Identity method: a new tool for the study of chemical fluctuations in particle production

(first look at the NA49 data)

M. Maćkowiak
Warsaw University of Technology
for the NA49 Collaboration

Outline

- Chemical fluctuations and their measures
- Identity method
- First look at NA49 data with identity method
- Advantages of the method

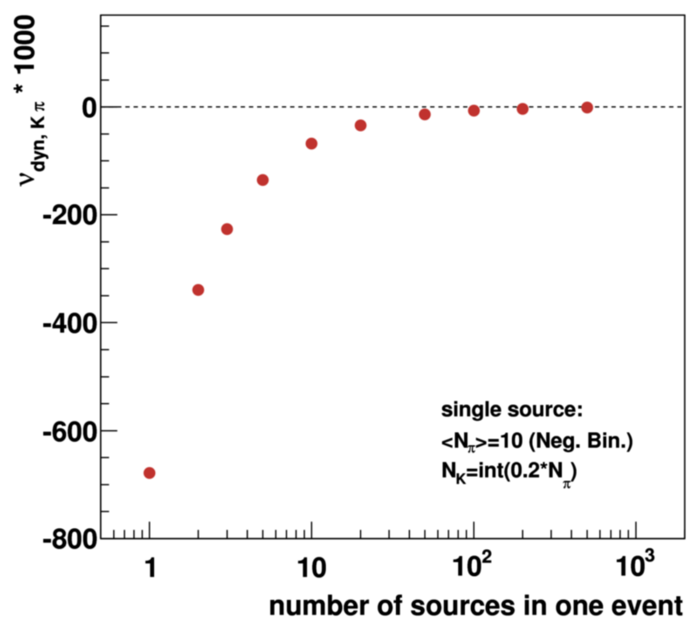
Chemical fluctuations and their measures

Identity method* was developed to study **event-by-event fluctuations of the chemical composition of the hadronic system produced in nuclear collisions**.

There are several measures which have already been developed to study chemical fluctuations:

- σ_{dyn} used by NA49 to quantify e-b-e particle ratio (e.g. K/π) fluctuations.
- ν_{dyn} used by STAR. Simple relation $\sigma_{dyn}^2 \approx \nu_{dyn}$ connects both measures.

Disadvantage of σ_{dyn} and ν_{dyn}:
for wounded nucleon and thermodynamical models they decrease as 1/<N_W> and 1/V, respectively



This disadvantage is not present for the ϕ measure of chemical fluctuations (next slide)

(*) *M. Gaździcki, K. Grebieszkow, M. Maćkowiak and S. Mrówczyński (to be published)*

Chemical fluctuations and their measures

- ϕ_x used already by NA49 to analyze p_T and charge fluctuations. There are two advantages of this measure:

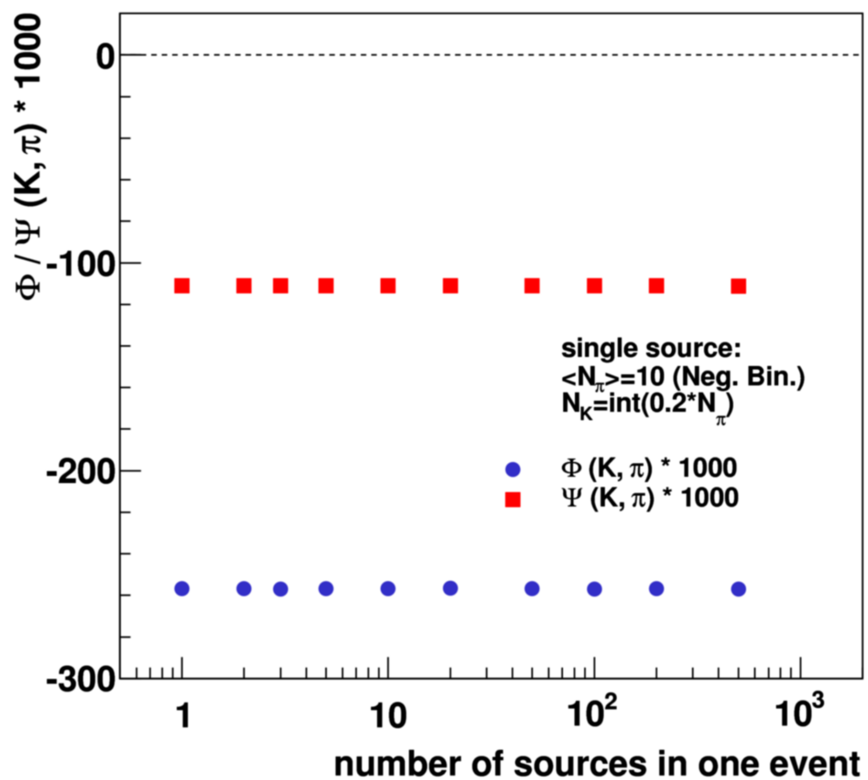
  $\phi_x(A+A)=\phi_x(N+N)$ if A+A is superposition of N+N and

  $\phi_x=0$ when inter-particle correlations in x are absent and single-particle x spectrum is independent of multiplicity

ϕ_x for chemical fluctuations:

$$x = \begin{cases} 1 & h_i = h_1 \\ 0 & h_i = h_2 \end{cases}$$

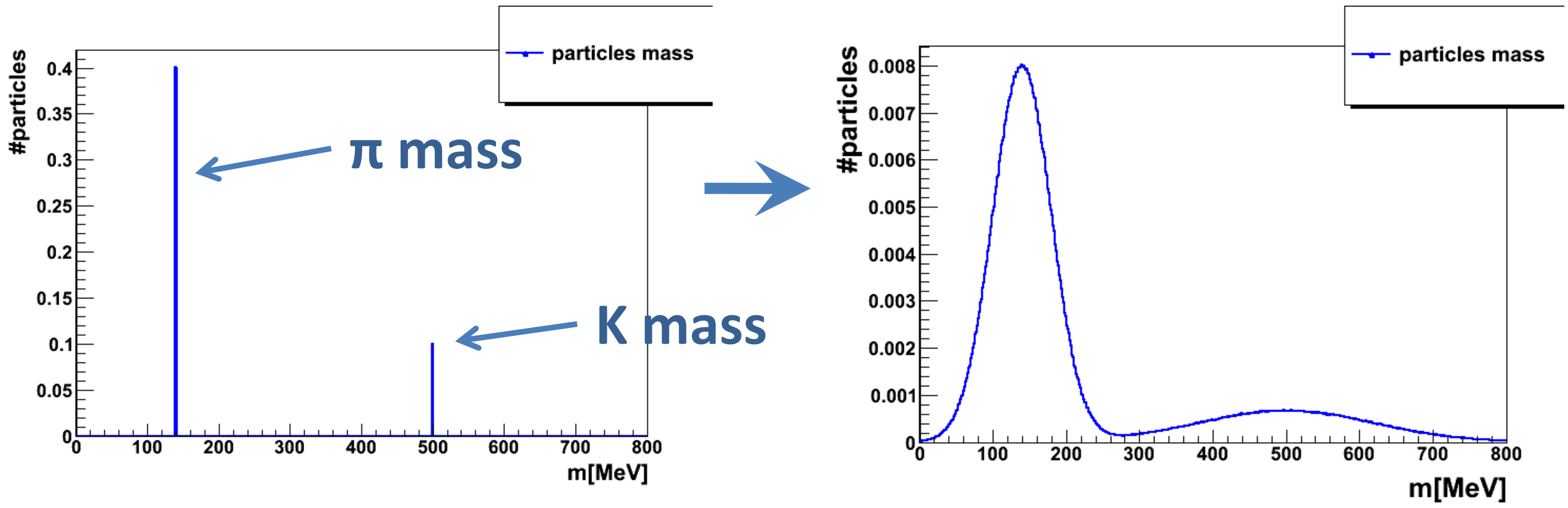
where h_i is a type of particle with index i and h_1 and h_2 are particle's types selected for analysis.



