1.4 Theory of Condensed Matter

Theoretical investigations in the Theory of Condensed Matter were performed in the framework of the following projects:

- Strongly correlated systems.
- Dynamical systems: chaos, integrability, and self-organization.
- Disordered structures: glasses, topological defects, nanostructures, and Josephson junction.
- Mesoscopic and coherent phenomena in quantum systems.

Major results of the investigations within these projects are presented below, while several topics are discussed in the attached brief reports.

Main results in the field of Strongly correlated systems were obtained in studies of electronic spectrum, superconductivity, and magnetic properties of materials with strong electron correlations like transition metal oxides. A number of important new results were obtained among which are the following.

A new method based on the dynamical mean-field theory for calculation of optical and transport properties of Ce, CrO_2, $LaTiO_3$ materials was proposed and implemented. It was found that $\alpha \rightarrow \gamma$ transition and optical data in Ce can be described by the Kondo collapse model better than a Mott transition picture. The full temperature dependence of Ce optical spectra was predicted.

An effective anisotropic t-J model for the "pseudo-ladder" compound $CaCu_2O_3$ was proposed based on recent experimental studies and band structure calculations. Superconducting pairing mediated by the exchange interaction in the model was investigated. It was shown that anisotropy in the electronic spectrum strongly suppresses superconducting T_c.

The interaction of strongly correlated electrons with acoustic phonons was investigated in the framework of the Hubbard-Holstein model. It was suggested that the polaron exchange by phonon clouds could lead to the polaron pairing and high-T_c superconductivity.

A valence bond crystal was considered within an effective spin-orbital Hamiltonian for an orbitally-degenerate spin-$1/2$ system on pyrochlore lattices. It was shown that the orbital degrees of freedom can modulate the spin exchange, removing the infinite spin-degeneracy of the ground state. The theory provides an explanation for the helical spin-singlet pattern in the B-spinel $MgTi_2O_4$. It was proposed to use the resonant X-ray spectroscopy for experimental observation of the orbital ordering.

A superexchange theory was developed for a spin-$1/2$ antiferromagnet Sr_2GaMno_{5+x} to explain a competition between 'vertical' and 'diagonal' interplane magnetic bonds. A natural explanation was obtained for an observed in neutron scattering experiments transition between two magnetic structures generated by a chemical substitution in $Sr_2GaMn(O,F)_6$ compound.

The state of itinerant charge carriers in complex magnetic materials such as the magnetic and diluted magnetic semiconductors was investigated. Within the framework of a spin-fermion ($s-d$ exchange) model, a detailed theory of quasiparticle excitations was developed using a reformulation of the Green function method.

Main results in the problem of Dynamical systems: chaos, integrability and self-organization were obtained for one-dimensional exactly solvable stochastic processes.

The normalization identity for the totally asymmetric exclusion process on an infinite lattice and on a ring is derived. The sum of conditional probabilities over all possible final positions

of particles at time t given arbitrary initial positions at time $t = 0$ is calculated. A method of derivation of the normalization identity can be used for evaluation of correlation functions of the exclusion process.

The Bethe ansatz solution for discrete time zero range and the asymmetric exclusion processes with fully parallel dynamics is presented. The model depends on two parameters: p, the probability of single particle hopping, and q, the deformation parameter, which in the general case obeys $|q| < 1$ and is responsible for the long range interaction between particles. The particular case $q = 0$ corresponds to the popular Nagel-Schreckenberg traffic model.

The largest eigenvalue of the equation for the generating function of the distance travelled by particles is evaluated. For $q = 0$ the result is obtained for an arbitrary size of the lattice and the number of particles. In the general case, $|q| < 1$, the model in the scaling limit is considered and the universal form specific for the Karadar-Parizi Zhang universality class is obtained. The phase transition occurring in the limit $p \to 1$ when $q < 0$ is described.

The discrete-time evolution of a finite number of particles obeying the totally asymmetric exclusion process (TASEP) with backward-ordered update on an infinite chain was considered. The first result in this direction is a determinant expression for the conditional probability of finding the particles at given initial and ending positions, provided they start and finish simultaneously. The expression has the same form as the one obtained by Schütz [J. Stat. Phys. **88**, 427 (1997)] for the continuous-time process. Next, it was proved that under some sufficient conditions the determinant expression can be generalized to the case when the particles start and finish at their own times. The latter result is used to solve a non-stationary zero-range process (ZRP) on a finite chain with open boundaries.

The investigations performed within the project Mesoscopic and Coherent Phenomena in Quantum Systems were mostly concentrated on the following topics: atomic interactions in superfluid ^{4}He and in atomic traps, squeezed light teleportation, quasiparticle spectra in quantum wells, electron-phonon interaction in polar and covalent materials, mesoscopic fluctuations and transient coherence in nonequilibrium systems, toroid polarizations and nonlinear phenomena.

The comprehensive description of coherent spin relaxation is given. The theory of nuclear spin superradiance is developed and the experimental observations of this phenomenon are considered. The intriguing problem of how coherence develops from initially incoherent quantum fluctuations is analysed. All main types of coherent radiation by nuclear spins are discussed, which are: free nuclear induction, collective induction, maser generation, pure superradiance, triggered superradiance, pulsing superradiance, punctuated superradiance, and induced emission. The influence of electron-nuclear hyperfine interactions and the role of magnetic anisotropy are studied. Conditions for realizing spin superradiance by magnetic molecules are investigated.

Main results in the problem of Disordered structures were obtained in investigation of electronic structure of graphite nanoparticles, tunnel splitting of classically degenerate ground states, in studies of the low-temperature thermal characteristics of topologically disordered materials, and in studies of some novel effects in Josephson junctions.

Within a gauge field-theory model, the local and total density of states (DOS) near the pentagonal defects (disclinations) were calculated (both analytically and numerically) for three geometries: sphere, cone, and hyperboloid. It was found that the low-energy total DOS had a cusp which drops to zero at the Fermi energy for disclinations with the Frank index $\nu < 1/2$. The appearance of an enhanced charge density near the Fermi level for nanocones with 60° opening angle (180° disclination at the apex) was predicted. The effect of pentagonal defects on the electronic structure at the tip of the carbon nanohorns was investigated within the continuum

gauge field-theory model. It was found that the existence of a localized electron state at the Fermi level (a true zero-mode state) results in the enhanced charge density near the tip.

An instanton approximation to the continuous-time spin coherent-state path integral was used to obtain the tunnel splitting of classically degenerate ground states. The method was applied for description of the molecular magnet Fe8 in a transverse field.

It was shown that the transformation properties of the mean-field slave boson/fermion order parameter under an action of the global SU(2) group imposed certain restrictions on their applications to describe the phase diagram of the t-J model.

The thermal conductivity and internal friction of plastically deformed metals (aluminium, niobium, tantalum and LiF) were investigated within the string model for dipoles of edge dislocation. It was found that the presence of dipoles led to a remarkable increase in the resonance frequency. This gives a possibility to obtain a good agreement with experiments.

