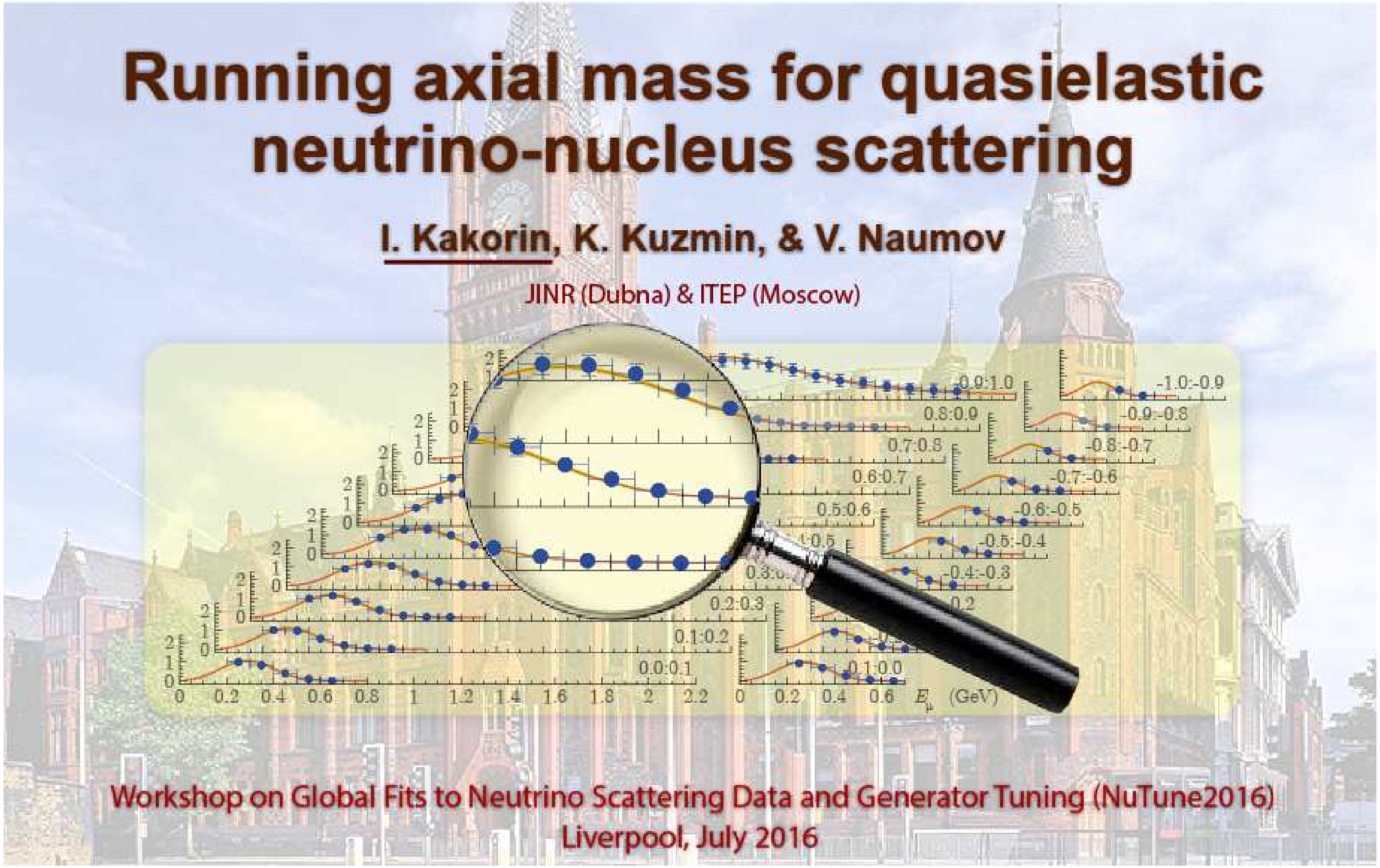
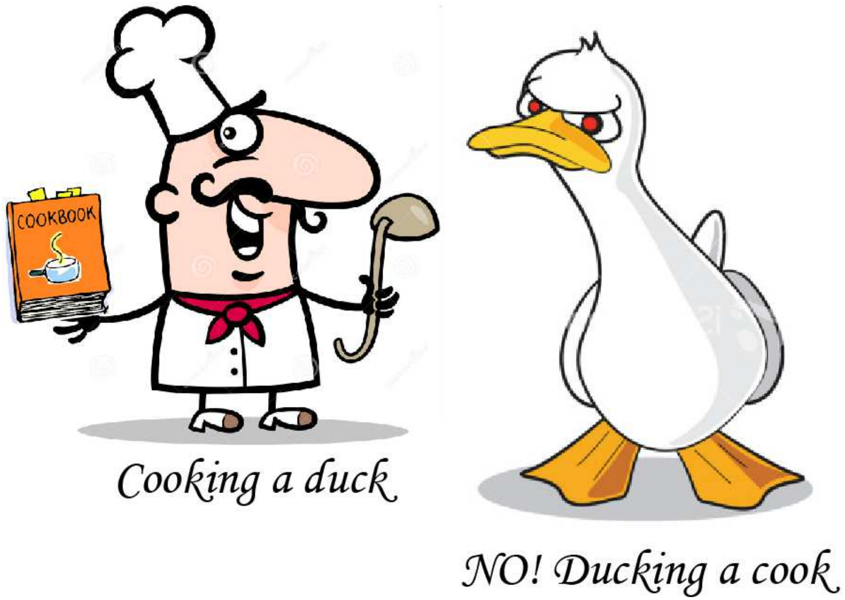

Running axial mass for quasielastic neutrino-nucleus scattering
I. Kakorin, K. Kuzmin, & V. Naumov
JINR (Dubna) & ITEP (Moscow)
E_μ (GeV)
Workshop on Global Fits to Neutrino Scattering Data and Generator Tuning (NuTune2016)
Liverpool, July 2016

The aim of our report is to provide a simple "cookbook recipe" for a precise description of the CCQE neutrino and antineutrino interactions with nuclei using only two adjustable parameters.



We do not intend discussing the physical meaning of the proposed recipe; instead we wish to demonstrate its usefulness as an empirical tool which could be used in the data processing of the neutrino oscillation experiments with accelerator and atmospheric (anti)neutrino beams.

Outline

- Introduction
- Total Cross Sections – Recent data
- FNAL MiniBooNE 2010-2013
- FNAL MINERνA 2013
- T2K ND280 2016
- Parameters of the Smith–Moniz RFG Model
- "Golden" dataset for the global fits
- Comaparison with the early golden data
- Comparison with certain datasets unclaimed in the global fit
- Conclusions

Introduction

One of the main sources of ambiguity in predicted neutrino event rates in the accelerator experiments at low and intermediate energies is caused by nuclear effects for the charged-current quasielastic (CCQE) $\nu/\overline{\nu}$ interactions with various detection targets.

Another closely-related problem arises from the experimental uncertainty of the nucleon axial mass parameter, M_A, in the dipole model of the nucleon axial form factor

$$F_A\left(Q^2\right) = F_A(0)\left(1+\frac{Q^2}{M_A^2}\right)^{-2}.$$

The most familiar Relativistic Fermi Gas (RFG) model cannot describe the up-to-date CCQE data (NOMAD, MiniBooNE, MINERνA, T2K, etc.) with a unique value of M_A.

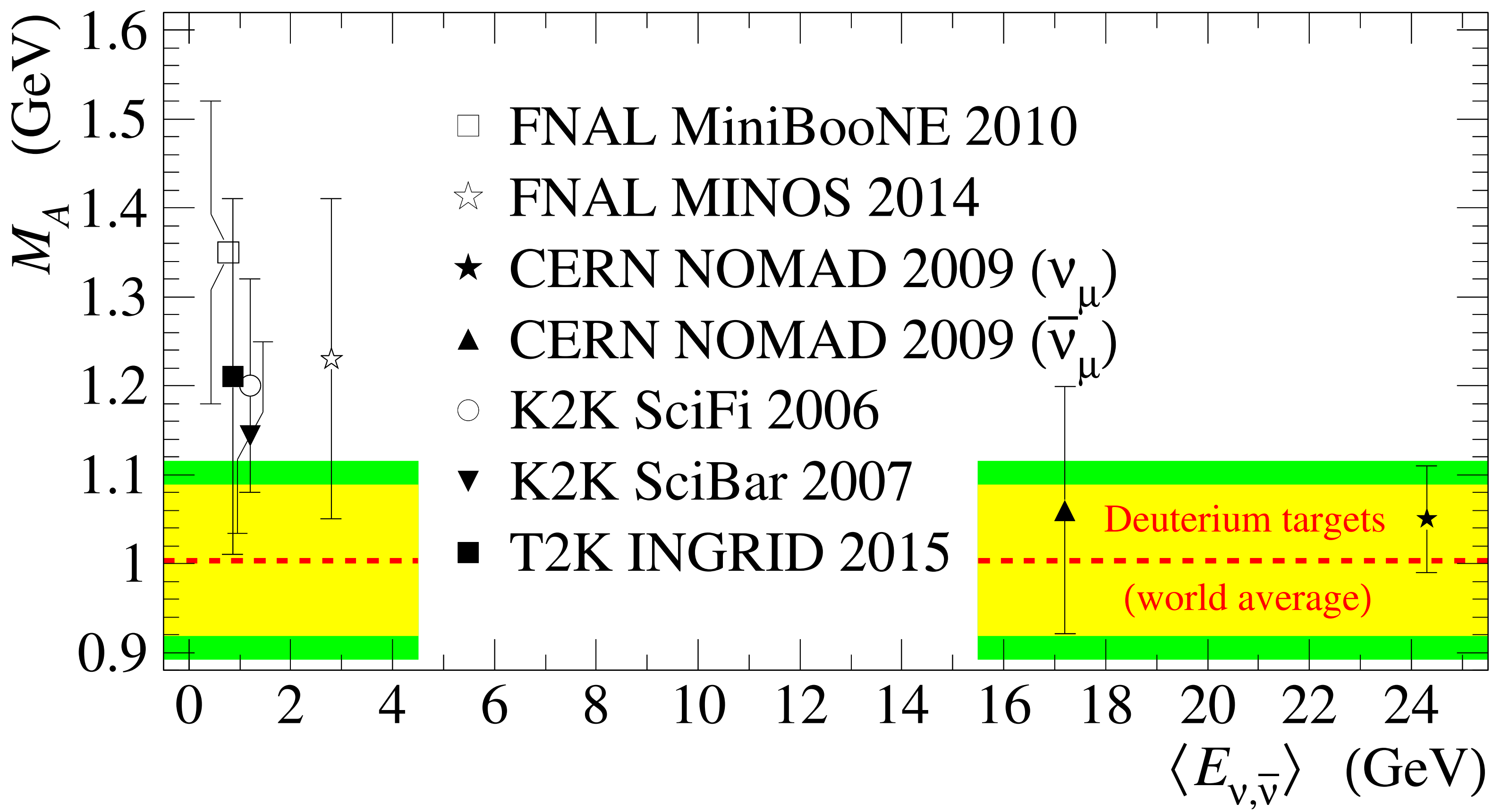
In fact, there are a lot of different models based on the Fermi-gas approach. One of the most popular models is the model by Smith and Moniz (SM).[a] Below, the term "RFG" is used for the slightly-modified SM model,[b] with an updated set of the input parameters.

[a] R.A. Smith and E.J. Moniz, *"Neutrino reactions on nuclear targets,"* Nucl. Phys. B **43**, 605 (1972); erratum – *ibid*. **101**, 547 (1975).

[b] K.S. Kuzmin, V.V. Lyubushkin and V.A. Naumov, *"Quasielastic axial-vector mass from experiments on neutrino-nucleus scattering,"* Eur. Phys. J. C **54**, 517 (2008) [arXiv:0712.4384 [hep-ph]].

Some nuclear models currently on the market.

- **RFG Extensions:**
  - **Local Fermi Gas (LFG)** or **Local Density Approximation (LDA)**
    [Fermi momenta and separation energies are functions of the position in the nucleus]
  - **Spectral Function (SF)**
    [correlations between removal energies and initial state momenta of the nucleons, Gaussian spectrum of each shell]
- **Scaling and Superscaling (SuSA) models.**
- **Relativistic Mean Field (RMF) models.**
- **Relativistic Green's Function (RGF) models.**
- **Transverse Enhancement Model (TEM).**
- **Many-body Currents:**
  - **Short-range correlations (SRC),**
  - **Meson Exchange Currents (MEC, 2p2h),** $\Longleftarrow$ (most popular and successful)
  - **Random Phase Approximation (RPA)** and **Quasiparticle RPA (QRPA).**
- **Hotchpotchs** (e.g., SuSA+MEC, MEC+RPA, etc.).



The nucleon axial mass vs. neutrino energy, obtained in several recent experiments. The dashed line and surrounding shaded double band represent the value of M_A and its 1σ and 2σ uncertainties extracted from the deuterium experiments (M_A^D hereafter).

This effect is now explained to an extent by the advanced nuclear models.

Besides, the much simpler RFG-based calculations can be somewhat fine-tuned by using a larger value of M_A in the lower energy range.[a]
This observation can be used for an empirical description of the experimental data within the RFG model. Namely, it is shown[b] that disadvantages of the RFG model can be effectively compensated by introducing the energy-dependent ("running") axial mass

$$M_A^{\text{run}} = M_0 \left(1 + \frac{E_0}{E_\nu}\right)$$

with the parameters values

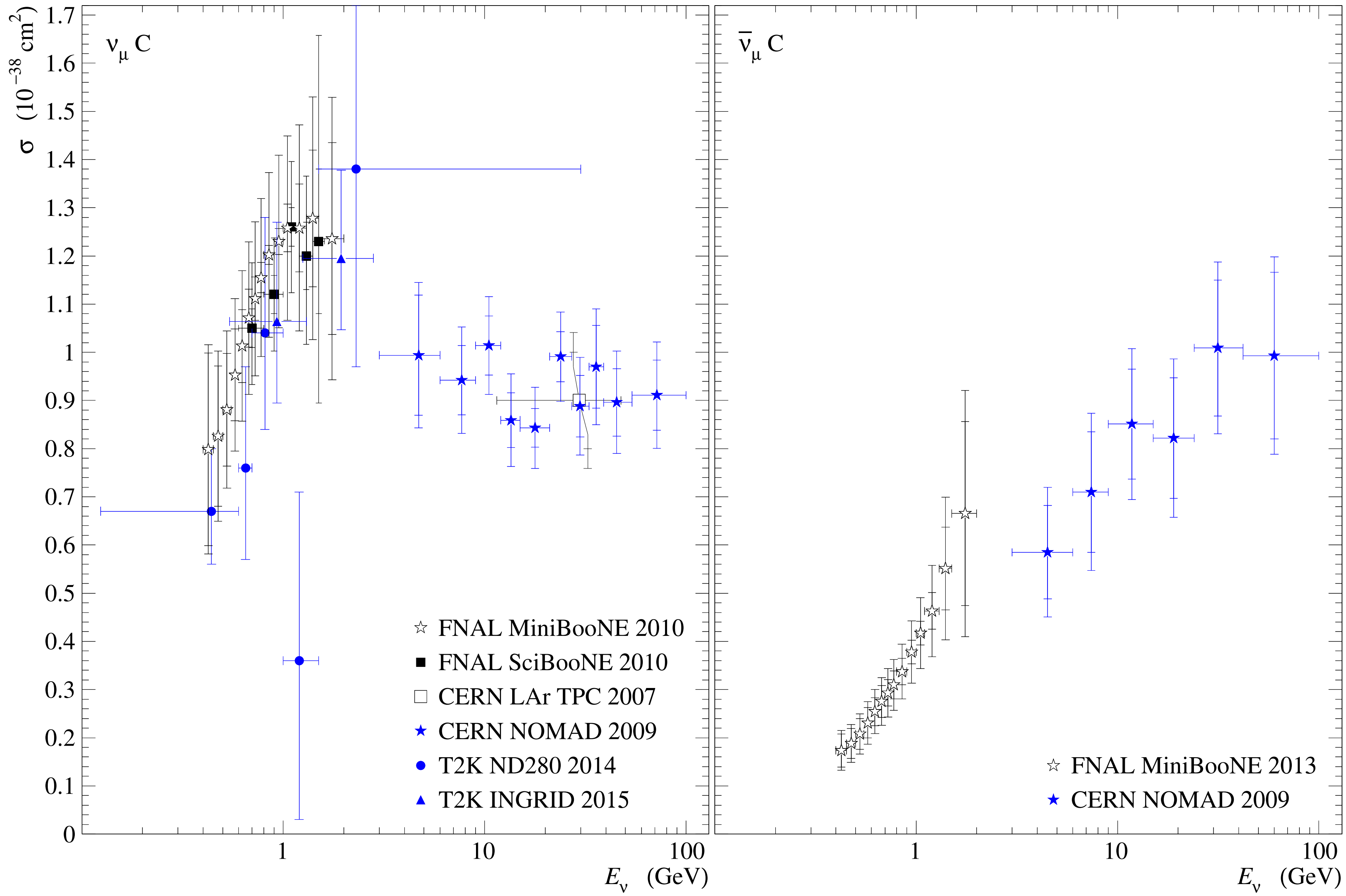
$$M_0 = 1.006 \pm 0.025\,(0.030)\ \text{GeV} \quad \text{and} \quad E_0 = 0.334^{+0.058\,(0.070)}_{-0.054\,(0.064)}\ \text{GeV}$$

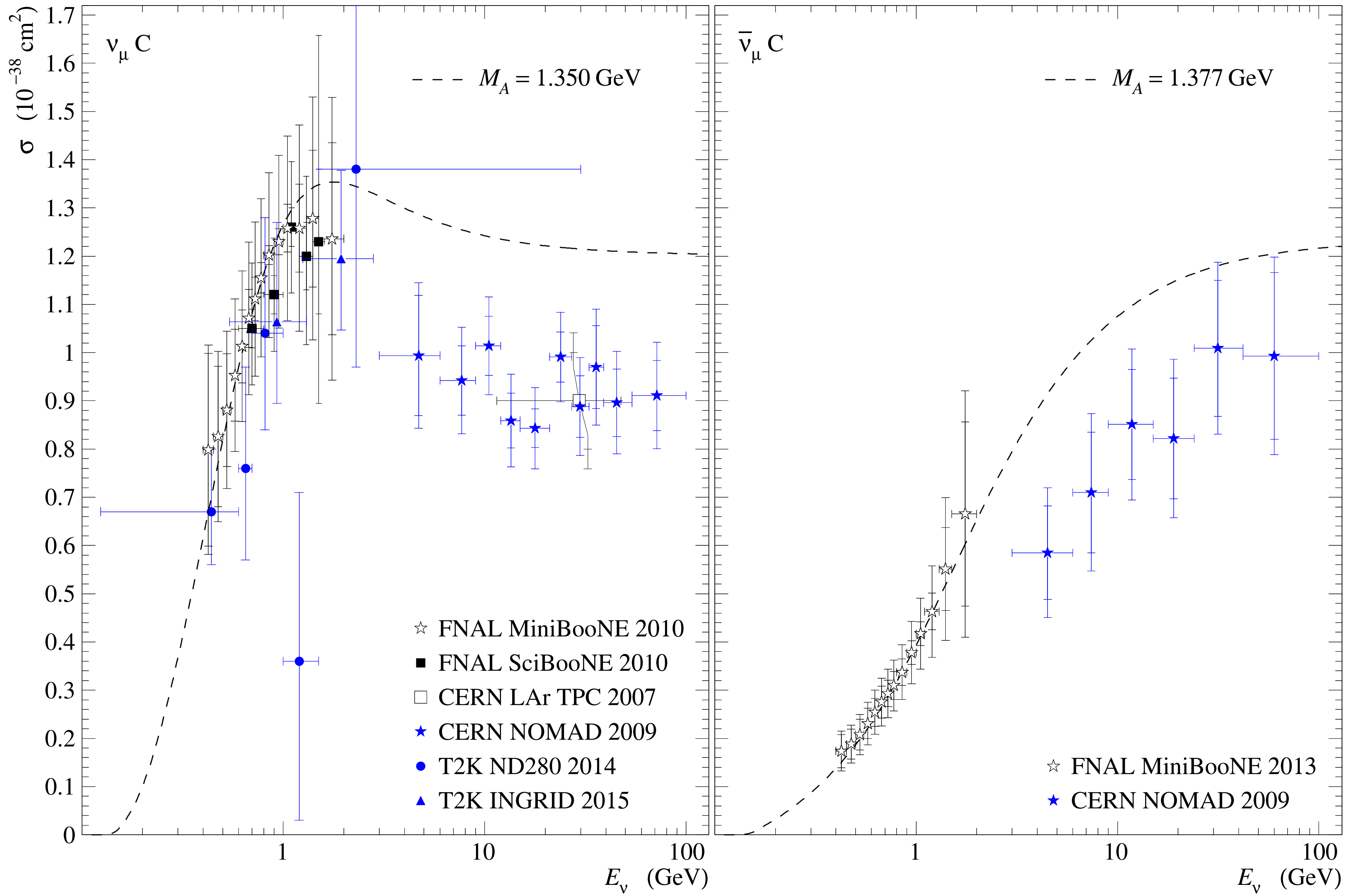
obtained from a global fit to all available data on neutrino-nucleus and antineutrino-nucleus CCQE scattering with $\chi^2/\text{NDF} = 300.4/(435\text{-}19) = 0.72$.
This allows to phenomenologically account for the intricate nuclear effects beyond RFG, significant at low and medium neutrino energies.

