Determination of the Finite Temperature Equation of State of Dense Matter

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The equation of state is calculated for temperatures less than 30 MeV and densities less than four times the saturation density of nuclear matter using a combined analysis of Auxiliarly Fields Diffusion Monte Carlo and Fermi Hypernetted Change methods.

1. INTRODUCTION

In order to understand the properties of matter at intermediate densities and temperatures, it is important to gather knowledge about the regime defined by densities of the order $0.5\,n_0 < n < 3\text{–}4\ n_0$, and temperatures between 0 and 10–20 MeV. This regime is particularly challenging. At such densities it is completely not obvious that the knowledge on the nucleon-nucleon and many-nucleon interactions, that has been developed essentially fitting properties of small nuclei, is still applicable. On the other hand, this regime is at present completely unaccesible to QCD.

The constraints coming from astrophysical observations (in particular on the properties of neutron stars) have been partly suplemented with information coming from heavy-ion

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collisions. However, the present situation does not allow for a definitive discrimination among the variety of proposed models that have been developed in the last two decades.

Recent methodological developments allowed for a more systematic approach to the study of the properties of high density nucleonic matter. In particular, by means of the Auxiliary Field Diffusion Monte Carlo (AFDMC) method [1], it is possible to solve with high accuracy the Schrödinger equation with realistic potentials for a number of nucleons A which might be of order 100 to make realistic predictions on the properties of nuclear [2] and neutron [3] matter.

This is a big step forward, because the limits on number of nucleons that can be treated efficiently has been moved forward by at least one order of magnitude. It is therefore possible to start discussing with no ambiguity the portability of current interactions (like Argonne-Urbana AV18+UIX or Illinois ILX potentials) to the high density regime, and to possibly develop alternatives that might at least give phenomenological input for attacking important problems.

In this context, for instance, we have recently developed a density dependent interaction (DDI) following the initial intuition of Lagaris and Pandharipande [4] which, after fitting three parameters on basic properties of symmetrical nuclear matter (saturation density, energy and compressibility at saturation density), yields an equation of state (EOS) which is by far softer than the celebrated Akmal, Pandharipande and Ravenhall [5] EOS, both for symmetrical nuclear matter and pure neutron matter, and consequently gives estimates of Neutron Star properties (such as mass/radius ratio, momentum of inertia, and others) that are much better reconciled with observations, and closer to extrapolated constraints.

The second issue concerns the temperature effects on the equation of state. From the point of view of microscopic calculations the neater approach would be the development of a code capable to compute the expectation of operators in the Quantum Canonical Ensemble (always neglecting relativistic effects) by means of Path Integral Monte Carlo techniques. In principle this would allow for a rigorous study of nuclear/neutron matter up to temperatures and densities very close to the phase transitions. However, approximate approaches based on the extrapolation of the temperature behavior from Fermi Hypernetted Chain calculations [6] can be used in order to include temperature dependence up to 20-30 MeV. This fact is important because it provides a guide to the corrections necessary to have a sensible comparison with $T = 0$ results yielded by AFDMC calculations.

In this Contribution we present a preliminary results of such combine analysis.

2. THE HAMILTONIAN

Recently we developed a realistic nuclear Hamiltonian which fits the available scattering data in S, P, D and F waves, and reproduces the ground state energy, density and compressibility of nuclear matter via Monte Carlo calculations [7]. This model contains two-nucleon and many-nucleon interactions.

For the two-body interaction, we take the Argonne AV6′ potential [8], which includes the four central spin-isospin components and the two tensor ones. The six components of the long range OPEP potential are fully included, whereas the first six of the 18 components $v_I^p(r_{ij})$ of the intermediate range part and $v_S^p(r_{ij})$ of the short range part of the AV18 interaction [9] are the only ones kept. The corresponding amplitudes I_p and S_p are re-fitted so as to correctly reproduce the deuteron binding energy and to give the best fit to NN scattering data. Those in S and $^1P_1$ waves are fitted equally as well as by AV18.