Within a 2D model of Josephson junction arrays (created by 2D network of twin boundary dislocations with strain fields acting as an insulating barrier between hole-rich domains in underdoped crystals), a few novel effects expected to occur in intrinsically granular material are predicted including: (i) Josephson chemomagnetism (chemically induced magnetic moment in a zero applied magnetic field) and its influence on a low-field magnetization (chemically induced paramagnetic Meissner effect), and (ii) magnetoconcentration effect (creation of oxygen vacancies in applied magnetic field) and its influence on a high-field magnetization (chemically induced analog of "fishtail" anomaly).

By improving resolution of home-made mutual-inductance measurement technique, a pronounced step-like structure (with the number of steps $n = 4$ for all AC fields) was observed in the temperature dependence of AC susceptibility in artificially prepared two-dimensional Josephson Junction Arrays (2D-JJA) of unshunted $Nb - AlO_x - Nb$ junctions with $\beta_L(4.2K) = 30$. Using a single-plaquette approximation of the overdamped 2D-JJA model, we were able to successfully fit our data assuming that steps are related to the geometric properties of the plaquette. The number of steps n corresponds to the number of flux quanta that can be screened by the maximum critical current of the junctions. The steps are predicted to manifest themselves in arrays with the inductance related parameter $\beta_L(T)$ matching a "quantization" condition $\beta_L(0) = 2\pi(n+1)$.

The magnetic field dependence of complex AC susceptibility $\chi = \chi' + i\chi''$ of artificially prepared highly ordered (periodic) two-dimensional Josephson junction arrays of unshunted $Nb - AlO_x - Nb$ junctions was studied experimentally. The observed anomalous behavior can be explained by assuming inhomogeneous critical current distribution within a single junction and using single-plaquette approximation of the overdamped 2D-JJA model.

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VALENCE-BOND CRYSTAL IN A PYROCHLORE ANTIFERROMAGNET WITH ORBITAL DEGENERACY

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Our work is motivated by the very recent synthesis and interesting experimental data on *B*-spinel $MgTi_2O_4$ [1, 2]. The physical behavior of this system, in which both orbital and spin degrees of freedom are active, is expected to be different from that with only spin degrees of freedom, as the occurrence of an orbital ordering (OO) can modulate the spin exchange and lift the geometrical degeneracy of the underlying lattice. The recent experiments on $MgTi_2O_4$ have shown that this compound undergoes a metal-to-insulator transition on cooling below 260 K with an associated cubic-to-tetragonal lowering of symmetry.[1] At the transition the magnetic susceptibility continuously decreases and saturates, in the insulating phase, to a value which is anomalously small for spin $1/2$ local moments: for this reason the insulating phase was interpreted as a spin-singlet phase. Subsequent synchrotron and neutron powder diffraction experiments revealed that the low-temperature crystal structure is made of alternating short and long Ti-Ti bonds forming a helix about the tetragonal *c*-axis [2] and can be regarded as a valence bond crystal (VBC) since the long-range order of spin-singlets extends throughout the whole pyrochlore lattice.

The aim of the present work is to discuss the microscopic mechanism behind the realization of this unusual and intriguing VBC structure on the pyrochlore lattice. We argue that the key role in this mechanism is played by orbital degeneracy. We consider the system in its cubic structure and look for possible instabilities towards symmetry reductions. The effective Hamiltonian can be written as [4]:

$$H_{\rm eff} = -J_1 \sum_{\langle ij \rangle} \left[\vec{S}_i \cdot \vec{S}_j + 3/4\right] O_{ij} + J_2 \sum_{\langle ij \rangle} \left[\vec{S}_i \cdot \vec{S}_j - 1/4\right] O_{ij} + J_3 \sum_{\langle ij \rangle} \left[\vec{S}_i \cdot \vec{S}_j - 1/4\right] \tilde{O}_{ij} \qquad (1)$$

where the sum is restricted to the NN sites on the pyrochlore lattice. As only the leading part of the hopping term, due to the largest $dd\sigma$ element, is taken into account, the orbital contributions to the effective Hamiltonian are Ising-like and can be expressed through projectors $P_{i,\alpha\beta}$ on the orbital state $|\alpha\beta\rangle$ as $O_{ij} = P_{i,\alpha\beta}(1-P_{j,\alpha\beta}) + P_{j,\alpha\beta}(1-P_{i,\alpha\beta})$ and $\tilde{O}_{ij} = P_{i,\alpha\beta}P_{j,\alpha\beta}$. The first and second terms in $H_{\rm eff}$ (1) describe the ferromagnetic (FM) $J_1 = t^2/(U_2 - J_H)$ and the antiferromagnetic (AFM) $J_2 = t^2/(U_2 + J_H)$ interactions, respectively, and are active only when the two sites involved are occupied by different orbitals. The last term is AFM, with $J_3 = \frac{4}{3}t^2\left[2/(U_2 + J_H) + 1/(U_2 + 4J_H)\right]$, and is nonzero only when the two sites have the same orbital occupancy.

The estimates for parameters that play a role are taken from Ref.[3] and they are: the NN hopping term $t \equiv t_\sigma \simeq 0.32$ eV, the Coulomb on-site repulsion among different orbitals $U_2 \simeq 4.1$ eV, and the Hund exchange $J_H \simeq 0.64$ eV . In what follows, it will be useful to introduce a ratio $\eta = J_H/U_2 \simeq 0.15 \ll 1$, and expand the exchange energies in η around $\eta = 0$. We get $J_1 \simeq J(1+\eta)$, $J_2 \simeq J(1-\eta)$ and $J_3 \simeq 4J(1-2\eta)$, where $J = t^2/U_2 \simeq 25$ meV represents the overall energy scale.

The main aspect of the spin-orbital model (1) is that due to $dd\sigma$-character of the hopping terms only some orbital configurations contribute to the energy. We can thus classify tetrahedral bonds in four types: **(i)** b_0 – both ions at sites i and j of the generic $\alpha\beta$-plane have $\alpha\beta$ orbital

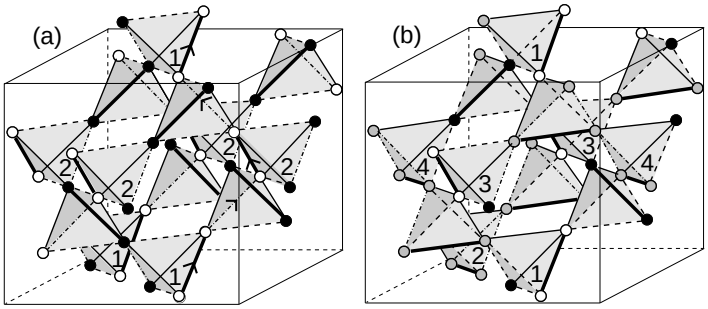

Figure 1: The ground state coverings of the unit cubic cell through dimers. Locations of singlets are represented by thick links. Different numbers correspond to inequivalent tetrahedra. (a) The helical dimerization pattern (indicated by arrows) is formed by alternating short b_0 and long b_3 bonds, Dimer phase B_3. (b) One of the possible coverings of the cubic unit cell by B_1B_2 tetrahedra.

