A formula for charmonium suppression

C. Peña[1,*] and D. Blaschke[1,2,**]

[1]*Institute for Theoretical Physics, University of Wrocław, Wrocław, Poland*

[2]*Bogoliubov Laboratory of Theoretical Physics, JINR Dubna, Russia*

In this work a formula for charmonium suppression obtained by Matsui in 1989 is analytically generalized for the case of complex $c\bar{c}$ potential described by a 3-dimensional and isotropic time-dependent harmonic oscillator (THO). It is suggested that under certain conditions the formula can be applied to describe J/ψ suppression in heavy-ion collisions at CERN-SPS, RHIC, and LHC with the advantage of analytical tractability.

1. INTRODUCTION

The modification of the charmonium production cross section has been studied using a schematic 3-dimensional harmonic oscillator for the intermediate and final $c\bar{c}$ pair in [1]. In that reference the distorted wave Born approximation was used for the two-gluon fusion model and suppression ratios were calculated. In the present paper, we consider a 3-dimensional THO with a complex and continuous time dependent frequency. For such a generalization, we derive the suppression ratio for charmonia states and present a formula for J/ψ suppression including feed-down contributions.

2. QUANTUM MECHANICAL EVOLUTION OF THE $C\bar{C}$ STATE

The Charmonium suppression ratio was defined as a ratio of two cross sections by the expression $S_\psi(t) = \frac{\sigma(2g\to\psi)}{\sigma_0(2g\to\psi)}$ and was calculated explicitly in Ref. [1]. From Eqs. (2.22) and (4.17) of that paper the survival probability for the s-wave can be written in the following form

* Electronic address: pena@ift.uni.wroc.pl

** Electronic address: blaschke@ift.uni.wroc.pl

$$S_\psi(t) = \left| \frac{\int_0^\infty \ dr \ r^2 \ \psi(r) \ U_{c\bar{c}}(r,t)}{\lim\limits_{t\to 0} \int_0^\infty \ dr \ r^2 \ \psi(r) \ U_{c\bar{c}}(r,t)} \right|^2 . \tag{1}$$

3. TIME EVOLUTION OPERATOR FOR THE THO MODEL

We make use of the standard path integral approach in order to calculate the time evolution operator $U_{c\bar{c}}(r,t)$. We start by considering a 3-dimensional isotropic THO model with the Hamiltonian $H = \frac{p^2}{2\mu} + \frac{\mu}{2} \ \omega^2(\tau) \ r^2(\tau)$, where r is the $c\bar{c}$ separation and the complex function of time $\omega(\tau)$ enters in the classical equation of motion for the heavy pair as

$$\ddot{r}(\tau) + \omega^2(\tau) \ r(\tau) = 0 \ . \tag{2}$$

The general solution of equation (2) is a linear combination given by $r(\tau) = \rho(\tau)\Big(A \ \cos\gamma(\tau) + B \ \sin\gamma(\tau)\Big)$, where $\gamma(\tau) = \int_0^\tau \ dt' \ \frac{1}{\rho^2(t')}$. Replacing these definitions into (2), clearly leads to the following Ermakov equation [2]

$$\ddot{\rho}(\tau) + \omega^2(\tau) \ \rho(\tau) - \frac{1}{\rho^3(\tau)} = 0 \ . \tag{3}$$

If $\tau \ \in \ [0 \ , \ t]$ then A and B can be easily obtained from the initial conditions as

$$A = \frac{r(0)}{\rho(0)} \ , \qquad B = \frac{1}{\sin\gamma(t)} \left[\frac{r(t)}{\rho(t)} - \frac{r(0)}{\rho(0)} \ \cos\gamma(t)\right] \ . \tag{4}$$

Where we have used that $\gamma(0) = 0$. By replacing A and B in the general solution, we obtain $r(\tau)$ and $\dot{r}(\tau)$. For a THO the classical action s_{cl} and the fluctuation factor $F(t)$ in the 3-dimensional isotropic space are defined in Ref. [3]. We calculate here their relationship with Ermakov function as[1]

$$\begin{aligned}
s_{cl} &= \frac{\mu}{2} \ \Big(r(t) \ \dot{r}(t) - r(0) \ \dot{r}(0)\Big) \\
&= \frac{\mu}{2} \ \frac{1}{\sin\gamma(t)} \times \Big[r(t)^2 \ \Big(\dot{\gamma}(t)\cos\gamma(t) + \frac{\dot{\rho}(t)}{\rho(t)} \ \sin\gamma(t)\Big) \\
&\quad + r(0)^2 \ \Big(\dot{\gamma}(0) \ \cos\gamma(t) - \frac{\dot{\rho}(0)}{\rho(0)} \ \sin\gamma(t)\Big) - r(t) \ r(0) \ \Big(\frac{\rho(t)}{\rho(0)} \ \dot{\gamma}(t) + \frac{\rho(0)}{\rho(t)} \ \dot{\gamma}(0)\Big)\Big] \ ,
\end{aligned} \tag{5}$$

$$F(t) = \left[\frac{\mu}{2\pi i}\Big(-\frac{\partial \dot{r}(t)}{\partial r(0)}\Big)\right]^{3/2} = \left[\frac{\mu}{2\pi i} \ \frac{\rho(t) \ \dot{\gamma}(t)}{\rho(0)\sin\gamma(t)}\right]^{3/2} \ . \tag{6}$$

---

[1] We use the notation $\dot{r}(t) = \frac{dr(\tau)}{d\tau}|_{\tau=t}$ for all functions of time.