Chemical fluctuations and their measures

The study event-by-event chemical fluctuations has to consider one more effect which affects all ‘chemical’ measures:

resolution effect



ϕ_x can be used only for perfect identification.

Identity method generalizes ϕ_x to account to experimental resolution case keeping advantages of ϕ_x.

Notation of the method

experimental mass resolution

m – measured particle mass;

ρ - mass distribution of all particles averaged over events;

$\int \rho(m)dm = N$ - average multiplicity of an event;

M – total number of particles in all events;
h – particle type selected for fluctuation analysis;
ρ_h - mass distribution of h particles averaged over events;

$\int \rho_h(m)dm = N_h$ - average multiplicity of h particles in an event;

$$w_h = \frac{\rho_h(m)}{\rho(m)}$$

- for measured particle its probability of being h ('identity').

It is defined by measured particle's mass m

perfect mass resolution

h – particle type selected for fluctuation analysis;

i – all particles;

$$w_h = \begin{cases} 1 & \text{for } i = h \text{ particle} \\ 0 & \text{for } i \neq h \text{ particle} \end{cases}$$

Ψ fluctuation measure

Ψ, which is defined analogous to ϕ measure, as:

single-particle variable $z = w_{hi} - \overline{w_h}$,

where $\overline{w_h} = \frac{N_h}{N}$ - average over single particle inclusive distribution

event variable $Z = \sum_{i=1}^{n} (w_{hi} - \overline{w_h})$,

where n – multiplicity of an event

$$\Psi_{w_h} = \frac{<Z^2>}{<N>} - \overline{z^2}$$

Let's denote:

Ψ_{res} – value of Ψ_{w_h} for experimental mass resolution case

Ψ_{corr} – value of Ψ_{w_h} for perfect mass resolution case

Statistical variance due to finite resolution

$$Var_{res} = \frac{1}{M}\int_0^\infty dm\ \rho(m)\cdot w_h(m)(1 - w_h(m))$$

$Var_A = 0$ for perfect mass resolution

$Var_B = w_h*(1-w_h)$ for no resolution in mass measurement

In the case of experimental data integral is replaced by sum:

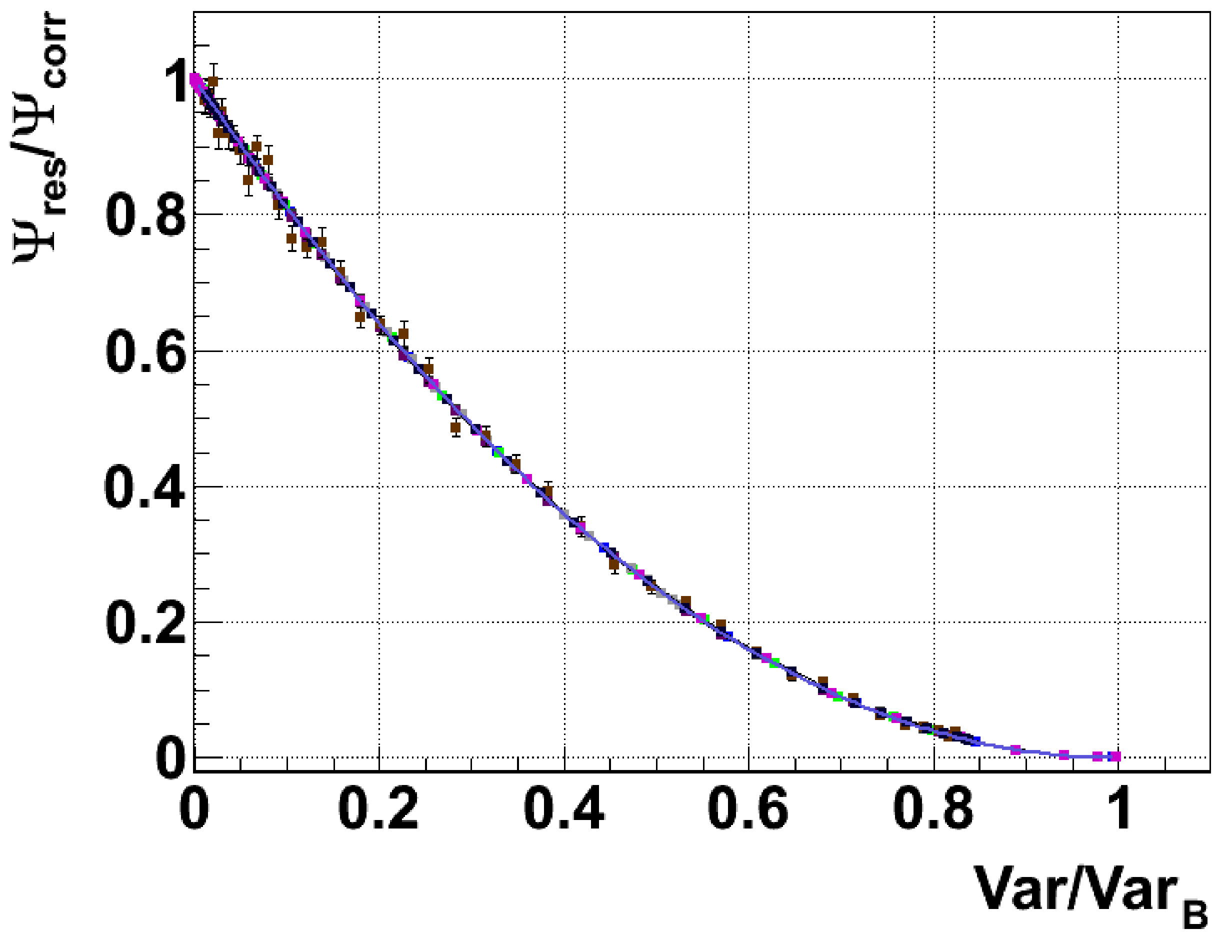
$$Var_{res} = \frac{1}{M}\sum_{i=1}^{M} w_{hi}(m_i)\cdot(1 - w_{hi}(m_i))$$