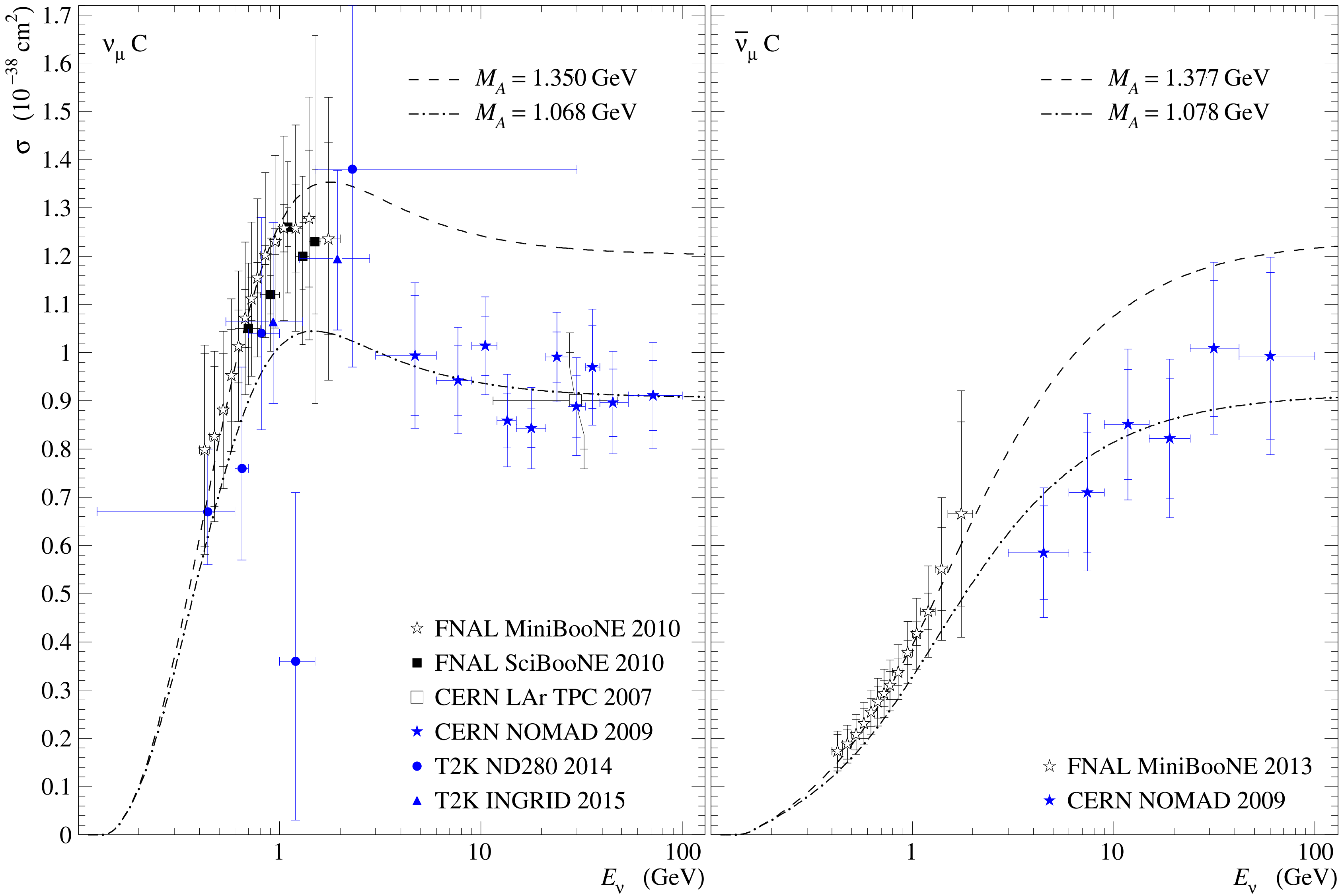
[a]A lot of papers; see, e.g., A.M. Ankowski, O. Benhar, C. Mariani and E. Vagnoni, Phys. Rev. D **93**, 113004 (2016) [arXiv:1603.01072 [hep-ph]] and references therein.

[b]K. S. Kuzmin and V. A. Naumov, in preparation. See also L. D. Kolupaeva, K. S. Kuzmin, O. N. Petrova and I. M. Shandrov, Mod. Phys. Lett. A **31**, 1650077 (2016) [arXiv:1603.07451 [hep-ph]]; K. S. Kuzmin, V. A. Naumov, O. N. Petrova, Accepted by Phys. Part. Nucl. (in press); K. S. Kuzmin, V. A. Naumov, O. N. Petrova, submitted to Acta Phys. Polon. B.

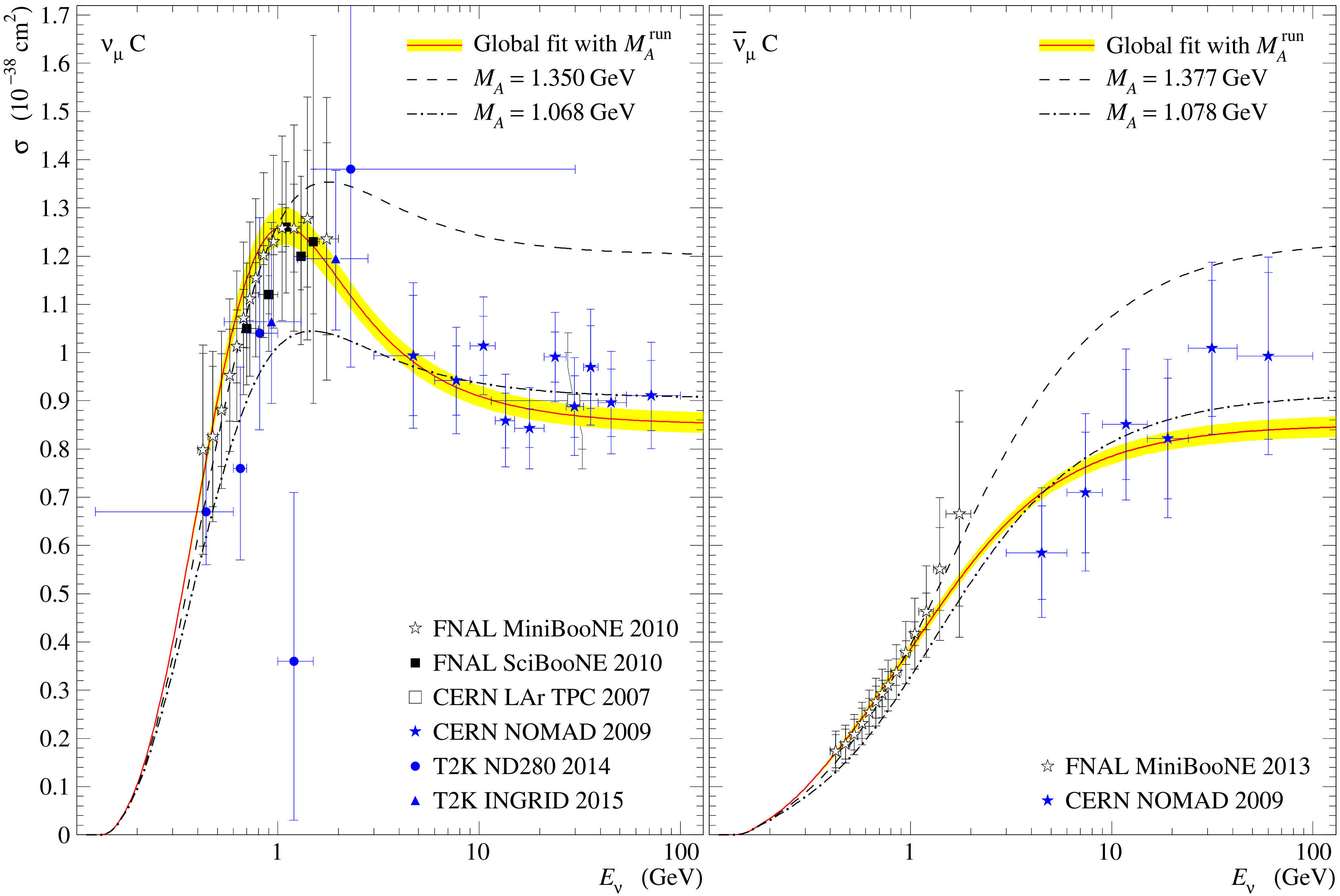
Total Cross Sections
Recent data


σ (10^−38 cm^2)
νμ C
ν̄μ C
FNAL MiniBooNE 2010
FNAL SciBooNE 2010
CERN LAr TPC 2007
CERN NOMAD 2009
T2K ND280 2014
T2K INGRID 2015
FNAL MiniBooNE 2013
CERN NOMAD 2009
Eν (GeV)
Eν (GeV)

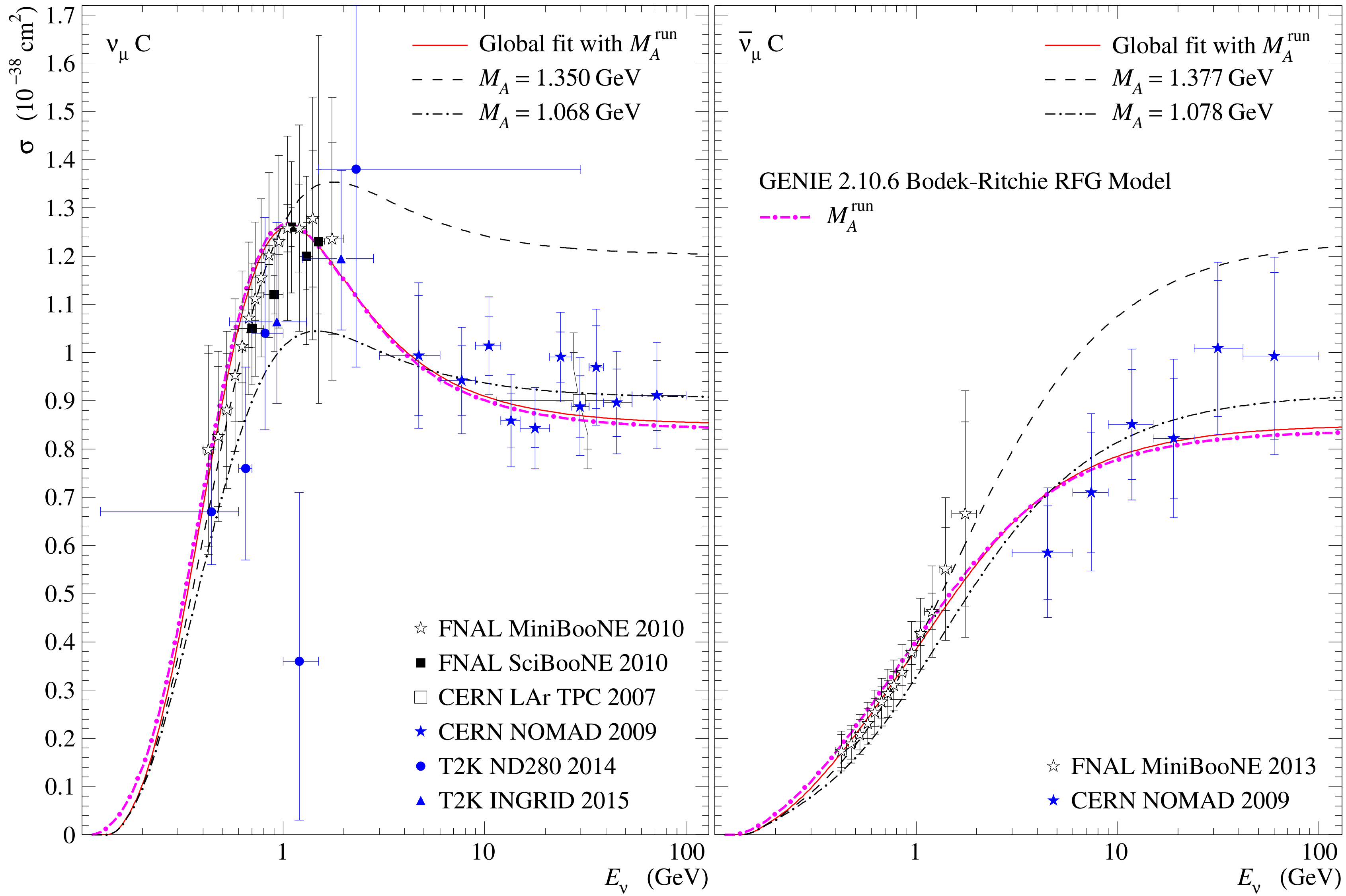

σ (10^−38 cm²)
νμ C
MA = 1.350 GeV
FNAL MiniBooNE 2010
FNAL SciBooNE 2010
CERN LAr TPC 2007
CERN NOMAD 2009
T2K ND280 2014
T2K INGRID 2015
Eν (GeV)
ν̄μ C
MA = 1.377 GeV
FNAL MiniBooNE 2013
CERN NOMAD 2009
Eν (GeV)



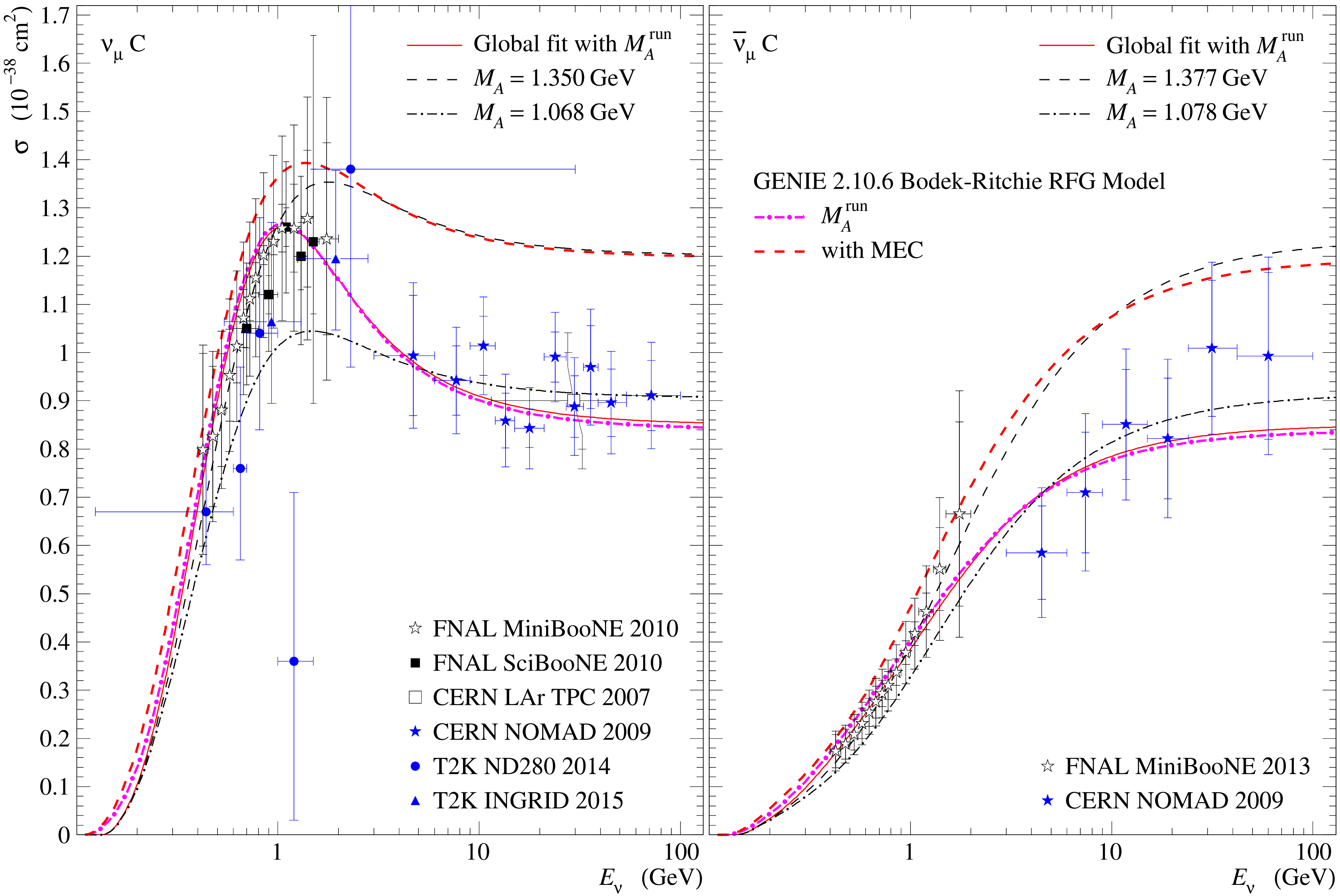
σ (10^−38 cm²)
ν_μ C
M_A = 1.350 GeV
M_A = 1.068 GeV
FNAL MiniBooNE 2010
FNAL SciBooNE 2010
CERN LAr TPC 2007
CERN NOMAD 2009
T2K ND280 2014
T2K INGRID 2015
E_ν (GeV)
$\bar{\nu}_\mu$ C
M_A = 1.377 GeV
M_A = 1.078 GeV
FNAL MiniBooNE 2013
CERN NOMAD 2009
E_ν (GeV)