occupancy. It is characterized by a Hamiltonian with strong AFM exchange: $H_{b_0} = -J(1-2\eta)(1-4\vec{S}_i\cdot\vec{S}_j)$; **(ii)** b_1 – the two sites of bond ij in $\alpha\beta$-plane are occupied by one $\alpha\gamma$ and one $\alpha\beta$ orbitals, $\gamma\neq\beta$. This bond has a weak FM exchange $\sim\eta J$ and the corresponding Hamiltonian is: $H_{b_1} = -J(1+\eta/2+2\eta\vec{S}_i\cdot\vec{S}_j)$; **(iii)** b_2 – the two sites of bond ij in $\alpha\beta$-plane occupied by one $\alpha\gamma$ and one $\beta\gamma$ orbitals. In this case there is no energy contribution: $H_{b_2}=0$; **(iv)** b_3 – both sites of bond ij in $\alpha\beta$-plane are occupied by two $\alpha\gamma$ ($\beta\gamma$) orbitals: $H_{b_3}=0$.

It is possible to show that only three energetically inequivalent tetrahedrons can be singled out of the four b_n bonds introduced above, which can be characterized by the number of b_0-bonds: *A*-type tetrahedra, with two b_0 bonds, *B*-type tetrahedra with one b_0 bond and *C* tetrahedra with no b_0-bonds.

(I) Heisenberg chains: If all sites of pyrochlore lattice are occupied by the same orbital, then the effective Hamiltonian (1) can be mapped into a set of one-dimensional decoupled AFM Heisenberg chains that lie within the plane of the chosen orbital. The ground-state energy per site can be evaluated exactly by using the results for an Heisenberg chain which give $E_A = -2.77(1-2\eta)J$. **(II) Dimer phase** *B*: This state is made of only *B*-type tetrahedra with one strong b_0-bond and two intermediate b_1-bonds. There are three energetically equivalent B_i tetrahedra and all possible coverings of pyrochlore lattice by B_i tetrahedra have the same energy (two possible coverings are shown in Fig. 1). In the limit $\eta\to 0$, the spin-only Hamiltonian can be solved exactly, as it can be decomposed into a sum of spin-uncoupled b_0 bonds. The energy minimum is reached when the Heisenberg term of the b_0-bond is the lowest, i.e., for a pure quantum spin-singlet ($\vec{S}_i\cdot\vec{S}_j=-3/4$). As $\eta\ll 1$, the dimer state is stable against the weak FM interdimer interaction. In this case, an energy per site is given by: $E_B = E_{b_0}/2 + E_{b_1} = -(3-\frac{7}{2}\eta)J$. Here $E_{b_{0(1)}}$ is the energy of the bond $b_{0(1)}$. **(III) FM order**: In this case, all tetrahedra are of *C*-type with four interacting b_1 bonds and two b_3 (or one b_2 and one b_3) noninteracting bonds. Thus, all nonzero spin-exchanges are ferromagnetic, and the ground state has an energy per site given by $E_C = 2E_{b_1} = -2(1+\eta)J$. **(IV) Frustrated AFM**: We consider for completeness the case where each orbital is occupied by a linear superposition with equal weight of the three orbitals: $\frac{1}{\sqrt{3}}(|xy\rangle+|xz\rangle+|yz\rangle)$. The energy in this phase is higher than that of the other phases, as it does not exploit at all the potential energy-gain contained in the orbital ordering.

For $\eta=0$ the lowest ground-state energy is that of spin-singlet degenerate manifold. It is

characterized by a static pattern of spin-singlets throughout the whole pyrochlore lattice and, thus, is different from RVB state. Each dimer covering is frozen in an exact eigenstate of the Hamiltonian (1) for $\eta = 0$. For finite η the different dimer patterns are not connected by the Hamiltonian, thus, a tunneling between different dimer states can not take place.

In this spin-singlet manifold the original spin degeneracy is removed. However there is still a remaining orbital degeneracy to be lifted. This degeneracy can be removed by magnetoelastic coupling. In a spin-Peierls system the magnetic energy gain due to the spin-singlet pairs outweights the increase in elastic energy due to the dimerization of the regular array. We have shown with qualitative [4] and quantitative [5] arguments that this mechanism selects the triplet-T normal mode of the tetrahedron group, leading to a distortion with one short and one long bonds located at the opposite edges and four undistorted bonds. Indeed, a reduction of the bond length increases the magnetic energy gain and, therefore, favors the shortening of the bond with the strongest superexchange, i.e., b_0, where the singlet is located. This mechanism determines also the position of the two b_1-bonds in the tetrahedron: in order to maximize the superexchange energy gain, the intermediate-strength b_1-bonds are not allowed to lie on the long bond opposite to the singlet b_0. The elongation of the weak bonds of b_3 type is energetically more favorable. It is easy to check [4, 5] that it corresponds to the covering when all tetrahedra are of B_3 kind, with a T-type tetragonal distortion. In this state all dimers are condensed in the ordered helical pattern shown in Fig. 1a and form a VBC. This dimerization pattern exactly reproduces the one observed in the insulating phase of $MgTi_2O_4$ [2].

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PHYSICS OF COMPLEX MAGNETIC MATERIALS: QUASIPARTICLE MANY-BODY DYNAMICS

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The existence and properties of localized and itinerant magnetism in metals, oxides, and alloys, and their interplay is an interesting but not yet fully understood problem of quantum theory of magnetism[1, 2]. The behavior and the true nature of the electronic and spin states, and their quasiparticle dynamics are of central importance to the understanding of physics of correlated systems such as magnetism and Mott-Hubbard metal-insulator transition in metals and oxides, magnetism and heavy fermions in rare-earths compounds, and anomalous transport properties in perovskite manganites. This class of systems is characterized by complex, many-branch spectra of elementary excitations. Moreover, the correlation effects (competition and interplay of Coulomb correlation, direct or indirect exchange, sp-d hybridization, electron-phonon interaction, disorder, etc.) are essential[3, 4]. These materials are systems of great interest both intrinsically and as a possible source of understanding the magnetism of matter generally [5, 6]. There has been considerable interest in identifying the microscopic origin of quasiparticle states [3, 4, 6] in these systems and a few model approaches have been proposed. Many magnetic and electronic properties of rare-earth metals and compounds, and

