The simplest and universal constituents for description of the leptons and hadrons

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Here I discus a possibility to connect quantum numbers of elementary particles with their structure on the base of four connected components of Lorentz group. The complete and closed (with respect to homogenous Lorentz group) series of equations for description of lepton sector forms the pattern and ground for present project.

1. INTRODUCTION

Unnatural situation take place in lepton sector. We consider leptons as unstructured particles, but they differ each other by quantum numbers. One can belief as pure theoretical defect internal lack of agreement, when some properties are observed, but the carriers of this properties are absent. Observable properties are reactions with participation of the leptons, which are allowed or forbidden on the base of lepton numbers coservation.

Significance and importance of structural notions increased after successes of quark model [1]. Shortly after it proposing, a set of articles was appeared noting a difficulties of coupling quark model with Poincare group. They received the common name "no-go theorems" [2]. The most strong statement of such kind is known as O'Raifeartaigh theorem [3]. Coclusion of theorem is following. There is no group which contains group internal symmetry and Poincare group as the subgroups in different way than direct product. In a such case all particles belong to the unitary multiplet have equal masses.

A. Pais ended his review [2] by the words: "Are there any alternatives left to the internal symmetry $\otimes$ Poincare group picture in face of the no-go theorems?"

Our structural approach is alternative based on rigorous relative theory. In spite of successes of quark model the whole lepton sector was left out of possibilities of quark model and did not fit in unitary scheme. Lepton sector can not be described beyond the frame-

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work of consistent relativity, while quarks does not satisfy this requirement. At the same time it is not excluded possibility, when quarks are more extensive and complex formations then constituents for leptons. Experience of the work [4–6] with the lepton sector creates prerequisites detailed elaboration and generalization most essential notions both the hadron and their constituents. These notions are quantum numbers, antiparticle and some other.

2. STRUCTURAL LEPTON CHARACTERISTICS

Necessary and sufficient conditions for description of lepton equations were obtained on the exhaustive analysis of Dirac equation group. It was found that Dirac equation quite informative for this aim [6]. Result looks thus.

1. The equations must be invariant and covariant under homogeneous Lorentz transformations taken into account all four connected components.
2. The equations must be formulated on the base of irreducible representations of the groups determining every lepton equation.
3. Conservation of four-vector of probability current must be fulfilled and fourth component of the current must be positively defined.
4. The lepton spin is supposed equal to 1/2.
5. Every lepton equation must be reduced to Klein-Gordon equation.

We see here different kinds of symmetries: with respect to the homogeneous Lorentz group, relativistic wave equations, quantum mechanics.

It was found in agreement with accepted conditions that one can obtain five Dirac-like groups and corresponding five lepton equations. Every group related with corresponding equation has own nonrecurrent composition. Structural compositions of the stable lepton groups (or groups of lepton equations) look thus [6]:

1. The Dirac equation — $D_\gamma(II)$: d_γ, b_γ, f_γ; structural invariant $\text{In}[D_\gamma(II)] = -1$.
2. The equation for a doublet of massive neutrinos— $D_\gamma(I)$: d_γ, c_γ, f_γ; structural invariant $\text{In}[D_\gamma(I)] = 1$.
3. The equation for massless neutrinos, (P)-conjugate quartet — $D_\gamma(III)$: d_γ, b_γ, c_γ, f_γ; structural invariant $\text{In}[D_\gamma(III)] = 0$.
4. The equation for a massless (T)-singlet — $D_\gamma(IV)$: b_γ; structural invariant $\text{In}[D_\gamma(IV)] = -1$.

5. The equation for a massless (PT)-singlet — $D_\gamma(V)$: c_γ; structural invariant $\mathrm{In}[D_\gamma(V)] = 1$.

Here $d_\gamma, b_\gamma, c_\gamma, f_\gamma$ are subgroups of lepton group which realize four irreducible representations of Lorentz group. $\mathrm{In}[D_\gamma(...)]$ is a numerical characteristic of $D_\gamma(...)$ irreducible matrix group which takes tree values ± 1 or 0 [7]. It was called structural invariant.

