > b^2/4d, \\ M &= \quad 0, \pm M_+, \pm M_-, \quad b^2/4d > a > 0, \\ M &= \quad 0, \pm M_+, \quad a < 0. \end{aligned} \tag{2}$$

Condn. of degeneracy of two minima
($\Omega(M = 0)$=$\Omega(M = M_+)$ or $a_c = 3b^2/(16d)$) determines T_c.
T_1 (T_2) is determined by $a = b^2/4d$ ($a = 0$).

GINZBURG LANDAU PHASE DIAGRAM

in the (b,a) space



Phase diagram in (b, a)-space for the GL free energy.

Non equllibrium system can probe the metastable and unstable region

DYNAMICAL EQUATIONS (TDGL EQNS)

Consider a system which is rendered thermodynamically unstable by a rapid quench from the disordered (symmetric) phase to the ordered (broken-symmetric) phase.
The unstable homogeneous state (with $M \simeq 0$) evolves via the emergence and growth of domains rich in the preferred phase (with $M \neq 0$).
Such far-from-equilibrium evolution, is termed *phase ordering dynamics* or *domain growth* or *coarsening*. Most problems in this area historically arise from condensed matter systems.
Equally fascinating is the kinetics of chiral transition!

TDGL CONTD. $\cdots$

Since coarsening system is inhomogeneous one includes a gradient term in the GL free energy

$$\Omega\left[M\right] = \int d\vec{r}\left[F\left(M\right) + \frac{K}{2}\left(\vec{\nabla}M\right)^2\right]$$

The evolution of the system is described by the time-dependent Ginzburg-Landau (TDGL) equation:

$$\frac{\partial}{\partial t}M\left(\vec{r},t\right) = -\Gamma\frac{\delta\Omega\left[M\right]}{\delta M} + \theta\left(\vec{r},t\right)$$

which models the relaxational dynamics of $M\left(\vec{r},t\right)$ to the minimum of $\Omega\left[M\right]$ (dissipative which damps the system towards the equillibrium configuration). Γ: inverse damping coefficient.

$\theta(\vec{r},t)$ represents the Langevin noise force assumed to be Gaussian and white satisfying the fluctuation-dissipation relation $\langle\theta\left(\mathbf{r},t\right)\rangle = 0$ and

$\langle\theta(\mathbf{r}',t')\theta(\mathbf{r}'',t'')\rangle = 2\Gamma T\delta(\mathbf{r}'-\mathbf{r}'')\delta\left(t'-t''\right)$

TDGL CONTD$\cdots$

Rescaling

$$\begin{aligned} M &= \sqrt{\frac{|a|}{|b|}} M' \\ \vec{r} &= \sqrt{\frac{K}{|a|}} \cdot \vec{r'} \\ t &= \frac{1}{\Gamma |a|} t' \\ \theta &= \frac{\Gamma |a|^{3/2}}{|b|^{1/2}} \theta'. \end{aligned} \qquad (3)$$

Dropping primes, we obtain the dimensionless TDGL equation:

$$\frac{\partial}{\partial t} M(\vec{r}, t) = -\mathrm{sgn}\,(\mathrm{a}) M - \mathrm{sgn}\,(\mathrm{b}) M^3 - \lambda M^5 + \nabla^2 M + \theta(\vec{r}, t),$$

where $\lambda = |a|d/|b|^2 > 0$.

Quenching through second order line $b > 0$

For $b > 0$, the chiral transition occurs when $a < 0$. The relevant TDGL equation is

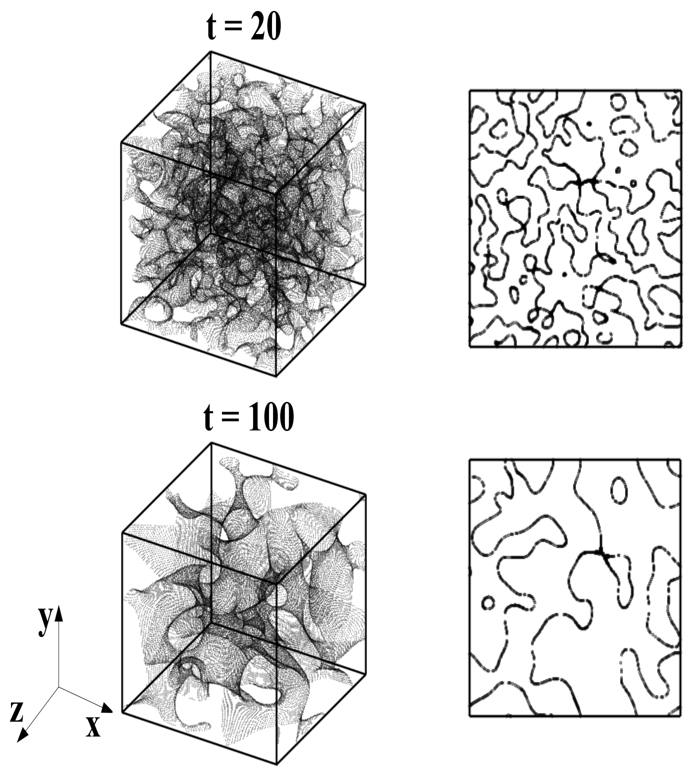
$$\frac{\partial}{\partial t} M\left(\vec{r}, t\right) = M - M^3 - \lambda M^5 + \nabla^2 M + \theta\left(\vec{r}, t\right),$$

