Physical mechanism of the (tri)critical point generation

K. A. Bugaev,[1,*] A. I. Ivanitskii,[1,**] E. G. Nikonov,[2,***]
V. K. Petrov,[1,****] A. S. Sorin,[3,*****] and G. M. Zinovjev[1,******]

[1] *Bogolyubov Institute for Theoretical Physics, Kiev, Ukraine*

[2] *Laboratory for Information Technologies, JINR Dubna, Russia*

[3] *Bogoliubov Laboratory of Theoretical Physics, JINR Dubna, Russia*

We discuss some ideas resulting from a phenomenological relation recently declared between the tension of string connecting the static quark-antiquark pair and surface tension of corresponding cylindrical bag. This relation analysis leads to the temperature of vanishing surface tension coefficient of the QGP bags at zero baryonic charge density as $T_\sigma = 152.9 \pm 4.5$ MeV. We develop the view point that this temperature value is not a fortuitous coincidence with the temperature of (partial) chiral symmetry restoration as seen in the lattice QCD simulations. Besides, we argue that T_σ defines the QCD (tri)critical endpoint temperature and claim that a negative value of surface tension coefficient recently discovered is not a sole result but is quite familiar for ordinary liquids at the supercritical temperatures.

1. INTRODUCTION.

The contemporary paradigm that at the deconfinement region the quark gluon plasma (QGP) is a strongly interacting plasma [1] seems to become a commonplace fact in the lattice QCD [2]. We, however, would like to point out that recently there appeared two almost revolutionary findings related to this paradigm. The first of them is found by the lattice QCD Wuppertal-Budapest (WB) group [3] and it states that at vanishing baryonic

* Electronic address: `Bugaev@th.physik.uni-frankfurt.de`

** Electronic address: `A_Iv_@ukr.net`

*** Electronic address: `E.Nikonov@jirn.dubna`

**** Electronic address: `Vkpetrov@yandex.ru`

***** Electronic address: `Sorin@theor.jinr.ru`

****** Electronic address: `Gennady.Zinovjev@cern.ch`

densities the (partial) chiral symmetry restoration temperature $T_{\rm chir}$ is between $146 \pm 2 \pm 3$ and $152 \pm 3 \pm 3$ MeV. This $T_{\rm chir}$ value is essentially smaller than the cross-over temperature $T_{\rm co} = 170 \pm 4 \pm 3$ MeV [3]. Indirectly this finding supports the possibility of the quarkyonic phase existence [4]. The second of them is that at $T_{\rm co}$ and vanishing baryonic densities the surface tension of quark gluon bags is negative [5]. As we argue the latter signals about new physics which, so far, was not investigated by the theory of ordinary liquids. Also here we discuss two kinds of exactly solvable statistical models, the quark gluon bags with surface tension models, which incorporate the existence of negative surface tension coefficient and use it to generate the tricritical [6, 7] and critical [8, 9] endpoint at some finite value of baryonic chemical potential.

2. RELATION BETWEEN STRING TENSION AND SURFACE TENSION.

In order to estimate the surface tension of QGP bags let us consider the static quark-antiquark pair connected by the unbreakable color tube of length L and radius $R \ll L$. In the limit of large L the free energy of the color tube is $F_{\rm str} \rightarrow \sigma_{\rm str} L$. The non-vanishing string tension coefficient $\sigma_{\rm str}$ signals about the color confinement, while its zero value is the usual measure for the deconfinement. Now we consider the same tube as an elongated cylinder of the same radius and length. For its free energy we use the standard parameterization [5]

$$F_{\rm cyl}(T,L,R) = -p_v(T)\pi R^2 L + \sigma_{\rm surf}(T) 2\pi R L + T\tau \ln\left[\frac{\pi R^2 L}{V_0}\right], \qquad (1)$$

where $p_v(T)$ is the bulk pressure inside a bag, $\sigma_{\rm surf}(T)$ is the temperature dependent surface tension coefficient, while the last term on the right hand side above is the Fisher topological term [10] which is proportional to the Fisher exponent $\tau = {\rm const} > 1$ [6–9] and $V_0 \approx 1$ fm^3 is a normalization constant. Since we consider the same object then its free energies calculated as the color tube and as the cylindrical bag should be equal to each other. Then for large separating distances $L \gg R$ one finds the following relation

$$\sigma_{\rm str}(T) = \sigma_{\rm surf}(T)\, 2\pi R \; - \; p_v(T)\pi R^2 + \frac{T\tau}{L} \ln\left[\frac{\pi R^2 L}{V_0}\right] \rightarrow \sigma_{\rm surf}(T)\, 2\pi R \; - \; p_v(T)\pi R^2 \,. \qquad (2)$$