magnetic semiconductors [7] and related complex magnetic materials may reasonably be interpreted in terms of combined spin-fermion models which include interacting spin and charge subsystems [8, 9, 10]. This approach permits one to describe significant and interesting physics, e.g., bound states and magnetic polarons [9], anomalous transport properties, etc.
The problem of adequate physical description within various types of spin-fermion model has intensively been studied during the last decades, especially in the context of magnetic and transport properties of rare-earth and transition metals and their compounds and magnetic semiconductors.
Substances which we refer to as magnetic semiconductors, occupy an intermediate position between magnetic metals and magnetic dielectrics. Magnetic semiconductors are characterized by the existence of two well defined subsystems, the system of magnetic moments which are localized at lattice sites, and a band of itinerant or conduction carriers (conduction electrons or holes). Typical examples are the Eu-chalcogenides, where the local moments arise from 4f electrons of the Eu ion, and the spinell chalcogenides containing Cr^{3+} as a magnetic ion [7]. There is experimental evidence of a substantial mutual influence of spin and charge subsystems in these compounds. This is possible due to the $sp-d(f)$ exchange interaction of the localized spins and itinerant charge carriers. More recent efforts have been directed to the study of the properties of diluted magnetic semiconductors (DMS)[10]. Further attempts have been made to study and exploit carriers which are exchange-coupled to the localized spins. The effect of carriers on the magnetic ordering temperature is found to be very strong in DMS . Diluted magnetic semiconductors are mixed crystals in which magnetic ions (usually Mn^{++}) are incorporated in a substitutional position of the host (typically a II-VI or III-V) crystal lattice. The diluted magnetic semiconductors offer a unique possibility for a gradual change of the magnitude and sign of exchange interaction by means of technological control of carrier concentration and band parameters. This field is very active and there are many aspects to the problem. A lot of materials were synthesized and tested. The new material design approach to fabrication of new functional diluted magnetic semiconductors resulted in producing a variety of compounds . The presence of the spin degree of freedom in DMS may lead to a new semiconductor spin electronics which will combine the advantages of the semiconducting devices with the new features due to the possibilities of controlling the magnetic state. However, the coexistence of ferromagnetism and semiconducting properties in these compounds require a suitable theoretical model which would describe well both the magnetic cooperative behavior and the semiconducting properties as well as a rich field of interplay between them. The majority of theoretical papers on DMS studied their properties mainly within the mean field approximation and continuous media terms. In a picture like this the disorder effects, which play an essential role, can be taken into account roughly only. Moreover, there are different opinions on the intrinsic origin and the nature of disorder in DMS. Thus, many experimental and theoretical investigations call for a better understanding of the relevant physics and the nature of solutions (especially magnetic) within the lattice spin-fermion model[8, 9, 10]. For treating the problems we used a nonperturbative many-body approach, the formalism of the method of Irreducible Green Functions (IGF) [4]. This IGF method allows one to describe quasiparticle spectra with damping for many-particle systems on a lattice with complex spectra and a strong correlation in a very general and natural way. This scheme differs from the traditional method of decoupling of an infinite chain of equations [1] and permits a construction of the relevant dynamic solutions in a self-consistent way at the level of the Dyson equation without decoupling the chain of equations of motion for the GFs.
In paper [9] the concepts of bound and scattering states were analysed and developed to eluci-

date the nature of itinerant charge carrier states in magnetic semiconductors and similar complex magnetic materials. By contrasting the scattering and bound states of carriers within the $s-d$ exchange model, the nature of bound states at finite temperatures was clarified. The free magnetic polaron at certain conditions is realized as a bound state of the carrier (electron or hole) with the spin wave. Quite generally, a self-consistent theory of a magnetic polaron was formulated within the IGF method which was used to describe the quasiparticle many-body dynamics at finite temperatures. Within the above approach we elaborated a self-consistent picture of dynamic behavior of two interacting subsystems, the localized spins and the itinerant charge carriers. In particular, it was shown that the relevant generalized mean fields emerges naturally within our formalism. At the same time, the correct separation of elastic scattering corrections permitted one to consider the damping effects (inelastic scattering corrections) self-consistently. The damping of magnetic polaron state, which is quite different from the damping of the scattering states, finds a natural interpretation within the present self-consistent scheme.
In paper [10], we applied the IGF formalism to consider quasiparticle spectra for the lattice spin-fermion model consisting of two interacting subsystems. It was the purpose of that paper to explore more fully the notion of Generalized Mean Fields (GMF) [4] which may arise in the system of interacting localized spins (including effects of disorder) and lattice fermions to justify and understand the nature of the relevant mean fields. Background and applications of the generalized spin-fermion (sp-d) exchange model to magnetic and diluted magnetic semiconductors were discussed in some detail. The capabilities of the model to describe quasiparticle spectra were investigated. The key problem of most of the work was the formation of spin excitation spectra under various conditions on the parameters of the model. In paper [10], we concentrated on the description of the magnetic excitation spectra and treated the disorder effects in the simplest VCA (virtual crystal approximation) to emphasize the need for a suitable definition of the relevant generalized mean fields and for internal self-consistency in the description of the spin quasiparticle many-body dynamics.
Thus, we were able to calculate the quasiparticle spectra and GMF of the magnetic semiconductors consisting of two interacting charge and spin subsystem within the lattice spin-fermion model to analyze the role and influence of the Coulomb correlation, exchange, and effects of disorder in a unified and coherent fashion.

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SELF-ORGANIZED CRITICALITY IN AN IMMUNE SYSTEM

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It has been a routine practice since the seminal work by Eigen [1] to use kinetic equations on complex graphs for description of the dynamics of competing macromolecular organisms [2, 3, 4]. The quasispecies model introduced by Eigen and Schuster [5] describes the mutation within a given genome space visualized as a graph with sites representing different types of organisms and bonds connecting neighboring sites which are mutants with closest biological affinity. Usually, the rates of replication for each site are assumed to be constant in time, so the system evolves to the relatively stable quasispecies on a static fitness landscape.

Recently, some extensions to dynamic fitness landscape have been made [6, 7]. The height of the fitness peak can change in time without changing its space location [6] or, vice versa, can move in the sequence space [7]. The dynamic fitness assumes an interaction of the evolving system with an adaptive environment. The closest example is the evolution of viruses in the presence of a changing immune system [8, 9, 10].

Effects of the interaction between an evolving system and a changing environment were considered in [11], where the evolving system was taken as a simple random walk on a lattice and the changing environment as a system of arrows pointing the direction of continuation of the walk at each site. After each visit, the walk changes the direction of an arrow in the visited site to the next position. It was demonstrated in [11] that the motion of a diffusive particle converges asymptotically to the Eulerian cycle which passes each bond of the graph in both directions exactly once. On the other hand, the configuration of arrows, initially random, tends to a set of strongly correlated configurations corresponding to spanning trees on the given graph.

The correlation functions of the spanning trees are well known [12] and decay with the distance by a power law. Moreover, the number of spanning trees exactly equals the number of recurrent configurations in the Abelian sandpile model [13] which is the basic model of self-organized criticality [14].