The following relation can be proven:

$$\frac{\Psi_{res}}{\Psi_{corr}} = (1 - Var_{res}/Var_B)^2 \quad (*)$$

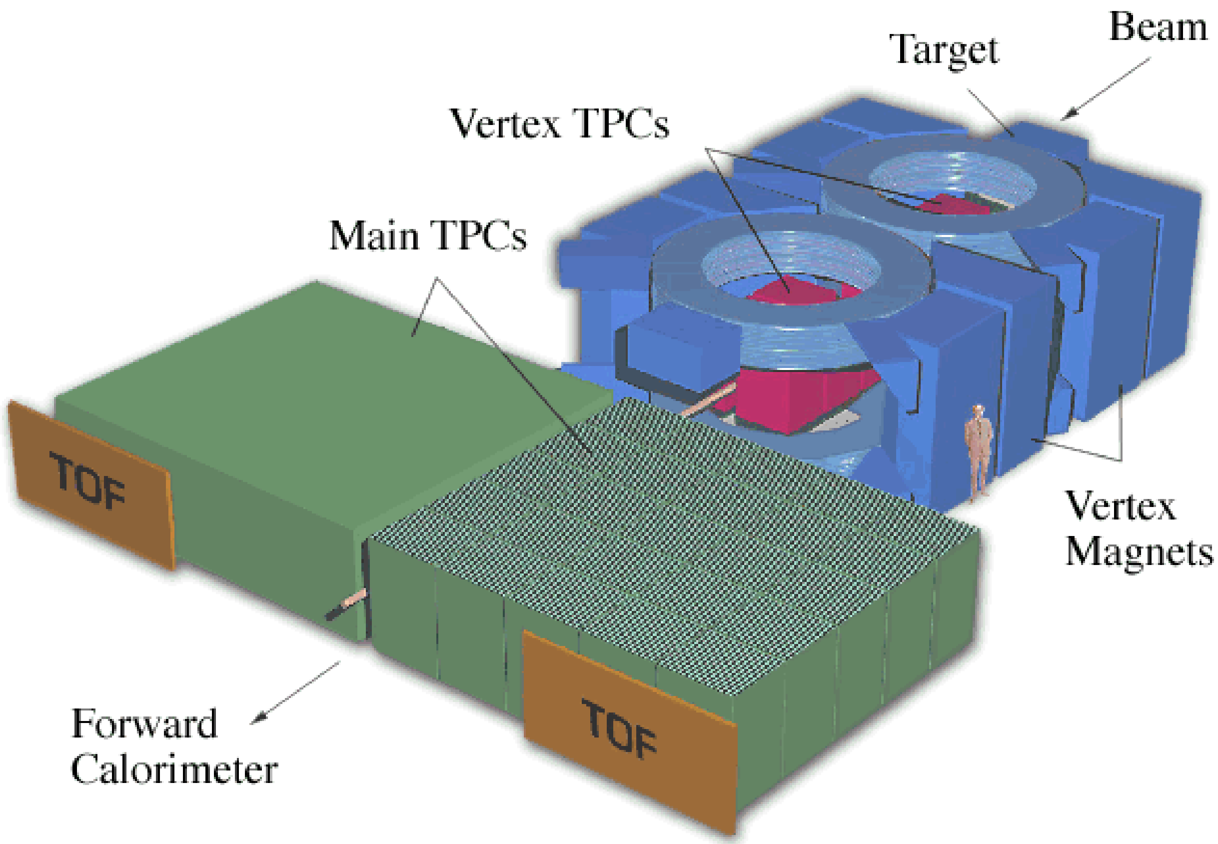
Resolution function and inclusive particle yields are included in Var_{res}/Var_B element. This relation is found to be this same for different types of correlations.

Monte Carlo check of the relation (*)



Points represent Monte Carlo simulations for different types of correlations, of mass resolution and of particle yields.

NA49 (fixed target) experiment at CERN SPS



Key features:

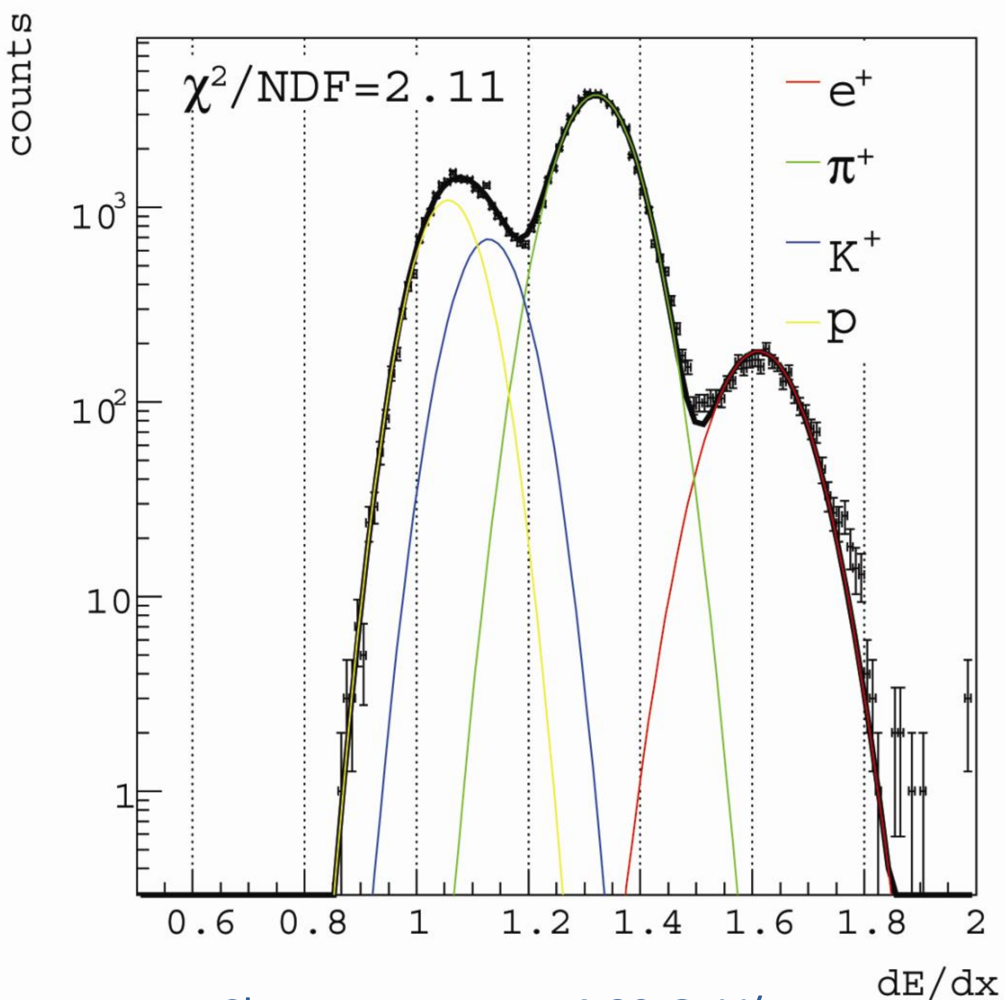
- **hadron spectrometer**
  4 large volume TPCs (two of them in B field)
- **good particle identification**
  by dE/dx, TOF, decay topology, invariant mass
- **Centrality determination: Forward Calorimeter**
  (energy of projectile spectators)

Operating **1994-2002**; **p+p, C+C, Si+Si** and **Pb+Pb** interactions at center of mass energy **6.3 – 17.3 GeV for N+N pair**

Example of application of identity method

In real data information about particle's mass is provided via dE/dx information.

