Off-shell neutrino oscillations in a covariant QFT approach

Vadim A. Naumov and Dmitry S. Shkirmanov
Bogoliubov Laboratory of Theoretical Physics, JINR, Dubna

Introduction

The standard quantum-mechanical approach to the neutrino oscillation phenomenon is not self-consistent. After pioneering papers [1] a lot of researches were devoted to treatment of the neutrino flavour transitions on the basis of the quantum field-theoretical (QFT) S-matrix approach. Present study is based on the formalism of Refs. [2], according to which

- the "ν-oscillation" phenomenon in QFT is nothing else than a result of interference of macroscopic Feynman diagrams with the neutrino mass eigenfields, ν_i ($i = 1, 2, 3, \ldots$), as internal lines (propagators);
- the external lines are wave packets (WP) constructed as covariant superpositions of one-particle Fock states and satisfying a correspondence principle (WP turns into the Fock state in the plane-wave limit).

The neutrino propagator

In the QFT approach, the long-distance neutrino field evolution is governed by the generalized (macroscopic) neutrino propagators

$$J(L) = \int \frac{d^4q}{(2\pi)^4} \frac{(\hat{q}+m)\,\mathcal{F}(q)e^{-iqL}}{q^2 - m^2 + i\epsilon} = \left(i\hat{\partial} + m\right) \int \frac{d^4q}{(2\pi)^4} \frac{\mathcal{F}(q)e^{-iqL}}{q^2 - m^2 + i\epsilon}$$

appiaring as multipliers with $m = m_i$ in each ν_i contribution into the full amplitude. Here L is the space-time 4-vector connecting the impact points in the source and detector vertices, $\mathcal{F}(q)$ is a function responsible for the ***approximate energy-momentum conservation***. In the plain wave limit

$$\mathcal{F}(q) \longmapsto \delta^4(q - q_s)\delta^4(q + q_d),$$

where q_s and q_d are the 4-momentum transfers in the source and detector vertices; in that case $J(L)$ turns into the ordinary QFT propagator.

Long-distance asymptotics and inverse-square law

The explicit form of the macroscopic propagator can be obtained by means of so-called Extended Grimus-Stockinger (EGS) theorem [3]:

$$\int \frac{d\boldsymbol{q}}{(2\pi)^3} \frac{\Phi(\boldsymbol{q})e^{i\boldsymbol{q}\boldsymbol{L}}}{\boldsymbol{q}^2 - \kappa^2 - i\epsilon} = \frac{\Phi(\kappa\boldsymbol{l})e^{i\kappa|\boldsymbol{L}|}}{4\pi|\boldsymbol{L}|}\left[1 + \sum_{n\geq 1} \frac{(-i)^n[D_n\Phi(\boldsymbol{q})]_{\boldsymbol{q}=\kappa\boldsymbol{l}}}{|\boldsymbol{L}|^n}\right].$$

Here D_n are certain differential operators, defined recursively [3],

$$\kappa = \sqrt{q_0^2 - m^2}, \quad \boldsymbol{l} = \boldsymbol{L}/|\boldsymbol{L}|, \quad \varSigma^2|\boldsymbol{L}|/E_\nu \gg 1.$$

- The inverse-square law (ISL) is approximate due to $1/|\boldsymbol{L}|^{2n}$ corrections:

$$\text{Flux} \propto \frac{1}{|\boldsymbol{L}|^2}\left(1 - \frac{\mathfrak{L}_0^2}{|\boldsymbol{L}|^2} + \ldots\right). \qquad (1)$$

- The leading-order ISL violating correction is **negative** and $\mathfrak{L}_0$ is an energy dependent parameter of dimension of length,

$$\mathfrak{L}_0 \sim \frac{E_\nu}{\varSigma^2(E_\nu)} \approx 20\left(\frac{E_\nu}{1\text{ MeV}}\right)\left[\frac{\varSigma(E_\nu)}{1\text{ eV}}\right]^{-2}\text{cm},$$

where $\varSigma(E_\nu)$ is an effective neutrino transverse momentum spread explicitly defined by the momenta, masses and momentum spreads of all in and out WPs in the source and detector vertices of the diagram.