A direct attempt to map a viral evolution model with dynamic fitness on the model of Eulerian walks fails as the state of immune system is characterized by site variables rather than bond variables. Activity of the immune system at a given site depends on the site distribution of a virus population: it increases with time if the site is in the vicinity of a local maximum of the virus probability distribution and decreases if the local maximum goes to another region of the graph. Thus, we need a site version of the model proposed in [11].

The second peculiarity of the model is the discrete time approximation. We assume that at each discrete moment of time, the population of viruses has a single peaked probability distribution located at one of the sites of a graph. The evolution of the distribution is reduced in this approximation to jumps between neighboring sites of the graph. In the context of the qusispecies evolution model this implies that a fitness peak moves in the sequence space by jumps and the time interval between jumps is sufficient for the virus population to reach a local stationary state.

In the absence of immune system, the trajectory of the peak of virus distribution is just a simple random walk on the graph provided that all sites of the graph outside of the peak of fitness are equivalent. The presence of the immune system changes this picture drastically. The activity of the immune system is a function of the coordinates of sites and the time passed from the last visit of the given site by the peak of the virus distribution.

We assume that activity at the given site grows rapidly when the peak of the virus distribution comes into this site and relaxes slowly when the peak leaves this site. As a result, the immune

system ”remembers” all visits and its activity at each site is a monotonically decreasing function of time from the last visit of this site. These assumptions are sufficient to formulate the model.

Given a graph G, whose vertices represent possible states of virus population, we associate the peak of virus distribution at a given moment of discrete time with the position of the random walk at the given site. The activity of the immune system at the initial moment of time is described by a set of random numbers uniformly distributed on an interval. Starting a motion, the random walk located at a site chooses among its nearest neighbors the site where activity of the immune system is minimal. During the initial period, the motion of the random walk is mostly random. The next period is characterized by appearance of more sites visited before among the nearest neighbors of each site. Using the rule of monotonic decrease of activity, the walker chooses the site visited at the earliest moment of time. During this period, the motion of the walker becomes more deterministic as the concentration of visited sites grows. At the last stage of evolution, when all sites of the graph are already visited at least once, the walk becomes purely deterministic.

We show that the motion of the walker at the last stage converges asymptotically to a cycle which passes all sites of the graph and has a property of the long-range correlations. Considering a specific graph (the Manhattan lattice), we prove that the limiting cycle is a Hamiltonian walk [15], that is the closed path which passes all sites of the graph exactly once. The elements of the Hamiltonian walk are strongly correlated objects. The correlations in the trajectory of the walk mean simultaneously the correlations in the environment, namely, the correlation of activity of the immune system at two sites separated by the Euclidean distance r decays as a power law of r.

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DIFFEOMORPHISMS INDUCED BY PRIME SERIES

Lubomir Alexandrov

The pair of one-to-one arithmetic functions

$$p(n):\mathbb{N}\to\mathbb{P},\quad p^{-1}(q):\mathbb{P}\to\mathbb{N},$$

can be extended to a pair of diffeomorphisms over the real semi-axis $(0,\infty)$.
Denotation: $\mathbb{N}$ and $\mathbb{P}$ are natural and prime series, respectively; $p(n)$ and p_n denote nth prime; $\hat{p}_n = p_{n+1}-p_n-1$ is the number of composite numbers in the interval (p_n,p_{n+1}) .

Definition 1 *The functions $f(x)\in C^{(1)}(0,\infty)$, $g(x)\in C^{(1)}(1,\infty)$ are called prime number diffeomorphic if the following conditions hold:*

$$\begin{aligned}
f(n) &= p_n,\quad n\in\mathbb{N} && (1)\\
f\left(n+\frac{1}{2}\right) &= \frac{1}{2}(p_n+p_{n+1}),\quad n\in\mathbb{N} && (2)\\
f(g(x)) &= x\quad \forall x\in(1,\infty), && (3)\\
g(f(x)) &= x\quad \forall x\in(2,\infty), && (4)\\
\pi(x) := \sum_{p\le x,\ p\in\mathbb{P}} 1 &= \lfloor g(x)\rfloor. && (5)
\end{aligned}$$

The diffeomorphisms $f(x)$ and $g(x)$ are called the prime curve and the prime counting curve, respectively.

The function $\pi_R(x)$ of Riemann–Von Mangoldt ([1], p. 34, (2), (3)) which can be expressed in terms of the zeros of the Riemann zeta function is the closest, among all known function, to the prime counting curve. However, the function $\pi_R(x)$ is not invertible and cannot be used for the correspondence to the appropriate prime curve. It turns out that one can take the opposite way: construct an invertible interpolation of the prime series and then obtain from it a continuous differentiable counting curve. This way leads to prime number diffeomorphisms based on a quadric spline.
Let the functions be given

$$\begin{aligned}
a_n^-(x) &= -2\hat{p}_{n-1}(x-n)^2+(x-n)+p_n,\quad x>0,\quad n=2,3,\ldots,\\
a_n^+(x) &= 2\hat{p}_n\left(x-n-\frac{1}{2}\right)^2+(2\hat{p}_n+1)\left(x-n-\frac{1}{2}\right)+\frac{p_n+p_{n+1}}{2}.
\end{aligned}$$

Their derivatives are

$$\frac{da_n^-(x)}{dx}=4\hat{p}_{n-1}(n-x)+1,\quad \frac{da_n^+(x)}{dx}=4\hat{p}_n(x-n)+1.$$

For any $n=2,3,\ldots$ the functions above are sewed together

$$\begin{aligned}
a_n^-(n) &= a_n^+(n)=p_n, && (6)\\
\left.\frac{da_n^-(x)}{dx}\right|_{x=n} &= \left.\frac{da_n^+(x)}{dx}\right|_{x=n}=1, && (7)\\
a_{n+1}^-\left(n+\frac{1}{2}\right) &= a_n^+\left(n+\frac{1}{2}\right)=\frac{1}{2}(p_n+p_{n+1}), && (8)\\
\left.\frac{da_{n+1}^-(x)}{dx}\right|_{x=n+\frac{1}{2}} &= \left.\frac{da_n^+(x)}{dx}\right|_{x=n+\frac{1}{2}}=2\hat{p}_n+1. && (9)
\end{aligned}$$

Equations (6), (7), (8) and (9) define the following continuously differentiable quadric spline:

$$p(x) = \begin{cases} x+1, & 0 < x \le \frac{3}{2}, \\ a_n^-(x), & n-\frac{1}{2} \le x \le n, \quad n = 2,3,\ldots, \\ a_n^+(x), & n \le x \le n+\frac{1}{2}, \quad n = 2,3,\ldots . \end{cases} \tag{10}$$

with the first derivative

$$\frac{dp(x)}{dx} = \begin{cases} 1, & 0 < x \le \frac{3}{2}, \\ 4\hat{p}_{n-1}(n-x)+1, & n-\frac{1}{2} \le x \le n, \quad n = 2,3,\ldots, \quad . \\ 4\hat{p}_n(x-n)+1, & n \le x \le n+\frac{1}{2}, \quad n = 2,3,\ldots . \end{cases} \tag{11}$$