Energy 40A GeV, example bin in q, p_{tot}, p_T, ϕ :



Charge +; <p_{tot}> = 4.82 GeV/c;
<p_T> = 0.3 GeV/c; <ϕ> = 1,375π

NA49 dE/dx information is stored in bins with specific q, p_{tot}, p_T, and ϕ. For every bin four Gaussian functions are fitted (ρ_e, ρ_π, ρ_K, ρ_p).

↓

N_h, ρ_h, ρ

↓

w_h

Identity method applied to NA49 data:

- using inclusive yields calculate statistical variance for no mass resolution

$$Var_B = \frac{N_h}{N} \cdot (1 - \frac{N_h}{N})$$

- for each particle calculate its probability of being h ('identity')

$$w_{hi}(<dE/dx>_i \,|q, p_{tot}, p_T, \phi) = \frac{\rho_h(<dE/dx>_i \,|q, p_{tot}, p_T, \phi)}{\rho(<dE/dx>_i \,|q, p_{tot}, p_T, \phi)}$$

- from all particles in all events using inclusive yields calculate statistical variance for experimental mass resolution

$$Var_{res} = \frac{1}{M} \sum_{i=1}^{M} w_{hi}(<dE/dx>_i \,|q, p_{tot}, p_T, \phi) \cdot (1 - w_h(<dE/dx>_i \,|q, p_{tot}, p_T, \phi))$$

- using the w_h calculate

$$\Psi_{res} = \frac{<Z^2>}{<N>} - \overline{z^2}$$

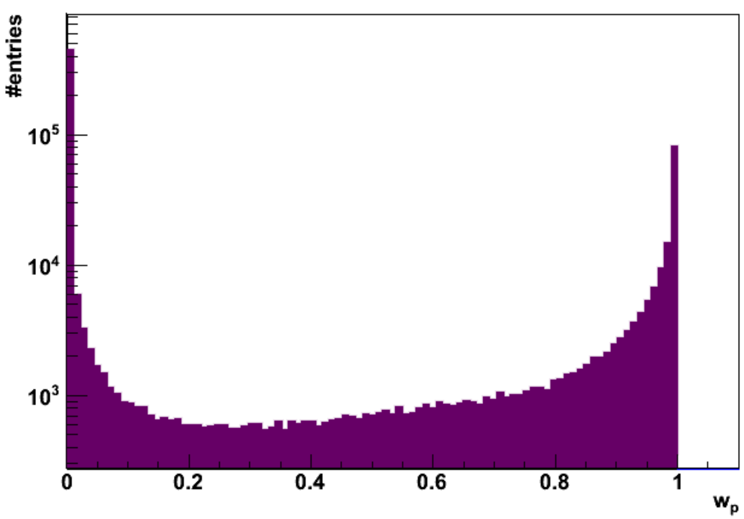
- correct Ψ_{res} for the bias due to the experimental mass resolution:

$$\Psi_{corr} = \Psi_{res} \cdot (1 - Var_{res} / Var_B)^{-2}$$

First look at the NA49 data

p fluctuations for Pb+Pb collisions at 40A GeV. Sample: 4k events.

Ranges of kinetic variables:
- q: neg. and pos. charge
- p_{tot}: 0-40 GeV/c
- p_T: 0-2 GeV/c
- ϕ from 0 to 2π



M = 661581

N = 165.40

N_p = 42.16 – value calculated from dE/dx fit

Var_B = 0.1899

Var_{res} = 0.0223

$\Psi_{res} \cdot 1000$ = -17.3823 ± 3.44916

correction Ψ_{corr}/Ψ_{res} = 1.2832

$\mathbf{\Psi_{corr} \cdot 1000}$ **= -22.3048 ± 4.4259**

Advantages of the identity method:

- Ψ is independent of volume and volume fluctuations for independent source models (strongly intensive fluctuation measure)

- event-by-event fits are not used (instead particle identity is used)

- mixed events are not used ($\Psi_{mixed} = 0$)

- correction for finite mass resolution is independent of event properties and has a simple analytical form

Thank you

Additional slides

σ_{dyn} and ν_{dyn} measures

σ_{dyn} is defined the following way:

$$\sigma_{dyn} = \mathrm{sign}(\sigma^2_{data} - \sigma^2_{mixed})\sqrt{\left|\sigma^2_{data} - \sigma^2_{mixed}\right|}$$

where σ_{data} is relative width (standard deviation divided by the mean) of the K/π distribution for the data and σ_{mixed} is relative width of the K/π distribution for mixed events.

ν_{dyn} is defined the following way:

$$\nu_{dyn,K\pi} = \frac{<N_K(N_K-1)>}{<N_K>^2} + \frac{<N_\pi(N_\pi-1)>}{<N_\pi>^2} - 2\frac{<N_K N_\pi>}{<N_K><N_\pi>}$$

where N_π is the number of π in each event and N_K is the number of kaons in each event.

ϕ_x measure

Φ is calculated as:

$$x = \begin{cases} 1 & i = h \\ 0 & i \neq h \end{cases}$$

where i is a particle and h is particle's type selected for analysis.

$$z_x = x - \bar{x},$$

single particle variable

where χ and $\bar{\chi}$ are single particle variable and average over single-particle inclusive distribution

