On the critical behavior of (2+1)-dimensional QED

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It is shown the analysis [1] for QED in 2+1 dimensions with N four-component fermions in the leading and next-to-leading orders of the $1/N$ expansion. As it was demonstrated in [1] the range of the admissible values N, where the dynamical fermion mass exist, decreases strongly with the increasing of the gauge charge. So, in Landau gauge the dynamical chiral symmetry breaking appears for $N < 3.78$, that is very close to the results of the leading order and in Feynman gauge dynamical mass is completely absent.

Quantum Electrodynamics in 2+1 dimensions (QED_3) has acquired increasing attention [1–7] because of its similarities to (3+1) dimensional QCD. Moreover, last years a new strong interest comes to QED_3 in the relation with graphene properties (see [8] and discussions and references therein). Graphene, a one-atom-thick layer of graphite, is a remarkable system with many unusual properties that was fabricated for the first five years ago [9]. Theoretically it was shown long time ago [10] that quasiparticle excitations in graphene are described by the massless Dirac equation in (2+1) dimension. This explains why the bilayer graphene in external fields is a subject of intensive recent study [11].

A number of investigations have been performed for the study of dynamical chiral symmetry breaking in QED_3 and very different results have been obtained. Using the leading order (LO) in the $1/N$ expansion of the Schwinger-Dyson (SD) equation, Appelquist *et al.* [2] showed that the theory exhibits a critical behavior as the number N of fermion flavors approaches $N_c = 32/\pi^2$; that is, a fermion mass is dynamically generated only for $N < N_c$. On the contrary, Pennington and collaborators [3], adopting a more general non-perturbative approach to the SD equations, found that the dynamically generated fermion mass decreases exponentially with N, vanishing only as $N \to \infty$. This conclusion was supported also by Pisarski [4] by the use of the other methods. On the other hand, an alternative non-perturbative study by Atkinson *et al.* [5] suggested that chiral symmetry is unbroken at sufficiently large

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N. The theory has also been simulated on the lattice [6, 7]. Remarkably, the conclusions of Ref. [6] are in the agreement with the existence of a critical N as predicted in the analysis of Ref. [2] while the second paper [7] contains the opposite results.

Because the critical value N_c is not large, the contribution of the higher orders in the $1/N$ expansion can be essential and may lead to better understanding of the problem. The purpose of this work is to consider the $1/N$ correction [1, 12] to LO result [2].

1. The Lagrangian of massless QED$_3$ with N flavors is

$$L = \overline{\Psi}(i\hat{\partial} - e\hat{A})\Psi - \frac{1}{4}F_{\mu\nu}^2,$$

where Ψ is taken to be a four component complex spinor. In massless case, which we are considering, the model contains infrared divergences, which can be canceled when the model is analyzed in a $1/N$ expansion [13]. Since the theory is massless, the mass scale is the dimensional coupling constant $a = Ne^2/8$ which is kept fixed as $N \to \infty$.

Following [2] we study the solution of the SD equation. The inverse fermion propagator has the form

$$S^{-1}(p) = -[1 + A(p)]\left[\hat{p} + \Sigma(p)\right],$$

where $A(p)$ is the wave-function renormalizable coefficient and $\Sigma(p)$ is a dynamical, parity-conserving mass taken to be the same for all the fermions.

The SD equation is

$$\Sigma(p) = \frac{2a}{N} Tr \int \frac{d^3k}{(2\pi)^3} \frac{\gamma^\mu D_{\mu\nu}(p-k)\left[1 + A(k)\right]\left(\hat{k} + \Sigma(k)\right)\Gamma^\nu(p,k)}{\left[1 + A(k)\right]^2 \left(k^2 + \Sigma^2(k)\right)}, \tag{1}$$

where[1]

$$D_{\mu\nu}(p) = \frac{g_{\mu\nu} - (1-\xi)p_\mu p_\nu/p^2}{p^2\left[1 + \Pi(p)\right]}$$

is the photon propagator and $\Gamma^\nu(p,k)$ is the vertex function.

2. The LO approximations in the $1/N$ expansion are

$$A(p) = 0, \quad \Pi(p) = a/\mid p \mid, \quad \Gamma^\nu(p,k) = \gamma^\nu,$$

[1] Following [12] we introduce a nonlocal gauge-fixing term. The detailed analysis of this possibility has been given in Ref. [14].

where we neglect the fermion mass in the calculation of $\Pi(p)$. The gap equation is

$$\Sigma(p) = \frac{8a(2+\xi)}{N} Tr \int \frac{d^3k}{(2\pi)^3} \frac{\Sigma(k)}{k^2\big[(p-k)^2 + a \mid p-k \mid\big]}, \tag{2}$$

where we ignore the term $\Sigma^2(k)$ in the denominator of r.h.s.

