AdS/QCD at finite density and temperature

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We review some basics of AdS/QCD following a non-standard path and list a few results from AdS/QCD or holographic QCD. The non-standard path here is to use the analogy of the way one obtains an effective model of QCD like linear sigma model and the procedure to construct an AdS/QCD model based on the AdS/CFT dictionary.

1. INTRODUCTION

Surely understanding many faces of non-perturbative QCD is important. The approaches based on the AdS/CFT [1–3] or, in general, gauge/gravity duality might be one of the promising analytic tools to explore strongly interacting QCD. Although the correspondence between QCD and gravity theory is not known, we can obtain much insights on QCD together with qualitatively robust numbers by using this correspondence.

We first discuss how to construct a bottom-up holographic QCD model at zero temperature and density in a non-standard way; for the standard, we refer to [4]. Then, we include the temperature and density. Finally, we show a few sample works based on the holographic QCD.

2. BUILDING A HOLOGRAPHIC QCD MODEL

We show how to establish a bottom-up holographic QCD model in 5D starting from low energy QCD in 4D. The analogy taken in this section is literally analogy for illustration.

Suppose we try to obtain an effective chiral model of QCD like linear sigma model in 4D. What we have to do first is to identify or determine relevant fields in our effective model. To

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do this, we consider composites of quark fields that have the same quantum numbers of the hadrons of interest. For instance we introduce pion-like and sigma-like fields: $\boldsymbol{\pi} \sim \bar{q}\boldsymbol{\tau}\gamma^5 q$ and $\sigma \sim \bar{q}q$. In AdS/CFT business, this procedure may be dubbed operator/field correspondence: one-to-one mapping between gauge-invariant local operators in gauge theory and bulk fields in AdS (or gravity) sides. A remarkable point here is that unlike the effective model of QCD, the 5D mass of the bulk field is not the parameter of the holographic QCD model. This bulk mass is determined by the dimension Δ and spin p of the dual 4D operator in AdS_{d+1}. For instance, consider a bulk field $X(x,z)$ dual to $\bar{q}(x)q(x)$. The bulk mass of $X(x,z)$ is given by $m_X^2 = (\Delta - p)(\Delta + p - d)$ with $\Delta = 3$, $p = 0$ and $d = 4$, where z is for the fifth coordinate.

To write down a Lagrangian of an effective chiral model, we consider (global) chiral symmetry of QCD. Since the mass of light quark $\sim$ 5–10 MeV is negligible compared to the relevant QCD energy scale $\Lambda_{\rm QCD} \sim 200$ MeV, we may consider the exact chiral symmetry of QCD and treat quark mass effect in a perturbative approach. Under the axial transformation, $q \to e^{-i\gamma_5 \boldsymbol{\tau}\cdot\boldsymbol{\theta}/2} q$, the pion-like and sigma-like states transform as $\boldsymbol{\pi} \to \boldsymbol{\pi} + \boldsymbol{\theta}\sigma$ and $\sigma \to \sigma - \boldsymbol{\theta}\cdot\boldsymbol{\pi}$. From this, we can obtain terms that respect chiral symmetry such as $\boldsymbol{\pi}^2 + \sigma^2$. Similarly we ask the holographic QCD model to respect chiral symmetry of QCD. In AdS/CFT, however, a global symmetry in gauge theory corresponds local symmetry in the bulk, and therefore the corresponding holographic QCD model should posses local chiral symmetry. This way vector and axial-vector fields naturally fit into chiral Lagrangian in the bulk as the gauge boson of the local symmetry.

In both cases, we retain the chiral symmetry in the Lagrangian since it will be spontaneously broken. Then we should ask how to realize the spontaneous chiral symmetry breaking in each case. In an effective chiral model, we have a term like $((\boldsymbol{\pi}^2 + \sigma^2) - c^2)^2$ that leads to spontaneous chiral symmetry due to a nonzero vacuum expectation value of the scalar field σ, $<\sigma>= c$. In this case the explicit chiral symmetry due to the small quark mass could be mimicked by adding a term $\epsilon\sigma$ which induces a finite mass of the pion, $m_\pi^2 \sim \epsilon/c$. In a holographic QCD model, the chiral symmetry breaking is encoded in the vacuum expectation value of a bulk scalar field dual to $\bar{q}q$. For instance in the hard wall model [5, 6], it is given by $<X>= m_q z + \zeta z^3$, where m_q and ζ are proportional to the quark mass and chiral condensate in QCD.