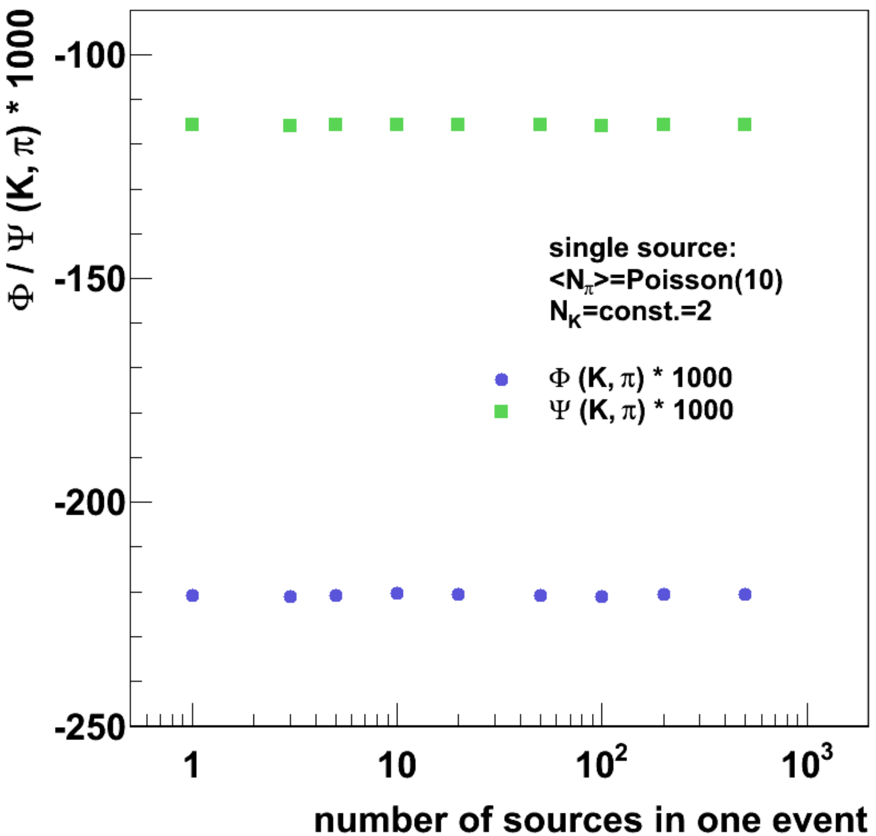
$$Z_x = \sum_{i=1}^{N} (x_i - \bar{x}),$$

where summation runs over particles in a given event

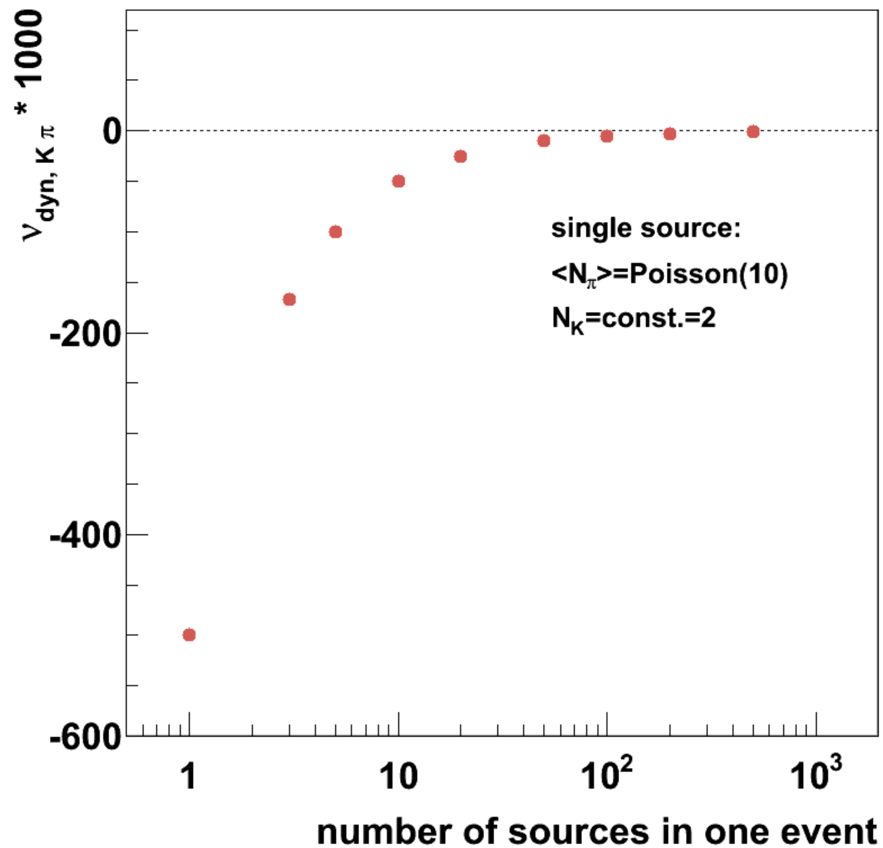
$$\Phi_x = \sqrt{\frac{< Z_x^2 >}{< N >}} - \sqrt{\overline{z^2}}$$

ϕ_x measure

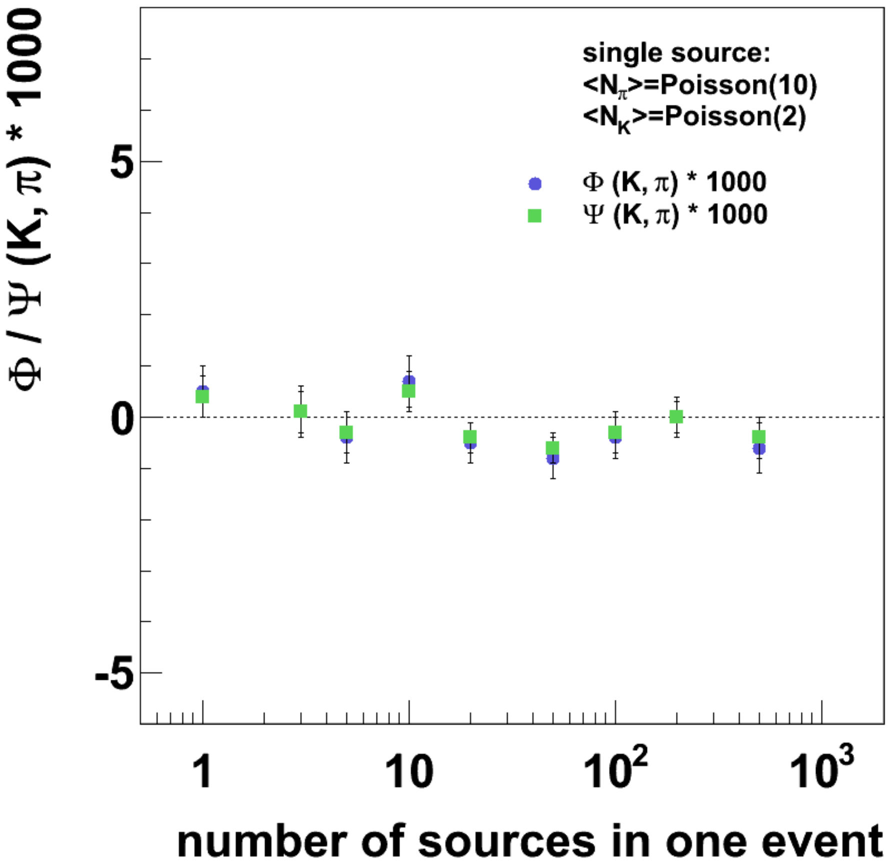
*M. Gaździcki and St. Mrówczyński, *A method to study `equilibration' in nucleus-nucleus collisions*, Zeischrift für Physik **C54** (1992) 127