Numerically solve this equation using a simple Euler discretization scheme on a 3d lattice of size 256^3 with periodic boundary condn.For stabilty,

$$\Delta t < \frac{2\Delta x^2}{4d + \alpha_1 \Delta x^2},$$

Initial cond. :Small amplitude random fluctuation about $M = 0$. The system rapidly evolves with domains with nonzero value of the order parameter. Interface of these domains have $M = 0$.

DOMAIN GROWTH ($b > 0$)

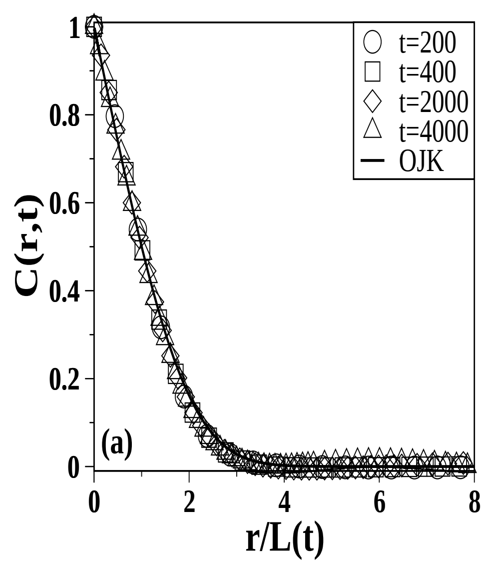


Evolution of domain in 3-d when $b > 0$ and $a < 0$. Here, we plot the interfaces between the domains ($M = 0$) only. Either side of the interface corresponds to equivalent domains with $M \simeq +M_+$ or $M \simeq -M_+$.

Correlation functions

Domains have a characteristic length scale $L(t)$, which grows with time.

$$C(\vec{r},t) \equiv \frac{1}{V}\int d\vec{R}\left[\left\langle M(\vec{R},t)M(\vec{R}+\vec{r},t)\right\rangle - \left\langle M(\vec{R},t)\right\rangle \left\langle M(\vec{R}+\vec{r},t)\right\rangle\right],$$



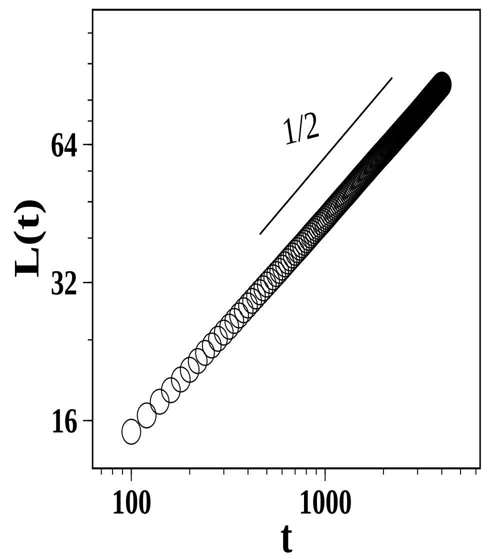
Scaling of correlation function for $\lambda = 0.14$ at four different time steps. OJK function (as for usual M^4-free energy) has good agreement with simulation data.

CORRELATION FUNCTIONS CONTD$\cdots$

The existence of characteristic scale results in the *dynamical scaling* of $C(\vec{r},t)$

$$C\left(\vec{r},t\right)= \ g\left(r/L\right)=\frac{2}{\pi}\sin^{-1}\left(e^{-r^2/L^2}\right).$$

Ohta-Jasnow-Kawasak (PRL49,1223 (1982)) scaling function.



log-log plot of domain size $L(t)$ vs t for $\lambda = 0.14$. The domain growth data is consistent with the AC growth law, $L(t) \sim t^{1/2}$

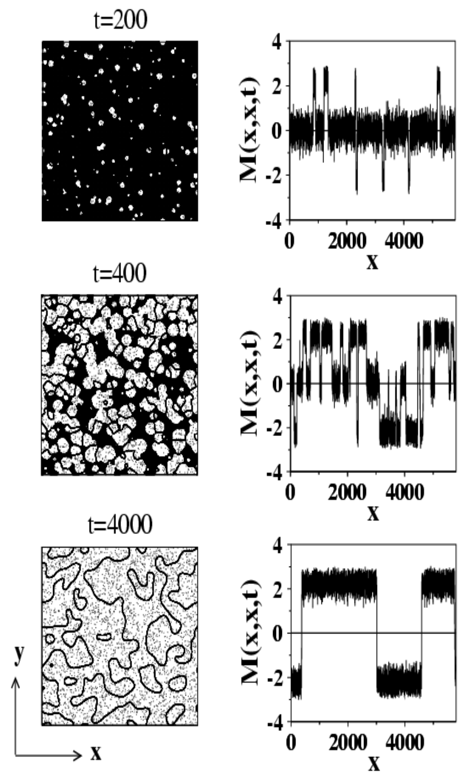
QUENCHING THROUGH FIRST ORDER LINE $b < 0$