In doing so, in fact, we match an ensemble of all string shapes of fixed L to a mean elongated cylinder, which according to the original Fisher idea [10] and the results of the Hills and Dales Model (HDM) [11, 12] represents a sum of all surface deformations of such a bag. The last equation allows one to determine the T-dependence of bag surface tension as

$$\sigma_{\rm surf}(T) = \frac{\sigma_{\rm str}(T)}{2\pi R} \ + \ \frac{1}{2} \, p_v(T) R \,, \tag{3}$$

if $R(T)$, $\sigma_{\rm str}(T)$ and $p_v(T)$ are known. This relation opens a principal possibility to determine the bag surface tension directly from the lattice QCD simulations for any T. Also it allows us to estimate the surface tension at $T = 0$. Thus, taking the typical value of the bag model pressure which is used in hadronic spectroscopy $p_v(T = 0) = -(0.25)^4$ GeV^4 and inserting into (3) the lattice QCD values $R = 0.5$ fm and $\sigma_{\rm str}(T = 0) = (0.42)^2$ GeV^2 [13], one finds $\sigma_{\rm surf}(T = 0) = (0.2229 \text{ GeV})^3 + 0.5 \, p_v \, R \approx (0.183 \text{ GeV})^3 \approx 157.4$ MeV fm^{-2} [5].

The found value of the bag surface tension at zero temperature is very important for the phenomenological equations of state of strongly interacting matter in two respects. Firstly, according to HDM the obtained value defines the temperature at which the bag surface tension coefficient changes the sign [11, 12, 14]

$$T_\sigma = \sigma_{\rm surf}(T = 0) \, V_0^{\frac{2}{3}} \cdot \lambda^{-1} \ \in \ [148.4; \ 157.4] \text{ MeV} \,, \tag{4}$$

where the constant $\lambda = 1$ for the Fisher parameterization of the T-dependent surface tension coefficient [10] or $\lambda \approx 1.06009$, if we use the parameterization derived within the HDM for surface deformations [11, 12, 14]. A straightforward evaluation of the entropy density of the elongated cylinder made from (1) in [5] shows that at the cross-over temperature the surface tension coefficient of bag should be negative otherwise its entropy density is negative.

The remarkable fact is that the value of temperature T_σ in (4) just matches that one of (partial) chiral symmetry restoration found by the WB group [3], i.e. $T_\sigma = T_{\rm chir}$. In other words, two different physical quantities, i.e. the chiral condensate and surface tension coefficient, which are obtained by entirely independent methods indicate that the properties of strongly interacting matter are qualitatively changed in the same temperature range. Such a 'coincidence' can be understood naturally, if we recall that the relevant degrees of freedom (=constituents), interaction between them together with the properties of their surface are qualitatively different in different phases of matter. Thus, one can expect that different physics is indicated by the sign change of the surface tension. This conclusion is supported by the results of quark gluon bags with surface tension models [6–9], by the Fisher droplet model (FDM) [10] and by the simplified statistical multifragmentation model (SMM) [15].

Secondly, according to the most successful models of liquid-gas phase transition, i.e. the FDM [10] and the SMM [16], the surface tension coefficient linearly depends on temperature. This conclusion is well supported by HDM and by microscopic models of vapor-liquid interfaces [17]. Therefore, the temperature T_σ in (4), at which the surface tension coefficient vanishes, is also the temperature of the (tri)critical endpoint $T_{\rm cep}$ of the liquid-gas phase diagram. On the basis of these results we conclude that the value of QCD critical endpoint temperature is $T_{\rm cep} = T_\sigma = 152.9 \pm 4.5$ MeV. Hopefully, the latter can be verified by the lattice QCD simulations using Eq. (3).