The many-body interactions are represented by density-dependent factors of the structural form given by the LP model [4]. The resulting potential, denoted as DD6′, is given by the following six two-body components

$$v^p_{\mathrm{DD6'}} = v^p_{\mathrm{OPEP}} + v^p_I e^{-\gamma_1 \rho} + v^p_S + \mathrm{TNA}(\rho)\,,$$
$$\mathrm{TNA}(\rho) = 3\gamma_2 \rho^2 e^{-\gamma_3 \rho}\left(1 - \frac{2}{3}\left(\frac{\rho_n - \rho_p}{\rho_n + \rho_p}\right)^2\right) \qquad (1)$$

with γ_1, γ_2 and γ_3 being fixed by means of AFDMC method [7] so as to reproduce the experimental values of the saturation density $\rho_0 = 0.16\ \mathrm{fm}^{-3}$, the binding energy per particle $E_0 = -16$ MeV and the compressibility $K = 9\rho_0^2\left(\partial^2 E(\rho)/\partial\rho^2\right)_{\rho_0} \approx 240$ MeV.

The $\gamma_1\rho$ term of the $\exp(-\gamma_1\rho)$ simulates the effect of the three-body repulsion. At neutron star densities ($\sim 1\ \mathrm{fm}^{-3}$) the ρ^2 term in the $\exp(-\gamma_1\rho)$, which corresponds to four-nucleon interaction gives small, but non-zero contribution. The four and more nucleon intereactions implied by the $\exp(-\gamma_1\rho)$ are theoretically plausible.

$\mathrm{TNA}(\rho)$ simulates an attractive many-body contribution via correlations. At its maximum TNA is ~ -6 MeV and ~ -2 MeV in nuclear and neutron matter, respectively. Moreover, the magnitudes of the many-body contributions appear to be reasonable from the point of view of microscopic calculations of multiple pion exchange, and pion rescattering in matter [10].

3. RESULTS

The constraned variational free energy $F_{\rm con}(\rho,T)$, defined as

$$\frac{F_{\rm con}(\rho,T)}{A}=\frac{F(\rho,T)}{A}+\rho\Lambda\left[(I_c-1)^2+(I_\tau/3+1)^2\right]\,, \tag{2}$$

where I_c and I_τ are sum rules on the conservation of mass and charge, is minimized by varying both the single particle spectrum $\epsilon(\mathbf{k},\rho,T)$ and the correlation operator $\mathcal{F}_{ij}$.

3.1. Equation of state of nuclear matter at finite temperature

It is well known that FHNC/0 overbinds symmetrical nuclear matter. The AFDMC density-dependent terms give more attraction than in FHNC/0. The TNA term is $\sim 30\%$ larger at ρ_0 and more than the double at $5\rho_0$, giving more attraction, and $\exp(\gamma_1\rho)-1$ is $\sim 30\%$ smaller over the whole range $(\rho_0, 5\rho_0)$, giving less repulsion.

$$E_{\rm SNM}(\rho)/A=E_0+a(\rho-\rho_0)^2+b(\rho-\rho_0)^3e^{\gamma(\rho-\rho_0)}\,, \tag{3}$$

where $E_0=-16.0$ MeV, $\rho_0=0.16$ fm^{-3}, $a=517\pm1$ MeV fm^6, $b=-1270\pm12$ MeV fm^9 and $\gamma=-2.19\pm0.02$ fm^3. This parametrisation was chosen to represent the EOS of nuclear matter, reproducing properties constrained by terrestrial experiments on nuclei [11].

The DD6$'$ Hamiltonian was then used to compute the EOS of pure neutron matter. The EOS for nuclear matter as a function of the proton fraction $x=\rho_p/\rho$ is then parametrized as

$$E(\rho,x)/A=E_{\rm SNM}(\rho)/A+C_s\left(\frac{\rho}{\rho_0}\right)^{\gamma_s}(1-2x)^2\,. \tag{4}$$

The two extra parameters of the symmetry energy term, C_s and γ_s, were obtained by fitting $E(\rho,x=0)/A$ to the AFDMC result for pure neutron matter. This gives $C_s=31.97\pm0.01$ MeV and $\gamma_s=0.6131\pm0.0003$. It should be noted that usually the symmetry energy is constrained over the range of densities typical of nuclei, whereas here it were fitted over a very wide density range well above ρ_0. This means that the parametrization of Eq. (4) is build to be accurate up to very high densities.

The FHNC/SOC and AFDMC results are compared in Fig. 1. At highest density ($\rho=0.56$ fm^{-3}) the energy differences are 5.8 MeV and 4 MeV for PNM and SNM, respectively.