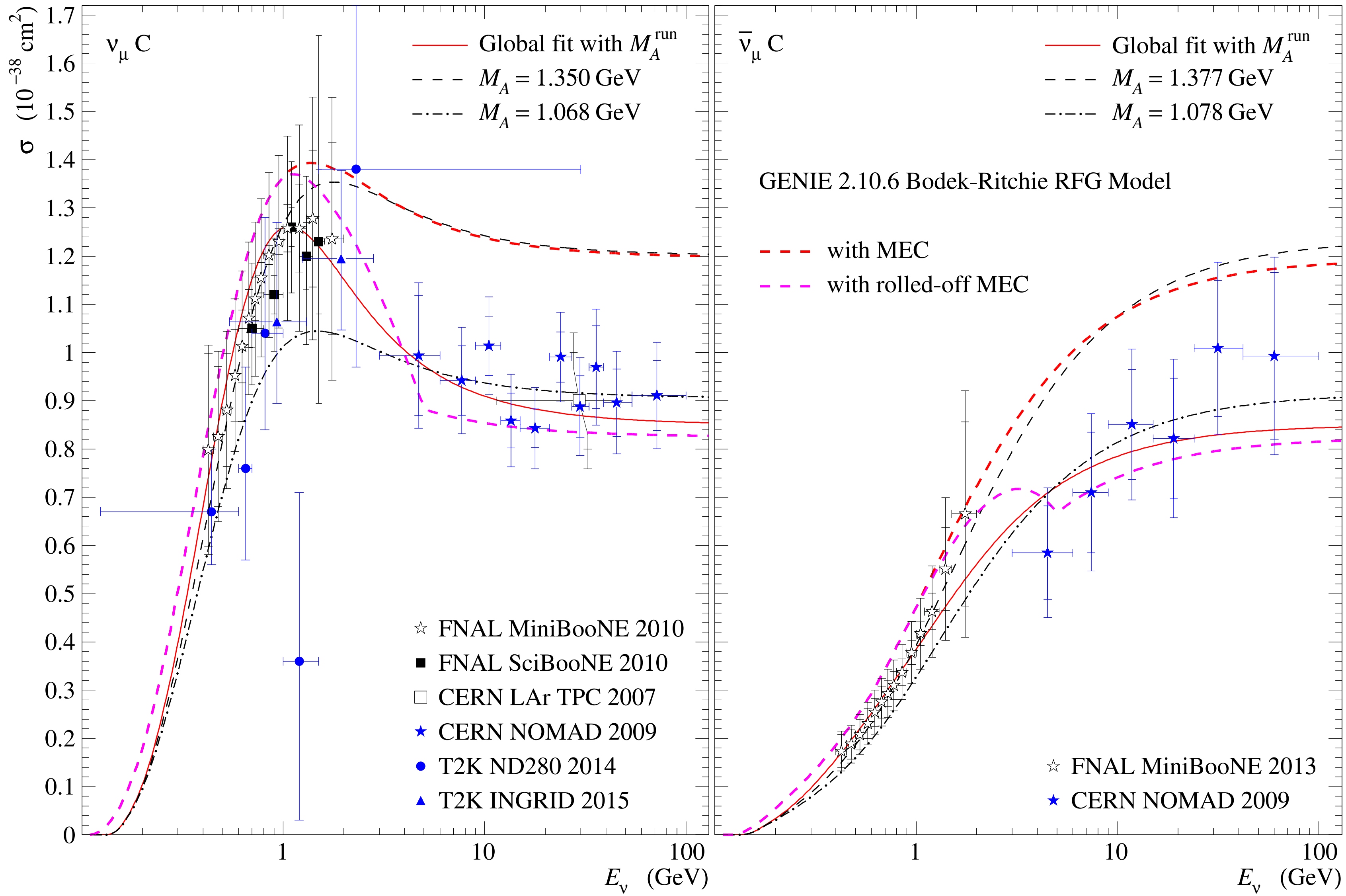
ν_μ C
Global fit with M_A^{run}
$M_A = 1.350$ GeV
$M_A = 1.068$ GeV
FNAL MiniBooNE 2010
FNAL SciBooNE 2010
CERN LAr TPC 2007
CERN NOMAD 2009
T2K ND280 2014
T2K INGRID 2015
σ $(10^{-38}$ cm$^2)$
E_ν (GeV)
$\overline{\nu}_\mu$ C
Global fit with M_A^{run}
$M_A = 1.377$ GeV
$M_A = 1.078$ GeV
FNAL MiniBooNE 2013
CERN NOMAD 2009
E_ν (GeV)



σ (10^-38 cm^2)
ν_μ C
Global fit with M_A^run
M_A = 1.350 GeV
M_A = 1.068 GeV
FNAL MiniBooNE 2010
FNAL SciBooNE 2010
CERN LAr TPC 2007
CERN NOMAD 2009
T2K ND280 2014
T2K INGRID 2015
E_ν (GeV)
ν̄_μ C
Global fit with M_A^run
M_A = 1.377 GeV
M_A = 1.078 GeV
GENIE 2.10.6 Bodek-Ritchie RFG Model
M_A^run
FNAL MiniBooNE 2013
CERN NOMAD 2009
E_ν (GeV)

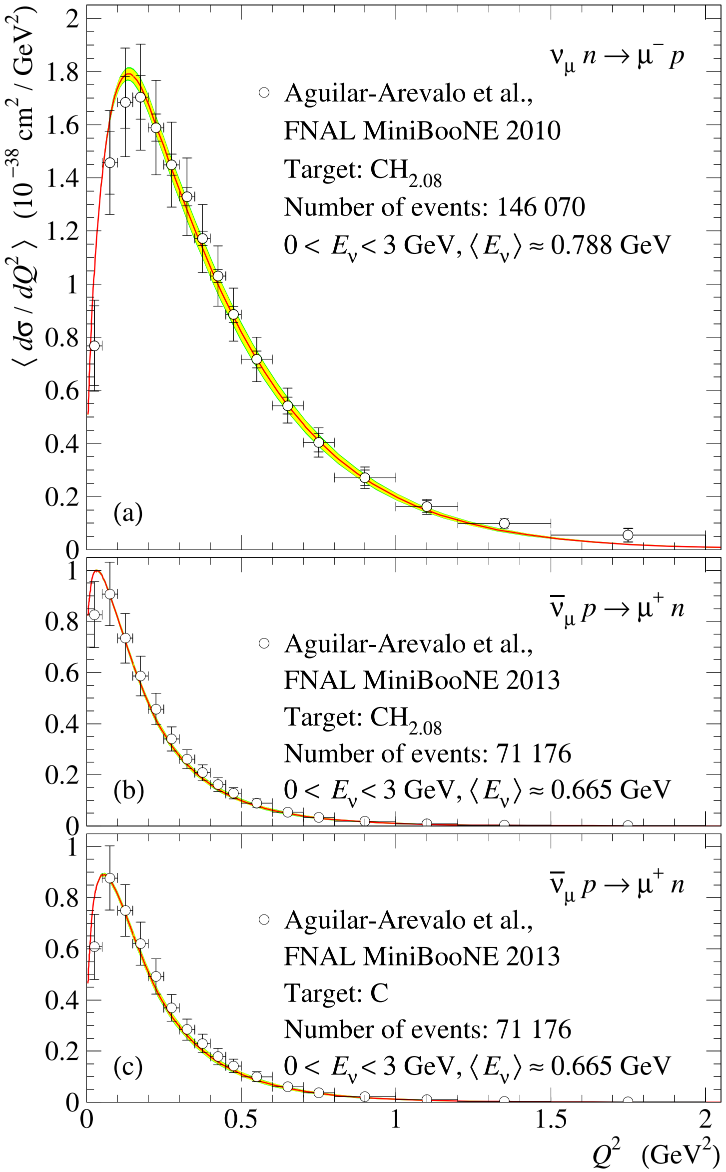


σ (10^{-38} cm^2)
ν_μ C
Global fit with M_A^{run}
M_A = 1.350 GeV
M_A = 1.068 GeV
FNAL MiniBooNE 2010
FNAL SciBooNE 2010
CERN LAr TPC 2007
CERN NOMAD 2009
T2K ND280 2014
T2K INGRID 2015
E_ν (GeV)
$\bar{\nu}_\mu$ C
Global fit with M_A^{run}
M_A = 1.377 GeV
M_A = 1.078 GeV
GENIE 2.10.6 Bodek-Ritchie RFG Model
M_A^{run}
with MEC
FNAL MiniBooNE 2013
CERN NOMAD 2009
E_ν (GeV)

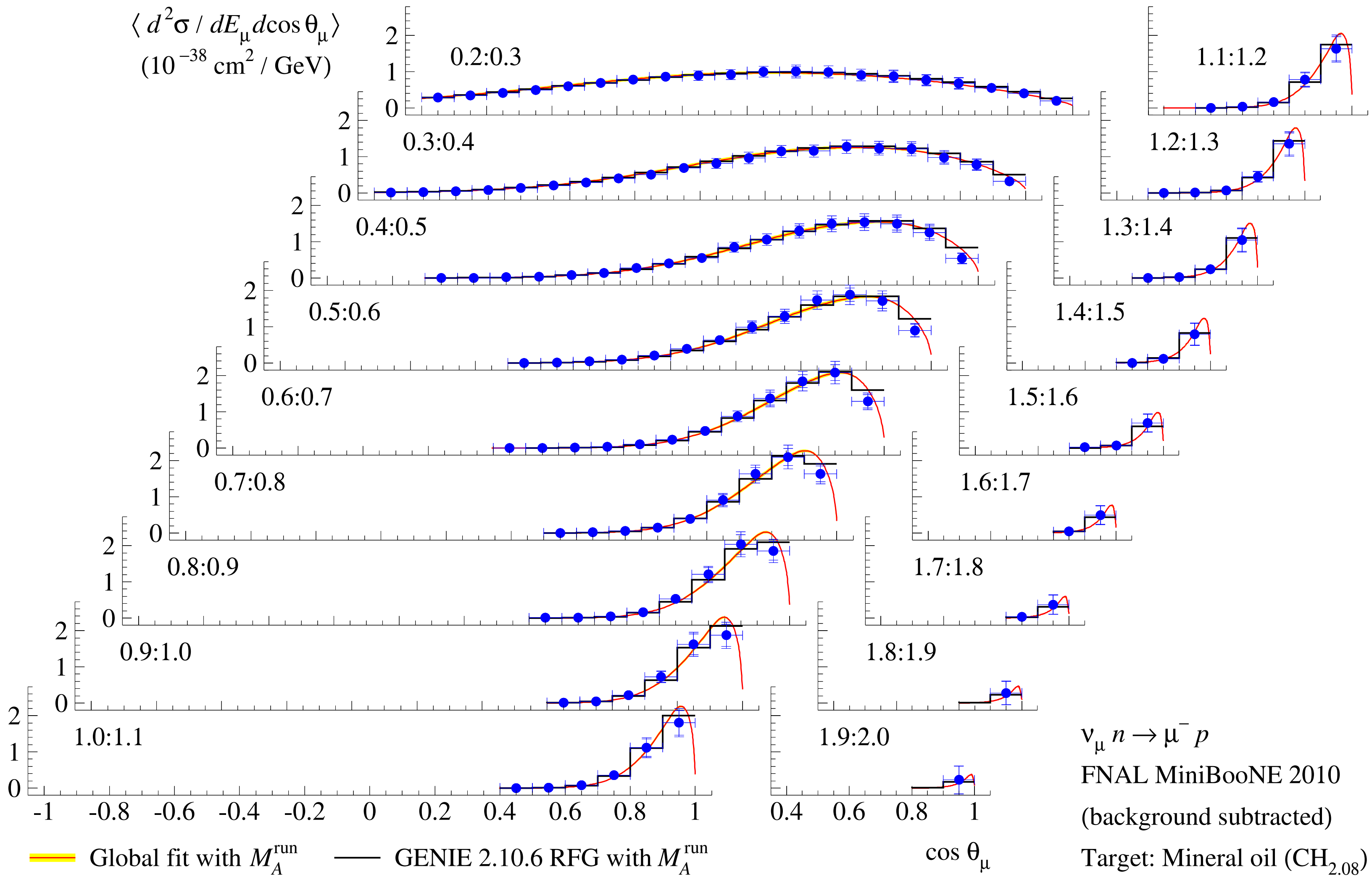


ν_μ C
Global fit with $M_A^{\rm run}$
$M_A = 1.350$ GeV
$M_A = 1.068$ GeV
FNAL MiniBooNE 2010
FNAL SciBooNE 2010
CERN LAr TPC 2007
CERN NOMAD 2009
T2K ND280 2014
T2K INGRID 2015
σ (10^{-38} cm^2)
E_ν (GeV)
$\bar{\nu}_\mu$ C
Global fit with $M_A^{\rm run}$
$M_A = 1.377$ GeV
$M_A = 1.078$ GeV
GENIE 2.10.6 Bodek-Ritchie RFG Model
with MEC
with rolled-off MEC
FNAL MiniBooNE 2013
CERN NOMAD 2009