First order transition occurs for $a < a_c = 3|b|^2/16d$ ($\lambda < \lambda_c = 3/16$)
For $a < 0$, double well structure for the free energy; the domain growth structure and ordering dynamics is similar to quenching through the 2nd order transition.
We confine our attention to $0 < a < a_c$ ($\lambda < \lambda_c$)

$$\frac{\partial}{\partial t} M\left(\vec{r}, t\right) = -M + M^3 - \lambda M^5 + \nabla^2 M + \theta\left(\vec{r}, t\right).$$

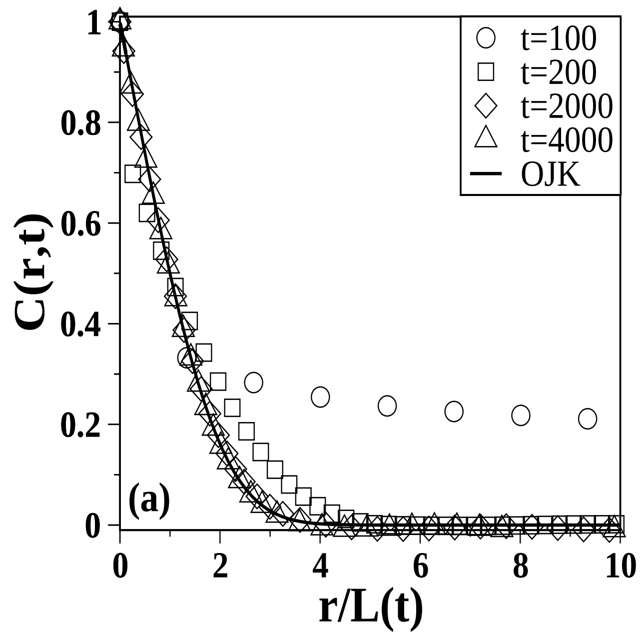
Evolve this equation with the initial state with $M = 0$ which is a metastable state. The chiral transition proceeds via the nucleation and growth of droplets of the preferred phase ($M = \pm M_+$). This nucleation results either from large fluctuations in the initial condition or thermal fluctuations during the evolution. Droplets grow with time and coalesce into domains.

NUCLEATION AND SPINODAL DECOMPOSITION

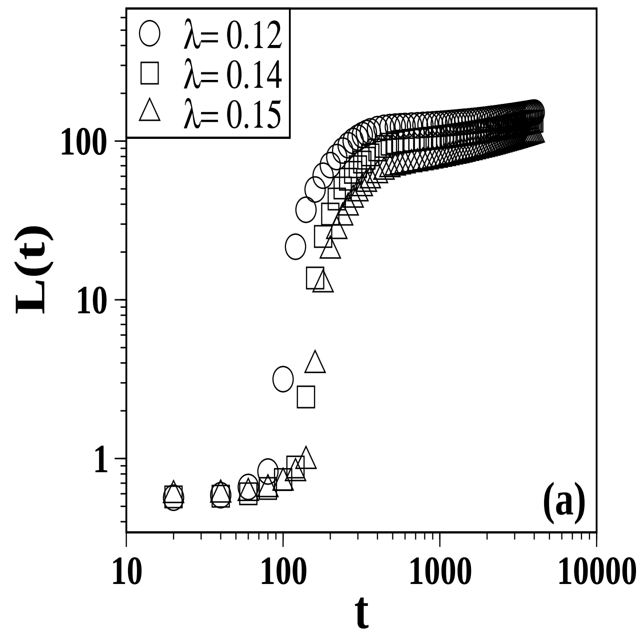


Shows regions with $M = 0$ only for $\lambda = 0.14$ at three different times $t = 200$, $t = 400$ and $t = 4000$ respectively. In the left of each evolution pattern we have shown the variation of order parameter profile along diagonal cross section.

Corrln. function and domain growth



Domain growth



Almost no growth in the early stages when droplets are being nucleated. Domain growth process begin once nucleation is over.

SUMMARY

- We considered the equllibrium phase diagram in a two flavor NJL model.
- In the mean field approximation and near the chiral phase transition, the thermodynamic potential can be Ginzburg Landau effective theory.
- The kinetics of the transition is considered using the TDGL equations.
- We studied the ordering dynamics resulting from a sudden temperature quench through both first order and second order transition lines. For quenches through the second order line the phase conversion is via spinodal decomposition. For quenches through the first order line, phase transition proceeds via nucleation and growth of droplets. Subsequent merger of these droplets results in late stage domain growth. Domain growth shows self similar dynamical scaling.