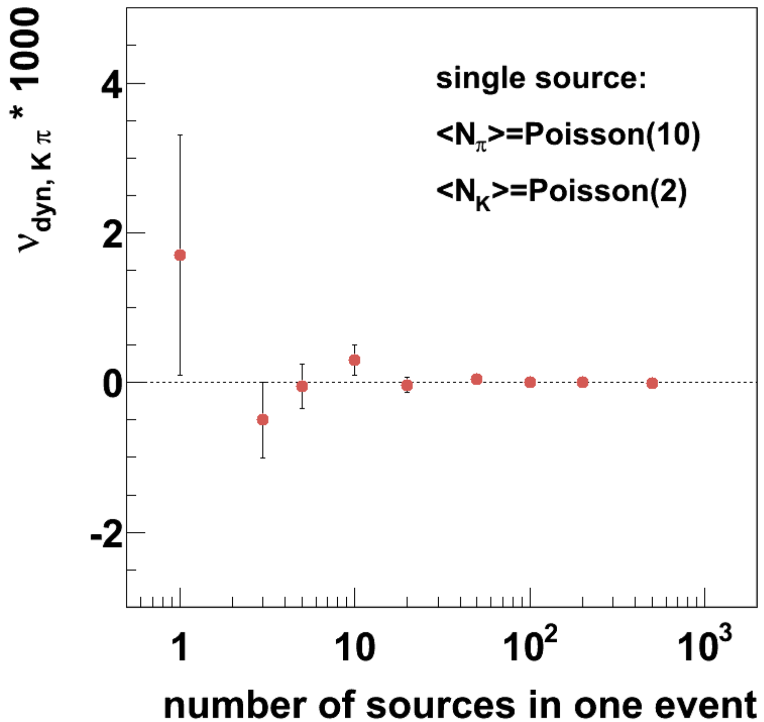


Φ / Ψ (K, π) * 1000
-100
-150
-200
-250
single source:
<N_π>=Poisson(10)
N_K=const.=2
Φ (K, π) * 1000
Ψ (K, π) * 1000
1
10
10^2
10^3
number of sources in one event



ν_dyn, K π * 1000
0
-200
-400
-600
single source:
<N_π>=Poisson(10)
N_K=const.=2
1
10
10^2
10^3
number of sources in one event


single source:
<Nπ>=Poisson(10)
<NK>=Poisson(2)
Φ (K, π) * 1000
Ψ (K, π) * 1000
Φ / Ψ (K, π) * 1000
number of sources in one event


single source:
<Nπ>=Poisson(10)
<NK>=Poisson(2)
νdyn, K π * 1000
number of sources in one event

Variance of the statistical fluctuation due to finite resolution

Let us quantify the experimental resolution of the mass measurements by the mean square deviation between the true number of particles h and the measured one using the identity method.

First divide the whole mass interval into M small intervals dM_i, i=1,..,M (for a moment i will be used instead of m).

The h type particles identity in an interval i is denoted: $w_h(m_i) = \frac{\rho_h(m_i)}{\rho(m_i)}$,

The expected number of particles in this interval is: $N_i = dM \cdot \rho_i$

Mixing between particles in this interval leads to binomial fluctuations (if particle id would be generated according to the identity value) around the real value of h type particles with variance:

$$Var_i = N_i \cdot w_{hi} \cdot (1 - w_{hi})$$

The bin-by-bin fluctuations due to particle mixing are independent and thus the variance for the whole event is equal to:

$$Var_{res} = \sum_{i=1}^{M} Var_i \rightarrow \int_0^\infty dm \rho(m) \cdot w_h(m)(1 - w_h(m))$$

Variance of the statistical fluctuation due to finite resolution

Let us consider Var in two limiting cases

A. perfect separation between particles, $\Delta m \to \infty$

$$w_{h_i} = \begin{cases} 1 \\ 0 \end{cases} \to Var_i = 0 \to Var(A) = 0$$

B. no separation between particles, $m_i = m$ and $\sigma_i = \sigma$

$$w_{h_i} = w_h = \frac{N_h}{N} = const(i)$$

Thus

$$Var(B) = Var_B = N \cdot w_h \cdot (1 - w_h)$$

is equal to the variance of the binomial distribution.

Toy model to study chemical fluctuations

We consider only two types of particles K and π. Their masses are noted m_K and m_π. The mass distribution after resolution effect is Gaussian shape defined as:

For K:

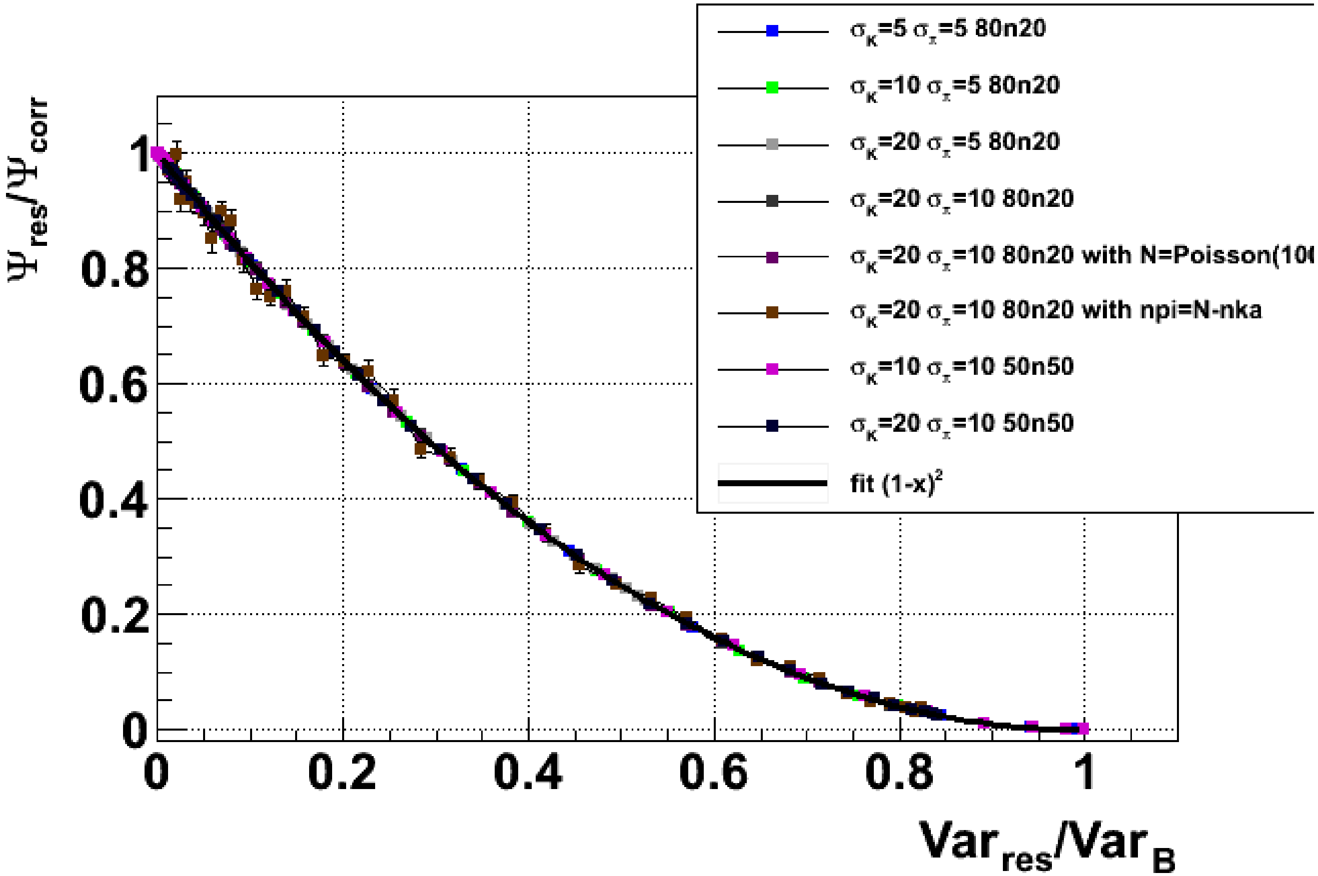
$$\rho_K(m_i) = \frac{P(K)}{\sqrt{2\pi}\sigma}\exp(-(m_i - m_K)^2/2\sigma^2),$$