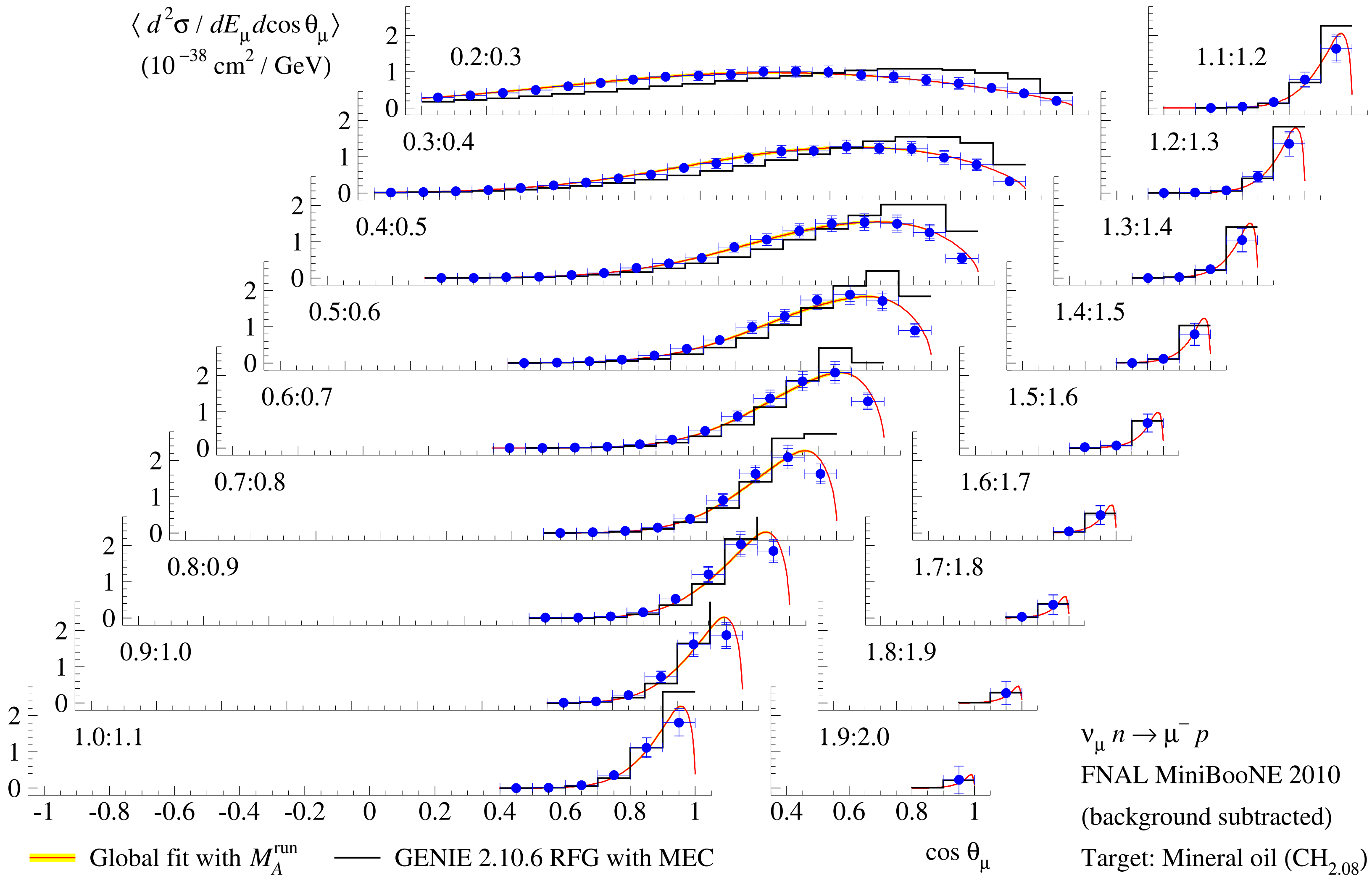
FNAL MiniBooNE 2010 – 2013

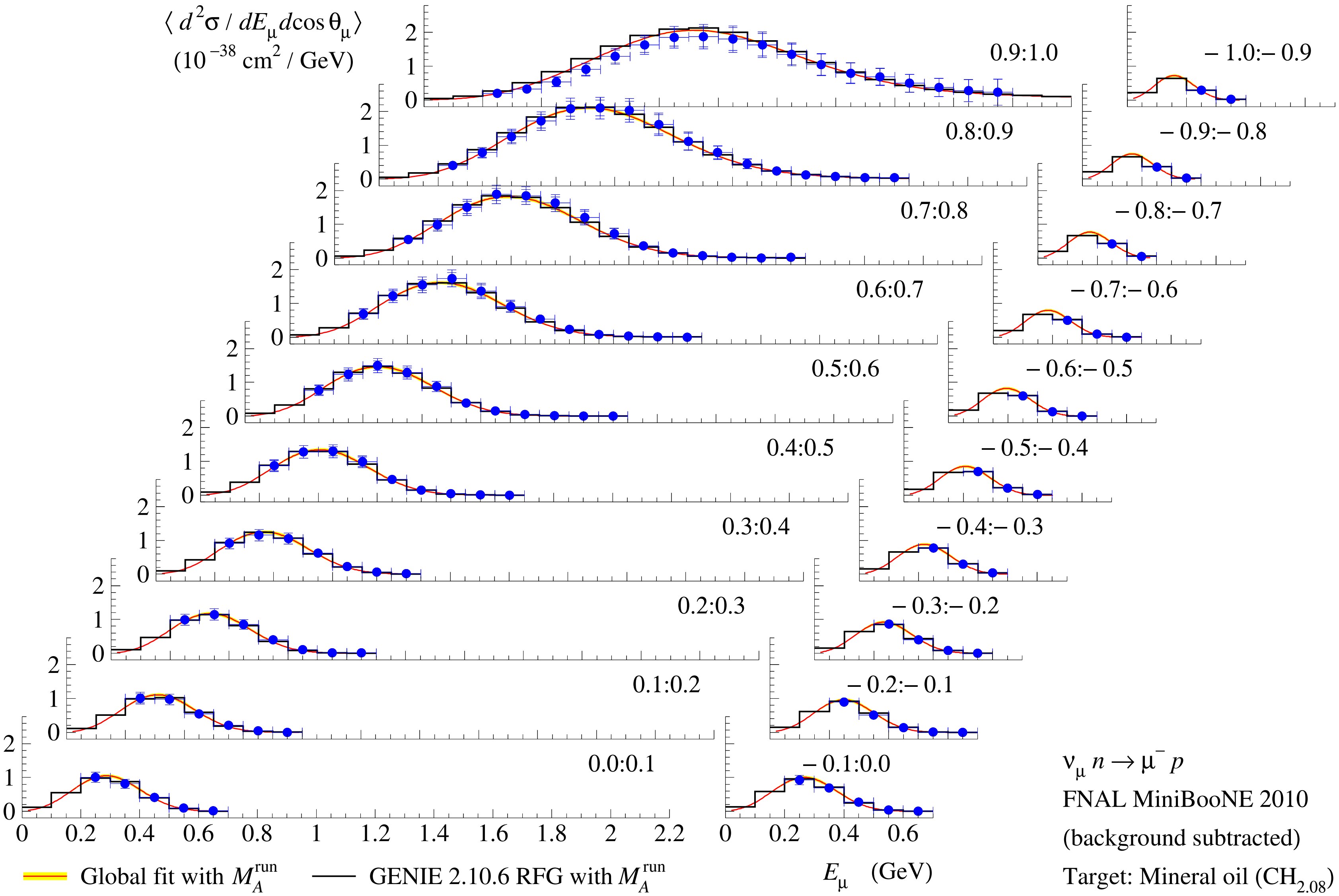


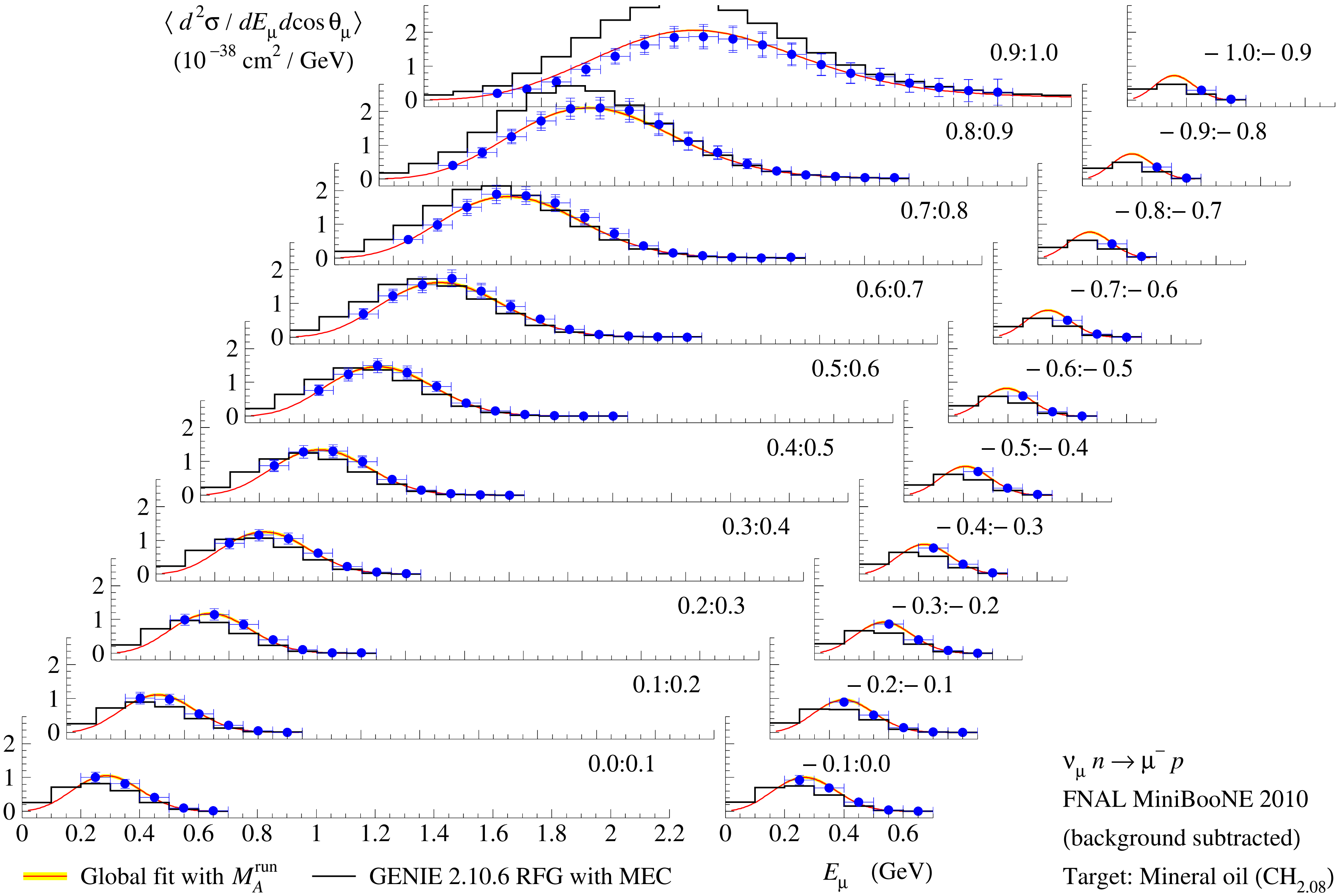
$\longleftarrow$ Recalculated by the MiniBooNE Co. from the "raw" data on the $CH_{2.08}$ target. The agreement is a little bit worse and we know why...



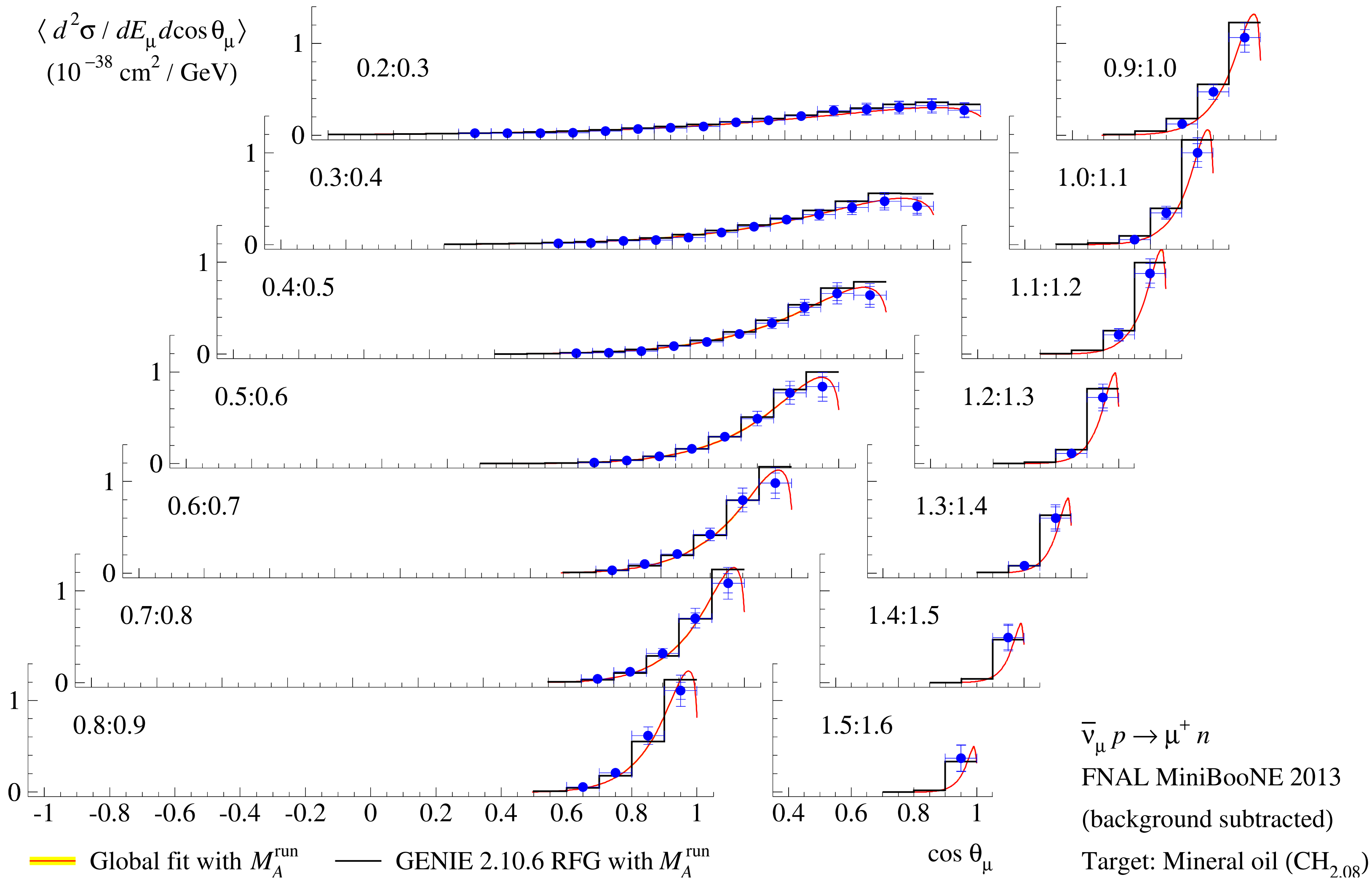
⟨ $d^2\sigma / dE_\mu d\cos\theta_\mu$ ⟩
(10^{-38} cm^2 / GeV)
0.2:0.3
0.3:0.4
0.4:0.5
0.5:0.6
0.6:0.7
0.7:0.8
0.8:0.9
0.9:1.0
1.0:1.1
1.1:1.2
1.2:1.3
1.3:1.4
1.4:1.5
1.5:1.6
1.6:1.7
1.7:1.8
1.8:1.9
1.9:2.0
cos θ_μ
Global fit with $M_A^{\rm run}$
GENIE 2.10.6 RFG with $M_A^{\rm run}$
$\nu_\mu n \to \mu^- p$
FNAL MiniBooNE 2010
(background subtracted)
Target: Mineral oil ($CH_{2.08}$)

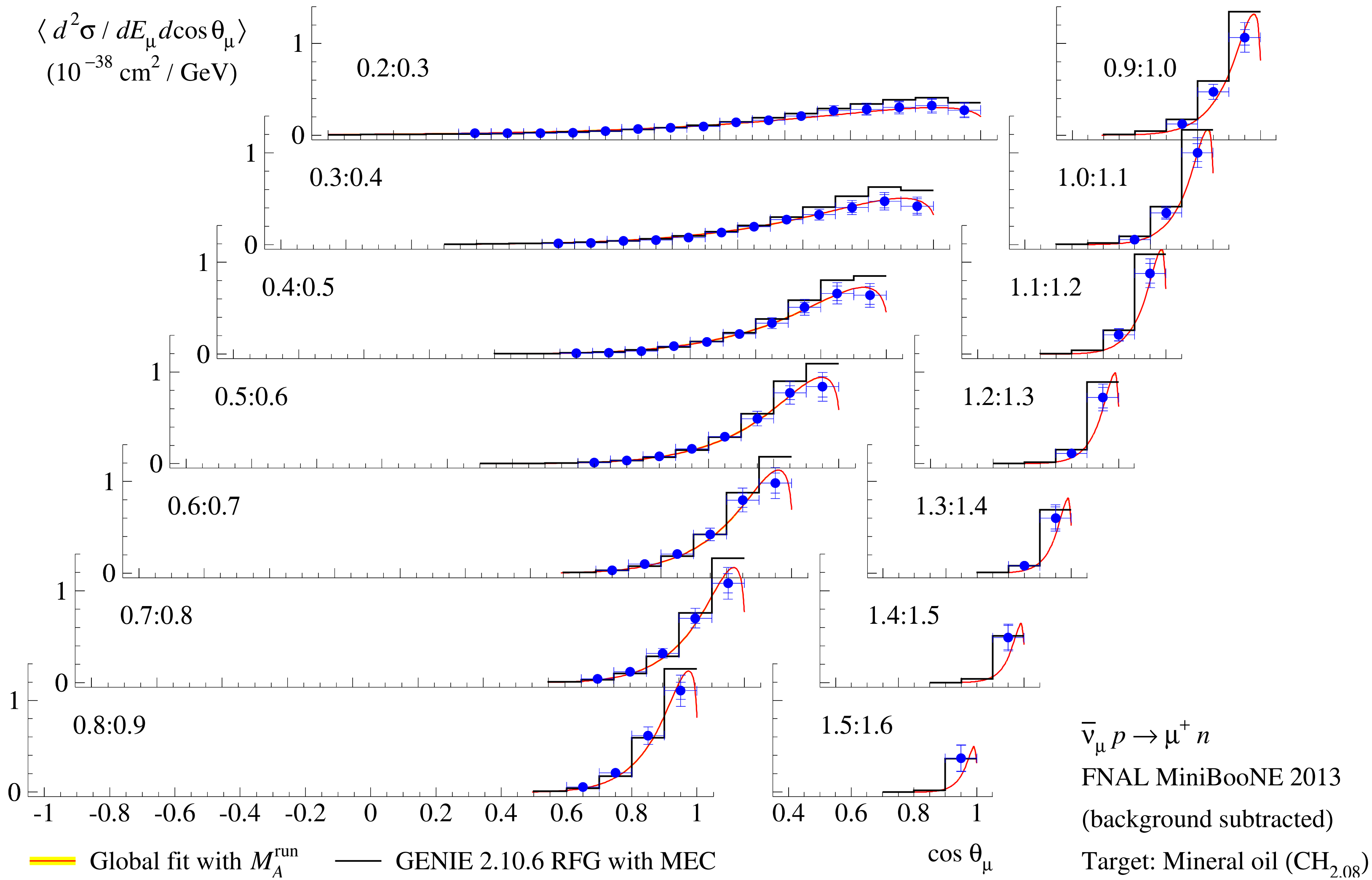

⟨ $d^2\sigma / dE_\mu d\cos\theta_\mu$ ⟩
(10^{-38} cm^2 / GeV)
0.2:0.3
0.3:0.4
0.4:0.5
0.5:0.6
0.6:0.7
0.7:0.8
0.8:0.9
0.9:1.0
1.0:1.1
1.1:1.2
1.2:1.3
1.3:1.4
1.4:1.5
1.5:1.6
1.6:1.7
1.7:1.8
1.8:1.9
1.9:2.0
-1 -0.8 -0.6 -0.4 -0.2 0 0.2 0.4 0.6 0.8 1
0.4 0.6 0.8 1
cos θ_μ
Global fit with M_A^{run}
GENIE 2.10.6 RFG with MEC
$\nu_\mu n \to \mu^- p$
FNAL MiniBooNE 2010
(background subtracted)
Target: Mineral oil ($CH_{2.08}$)

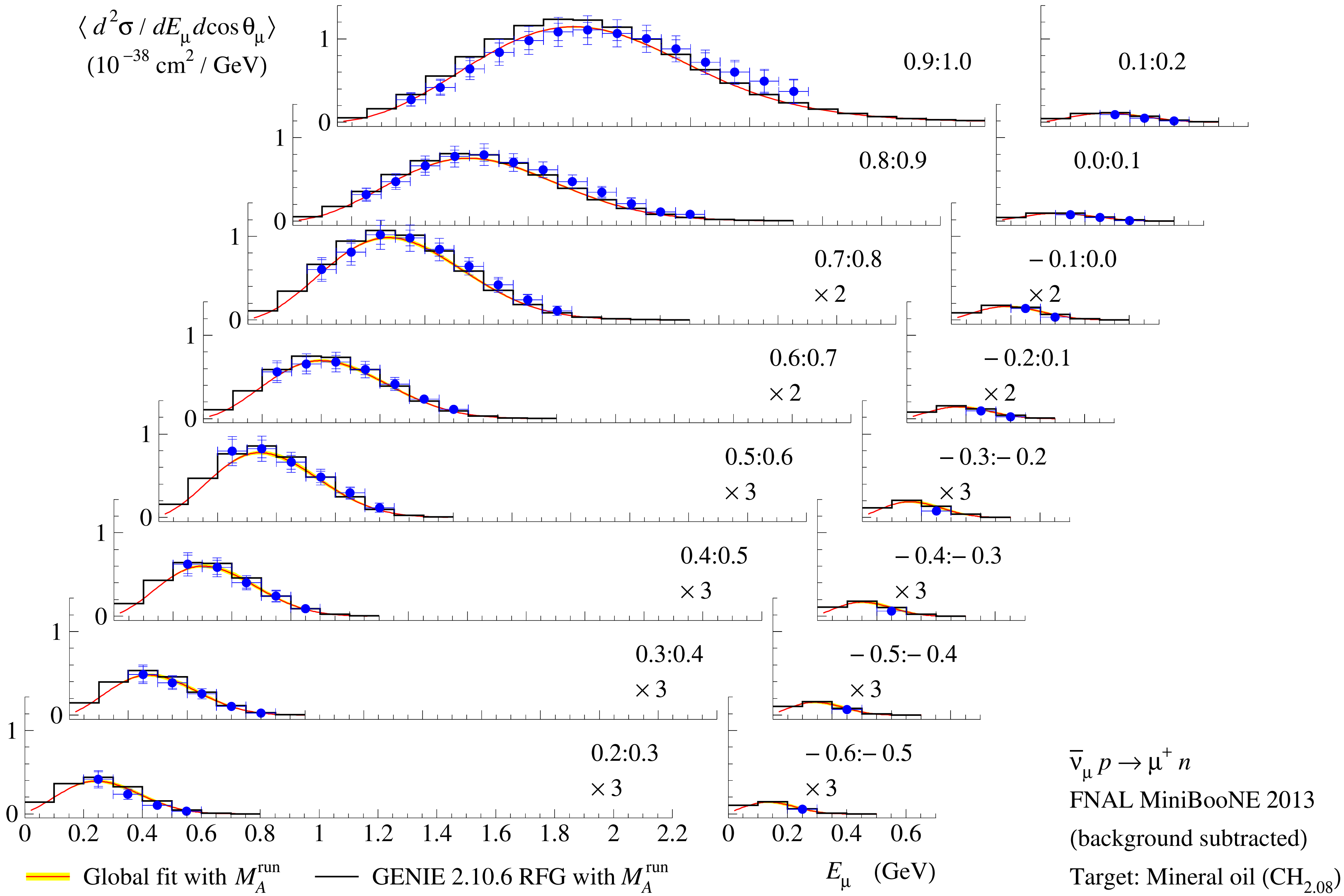

⟨ $d^2\sigma / dE_\mu d\cos\theta_\mu$ ⟩
(10^{-38} cm^2 / GeV)
0.9:1.0
0.8:0.9
0.7:0.8
0.6:0.7
0.5:0.6
0.4:0.5
0.3:0.4
0.2:0.3
0.1:0.2
0.0:0.1
− 1.0:− 0.9
− 0.9:− 0.8
− 0.8:− 0.7
− 0.7:− 0.6
− 0.6:− 0.5
− 0.5:− 0.4
− 0.4:− 0.3
− 0.3:− 0.2
− 0.2:− 0.1
− 0.1:0.0
E_μ (GeV)
$\nu_\mu n \to \mu^- p$
FNAL MiniBooNE 2010
(background subtracted)
Target: Mineral oil ($CH_{2.08}$)
Global fit with $M_A^{\rm run}$
GENIE 2.10.6 RFG with $M_A^{\rm run}$