Short-distance asymptotics

For short distances, by using the explicit form of $\mathcal{F}(q)$ we obtained the Lorenz-invariant asymptotic expansion for the neutrino propagator:

$$J(L) = -8i\pi^2\sqrt{\frac{\pi}{\det(\widetilde{\mathfrak{R}})\mathcal{G}}}\exp\left[\Omega + \sum_{abc} F_{abc}\,(i\delta)^a\,\eta^b\,\Delta^c\right], \quad \sigma|\boldsymbol{L}| \gtrsim 1, \quad (2)$$

- where F_{abc} are the coefficient functions, δ, η, and Δ are independent small parameters, $\Delta, \eta \sim \sigma^2/E_\nu^2$, $\delta \sim (\sigma/E_\nu)(|\boldsymbol{L}|\sigma)$, and σ is an invariant function – another effective spread generally different from $\varSigma$, but also explicitly defined by the kinematics and spreads of the external WPs.
- The complex phase Ω contains the oscillation and suppressing contributions caused by considerable neutrino virtuality (see below).
- The function $\mathcal{G}$ and matrix $\widetilde{\mathfrak{R}}$ are defined by WP overlap tensors.

At first sight, it may seem from Eq. (2) that ISL is fully destroyed.
But! It will be shown below that it is not the case.

Short-distance asymptotics and its features

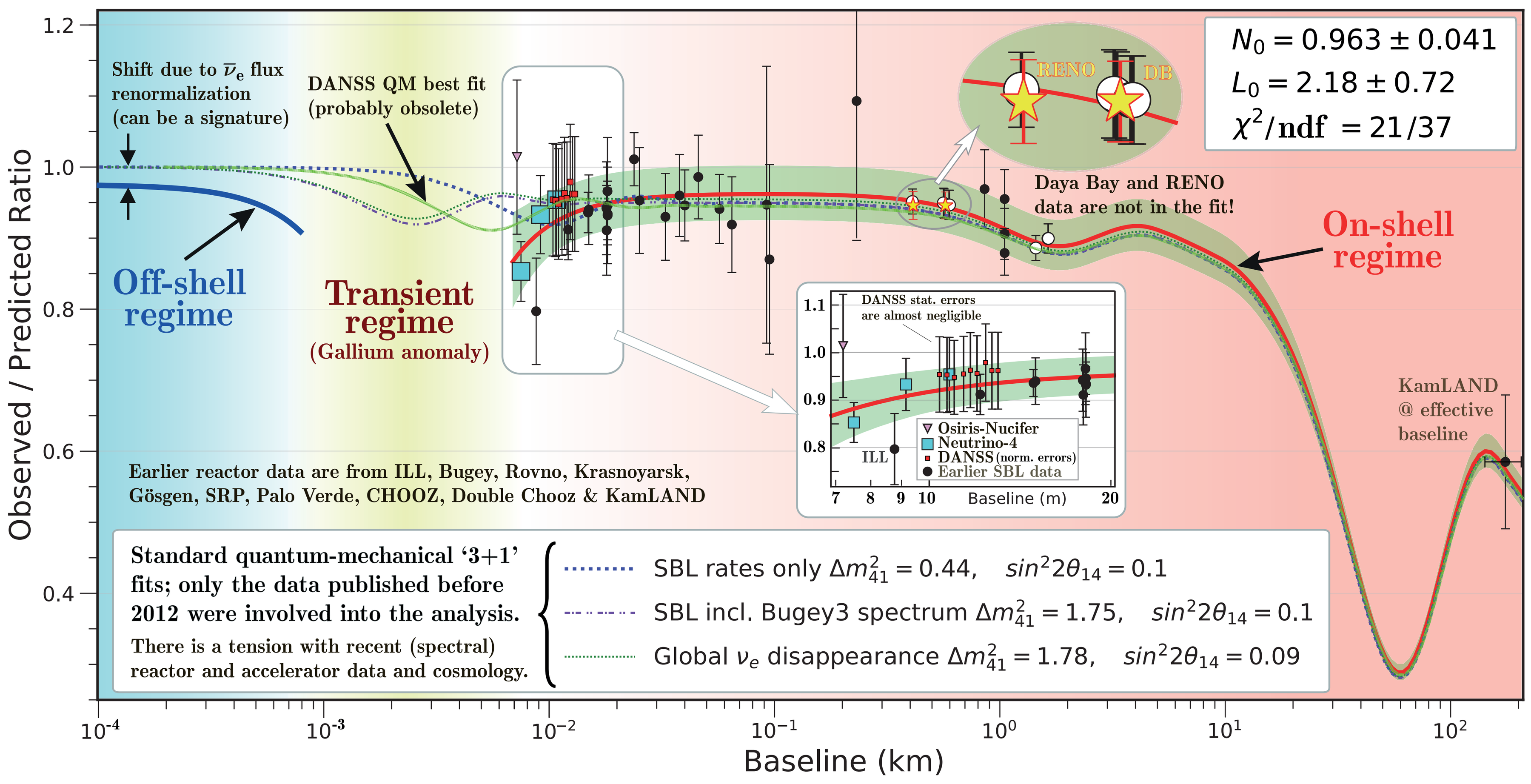
- According to the EGS theorem, at long distances the ν_is are on-shell, while at short (but still macroscopic) distances, the ν_is have off-shell 4-momenta, p, and virtualities, $p^2 = p_0^2 - \boldsymbol{p}^2$, restricted by the inequalities $m_i^2 \leq p^2 \lesssim 4|\boldsymbol{p}|\sigma\sqrt{\rho_\nu}$, where $\rho_\nu > 0$ is an invariant function of the external kinematic variables and momentum spreads, explicitly defined by the WP overlap tensors.
- **Key point:** By using the saddle-point approximation, the modulus squared amplitude can be represented as a 4d integral in neutrino 4-momentum $q = (q_0, \boldsymbol{q})$, where $q_3 = \boldsymbol{q}\boldsymbol{l}$. Then (and it is a bit tricky way) integrating in q_1, q_2, and q_0 we again come to the classical ISL + small corrections (from here on we may assume $|\boldsymbol{q}| = q_3 \gg m_i$):