For π:

$$\rho_\pi(m_i) = \frac{P(\pi)}{\sqrt{2\pi}\sigma}\exp(-(m_i - m_\pi)^2/2\sigma^2),$$

where m_i is measured mass.

Toy models in Monte Carlo check



Different Monte Carlo simulations are named after its color in the legend.

For blue(B), green(GR), grey(G), black(BL), pink(P) and dark blue(N) number of kaons and pions in an event is constant. For B, GR, G, BL it is 80 π and 20 K. For P and N it is 50 π and 50 K. σ parameter represents mass resolution in Gausian distributions (previous slide).
In Dark Pink(DP) total multiplicity N is generated from Poisson distirbution with N=100. 20% of generated particles are kaons.

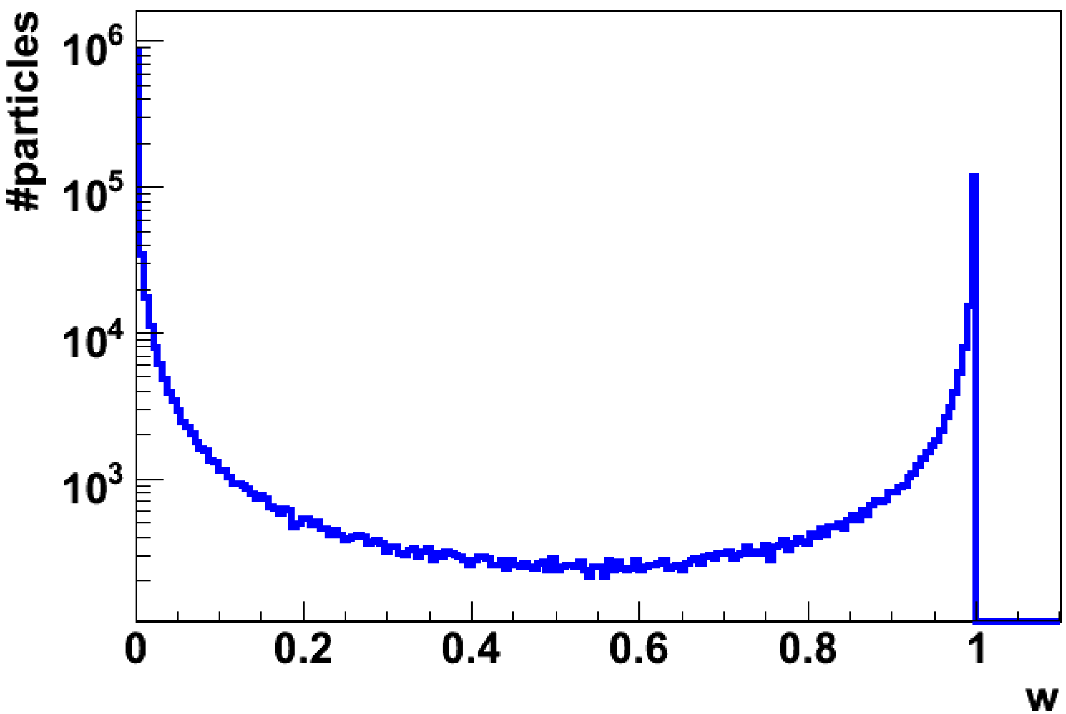
Identity for toy model

For every particle we can define quantity that it is a given type particle basing on its measured mass m_i:

$$w(m_i) = \frac{\rho_K(m_i)}{\rho_K(m_i) + \rho_\pi(m_i)},$$

where $\rho_K(m_i)$ and $\rho_\pi(m_i)$ are normalized K and π distributions for measured mass m_i.

w distribution for toy model from slide nr 12.