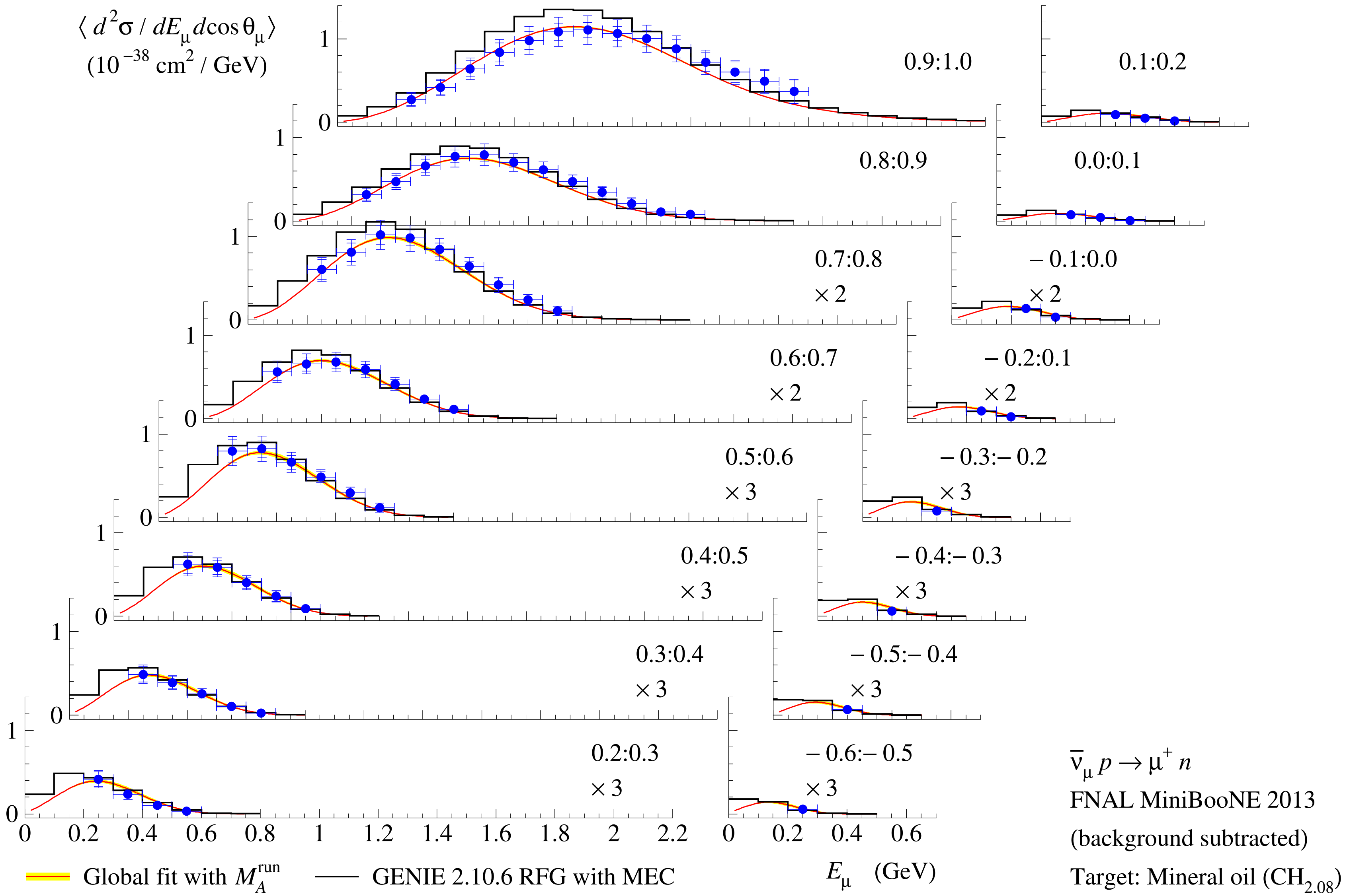
⟨ d²σ / dE_μ dcos θ_μ ⟩
(10⁻³⁸ cm² / GeV)
0.9:1.0
0.8:0.9
0.7:0.8
0.6:0.7
0.5:0.6
0.4:0.5
0.3:0.4
0.2:0.3
0.1:0.2
0.0:0.1
– 1.0:– 0.9
– 0.9:– 0.8
– 0.8:– 0.7
– 0.7:– 0.6
– 0.6:– 0.5
– 0.5:– 0.4
– 0.4:– 0.3
– 0.3:– 0.2
– 0.2:– 0.1
– 0.1:0.0
0 0.2 0.4 0.6 0.8 1 1.2 1.4 1.6 1.8 2 2.2
E_μ (GeV)
Global fit with M_A^run
GENIE 2.10.6 RFG with MEC
ν_μ n → μ⁻ p
FNAL MiniBooNE 2010
(background subtracted)
Target: Mineral oil (CH_2.08)


⟨ $d^2\sigma / dE_\mu d\cos\theta_\mu$ ⟩
(10^{-38} cm^2 / GeV)
0.2:0.3
0.3:0.4
0.4:0.5
0.5:0.6
0.6:0.7
0.7:0.8
0.8:0.9
0.9:1.0
1.0:1.1
1.1:1.2
1.2:1.3
1.3:1.4
1.4:1.5
1.5:1.6
cos θ_μ
$\bar{\nu}_\mu p \to \mu^+ n$
FNAL MiniBooNE 2013
(background subtracted)
Target: Mineral oil ($CH_{2.08}$)
Global fit with M_A^{run}
GENIE 2.10.6 RFG with M_A^{run}


⟨ d²σ / dE_μ dcos θ_μ ⟩
(10⁻³⁸ cm² / GeV)
0.2:0.3
0.3:0.4
0.4:0.5
0.5:0.6
0.6:0.7
0.7:0.8
0.8:0.9
0.9:1.0
1.0:1.1
1.1:1.2
1.2:1.3
1.3:1.4
1.4:1.5
1.5:1.6
-1 -0.8 -0.6 -0.4 -0.2 0 0.2 0.4 0.6 0.8 1
0.4 0.6 0.8 1
cos θ_μ
ν̄_μ p → μ⁺ n
FNAL MiniBooNE 2013
(background subtracted)
Target: Mineral oil (CH_2.08)
Global fit with M_A^run
GENIE 2.10.6 RFG with MEC

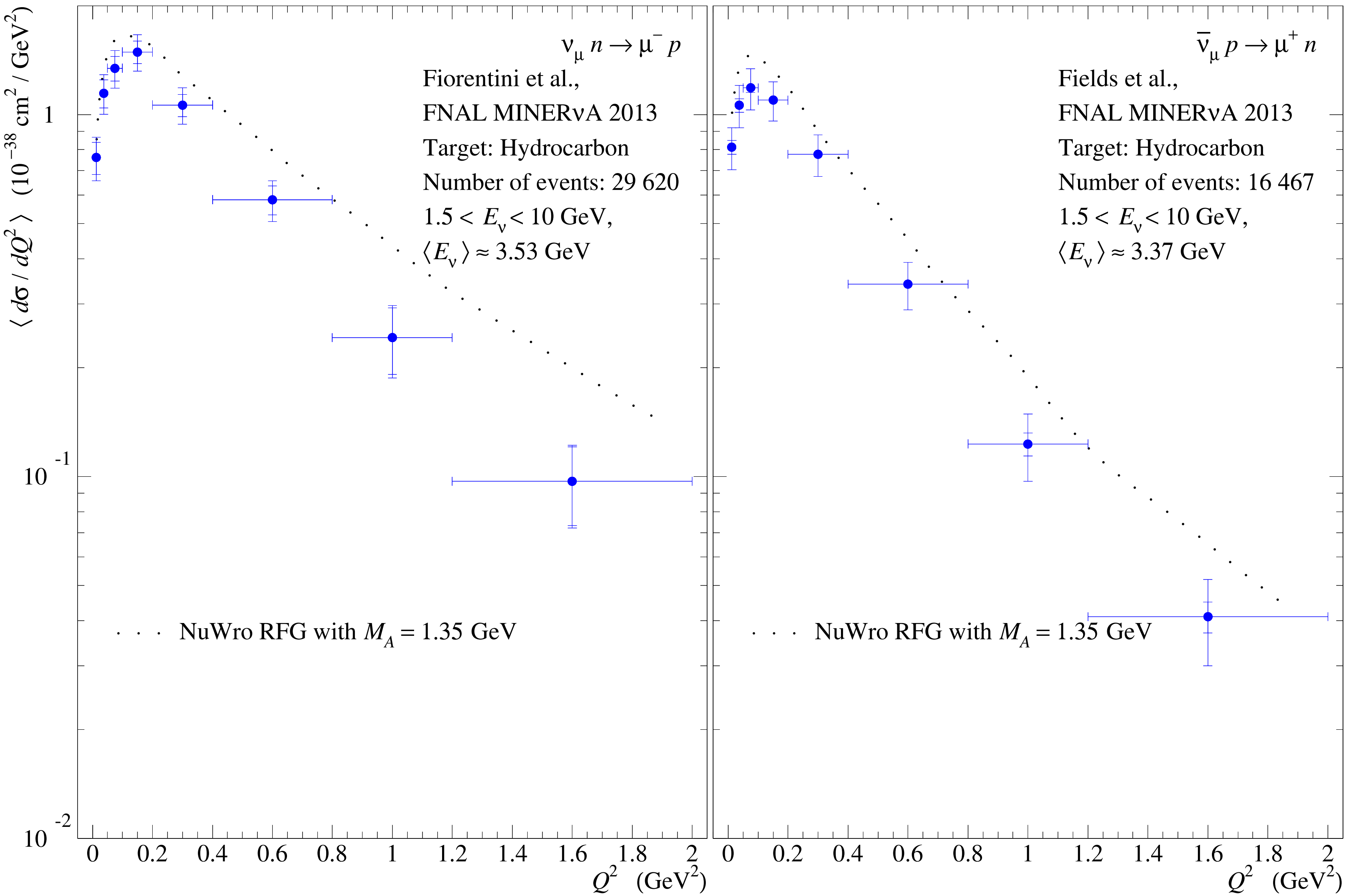


$\langle d^2\sigma / dE_\mu d\cos\theta_\mu \rangle$
$(10^{-38}\ \mathrm{cm}^2 / \mathrm{GeV})$
0.9:1.0
0.1:0.2
0.8:0.9
0.0:0.1
0.7:0.8 × 2
− 0.1:0.0 × 2
0.6:0.7 × 2
− 0.2:0.1 × 2
0.5:0.6 × 3
− 0.3:− 0.2 × 3
0.4:0.5 × 3
− 0.4:− 0.3 × 3
0.3:0.4 × 3
− 0.5:− 0.4 × 3
0.2:0.3 × 3
− 0.6:− 0.5 × 3
$\bar{\nu}_\mu p \to \mu^+ n$
FNAL MiniBooNE 2013
(background subtracted)
Target: Mineral oil ($CH_{2.08}$)
E_μ (GeV)
Global fit with M_A^{run}
GENIE 2.10.6 RFG with M_A^{run}

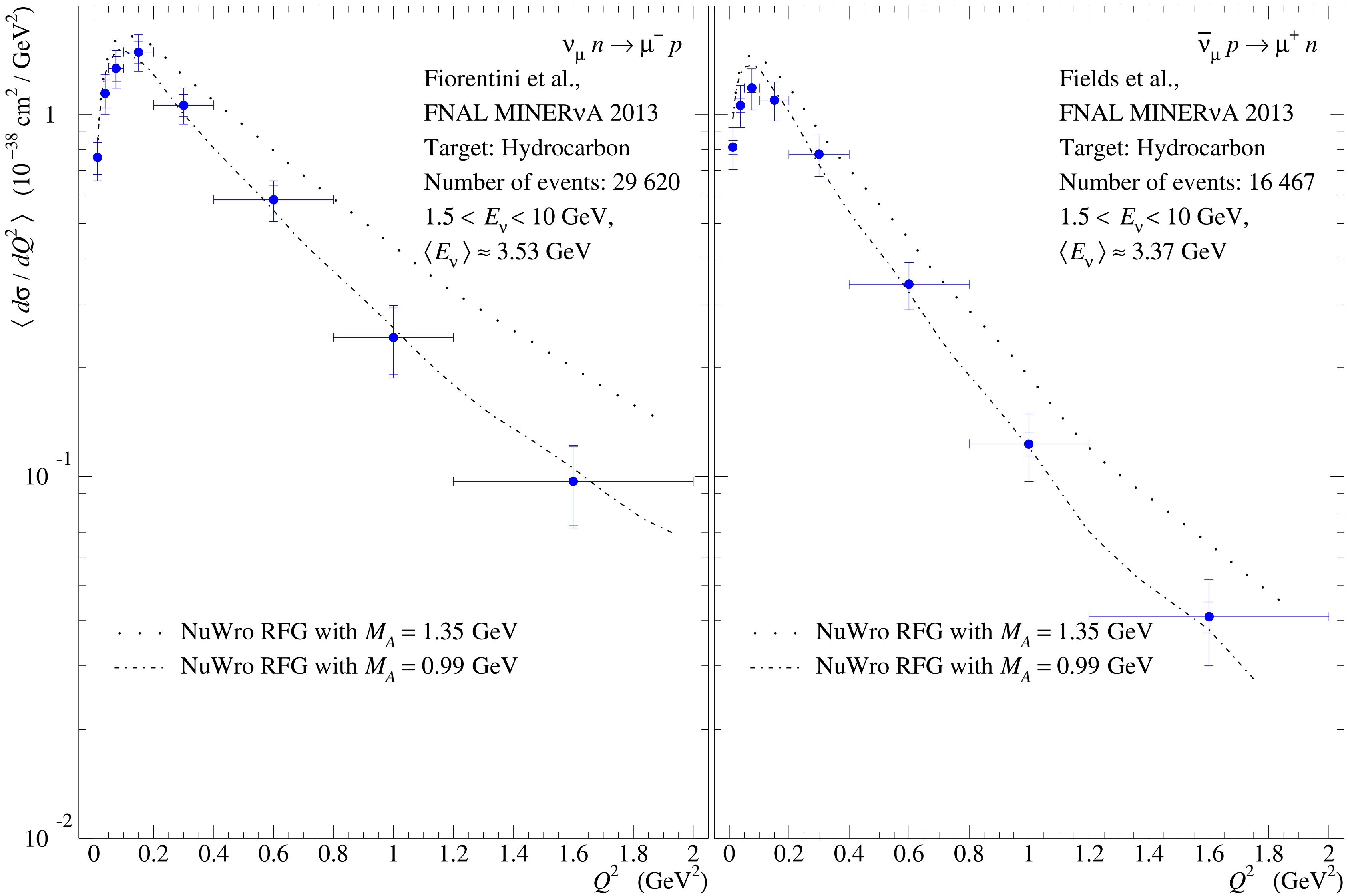


⟨ $d^2\sigma / dE_\mu d\cos\theta_\mu$ ⟩
(10^{-38} cm^2 / GeV)
0.9:1.0
0.8:0.9
0.7:0.8
× 2
0.6:0.7
× 2
0.5:0.6
× 3
0.4:0.5
× 3
0.3:0.4
× 3
0.2:0.3
× 3
0.1:0.2
0.0:0.1
− 0.1:0.0
× 2
− 0.2:0.1
× 2
− 0.3:− 0.2
× 3
− 0.4:− 0.3
× 3
− 0.5:− 0.4
× 3
− 0.6:− 0.5
× 3
E_μ (GeV)
Global fit with $M_A^{\rm run}$
GENIE 2.10.6 RFG with MEC
$\bar{\nu}_\mu p \to \mu^+ n$
FNAL MiniBooNE 2013
(background subtracted)
Target: Mineral oil ($CH_{2.08}$)