Inverting the function $a_n^-(x)$ in the interval $n-\frac{1}{2} \le x \le n$ and $a_n^+(x)$ in the interval $n \le x \le n+\frac{1}{2}$ gives the following inverse functions and their derivatives:

$$\begin{aligned} b_n^-(x) &= n + \frac{1-(8\hat{p}_{n-1}(p_n-x)+1)^{\frac{1}{2}}}{4\hat{p}_{n-1}}, \\ b_n^+(x) &= n + \frac{(8\hat{p}_n(x-p_n)+1)^{\frac{1}{2}}-1}{4\hat{p}_n}, \\ \frac{db_n^-(x)}{dx} &= (8\hat{p}_{n-1}(p_n-x)+1)^{-\frac{1}{2}}, \\ \frac{db_n^+(x)}{dx} &= (8\hat{p}_n(x-p_n)+1)^{-\frac{1}{2}}. \end{aligned}$$

The functions $b_n^-(x)$, $b_n^+(x)$ and their derivatives are sewed together in a similar way like Eqs. (6), (7), (8) and (9)

$$b_n^-(p_n) = b_n^+(p_n) = n, \tag{12}$$

$$\left.\frac{db_n^-(x)}{dx}\right|_{x=p_n} = \left.\frac{db_n^+(x)}{dx}\right|_{x=p_n} = 1, \tag{13}$$

$$b_{n+1}^-\left(\frac{p_n+p_{n+1}}{2}\right) = b_n^+\left(\frac{p_n+p_{n+1}}{2}\right) = n+\frac{1}{2}, \tag{14}$$

$$\left.\frac{db_{n+1}^-(x)}{dx}\right|_{x=\frac{p_n+p_{n+1}}{2}} = \left.\frac{db_n^+(x)}{dx}\right|_{x=\frac{p_n+p_{n+1}}{2}} = \frac{1}{2\hat{p}_n+1}. \tag{15}$$

Finally Eqs. (12), (13), (14) and (15) define the continuously differentiable inverse spline

$$p^{-1}(x) = \begin{cases} x-1, & 1 < x \le \frac{5}{2}, \\ n+\frac{1-(8\hat{p}_{n-1}(p_n-x)+1)^{\frac{1}{2}}}{4\hat{p}_{n-1}}, & \frac{p_{n-1}+p_n}{2} \le x \le p_n, \quad n = 2,3,\ldots, \\ n+\frac{(8\hat{p}_n(x-p_n)+1)^{\frac{1}{2}}-1}{4\hat{p}_n}, & p_n \le x \le \frac{p_n+p_{n+1}}{2}, \quad n = 2,3,\ldots, \end{cases} \tag{16}$$

with the first derivative

$$\frac{dp^{-1}(x)}{dx} = \begin{cases} 1, & 1 < x \le \frac{5}{2}, \\ (8\hat{p}_{n-1}(p_n-x)+1)^{-\frac{1}{2}}, & \frac{p_{n-1}+p_n}{2} \le x \le p_n, \quad n = 2,3,\ldots, \\ (8\hat{p}_n(x-p_n)+1)^{-\frac{1}{2}}, & p_n \le x \le \frac{p_n+p_{n+1}}{2}, \quad n = 2,3,\ldots . \end{cases} \tag{17}$$

Lemma 1 *The derivatives of $p(x)$ and $p^{-1}(x)$ satisfy the inequalities*

$$1 \leq \frac{dp(x)}{dx} < \infty, \quad x > 0, \tag{18}$$

$$0 < \frac{dp^{-1}(x)}{dx} \leq 1, \quad x > 1. \tag{19}$$

The following assertions are valid [2]:

Theorem 1

(i) *The pair $(p(x), p^{-1}(x))$ is prime number diffeomorphic.*

(ii) *The specific behavior of the prime and counting curves are traced by the invariants:*

$$1 = \left.\frac{dp(x)}{dx}\right|_{x=n} = \left.\frac{dp^{-1}(x)}{dx}\right|_{x=p_n}, \quad n = 2,3,\ldots, \tag{20}$$

$$-1 = \operatorname{sign}\left(\left.\frac{d^2p(x)}{dx^2}\right|_{x=n-0}\right)\operatorname{sign}\left(\left.\frac{d^2p(x)}{dx^2}\right|_{x=n+0}\right), \quad n = 3,4,\ldots, \tag{21}$$

$$-1 = \operatorname{sign}\left(\left.\frac{d^2p^{-1}(x)}{dx^2}\right|_{x=p_n-0}\right)\operatorname{sign}\left(\left.\frac{d^2p^{-1}(x)}{dx^2}\right|_{x=p_n+0}\right), \quad n = 3,4,\ldots . \tag{22}$$

Now one can compare the existent forms of the prime counting function in terms of splines as follows:
the Legandre-Gauss function $\pi(x)$ is a discontinuously broken-line spline-interpolation;
the Riemann–von Mangoldt function $\pi_R(x)$ is a continuously broken-line spline-interpolation;
the counting curve $p^{-1}(x)$ is a continuously differentiable curvilinear-spline double-interpolation.
In [2] are shown both numerical and analytical applications of the function $p^{-1}(x)$.

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THE EXTRA PHASE CORRECTION TO THE SEMICLASSICAL SPIN PROPAGATOR

E.A. Kochetov

As is known a correct representation of the semiclassical spin propagator beyond the leading exponential order includes the extra phase correction usually referred to as the Solari-Kochetov (SK) phase [1]. The relevance of the SK phase has been justified in the application of the spin coherent-state path integral to the calculation of the tunnel splitting of the classically degenerate ground state for a family of models that includes a realistic approximation to the molecular magnet Fe_8 [2]. It was also shown that this phase contributed to the Bohr-Sommerfeld quantization rule for a spin system [3]. Besides, it was noticed in [4] that the modification of the Gutzwiller trace formula for systems with a coupling of the translational and spin degrees of freedom should also contain this extra phase in the combined limit $\hbar \to 0$, $S \to \infty$, $\hbar S$=const.

The SK phase arises in the semiclassical expansion of the spin propagator in the next-to-leading order in $1/S$. Within the spin path-integral representation, the classical spin trajectory for which the action is stationary is responsible for the leading exponential factor. The prefactor results in the next-to-leading order from integrating out the Gaussian fluctuations around the classical path. Regulating the fluctuation determinant in a manner consistent with the discrete form of the spin path integral results in the SK correction to the naive expression for the prefactor.