The time evolution operator for THO is given exactly by $U(r,t) = F(t)\ \exp(i\ s_{cl})$. In the present context, it will represent the quantum mechanical evolution of a $c\bar{c}$ state for a medium-modified (distorted) interaction up to the time t when it gets projected onto the asymptotic bound state spectrum. Thus we define $U_{c\bar{c}}(r,t) = U(r,t)$. In fact formula (1) is independent of the initial condition which may be taken as $r(0) = 0$.

4. THE THO FORMULA FOR CHARMONIUM SUPPRESSION

The ground state of charmonium J/ψ can be identified with the 1s-wave of the harmonic oscillator given by $\psi(r) = \psi(0)\ \exp\Big(\frac{-r^2}{2\ r_\psi^2}\Big)$ with $r_\psi = \sqrt{\frac{1}{\mu\ \omega_\psi}}$ [4]. Thus we integrate the gaussian shape over r appearing in (1) which leads to the following suppression

$$S_{J/\psi}(t) \ = \ \left|\frac{\rho(t)}{\rho(0)}\right|^3 \times \left|\cos\gamma(t) + \Big(\frac{\dot{\rho}(t)\rho(t)^{-1}}{\dot{\gamma}(t)} + i\ \frac{\omega_\psi}{\dot{\gamma}(t)}\Big)\ \sin\gamma(t)\right|^{-3} . \tag{7}$$

The formula (7) depends on $\gamma(t)$, the frequency ω_ψ and the Ermakov function $\rho(t)$. For the case of the charmonium state ψ' we take the 2s-wave given by $\varphi(r) = \frac{2}{3}\ \varphi(0)\ \Big(\frac{3}{2} - \frac{r^2}{r_\psi^2}\Big)\ \exp\Big(\frac{-r^2}{2\ r_\psi^2}\Big)$. Applying the formula (1) we obtain

$$S_{\psi'}(t) \ = \ S_{J/\psi}(t)\ \left|\ 1\ - \ \frac{2\ i\ \omega_\psi\ \sin\gamma(t)}{\Big(i\ \omega_\psi + \frac{\dot{\rho}(t)}{\rho(t)}\Big)\sin\gamma(t) + \dot{\gamma}(t)\cos\gamma(t)}\ \right|^2 . \tag{8}$$

For the Charmonium state χ_c we take the 2p-wave given by $\chi(r) = \chi'(0)\ r\ \exp\Big(\frac{-r^2}{2\ r_\psi^2}\Big)$. However, in this case there is a contribution of the angular momentum and it was shown in Ref. [1] that for such waves the formula (1) vanishes and the next-to-leading order term in momentum O(p/m) must be considered leading to the expression

$$S_\chi(t) \ = \ \left|\frac{\int_0^\infty\ dr\ r^2\ \chi(r)\ U'_{c\bar{c}}(r,t)}{\lim\limits_{t\to 0}\int_0^\infty\ dr\ r^2\ \chi(r)\ U'_{c\bar{c}}(r,t)}\right|^2 = S_{J/\psi}^{\frac{5}{3}}(t)\ , \tag{9}$$

with $U'_{c\bar{c}} = -\frac{\mu\ r}{2\sin\gamma(t)}\Big(\rho(t)\ \dot{\gamma}(t)\ \rho(0)^{-1} + \rho(0)\ \dot{\gamma}(0)\ \rho(t)^{-1}\Big)U_{c\bar{c}}$. The observable J/ψ suppression ratio is influenced by feed-down from the higher charmonia states and we shall assume the following composition of the total contribution

$$S(t) \ = \ 0.6\ S_{J/\psi}(t)\ +\ 0.3\ S_\chi(t)\ +\ 0.1\ S_{\psi'}(t)\ . \tag{10}$$

The case of no feed-down is described by the expression $S_{no}(t) = S_{J/\psi}(t)$. Since we have already shown that $S_\chi(t) < S_{J/\psi}(t)$ and $S_{\psi'}(t) < S_{J/\psi}(t)$ for $S_{J/\psi}(t) < 1$ it is clear that $S(t) < S_{no}(t)$.

5. SUMMARY

We have generalized Matsui's harmonic oscillator model for charmonium suppression to the case of time-dependent complex oscillator strengths and included the effects of feed-down on the J/ψ suppression ratio. Preliminary results for the comparison with experimental results from CERN SPS and RHIC can be found in [5].

ACKNOWLEDGMENTS

The authors acknowledge support from the Polish Ministry for Science and Higher Education MNiSW. D.B. has been supported in part by the Russian Fund for Basic Research RFBR under grant No. 11-02-01538-a.

---

1. T. Matsui, Annals Phys. **196**, 182 (1989).
2. I. Gjaja and A. Bhattacharjee, Phys. Rev. Lett. **68**, 2413 (1992).
3. H. Kleinert, *Path integrals in Quantum Mechanics, Statistics, Polymer Physics, and Financial Markets (3rd Edition)* (World Scientific, Singapore, 2004), pp. 114-115.
4. B. Z. Kopeliovich, I. K. Potashnikova, H. J. Pirner, I. Schmidt, Phys. Rev. **C83**, 014912 (2011).
5. D. Blaschke, C. Peña, Nucl. Phys. Proc. Suppl. **214**, 137 (2011).