A huge difference between the two approaches is the background metric. In the effective chiral model, the metric is flat in 4D, while it is curved in the holographic QCD model with extra dimensions. For instance, in the hard wall model, the background, AdS, is given by

$$ds^2 = \frac{1}{z^2}(dt^2 - dx_i dx^i - dz^2)\,, \tag{1}$$

where $i = 1, 2, 3$.

3. FINITE TEMPERATURE AND DENSITY

In this section we show how to extend the holographic QCD model to finite temperature and density with a few sample works. Here we discuss mainly a bottom-up approach such as the hard wall model.

The finite temperature could be neatly introduced by a black hole in AdS$_{d+1}$, where d is the dimension of the boundary gauge theory. The background is given by

$$ds^2 = \frac{1}{z^2}\left(f(z)dt^2 - d\mathbf{x}^2 - \frac{dz^2}{f(z)}\right)\,, \tag{2}$$

where $f(z) = 1 - z^d/z_h^d$. The temperature of the boundary gauge theory is identified as $T = d/(4\pi z_h)$. It seems that it is a piece of cake to deal with temperature in holographic QCD. However, it is not true at least in confined phase.

In holographic QCD, the confinement to deconfinement phase transition is described by the Hawking-Page transition. In low-temperature confined phase, thermal AdS, which is nothing but the AdS metric in Euclidean space, dominates the partition function, while at high temperature, AdS-black hole geometry does. This was first discovered in the finite volume boundary case in [7]. In the bottom-up model, it is shown that the same phenomena happen also for infinite boundary volume if there is a finite scale along the fifth direction [8]. Due to this Hawking-Page transition, we are not to use the black hole in the confined phase and so are not to obtain the temperature dependence of any hadronic observables. This might be ok with large N_c QCD as long as we don't consider any large N_c corrections. But, in reality we observe temperature dependence of hadronic quantities, and therefore we need to include large N_c corrections in holographic QCD in a consistent way. A quick fix-up for this might be to use the temperature dependent chiral condensate as an input in the hard wall model and study how this temperature dependence conveys into other hadronic quantities like the pion decay constant [9].

Now we move on to dense matter. For this, we introduce an AdS/CFT dictionary. Suppose we have a term like $\int d^4x J\cdot\mathcal{O}$. Then we will have a bulk field ϕ dual to $\mathcal{O}$. The AdS/CFT dictionary dictates that the source term J is nothing but the boundary ($z \to 0$) value of ϕ_0, where ϕ_0 is the solution of the classical equation of motion of the bulk field. According to this, a chemical potential in boundary gauge theory is encoded in the boundary value of the time component of the 5D bulk U(1) gauge field. To be more specific on this, we first consider the chemical potential term in gauge theory,

$$\mathcal{L}_\mu = \mu_q q^\dagger q\,. \tag{3}$$

Then, we introduce a bulk U(1) gauge field A_μ which is dual to $\bar{q}\gamma_\mu q$. According to the dictionary, we have $A_t(z \to 0) \sim \mu_q$. This procedure is also applicable to the quark mass discussed in the previous section. In the hard wall model, the solution of the bulk vector field is given by

$$A_t(z) = \mu + \rho z^2\,, \tag{4}$$

where μ and ρ are related quark chemical potential and quark (or baryon) number density in QCD.

There are huge number of works on holographic QCD at finite temperature and/or density. Due to limitation in space, we give only one example here. In [10], self-bound dense objects are studied in the hard wall model. By studying classical equation of motion for the time component bulk U(1) gauge field and a scalar field dual to $\bar{q}q$, they obtain density distribution and chiral condensate variation along the radial direction of the boundary gauge theory. Their results were applied to the density distribution in nuclei to reach a conclusion that the confinement scale might change from a free nucleon $\sim$ 300 MeV to a nucleus $\sim$ 100 MeV.

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