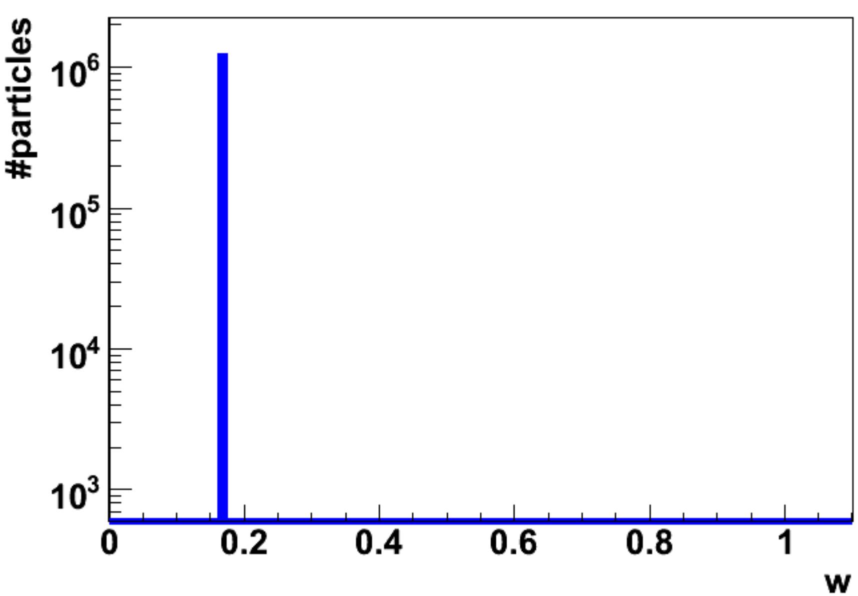


Method to study effect resolution

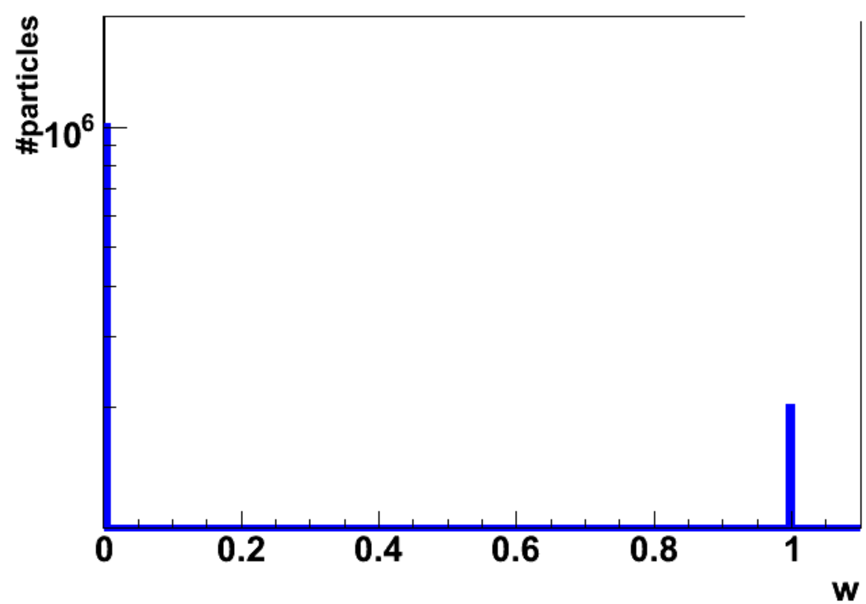
In order to study effect resolution we change distance between K and π masses, keeping constant σ.

$$\Delta m = m_K - m_\pi$$

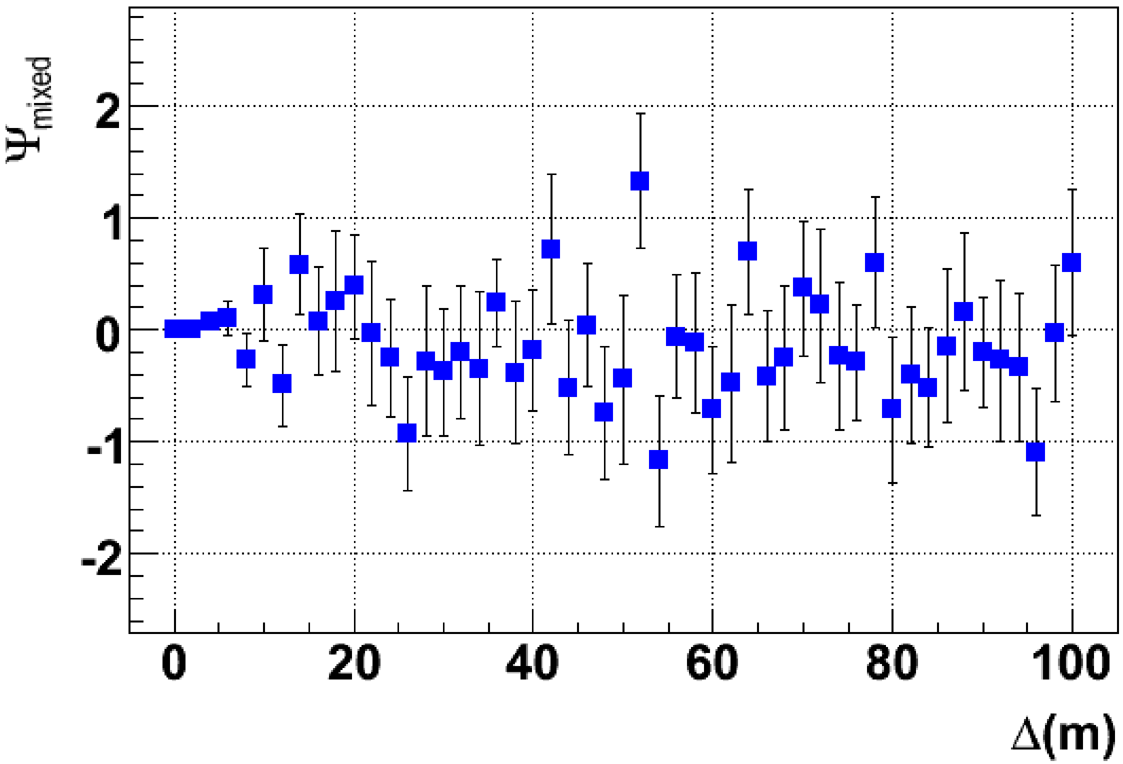
For $\Delta m=0$ w_h distributions for toy model defined on slide 8 is:



For $\Delta m=\infty$ w_h distributions is:



Ψ_{res} for mixed events



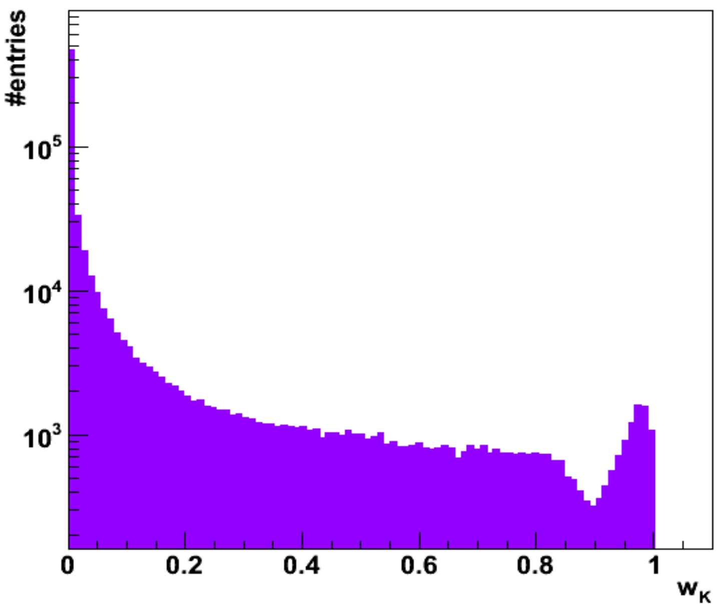
For mixed events Ψ_{res} (noted as Ψ_{mixed}) is consistent with 0.

First look at the NA49 data

K fluctuations for Pb+Pb collisions at 40A GeV. Sample: 4k events.

Ranges of kinetic variables:

- q: neg. and pos. charge
- p_{tot}: 0-40 GeV/c
- p_T: 0-2 GeV/c
- ϕ from 0 to 2π



M = 661581

N = 165.40

N_K = 11.62 – value calculated from dE/dx fit

Var_B = 0.065

Var_{res} = 0.031

$\Psi_{res} \cdot 1000 = 1.91111 \pm 0.88088$

correction $\Psi_{corr} / \Psi_{res} = 3.65$

$\mathbf{\Psi_{corr} \cdot 1000 = 6.98 \pm 3.22}$