As is known, regulating a functional determinant in a path-integral action amounts to fixing an operator ordering ambiguity in the hamiltonian formalism. Therefore, a freedom in choosing a regularization scheme in a path integral shows up in the hamiltonian formulation as an ordering ambiguity when classical variables are replaced by their quantum counterparts. This "duality" implies that the origin of the SK phase can be related to a specific choice of the spin operator ordering. Indeed, it was shown in [5] that the extra SK correction appeared as a difference between the principal and the Weyl symbol of a quantum spin hamiltonian.

From the geometrical viewpoint, the SK phase is known to naturally split into two parts of different geometrical origin: the kinetic term and the dynamical one, the Laplacian of the spin energy on the unit sphere. The purely topological (metric independent) part of the SK phase appears as a correction to the classical Wess-Zumino or kinetic term in the spin path-integral action. Geometrically, this part of the SK phase appears in the trace calculations as the area enclosed by a closed spin path on the unit sphere. Physically, it justifies the so-called Weyl shift, $S \to S + 1/2$, of the quasiclassical expansion parameter and explains the experimentally observed oscillations in the spin tunnel energy splitting as a function of an external magnetic field [2].

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MANIFESTATION OF NOVEL GEOMETRIC EFFECTS IN ORDERED 2D JOSEPHSON JUNCTION ARRAYS

S.A. Sergeenkov

Many unusual and still not completely understood magnetic properties of Josephson Junction Arrays (JJAs) continue to attract attention of both theoreticians and experimentalists alike (for a recent review on the subject, see, e.g. [1, 2] and further references therein). In particular, among the numerous spectacular phenomena recently discussed and observed in JJAs we would like to mention the dynamic temperature reentrance of AC susceptibility (closely related to paramagnetic Meissner effect) and avalanche-like magnetic field behavior of magnetization.

In this report we present experimental evidence for manifestation of novel geometric effects in magnetic response of high-quality ordered 2D-JJA [3]. By increasing the resolution of our home-made mutual-inductance measurements technique [4], we were able to observe for the first time a fine, step-like structure (with the number of steps $n = 4$ for all AC fields used in our experiments) in the temperature behavior of AC susceptibility in artificially prepared 2D-JJA of unshunted $Nb - AlO_x - Nb$ junctions. Using a single-plaquette approximation of the overdamped 2D-JJA model, we show that the number of steps n corresponds to the number of flux quanta that can be screened by the maximum critical current of the junctions and as a result steps will manifest themselves in arrays with the inductance related parameter $\beta_L(T)$ matching a "quantization" condition $\beta_L(0) = 2\pi(n+1)$.

The unshunted 2D-JJAs used in our study [3] are formed by loops of niobium islands linked through $Nb - AlO_x - Nb$ Josephson junctions and consist of 100×150 tunnel junctions. The unit cell has square geometry with lattice spacing $a = 46\mu m$ and a single junction area of $5 \times 5\mu m^2$. Since the inductance of each loop is $L = \mu_0 a = 64pH$ and the critical current of each junction is $I_C(4.2K) = 150\mu A$, we have $\beta_L(4.2K) = 30$. Recall that $\beta_L(T) = 2\pi L I_C(T)/\Phi_0$ where Φ_0 is the magnetic flux quantum. The observed temperature dependence of the real part of AC susceptibility for different AC fields is shown in Fig.1. A pronounced step-like structure is clearly seen at higher temperatures. Given the well-defined periodic structure of our arrays, it is reasonable to assume that this phenomenon could be understood by analyzing the dynamics of just a single plaquette containing four identical junctions. If we apply an AC external field $H_{ac}(t) = h_{ac}\cos\omega t$ normally to the 2D-JJA, then the total magnetic flux $\Phi(t)$ in the loop is given by $\Phi(t) = \Phi_{ext}(t) + LI(t)$ where $\Phi_{ext}(t) = SH_{ac}(t)$ with $S \simeq a^2$ being the projected area and $I(t) = I_C(T)\sin\phi(t)$ the circulating current in the loop. Here $\phi(t)$ is the gauge-invariant superconducting phase difference across the ith junction. In the case of four junctions, the flux quantization condition reads $\phi = \frac{\pi}{2}\left(n + \frac{\Phi}{\Phi_0}\right)$ where $n = 0, 1, 2...$ Notice that the current circulating in the loop passes through its maximum value whenever $\phi(t)$ reaches the value of $\frac{\pi}{2}(2n+1)$. As a result, the maximum number of fluxons threading a single plaquette over the period $2\pi/\omega$ becomes equal to $< \Phi(t) >= (n+1)\Phi_0$. In turn, the latter equation is equivalent to the following geometric "quantization" condition $\beta_L(0) = 2\pi(n+1)$. Recall that in our array $\beta_L(0) = 31.6$, which is a perfect match for the above "quantization" condition predicting $n = 4$ for the number of steps in a single plaquette. The solid lines in Fig.1 present fits of the observed temperature dependence of the normalized susceptibility $\chi'(T, h_{ac})/\chi_0$ for different magnetic fields h_{ac}, according to the above-described theoretical model using $\beta_L(0) = 10\pi$. As is seen, our simplified model based on a single-plaquette approximation demonstrates excellent agreement with the observations.