Following [2], we set

$$\Sigma(k) = (k^2)^{\alpha}\,. \tag{3}$$

One can see, that for large a the r.h.s. of (2) together with condition (3) (and the contributions of higher orders) can be calculated by the standard rules for massless diagrams of the perturbation theory (see, for example, [15]). Thus, we have for large a

$$1 = \frac{2+\xi}{\beta L} \tag{4}$$

with $\beta = (-\alpha)(\alpha + 1/2)$ and $L \equiv \pi^2 N$, or

$$\alpha_{\pm} = \big(-1 \pm [1 - 16(2+\xi)/L]^{1/2}\big)/4\,. \tag{5}$$

We reproduce the solution given in Ref. [2]. That analysis yields a critical number of fermions $N_c = 16(2+\xi)/\pi^2 \approx 1.62(2+\xi)$ (i.e. $L_c = 16(2+\xi)$), such that for $N > N_c$ $\Sigma(p) = 0$ and

$$\Sigma(0) \simeq exp\big[-2\pi/(N/N_c - 1)^{1/2}\big]$$

for $N < N_c$. Thus, chiral-symmetry breaks when α becomes complex, that is for $N < N_c$.

3. The next-to-leading order (NLO) approximation has been included in [1, 16] using the differential equation method [17]. The results have a cumbersome form [16], which is similar to results for complicated massless diagrams obtained using Gegenbauer polynomials [18]. In [1] we have analyzed simplified form, which contains only the terms $\sim (-\alpha)^{-k}$ and $\sim (\alpha + 1/2)^{-k}$ $(k = 1, 2, 3)$ from the series given in [16]. These terms are most important in the neighborhood of the critical point N_c. The Eq. (4) is replaced now by

$$1 = \frac{(2+\xi)}{\beta L} + \big[f(\xi) + \beta\varphi(\xi)\big]\frac{1}{(\beta L)^2}, \tag{6}$$

where $f(\xi) = 4(1-\xi)/3 - \xi^2$, $\varphi(\xi) = 176/9 - 4\pi^2 - (16/3)\xi + 4\xi^2$.

Let us get the exact critical value N_c from Eq. (6). Supposing $\alpha = \alpha_c \equiv -1/4$ we obtain the critical values in the following form

$$N_{c,\pm} = \frac{8}{\pi^2}\big[(2+\xi) \pm \big((2+\xi)^2 + 4f(\xi) + \varphi(\xi)/4\big)^{1/2}\big], \tag{7}$$

i.e.

$$N_{c,+}(\xi) = (3.31, 3.35, 3.09, 2.81), \quad N_{c,-}(\xi) = (-0.07, 0.38, 1.29, 1.88)$$

for $\xi = (0.0, 0.3, 0.7, 0.9)$, respectively.

Notice the intriguing fact that follows from (7). The addition of $1/N$ correction leads to the occurrence of the second critical point (for $0.05 \leq \xi \leq 0.95$) such that for $N < N_{c,-}$ the chiral symmetry does not break. The dynamical mass generation exists in the interval between the critical points $N_{c,-}$ and $N_{c,+}$. For $\xi \geq 0.95$ this interval disappears and the chiral symmetry breaking is absent. For small values of gauge parameter ξ ($\xi \leq 0.05$) new critical point does not occur.

The solution of the Eq. (6) is

$$\beta_{\pm} = \frac{1}{2L}\left[2 + \xi + \frac{\varphi(\xi)}{L} \pm \left((2+\xi)^2 + 4f(\xi) + 2(2+\xi)\frac{\varphi(\xi)}{L} + \frac{\varphi^2(\xi)}{L^2}\right)^{1/2}\right]$$

has the simple form in Landau gauge

$$\beta_{\pm}(\xi = 0) = \frac{1}{L}\left[1 + \frac{\varphi(0)}{2L} \pm \sqrt{7/3}\left(1 + \frac{3}{14}\frac{\varphi(0)}{L}\right)\left(1 + \frac{\frac{3}{49}\varphi^2(0)/L^2}{\left(1 + \frac{3}{14}\frac{\varphi(0)}{L}\right)^2}\right)^{1/2}\right], \tag{8}$$

where the last term in r.h.s. of Eq. (8) is very small for $L \sim L_c$. Leaving it out we get the following equation for β_+

$$\beta_+(\xi = 0) \approx 1 + \sqrt{7/3}\frac{1}{L} + \left(1 + \sqrt{3/7}\varphi(0)\right)\frac{1}{2L^2} \approx \frac{2.52}{L}\left(1 - \frac{6.52}{L}\right),$$

which has coefficients are close to those from the paper [12].

Resume. We reviewed the results of [1], where the $O(1/N^2)$ terms have been included into SD equation and the strong gauge dependence of the result has been found. Hence, the addition of $1/N$ correction does not lead to the essential improvement in the understanding of dynamical chiral symmetry breaking. However, as it was shown in [1], in the Landau gauge the inclusion of $O(1/N^2)$ terms slightly changes only quantitative (but not qualitative) properties of the LO results. Thus, in the Landau gauge the analysis in [1] gives further evidence in favor of the solution has been given by Appelquist *et al.* [2].

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