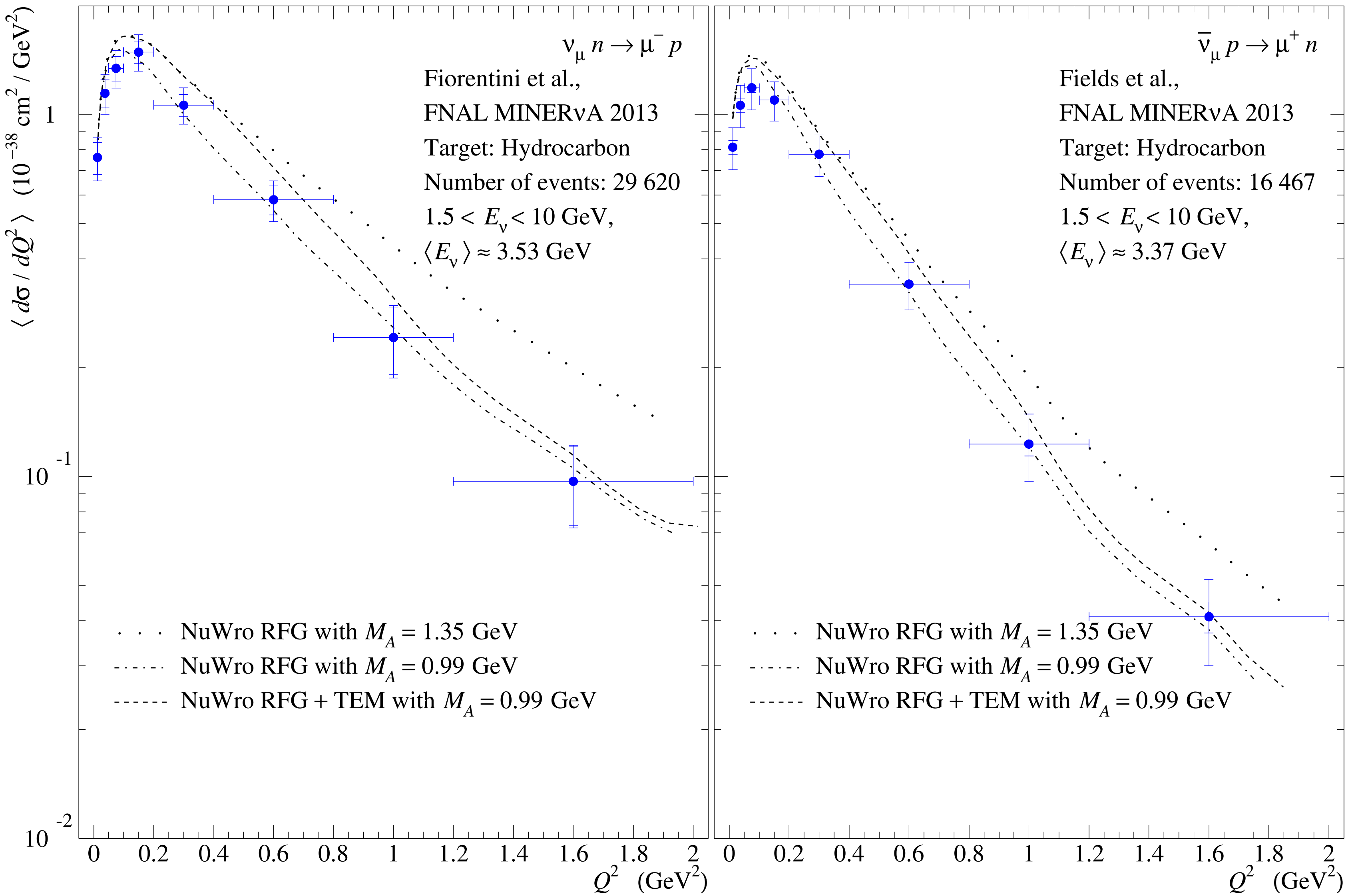
FNAL MINERνA 2013



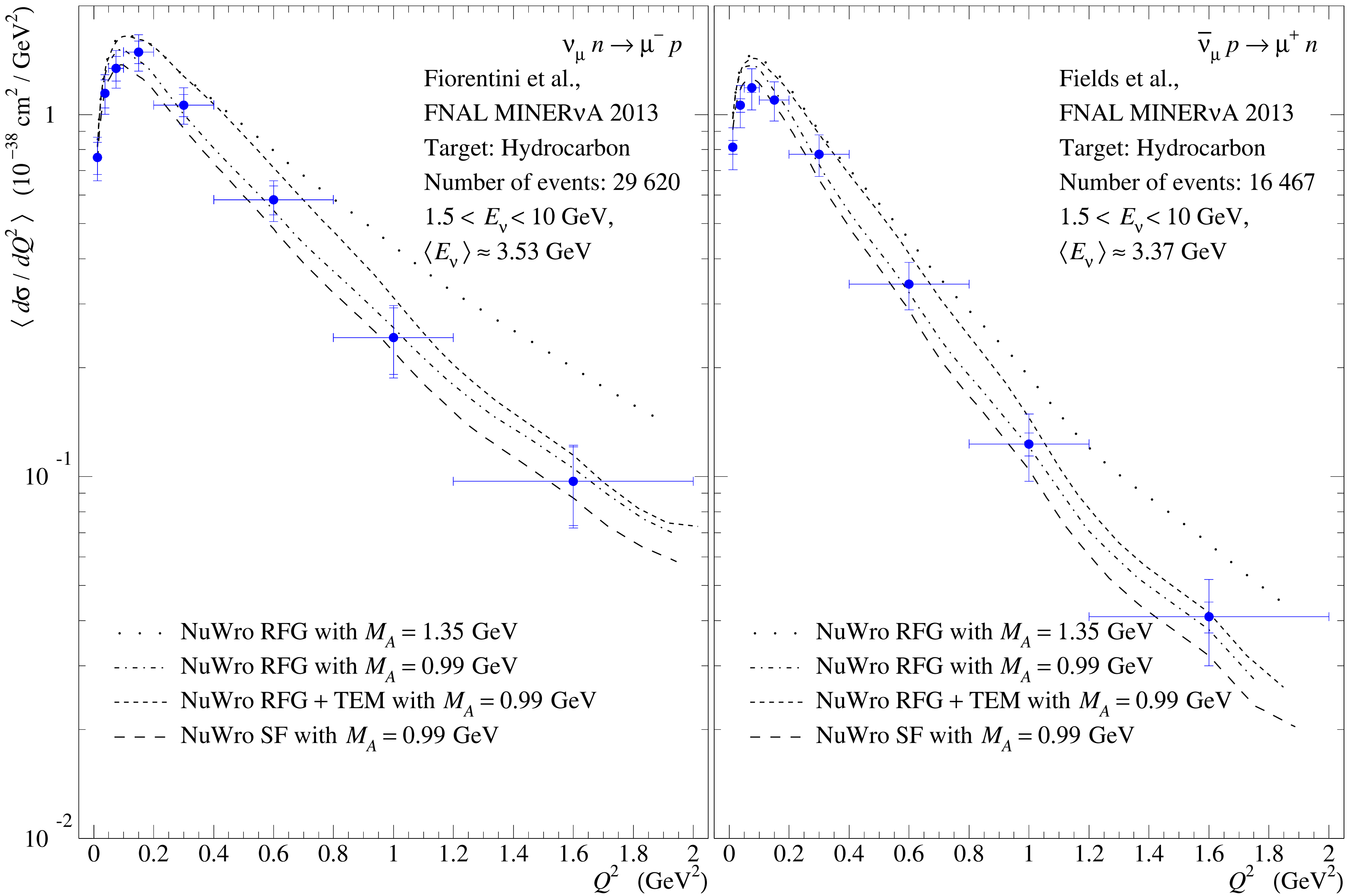
⟨dσ / dQ²⟩ (10⁻³⁸ cm² / GeV²)
1
10⁻¹
10⁻²
0 0.2 0.4 0.6 0.8 1 1.2 1.4 1.6 1.8 2
Q² (GeV²)
ν_μ n → μ⁻ p
Fiorentini et al.,
FNAL MINERνA 2013
Target: Hydrocarbon
Number of events: 29 620
1.5 < E_ν < 10 GeV,
⟨E_ν⟩ ≈ 3.53 GeV
NuWro RFG with M_A = 1.35 GeV
ν̄_μ p → μ⁺ n
Fields et al.,
FNAL MINERνA 2013
Target: Hydrocarbon
Number of events: 16 467
1.5 < E_ν < 10 GeV,
⟨E_ν⟩ ≈ 3.37 GeV
NuWro RFG with M_A = 1.35 GeV



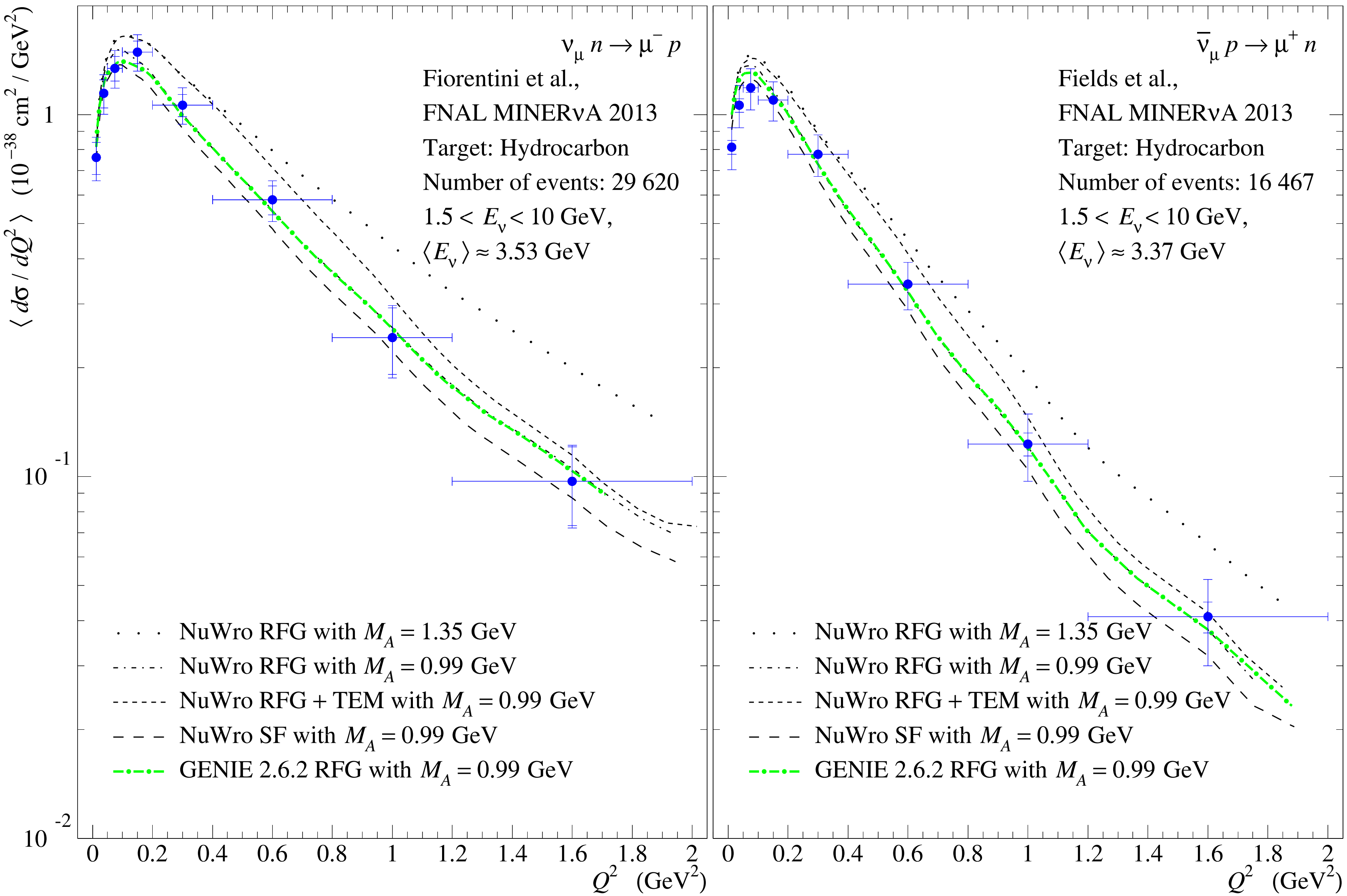
$\langle d\sigma / dQ^2 \rangle$ (10^{-38} cm^2 / GeV^2)
1
10^{-1}
10^{-2}
0
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
1.8
2
Q^2 (GeV^2)
$\nu_\mu n \rightarrow \mu^- p$
Fiorentini et al.,
FNAL MINERνA 2013
Target: Hydrocarbon
Number of events: 29 620
$1.5 < E_\nu < 10$ GeV,
$\langle E_\nu \rangle \approx 3.53$ GeV
NuWro RFG with $M_A = 1.35$ GeV
NuWro RFG with $M_A = 0.99$ GeV
$\bar{\nu}_\mu p \rightarrow \mu^+ n$
Fields et al.,
FNAL MINERνA 2013
Target: Hydrocarbon
Number of events: 16 467
$1.5 < E_\nu < 10$ GeV,
$\langle E_\nu \rangle \approx 3.37$ GeV
NuWro RFG with $M_A = 1.35$ GeV
NuWro RFG with $M_A = 0.99$ GeV



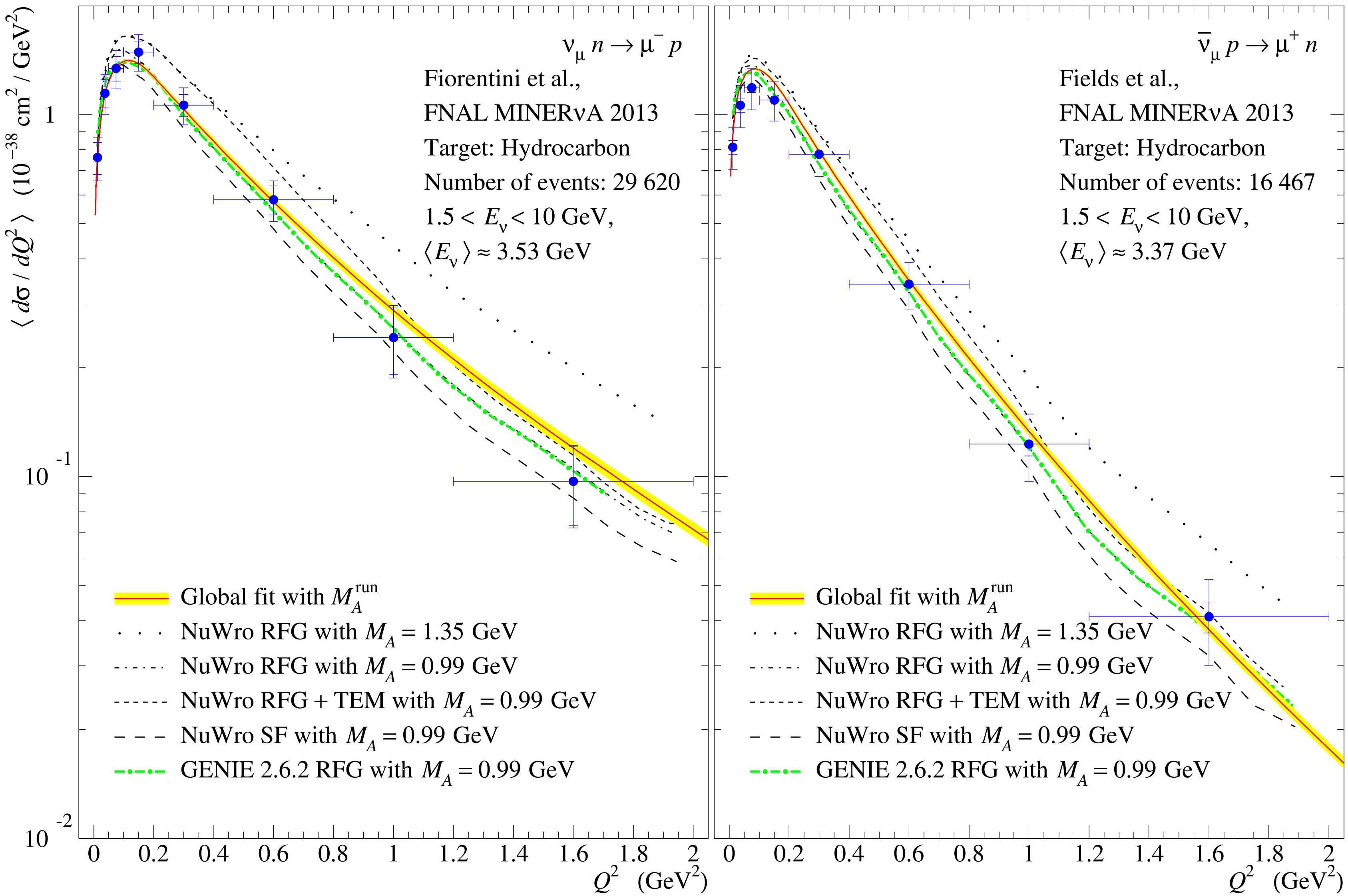
$\nu_\mu n \to \mu^- p$
Fiorentini et al.,
FNAL MINERνA 2013
Target: Hydrocarbon
Number of events: 29 620
$1.5 < E_\nu < 10$ GeV,
$\langle E_\nu \rangle \approx 3.53$ GeV
$\bar{\nu}_\mu p \to \mu^+ n$
Fields et al.,
FNAL MINERνA 2013
Target: Hydrocarbon
Number of events: 16 467
$1.5 < E_\nu < 10$ GeV,
$\langle E_\nu \rangle \approx 3.37$ GeV
NuWro RFG with $M_A = 1.35$ GeV
NuWro RFG with $M_A = 0.99$ GeV
NuWro RFG + TEM with $M_A = 0.99$ GeV
$\langle d\sigma / dQ^2 \rangle$ (10^{-38} cm^2 / GeV^2)
Q^2 (GeV^2)



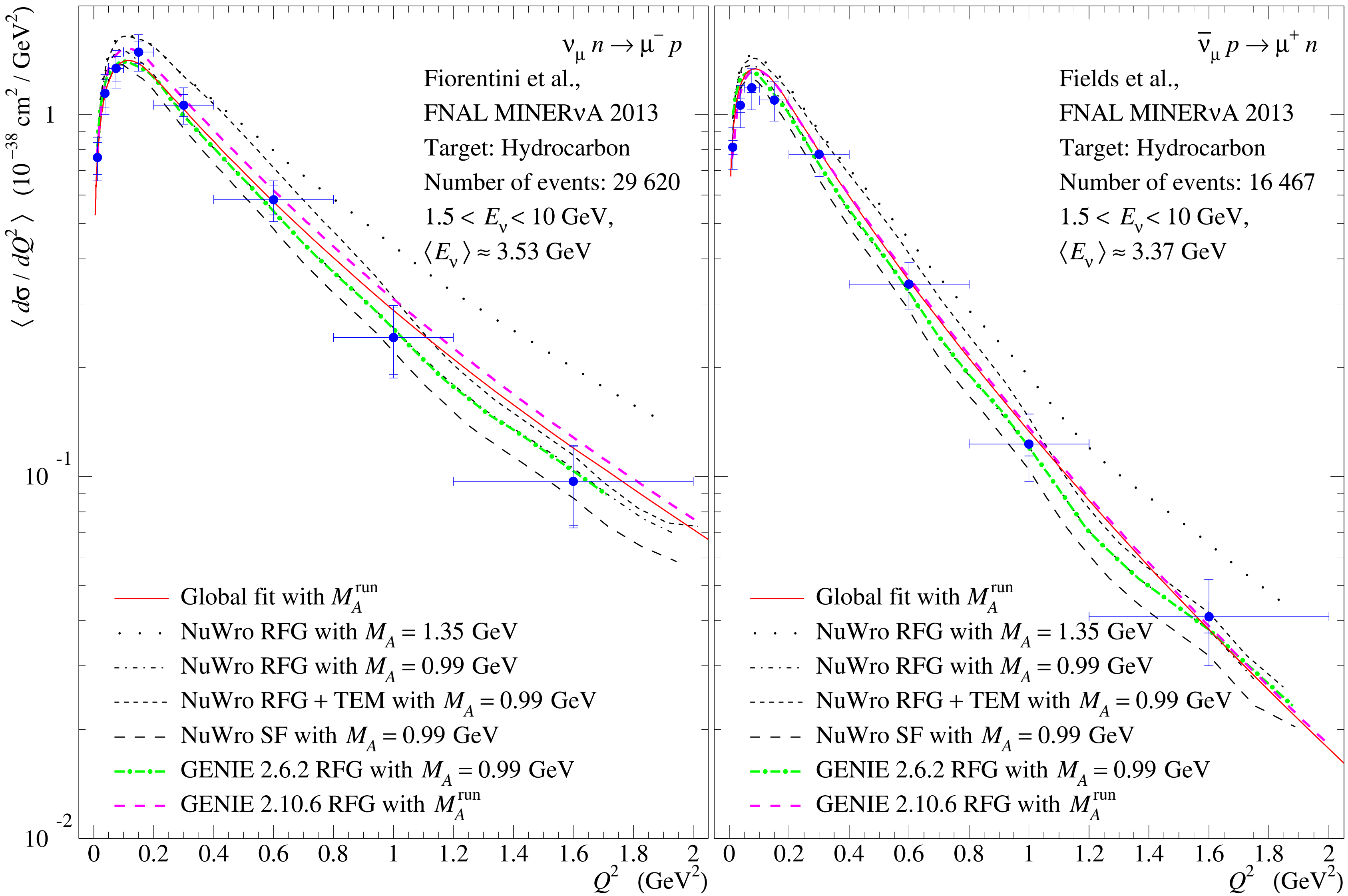
$\nu_\mu\, n \to \mu^-\, p$
Fiorentini et al.,
FNAL MINERνA 2013
Target: Hydrocarbon
Number of events: 29 620
$1.5 < E_\nu < 10$ GeV,
$\langle E_\nu \rangle \approx 3.53$ GeV
$\bar{\nu}_\mu\, p \to \mu^+\, n$
Fields et al.,
FNAL MINERνA 2013
Target: Hydrocarbon
Number of events: 16 467
$1.5 < E_\nu < 10$ GeV,
$\langle E_\nu \rangle \approx 3.37$ GeV
NuWro RFG with $M_A = 1.35$ GeV
NuWro RFG with $M_A = 0.99$ GeV
NuWro RFG + TEM with $M_A = 0.99$ GeV
NuWro SF with $M_A = 0.99$ GeV
$\langle d\sigma / dQ^2 \rangle$ $(10^{-38}\,\mathrm{cm}^2 / \mathrm{GeV}^2)$
Q^2 (GeV^2)