We find that the following functional forms provide excelent parametrizations of the new numerical results in the required ranges of density and temperature

$$F(\rho,x,T)/A = E(\rho,x)/A + \Delta F_0(\rho,T)/A + (1-2x)^2 \Delta F_S(\rho,T)/A$$
$$- \alpha \left(\frac{\rho_0}{\rho}\right)^{\beta} \left[x^{1/3} + (1-x)^{1/3}\right] T^2 \,. \quad (5)$$

α and β are expected to be almost independent on isospin x. The fit is inspired by the Sommerfeld expansion, and resembles the hot non-interacting excitation energy, which comes from the kinetic energy term

$$\left(F - F_0\right)/A = -\frac{3\pi^2}{8\mu_F}(k_B T)^2 \left[x^{1/3} + (1-x)^{1/3}\right] + O(T^4)$$
$$\approx -\alpha_0 \left(\frac{\rho_0}{\rho}\right)^{2/3} \left[x^{1/3} + (1-x)^{1/3}\right] T^2 \,. \quad (6)$$

At normal density $\alpha_0 = a_e = 3\pi^2/(8\mu_F) = 0.03315$ MeV^{-1}.

Other functions, entering in defintition of (5), are the following:

$$\Delta F_0(\rho,T)/A = \left[a \log\rho + b\left(\frac{\rho_0}{\rho}\right)\right] T + \left[c \log^2\rho + d\left(\frac{\rho_0}{\rho}\right)\right] T^2 \,,$$
$$\Delta F_S(\rho,T)/A = e\left(\frac{\rho_0}{\rho}\right) T^2 \,. \quad (7)$$

All parameters are fixed by means of the FHNC method: $a = -0.15 \pm 0.02$, $b = -0.38 \pm 0.04$, $c = -0.008 \pm 0.0013$, $d = 0.06 \pm 0.03$, $e = -0.016 \pm 0.013$, $\alpha = 0.047 \pm 0.023$, $\beta = 0.72 \pm 0.14$ with $\chi^2/n.d.f = 0.54$.

The free energy is shown in Figs. 2 and 3 for symmetric nuclear matter and pure neutron matter, respectively, for temperatures up to $T = 30$ MeV.

We notice that the free energy of symmetric matter shows a typical Van der Waals behavior and is monotonically decreasing function of the temperature.

It turns out that the dependence of the free energy on the proton fraction is not a usual quadratic, as at zero temperature, but exhibits a more comlex behavior, which could be important in the treatment of the case of neutron stars at beta-equilibrium.

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1. K. E. Schmidt and S. Fantoni, Phys. Lett. B **446**, 99 (1999).
2. S. Gandolfi, F. Pederiva, S. Fantoni, K. E. Schmidt, Phys. Rev. Lett. **98**, 102503 (2007).
3. S. Gandolfi, A. Yu. Illarionov, K. E. Schmidt, F. Pederiva, S. Fantoni, Phys. Rev. C **79**, 054005 (2009).
4. I. E. Lagaris and V. R. Pandharipande, Nucl. Phys. A **359**, 349 (1981).
5. A. Akmal, V. R. Pandharipande, and D. G. Ravenhall, Phys. Rev. C **58**, 1804 (1998).
6. K. E. Schmidt and V. R. Pandharipande, Phys. Lett. B **87**, 11 (1979).
7. S. Gandolfi, A. Yu. Illarionov, S. Fantoni *et al.*, Mon. Not. R. Astron. Soc. **404**, L35–L39 (2010).
8. R. B. Wiringa and S. C. Pieper, Phys. Rev. Lett. **89**, 182501 (2002).
9. R. B. Wiringa, V. G. J. Stoks, and R. Schiavilla, Phys. Rev. C **51**, 38 (1995).
10. A. Lovato, O. Benhar, S. Fantoni, A. Yu. Illarionov, K. E. Schmidt, arXiv: 1011.3784 [nucl-th].
11. P. Danielewicz, R. Lacey, and W. G. Lynch, Science **298**, 1592 (2002).