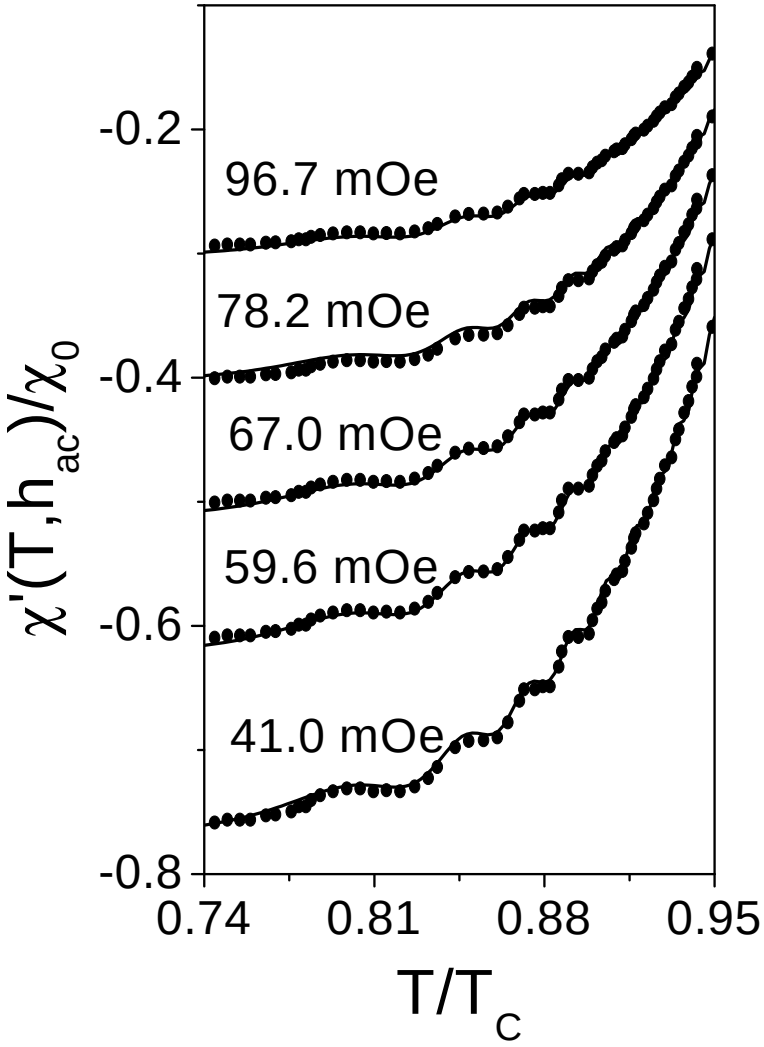

Figure 1: Experimental results for temperature dependence of the real part of AC susceptibility $\chi'(T, h_{ac})$ for different AC field amplitudes $h_{ac} = 41.0$, 59.6, 67.0, 78.2 and 96.7*mOe* and the fits (solid lines), according to our theoretical model with $\beta_L(0) = 10\pi$.

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PHYSICS OF COLD TRAPPED ATOMS

V.I. Yukalov

A survey has been given of the present state of the art in studying Bose-Einstein condensation of dilute atomic gases. The bulk of attention is focused on the principal theoretical problems, though the related experiments are also mentioned. Both uniform and nonuniform trapped

gases are considered. Existing theoretical contradictions are critically analysed. A correct understanding of the principal theoretical problems is necessary for gaining a more penetrating insight into experiments with trapped atoms and for their proper interpretation.

A mixture of the multicomponent Bose-Einstein condensate is considered, where each component moves with its own velocity. As a result of the relative motion, the mixture stratifies when the relative velocity reaches a critical value. Stability conditions for a binary moving mixture are derived and the critical velocity is found.

The possibility of generating multiple coherent modes in trapped Bose gases is advanced. This requires the usage of several driving fields whose frequencies are tuned close to the corresponding transition frequencies. A general criterion is derived explaining when the driving fields, even being in perfect resonance, cannot generate the topological coherent modes. This criterion is termed the theorem of shape conservation. Bose-Einstein condensates with generated coherent modes display a number of interesting effects, such as: interference fringes, interference current, mode locking, dynamic transition, critical phenomena, chaotic motion, harmonic generation, parametric conversion, atomic squeezing, and entanglement production. Approximate solutions, based on the averaging techniques, are found to be in good agreement with direct numerical calculations for the Gross-Pitaevskii equation.

Trapped Bose atoms cooled down to temperatures below the Bose-Einstein condensation temperature are considered. Stationary solutions to the Gross-Pitaevskii equation (GPE) define the topological coherent modes, representing nonground-state Bose-Einstein condensates. These modes can be generated by means of alternating fields whose frequencies are in resonance with the transition frequencies between two collective energy levels corresponding to two different topological modes. The theory of resonant generation of these modes is generalized in several aspects: Multiple-mode formation is described; a shape-conservation criterion is derived, imposing restrictions on the admissible spatial dependence of resonant fields; evolution equations for the case of three coherent modes are investigated; the complete stability analysis is accomplished; the effects of harmonic generation and parametric conversion for the topological coherent modes are predicted. All considerations are realized by both employing approximate analytical methods and numerically solving the GPE. Numerical solutions confirm all conclusions following from analytical methods.

Fluctuations of the number of particles for the dilute interacting gas with Bose-Einstein condensate are considered. It is shown that in the Bogolubov theory these fluctuations are normal. The fluctuations of condensed as well as noncondensed particles are also normal in both canonical and grand canonical ensembles.

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COHERENT EFFECTS IN COLLECTIVE RADIATION

V.I. Yukalov

A review of coherent phenomena has been published. The main part of this review is devoted to the comprehensive description of coherent radiation by nuclear spins. The theory of nuclear spin superradiance is developed and the experimental observations of this phenomenon

are considered. The intriguing problem of how coherence develops from initially incoherent quantum fluctuations is analysed. All main types of coherent radiation by nuclear spins are discussed, which are: free nuclear induction, collective induction, maser generation, pure superradiance, triggered superradiance, pulsing superradiance, punctuated superradiance, and induced emission. The influence of electron-nuclear hyperfine interactions and the role of magnetic anisotropy are studied. Conditions for realizing spin superradiance by magnetic molecules are investigated. The possibility of nuclear matter lasing accompanied by pion or dibaryon radiation is briefly touched.

The relation is studied between the entanglement production and collective radiation by an ensemble of atoms. Entanglement production is quantified by means of a general measure introduced earlier by the author. Primary emphasis is placed on the entanglement generated by pseudospin density matrices. The problem of collective atomic radiation can be described by the pseudospin evolution equations. These equations define the evolutional entanglement generated by the related density matrices. Under conditions of superradiant emission, the entanglement production exhibits sharp peaks at the delay time, where the intensity of radiation is maximal. The possibility of regulating the occurrence of such peaks by punctuated superradiance is discussed, which suggests the feasibility of *punctuated entanglement production*.

Atomic squeezing is studied for the case of large systems of radiating atoms when collective effects are well developed. All temporal stages are analyzed starting with the quantum stage of spontaneous emission, passing through the coherent stage of superradiant emission, and going to the relaxation stage ending with stationary solutions. A method of governing the temporal behaviour of the squeezing factor is suggested. The influence of a squeezed effective vacuum on the characteristics of collective emission is also investigated.

A spin system is considered with a Hamiltonian typical of molecular magnets, having dipole-dipole interactions and a single-site magnetic anisotropy. In addition, spin interactions through the common radiation field are included. A fully quantum-mechanical derivation of the collective radiation rate is presented. An effective narrowing of the dipole-dipole attenuation due to high spin polarization is taken into account. The influence of the radiation rate on spin dynamics is carefully analysed. It is shown that this influence is completely negligible. No noticeable collective effects such as superradiance can appear in molecular magnets being caused by electromagnetic spin radiation. Spin superradiance can arise in molecular magnets only when these are coupled to a resonant electric circuit, as was suggested earlier by one of the authors in Laser Phys. **12**, 1089 (2002).

A comparative analysis is given of spin superradiance and atomic superradiance. Their similarities and distinctions are emphasized. It is shown that despite a close analogy these phenomena are fundamentally different. In atomic systems, superradiance is a self-organized process, in which both the initial cause, being spontaneous emission, as well as the collectivizing mechanism of their interactions through the common radiation field are of the same physical nature. Contrary to this, in actual spin systems with dipole interactions, the latter are the major reason for spin motion. Electromagnetic spin interactions through radiation are negligible and can never produce collective effects. The possibility of realizing superradiance in molecular magnets by coupling them to a resonant circuit is discussed.

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