⟨dσ / dQ²⟩ (10⁻³⁸ cm² / GeV²)
Q² (GeV²)
ν_μ n → μ⁻ p
Fiorentini et al.,
FNAL MINERνA 2013
Target: Hydrocarbon
Number of events: 29 620
1.5 < E_ν < 10 GeV,
⟨E_ν⟩ ≈ 3.53 GeV
ν̄_μ p → μ⁺ n
Fields et al.,
FNAL MINERνA 2013
Target: Hydrocarbon
Number of events: 16 467
1.5 < E_ν < 10 GeV,
⟨E_ν⟩ ≈ 3.37 GeV
NuWro RFG with M_A = 1.35 GeV
NuWro RFG with M_A = 0.99 GeV
NuWro RFG + TEM with M_A = 0.99 GeV
NuWro SF with M_A = 0.99 GeV
GENIE 2.6.2 RFG with M_A = 0.99 GeV



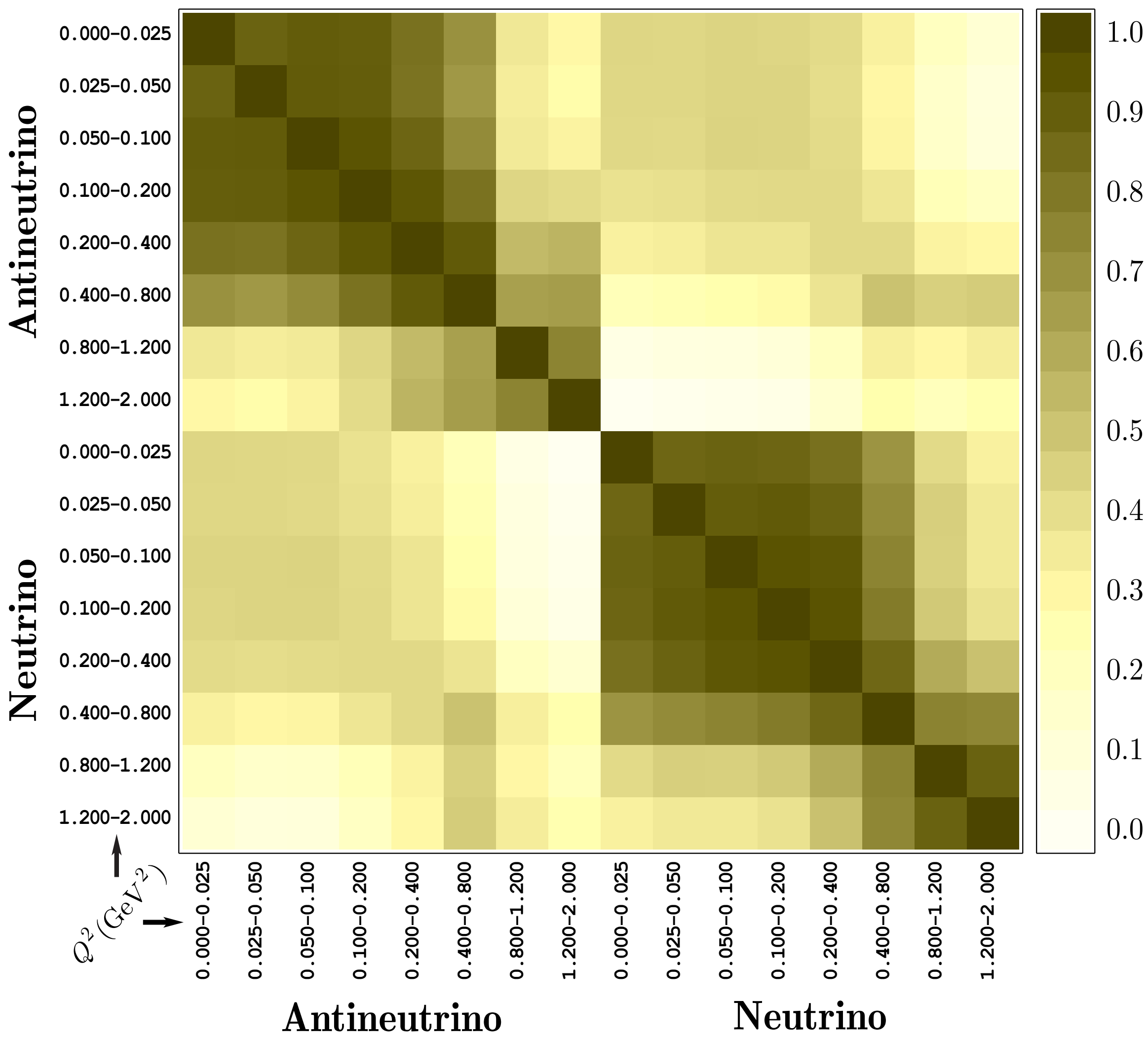
$\nu_\mu\, n \to \mu^- p$
Fiorentini et al.,
FNAL MINERνA 2013
Target: Hydrocarbon
Number of events: 29 620
$1.5 < E_\nu < 10$ GeV,
$\langle E_\nu \rangle \approx 3.53$ GeV
$\overline{\nu}_\mu\, p \to \mu^+ n$
Fields et al.,
FNAL MINERνA 2013
Target: Hydrocarbon
Number of events: 16 467
$1.5 < E_\nu < 10$ GeV,
$\langle E_\nu \rangle \approx 3.37$ GeV
Global fit with $M_A^{\rm run}$
NuWro RFG with $M_A = 1.35$ GeV
NuWro RFG with $M_A = 0.99$ GeV
NuWro RFG + TEM with $M_A = 0.99$ GeV
NuWro SF with $M_A = 0.99$ GeV
GENIE 2.6.2 RFG with $M_A = 0.99$ GeV
$\langle d\sigma / dQ^2 \rangle$ (10^{-38} cm^2 / GeV^2)
Q^2 (GeV^2)



$\nu_\mu n \to \mu^- p$
Fiorentini et al.,
FNAL MINERvA 2013
Target: Hydrocarbon
Number of events: 29 620
$1.5 < E_\nu < 10$ GeV,
$\langle E_\nu \rangle \approx 3.53$ GeV
$\bar{\nu}_\mu p \to \mu^+ n$
Fields et al.,
FNAL MINERvA 2013
Target: Hydrocarbon
Number of events: 16 467
$1.5 < E_\nu < 10$ GeV,
$\langle E_\nu \rangle \approx 3.37$ GeV
Global fit with M_A^{run}
NuWro RFG with $M_A = 1.35$ GeV
NuWro RFG with $M_A = 0.99$ GeV
NuWro RFG + TEM with $M_A = 0.99$ GeV
NuWro SF with $M_A = 0.99$ GeV
GENIE 2.6.2 RFG with $M_A = 0.99$ GeV
GENIE 2.10.6 RFG with M_A^{run}
$\langle d\sigma / dQ^2 \rangle$ (10^{-38} cm^2 / GeV^2)
Q^2 (GeV^2)

As it is seen from the correlation matrix, the MINERνA data are highly correlated. Therefore the *by-eye* comparison between the data and the model predictions is not very conclusive.

The values of χ^2/N (where N is the number of data-points) listed in next slide allow us to conclude that our model provides the best description of the MINERνA results among the models shown in slides 28–34.



Model	Author's estimation (by bin medians)	Our estimation (by bin medians)	Our estimation (by bin averages)
ν_μ FNAL MINERνA 2013 [27], $N = 8$			
NuWro RFG with $M_A = 1.35$	3.7	$29.26/8 = 3.66$	$31.13/8 = 3.89$
NuWro RFG with $M_A = 0.99$ + TEM	2.4	$18.40/8 = 2.30$	$20.93/8 = 2.62$
NuWro RFG with $M_A = 0.99$	3.5	$28.19/8 = 3.52$	$23.24/8 = 2.91$
NuWro SF with $M_A = 0.99$	2.8	$22.66/8 = 2.83$	$18.69/8 = 2.34$
GENIE 2.6.2 RFG with $M_A = 0.99$	–	$16.04/8 = 2.00$	$15.27/8 = 1.91$
Global fit with M_A^{run}	–	$7.19/8 = 0.90$	$8.04/8 = 1.00$
GENIE 2.10.6 with M_A^{run}	–	$11.16/8 = 1.39$	$10.77/8 = 1.35$
$\overline{\nu}_\mu$ FNAL MINERνA 2013 [28], $N = 8$			
NuWro RFG with $M_A = 1.35$	2.90	$23.50/8 = 2.94$	$23.20/8 = 2.90$
NuWro RFG with $M_A = 0.99$ + TEM	1.06	$8.35/8 = 1.04$	$9.50/8 = 1.19$
NuWro RFG with $M_A = 0.99$	2.64	$21.39/8 = 2.67$	$17.80/8 = 2.23$
NuWro SF with $M_A = 0.99$	2.14	$17.37/8 = 2.17$	$14.59/8 = 1.82$
GENIE 2.6.2 RFG with $M_A = 0.99$	–	$19.96/8 = 2.50$	$18.23/8 = 2.28$
Global fit with M_A^{run}	–	$7.29/8 = 0.91$	$5.51/8 = 0.69$
GENIE 2.10.6 with M_A^{run}	–	$19.31/8 = 2.41$	$15.92/8 = 1.99$
$\nu_\mu \oplus \overline{\nu}_\mu$ FNAL MINERνA 2013 [27, 28], $N = 16$			
NuWro RFG with $M_A = 1.35$	–	$59.15/16 = 3.70$	$59.34/16 = 3.71$
NuWro RFG with $M_A = 0.99$ + TEM	–	$28.33/16 = 1.77$	$30.74/16 = 1.92$
NuWro RFG with $M_A = 0.99$	–	$48.11/16 = 3.01$	$40.59/16 = 2.54$
NuWro SF with $M_A = 0.99$	–	$38.12/16 = 2.38$	$32.39/16 = 2.02$
GENIE 2.6.2 RFG with $M_A = 0.99$	–	$34.87/16 = 2.18$	$33.04/16 = 2.07$
Global fit with M_A^{run}	–	$19.93/16 = 1.25$	$17.90/16 = 1.10$
GENIE 2.10.6 with M_A^{run}	–	$34.77/16 = 2.17$	$30.02/16 = 1.88$

T2K ND280 2016

A caveat

We have not yet included the new (2016) T2K ND280 data [69] into the global fit due to the following reasons:

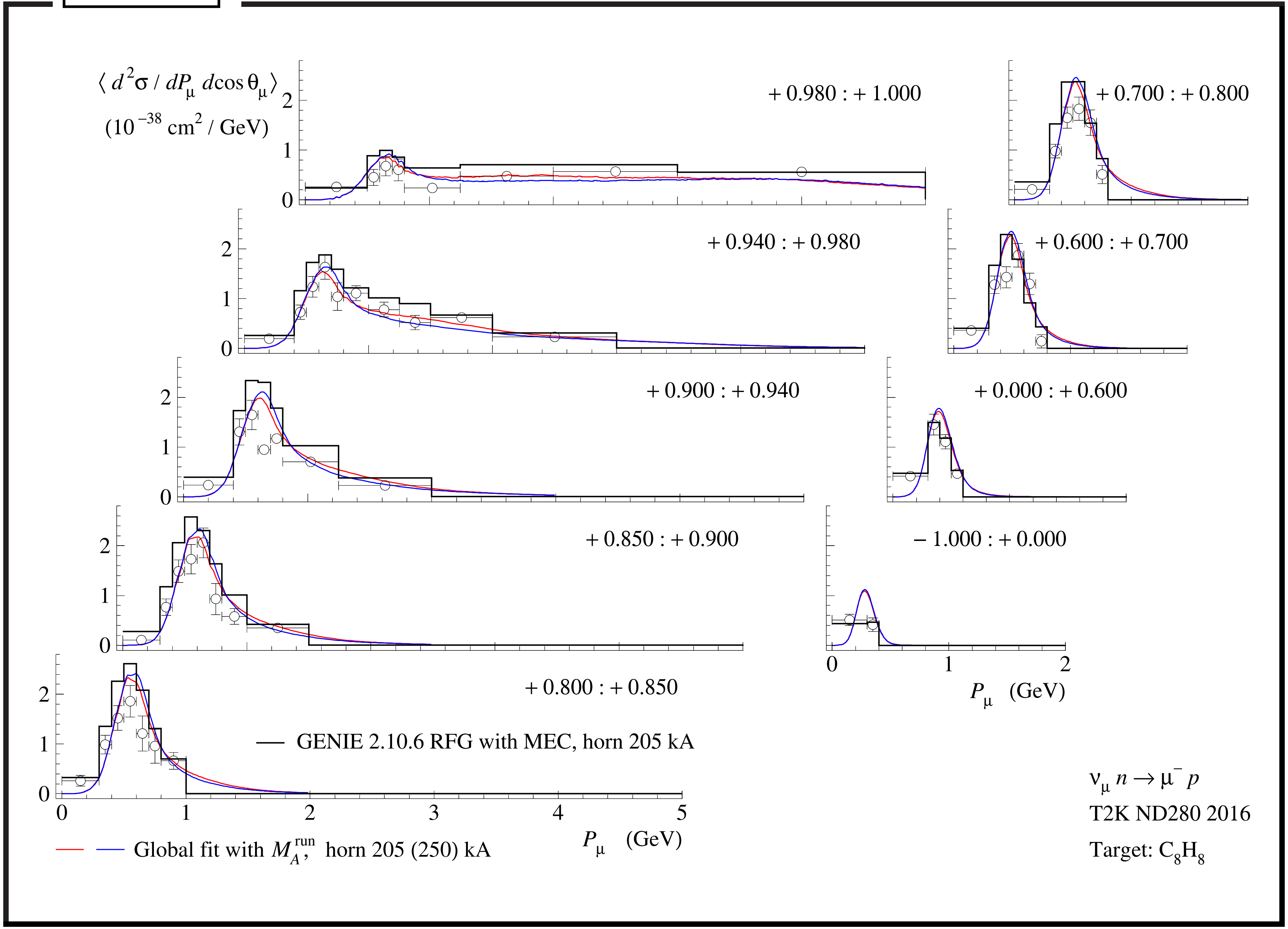
- We don't know the actual ν_μ spectrum since in several runs of the experiment, the three magnetic horns were excited by different current pulses.
- We don't know which of two data analyses (or both with certain weights) should be used in the fit.
- There is some small additional uncertainty related to complicated elemental composition of the detector target (fine-grained detector FGD1).[a] The measurement has been effectively considered as if on the FGD1 scintillator (C_8H_8). But our experience suggests that such simplification is not quite harmless...

Here we only compare our calculations with the data from Analyses 1 & 2 [69] by using the two ν_μ spectra corresponding to the horn currents of 250 and 205 kA.

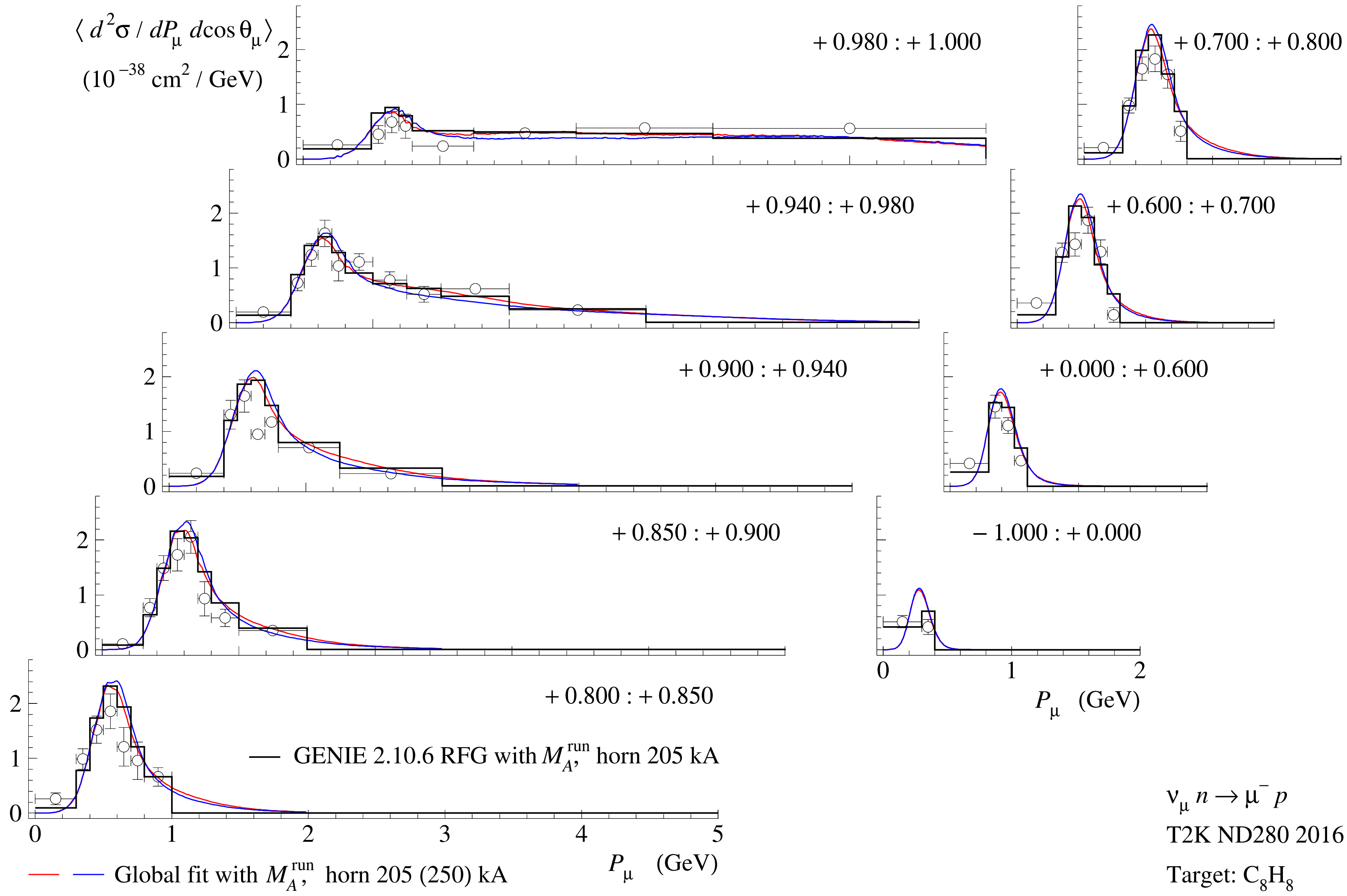
We find a reasonable (within the uncertainties) agreement between the ND280 results and those predicted by our model.

Therefore, it can be preliminary concluded that adding the T2K ND280 2016 data into the global fit would not drastically change the result.