3. THE ROLE OF NEGATIVE SURFACE TENSION COEFFICIENT.

The quark gluon bags with surface tension models with the tricritical [6, 7] and critical [8, 9] point employ the same physical mechanism of the endpoint generation as FDM [10] and SMM [15, 16] which is typical for simple liquids [10]: at the phase coexistence line the difference of bulk parts of free energy of two phases vanishes due to Gibbs criterion, whereas at the endpoint, additionally, the surface part of free energy of liquid phase disappears. However, in contrast to FDM and SMM, in which the surface tension coefficient is zero above the endpoint temperature, the quark gluon bags with surface tension models from the very beginning employ negative values of surface tension coefficient above $T_{\rm cep}$. So far, an existence of negative surface tension coefficient above $T_{\rm cep}$ is the only know physical reason explaining why the first order phase transition degenerates into a cross-over [6]. Now the question is whether negative surface tension exists in the usual liquids. The experimental data on negative surface tension coefficient of usual liquids are, of course, unknown. However, if one takes highly accurate experimental data in the critical endpoint vicinity, then one finds not only that the surface tension coefficient approaches zero, but, in contrast to the wide spread belief, its full T derivative does not vanish and remains finite at $T_{\rm cep}$: $\frac{d\sigma_{\rm surf}}{d\,T} < 0$ [18]. Therefore, just the naive extension of these data to the temperatures above $T_{\rm cep}$ would lead to negative values of surface tension coefficient at the supercritical temperatures. On the other hand, if one, as usually, believes that $\sigma_{\rm surf} = 0$ for $T > T_{\rm cep}$, then it is absolutely unclear what physical process can lead to simultaneous existence of the discontinuity of $\frac{d\sigma_{\rm surf}}{d\,T}$ at $T_{\rm cep}$ and the smooth behavior of the pressure's first and second derivatives at the cross-over. Therefore, we conclude that negative surface tension coefficient at supercritical temperatures is also necessary for ordinary liquids although up to now this question has not been investigated. The quark gluon bags with surface tension models tell us that the surface tension coefficient is the natural order parameter allowing one to distinguish the **quark gluon liquid** phase which is represented by a single bag of infinite size with $\sigma_{\rm surf} \geq 0$ from the **QGP** that is the mixture of bags of all sizes which due to $\sigma_{\rm surf} < 0$ has the finite mean volume. Also it is clear that the line $\sigma_{\rm surf} = 0$ is the natural border between the QGP ($\sigma_{\rm surf} < 0$) and hadron gas ($\sigma_{\rm surf} > 0$) at the cross-over region.

4. CONCLUSIONS.

Here we discuss the relation between the tension of the color string connecting the static quark-antiquark pair and the surface tension of the corresponding cylindrical bag. Such a relation allows us to determine the temperature of vanishing surface tension coefficient of QGP bags at zero baryonic charge density as $T_\sigma = 152.9 \pm 4.5$ MeV. We are arguing that just this range of temperatures does not randomly matches the range of the (partial) chiral symmetry restoration temperatures found by the WB collaboration [3]. Using the exact results for the partition of surface deformations [11, 12, 14] and Fisher conjecture for temperature dependence of surface tension coefficient [10] we conclude that the same temperature range corresponds to the value of QCD (tri)critical endpoint temperature, i.e. $T_{\rm cep} = T_\sigma = 152.9 \pm 4.5$ MeV. Furthermore, we claim that negative values of surface tension coefficient of QGP bags found recently in [5] are not unique, but also should exist for the supercritical temperatures of usual liquids.

Acknowledgement

E.G.N. and A.S.S. have been supported in part by the Russian Fund for Basic Research under grant number 11-02-01538-a.

---

1. E. V. Shuryak, Prog. Part. Nucl. Phys. **62**, 48 (2009).
2. Z. Fodor, PoS Lattice 2007, 011 (2007); F. Karsch, Prog. Theor. Phys. Suppl. **168**, 237 (2007).
3. Y. Aoki *et al.*, JHEP **0906**, 088 (2009).
4. A. Andronic *et al.*, Nucl. Phys. A **837**, 65 (2010).
5. K. A. Bugaev and G. M. Zinovjev, Nucl. Phys. A **848**, 443 (2010).

6. K. A. Bugaev, Phys. Rev. C **76**, 014903 (2007).

7. K. A. Bugaev, Phys. Atom. Nucl. **71**, 1585 (2008).

8. K. A. Bugaev, V. K. Petrov, and G. M. Zinovjev, arXiv: 0904.4420 (2009).

9. K. A. Bugaev, V. K. Petrov, and G. M. Zinovjev, Europhys. Lett. **85**, 22002 (2009).

10. M. E. Fisher, Physics **3**, 255 (1967).

11. K. A. Bugaev, L. Phair, and J. B. Elliott, Phys. Rev. E **72**, 047106 (2005).

12. K. A. Bugaev and J. B. Elliott, Ukr. J. Phys. **52**, 301 (2007).

13. A. Mocsy and P. Petreczky, PoS LAT2007, 216 (2007).

14. see an extended discussions in K. A. Bugaev, arXiv: 1012.3400 (2010).

15. K. A. Bugaev *et. al.*, Phys. Rev. C **62**, 044320 (2000); Phys. Lett. B **498**, 144 (2001).

16. J. P. Bondorf *et al.*, Phys. Rep. **257**, 133 (1995).

17. J. Gross, J. Chem. Phys. **131**, 204705 (2009).

18. J. B. Elliott, K. A. Bugaev, L. G. Moretto, and L. Phair, arXiv: nucl-ex/0608022.