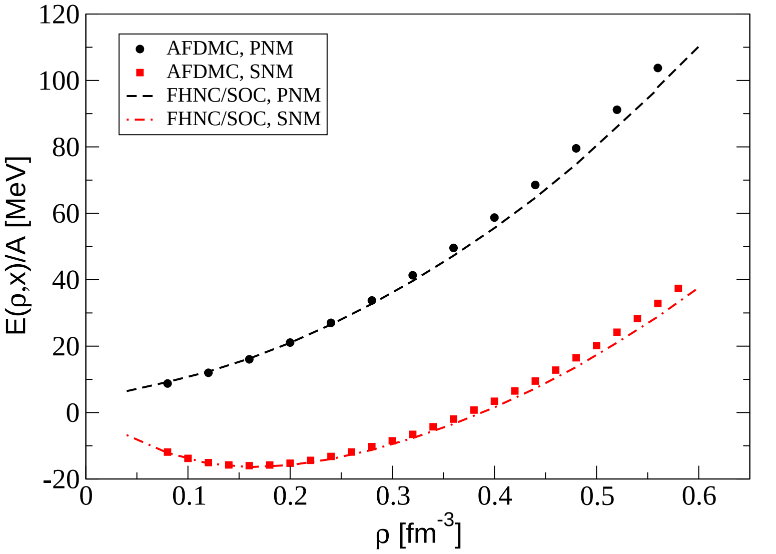

Figure 1. The variational equation of state for pure neutron matter (PNM, $x = 0$, dashed line) and symmetric nuclear matter (SNM, $x = 1/2$, dot-dashed line). The points show the AFDMC results for PNM (circles) and SNM (squares).

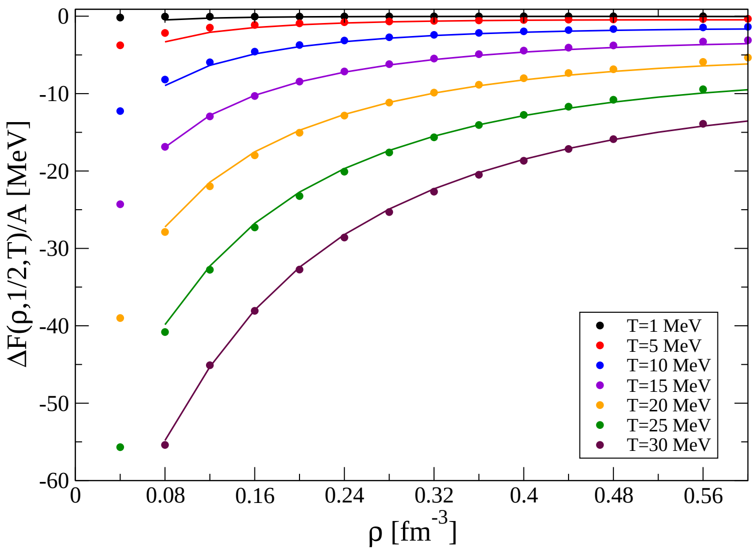

Figure 2. The free energy difference per nucleon, $(F(\rho, 1/2, T) - F(\rho, 1/2, 0))/A$, as a function of density

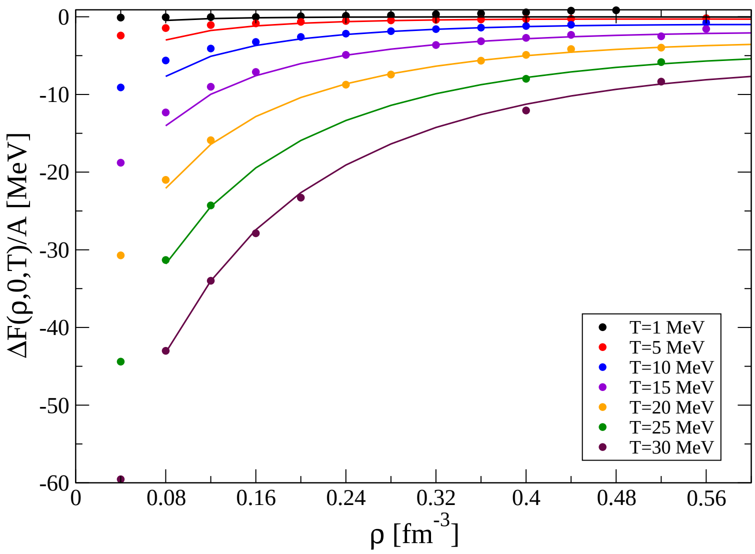

Figure 3. The free energy difference per nucleon, $(F(\rho, 0, T) - F(\rho, 0, 0))/A$, as a function of density in pure neutron matter for different temperatures.

FIGURE CAPTIONS

Fig.1: The variational equation of state for pure neutron matter (PNM, $x = 0$, dashed line) and symmetric nuclear matter (SNM, $x = 1/2$, dot-dashed line). The points show the AFDMC results for PNM (circles) and SNM (squares).

Fig.2: The free energy difference per nucleon, $(F(\rho, 1/2, T) - F(\rho, 1/2, 0))/A$, as a function of density in symmetric nuclear matter for different temperatures.

Fig.3: The free energy difference per nucleon, $(F(\rho, 0, T) - F(\rho, 0, 0))/A$, as a function of density in pure neutron matter for different temperatures.