$$\text{Flux} \propto \frac{1}{|\boldsymbol{L}|^2}\left(1 - 2F_{200}\frac{\sigma^4|\boldsymbol{L}|^2}{|\boldsymbol{q}|^2} + \ldots\right), \qquad F_{200} > 0 \text{ (theorem!)} \qquad (3)$$

- The range of applicability of the short baseline asymptotics (3) is opposite to that for the long baseline asymptotics (1):
  - ⋆ long baseline asymptotics: $|\boldsymbol{L}| \gg E_\nu/\varSigma^2$,
  - ⋆ short baseline asymptotics: $1/\sigma \lesssim |\boldsymbol{L}| \ll |\boldsymbol{q}|/\sigma^2$.
- Remind that the effective spreads σ and $\varSigma$ are generally different in magnitude but both are small in comparison with $|\boldsymbol{q}| \simeq |\boldsymbol{q}_s| \simeq |\boldsymbol{q}_d|$.
- At extremely short distances, $|\boldsymbol{L}| \lesssim 1/\sigma$, ISL may be destroyed again due to another (currently neglected) contribution to the amplitude.
- **Last but not least:** Strange as it may seem, the oscillation pattern remains exactly the same as in the long-distance (EGS) case and the decoherence factors are almost similar in form. However these effects are practically unmeasurable for the SM neutrinos as $|\boldsymbol{L}|/|\boldsymbol{q}| \ll 1/\sigma^2$. Thus the only experimental signature could be the ISL violation.

Reactor anomaly and inverse-square law violation

For a possible application of our result, let's assume that the ISL violation is responsible for the so-called reactor antineutrino anomaly (RAA) [4]. Then, using Eq. (1) as a theoretical basis, we can adjust the parameter $L_0 = \langle\mathfrak{L}_0\rangle$ along with the (yet uncertain) reactor antineutrino flux normalization, N_0, to the world data (see Refs. [5] for earlier analysis). The result is shown below along with the quantum-mechanical fits performed under assumption of mixing between the SM neutrinos and hypothetical light sterile neutrino within the 3+1 mixing scheme [6]. Since the real scale of the function σ is unknown (due to lack of the relevant data), the off-shell regime curve is plotted by hand and normalized by the same factor N_0.



Conclusions

Long- and short-baseline asymptotics of the macroscopic neutrino propagator lead, respectively, to on-shell and off-shell regimes. Deviations from the classical ISL are attributed to both these regimes. New experiments with artificial radioactive sources are desirable to test these predictions.

References

[1] I.Y. Kobzarev, B.V. Martemyanov, L.B. Okun & M.G. Shchepkin, Yad. Fiz. 35 (1982) 1210 [Sov. J. Nucl. Phys. 35 (1982) 708]; C. Giunti, C.W. Kim & U.W. Lee, Phys. Rev. D 44 (1991) 3635; W. Grimus & P. Stockinger, Phys. Rev. D 54 (1996) 3414.

[2] D.V. Naumov & V.A. Naumov, Russ. Phys. J. 53 (2010) 549; J. Phys. G 37 (2010) 105014; Phys. Part. Nucl. 51 (2020) 1.

[3] V.A. Naumov & D.S. Shkirmanov, Eur. Phys. J. C73 (2013) 2627.

[4] G. Mention *et al.*, Phys. Rev. D 83 (2011) 073006.

[5] D. V. Naumov, V. A. Naumov & D. S. Shkirmanov, Phys. Part. Nucl. 48 (2017) 12; Phys. Part. Nucl. 48 (2017) 1007.

[6] C. Giunti & M. Laveder, Phys. Rev. D 85 (2012) 031301(R).

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