The simple enumeration of all possibilities based on structure invariants of the four conjugate components and possible invariants of equations in the whole provides the statement that there are no other stable leptons in the frame work of the assumptions and requirements formulated above.

Further study of the lepton problem showed that one can obtain additional Dirac-like equations in the framework of the previous five assumptions. This problem is solved by introducing an additional (fifth) generator for obtaining new groups of the wave equations [5]. It was found that there are three and only three possibilities $(\Delta_1, \Delta_3, \Delta_2)$ admitting the interpretation as new wave equations with additional characteristics. In this case the new groups contain substructures coinciding with group having a physical interpretation in terms of stable leptons. This is the difference from groups of stable leptons that provide their interpretation as unstable if the particle mass turns out to be large than the sum of masses of decay particle.

Composition of the unstable lepton groups.

1. $\Delta_1\{D_\gamma(II), \quad D_\gamma(III), \quad D_\gamma(IV)\}, \qquad \mathrm{In}[\Delta_1] = -1.$
2. $\Delta_3\{D_\gamma(II), \quad D_\gamma(I), \quad D_\gamma(III)\}, \qquad \mathrm{In}[\Delta_3] = 0.$
3. $\Delta_2\{D_\gamma(I), \quad D_\gamma(III), \quad D_\gamma(V)\}, \qquad \mathrm{In}[\Delta_2] = 1.$

All three groups have its own structures as the stable one. We see that four conjugate components of Lorentz group allowed to describe different leptons due to complication of structural constituents.

3. SOME CONSEQUENCES AND PERSPECTIVES

Obtained results allow to make following conclusions. They are statement of the facts.

I. All leptons possess structural individual characteristics. They are related with properties of lepton relativistic wave equations.

II. Simplest constituents of these structures are four connected components of homogeneous

Lorentz group. Four groups, which realized these connected components was obtained in explicit form. These groups and representations corresponding them are:
group $d_\gamma \rightarrow$ proper orthochronous representation;
group $f_\gamma \rightarrow$ improper orthochronous, P-conjugate representation;
group $b_\gamma \rightarrow$ proper antichronous, T-conjugate representation;
group $c_\gamma \rightarrow$ improper antichronous, (PT)-conjugate representation.
III. So-called intrinsic symmetries of the leptons are reflection or conversion space-time symmetries taking into account (P)-, (T)-, (PT)-inversion and symmetries of relativistic quantum mechanics. Their totality one can call condition to be observable particle.
IV. All distinctions between leptons and possibilities of their specification by means of quantum numbers are determined by individual structure of every lepton equation. The complication of the internal structures leads to additional characteristics.

It should be noted the completeness of the whole set of the represented solutions. It is impossible to add any solution above obtained one in the framework of accepted suppositions. This is a corollary simple but rigorous group theoretical requirements.

Description of hadrons and leptons on the common foundation requires generalization and specification of some base notions. For example, it is necessary to differentiate by mathematical language observable particles and unobservable particle-like objects having definite quantum numbers, necessary to generalize notion particle-antiparticle and so on.

Quantum numbers according to Weyl definition are indexes characterizing group representations [8]. All stable and unstable leptons have own individual structures and therefore have proper quantum numbers. One can see on example of unstable leptons how complication of structure leads to additional properties or to additional quantum numbers. The presence of individual structure is equivalent of the existence of quantum numbers.

Antiparticles. Exhaustive analysis of the lepton equation groups shows, that necessary condition for description particle and antiparticle in the frame of the same equation is a presence of T-conjugate representations in the group of this equation. Conjugate components allowed to generalize this notion for any structural combination of the components including any components of hadron. All conjugate components of Lorentz group are connected between themselves by means of (P)-, (T)- and (PT)-inversions [6].

The common foundation of the leptons and the hadrons will serve for understanding generality and distinction between wick and strong interactions.

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