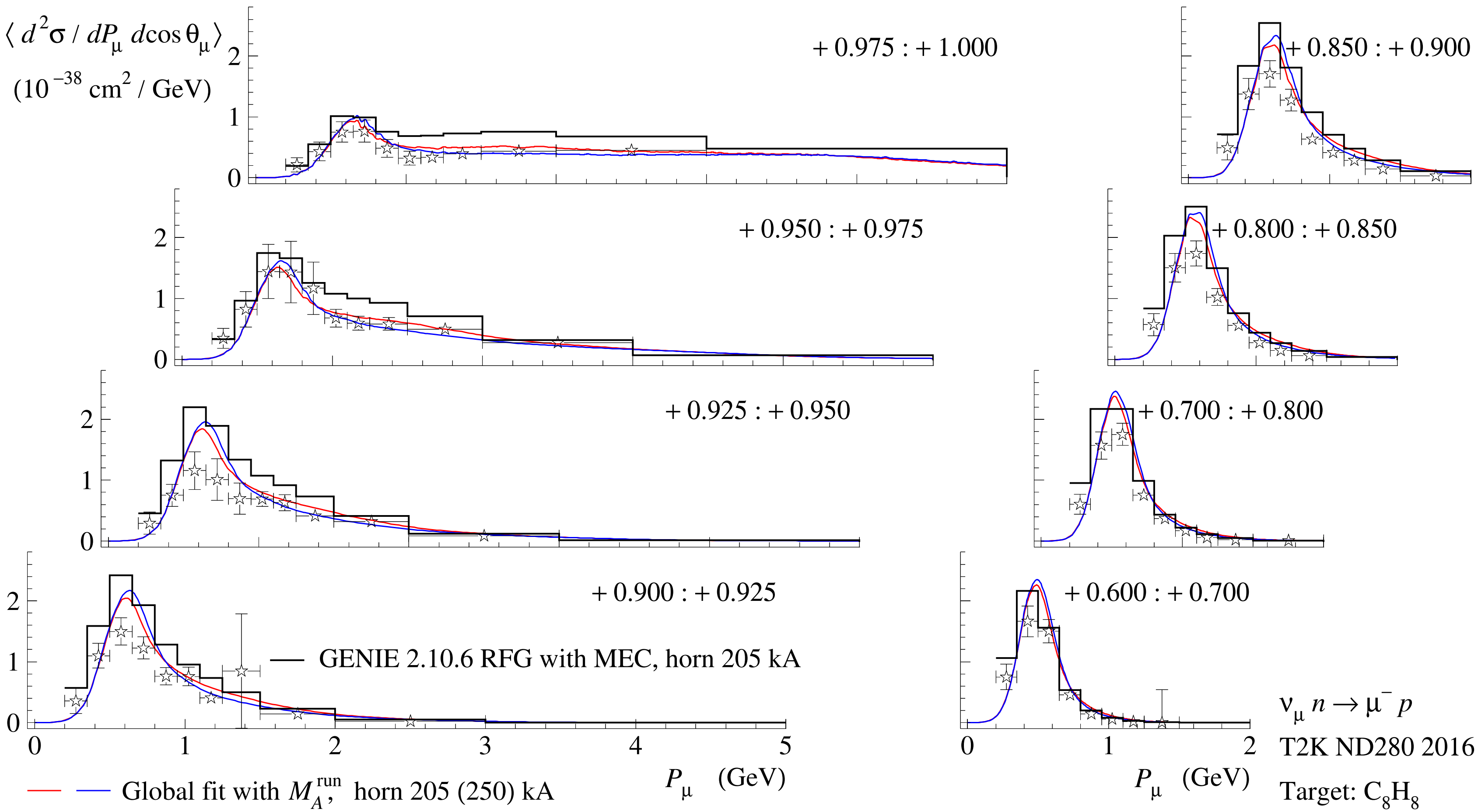
[a] The FGD1 composition: carbon (86.1%), hydrogen (7.35%), oxygen (3.70%), and small quantities of other elements such as titanium, silicon, and nitrogen (2.85%) [69].



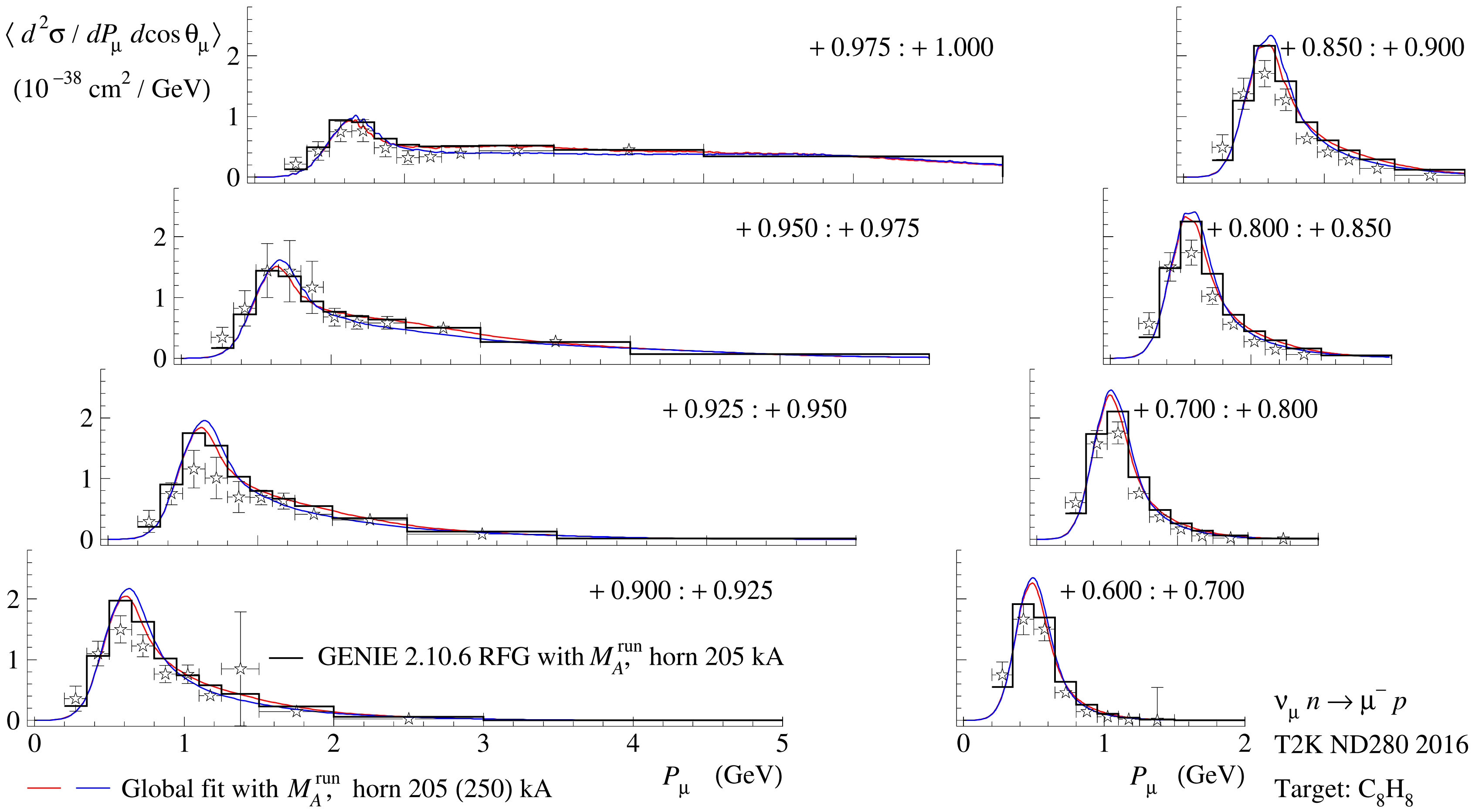
⟨ d²σ / dPμ dcos θμ ⟩
(10⁻³⁸ cm² / GeV)
+ 0.980 : + 1.000
+ 0.940 : + 0.980
+ 0.900 : + 0.940
+ 0.850 : + 0.900
+ 0.800 : + 0.850
+ 0.700 : + 0.800
+ 0.600 : + 0.700
+ 0.000 : + 0.600
− 1.000 : + 0.000
GENIE 2.10.6 RFG with MEC, horn 205 kA
Global fit with M_A^run, horn 205 (250) kA
Pμ (GeV)
νμ n → μ⁻ p
T2K ND280 2016
Target: C8H8



⟨ d²σ / dPμ dcos θμ ⟩
(10⁻³⁸ cm² / GeV)
+ 0.980 : + 1.000
+ 0.940 : + 0.980
+ 0.900 : + 0.940
+ 0.850 : + 0.900
+ 0.800 : + 0.850
+ 0.700 : + 0.800
+ 0.600 : + 0.700
+ 0.000 : + 0.600
− 1.000 : + 0.000
GENIE 2.10.6 RFG with M_A^run, horn 205 kA
Global fit with M_A^run, horn 205 (250) kA
Pμ (GeV)
νμ n → μ⁻ p
T2K ND280 2016
Target: C8H8



⟨ $d^2\sigma / dP_\mu\, d\cos\theta_\mu$ ⟩
(10^{-38} cm^2 / GeV)
+ 0.975 : + 1.000
+ 0.950 : + 0.975
+ 0.925 : + 0.950
+ 0.900 : + 0.925
+ 0.850 : + 0.900
+ 0.800 : + 0.850
+ 0.700 : + 0.800
+ 0.600 : + 0.700
GENIE 2.10.6 RFG with MEC, horn 205 kA
Global fit with M_A^{run}, horn 205 (250) kA
P_μ (GeV)
$\nu_\mu n \to \mu^- p$
T2K ND280 2016
Target: C_8H_8



⟨ d²σ / dPμ dcos θμ⟩
(10⁻³⁸ cm² / GeV)
+ 0.975 : + 1.000
+ 0.950 : + 0.975
+ 0.925 : + 0.950
+ 0.900 : + 0.925
+ 0.850 : + 0.900
+ 0.800 : + 0.850
+ 0.700 : + 0.800
+ 0.600 : + 0.700
GENIE 2.10.6 RFG with M_A^run, horn 205 kA
Global fit with M_A^run, horn 205 (250) kA
Pμ (GeV)
νμ n → μ⁻ p
T2K ND280 2016
Target: C8H8

Parameters of the SM RFG Model

Summary

The binding (separation) energies and Fermi momenta for isoscalar nuclei are calculated by using the paramerizations

$$p_F = 270[1 - 4.2/A + (6.0/A)^2 - (5.3/A)^3]$$

and

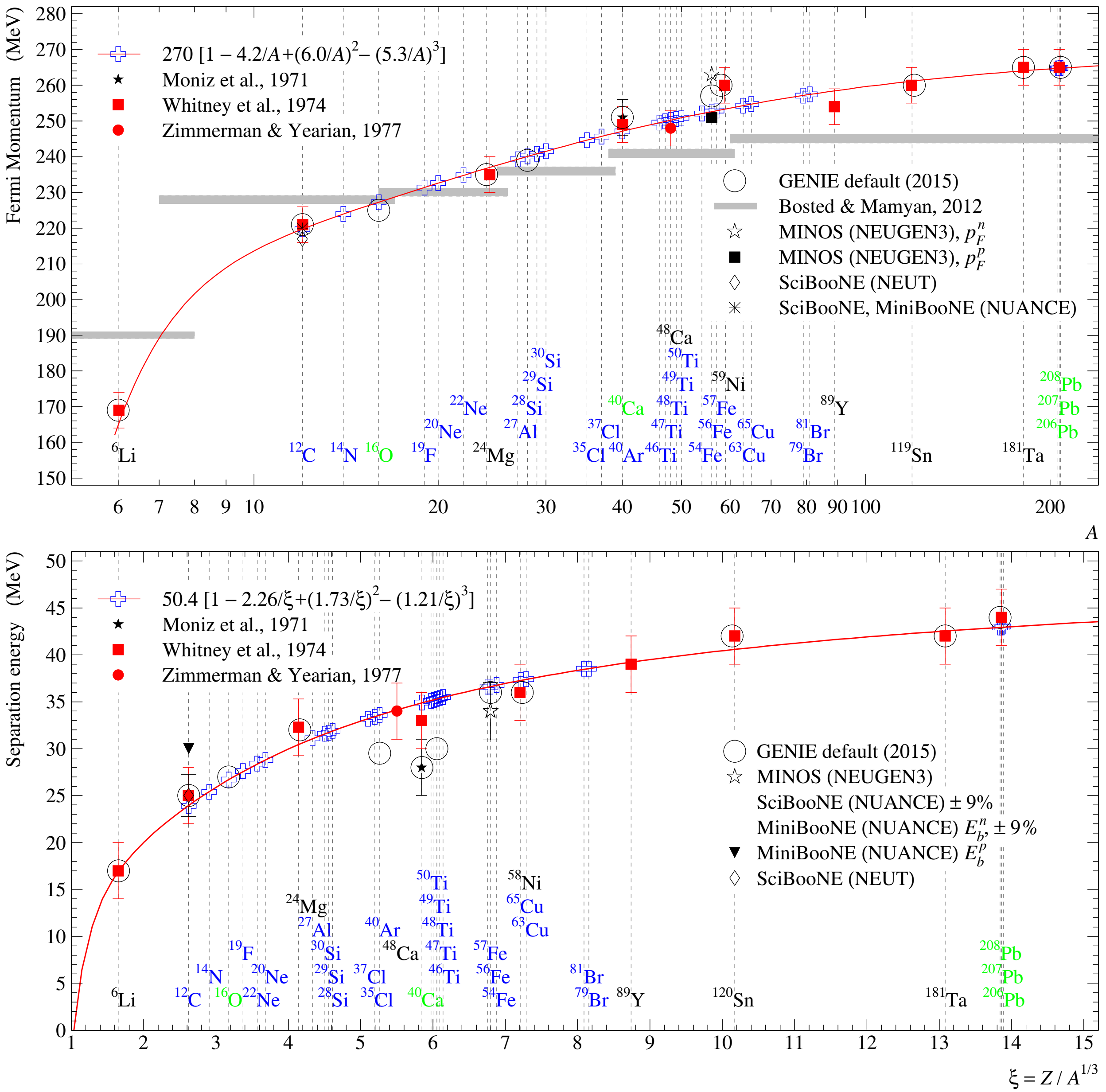
$$\varepsilon_b = 50.4 \left[1 - 2.26/\xi + (1.73/\xi)^2 - (1.21/\xi)^3\right], \quad \xi = Z/A^{1/3},$$

obtained from available (unfortunately, rather old) data on electron-nucleus scattering.[a] The proton and neutron Fermi momenta (p_F^p and p_F^n, respectively) are then calculated in the conventional way:[b]

$$p_F^p = (2Z/A)^{1/3} p_F \quad \text{and} \quad p_F^n = (2N/A)^{1/3} p_F \qquad (N = A - Z).$$

[a] E.J. Moniz, I. Sick, R.R. Whitney, J.R. Ficenec, R.D. Kephart, and W.P. Trower, *"Nuclear Fermi momenta from quasielastic electron scattering,"* Phys. Rev. Lett. **26**, 445 (1971); R.R. Whitney, I. Sick, J.R. Ficenec, R.D. Kephart and W.P. Trower, *"Quasielastic electron Scattering,"* Phys. Rev. C **9**, 2230 (1974); P.D. Zimmerman and M.R. Yearian, *"Fermi momenta and separation energies obtained from the quasi-elastic scattering of electrons from ^{48}Ca and ^{40}Ca,"* Z. Phys. A **278**, 291 (1976).

[b] These relations are based on the simplest assumption that the density of nuclear matter is approximately constant irrespective of the proton-to-neutron ratio Z/N.



Fermi Momentum (MeV)
270 [1 – 4.2/A+(6.0/A)^2– (5.3/A)^3]
Moniz et al., 1971
Whitney et al., 1974
Zimmerman & Yearian, 1977
GENIE default (2015)
Bosted & Mamyan, 2012
MINOS (NEUGEN3), p_F^n
MINOS (NEUGEN3), p_F^p
SciBooNE (NEUT)
SciBooNE, MiniBooNE (NUANCE)
A
Separation energy (MeV)
50.4 [1 – 2.26/ξ+(1.73/ξ)^2– (1.21/ξ)^3]
Moniz et al., 1971
Whitney et al., 1974
Zimmerman & Yearian, 1977
GENIE default (2015)
MINOS (NEUGEN3)
SciBooNE (NUANCE) ± 9%
MiniBooNE (NUANCE) E_b^n, ± 9%
MiniBooNE (NUANCE) E_b^p
SciBooNE (NEUT)
ξ = Z / A^1/3

“Golden” dataset for the global fits

Accountancy & taxonomy

Experiments with deuterium and hydrogen targets:
ANL 1977 [5] (σ_ν, 8 data points),
ANL 1982 [7] ($\langle dN_\nu/dQ^2\rangle$, 39 data points),
BNL 1980 [10] ($\sigma_{\overline{\nu}}$, 5 data point),
BNL 1990 [14, 15] ($\langle dN_\nu/dQ^2\rangle$, 37 data points),
FNAL 1983 [16] ($\langle dN_\nu/dQ^2\rangle$, 20 data points), and
CERN BEBC 1990 [44] ($\langle d\sigma_\nu/dQ^2\rangle$, 8 data points).

Experiment with Ne-H_2 mixture
FNAL 1984 [17] ($dN_{\overline{\nu}}/dQ^2$, 14 data points).

Experiment with aluminium target
IHEP–ITEP 1985 [51] (σ_ν, $\sigma_{\overline{\nu}}$, $\langle d\sigma_{\nu+\overline{\nu}}/dQ^2\rangle$, 8 data points in each set).

Experiments with carbonaceous targets
FNAL MiniBooNE 2010 [24] ($\langle d^2\sigma_\nu/dE_\mu d\cos\theta_\mu\rangle$, 137 data points),
FNAL MiniBooNE 2013 [25] ($\langle d^2\sigma_{\overline{\nu}}/dE_\mu d\cos\theta_\mu\rangle$, 75 data points),
CERN NOMAD 2009 [47, 48] (σ_ν and $\sigma_{\overline{\nu}}$, 10 and 6 data points, respectively), and
FNAL MINERνA 2013 [27, 28] ($\langle d\sigma_\nu/dQ^2\rangle$, $\langle d\sigma_{\overline{\nu}}/dQ^2\rangle$, 8 data points in each set).

Experiments with carbon-containing mixtures
CERN GGM 1979 [43, 45] ($\langle dN_{\overline{\nu}}/dQ^2\rangle$, 13 data points),
CERN LAr-TPC 2007 [46] (σ_ν, 1 data point),
IHEP SKAT 1990 [56] ($\langle d\sigma_\nu/dQ^2\rangle$ and $\langle d\sigma_{\overline{\nu}}/dQ^2\rangle$, 8 and 7 data points, respectively).

Experiment with iron target T2K INGRID 2015 [63] (σ_ν, 2 data points).

- Thus the fitted dataset consists of 430 data points with
  286 data points for the ν_μ cross sections (66.5% of the full dataset),
  136 data points for the $\overline{\nu}_\mu$ cross sections (31.6%), and
  8 data points for the summarized cross sections of $\nu_\mu + \overline{\nu}_\mu$ (1.9%).
- The experimental data are presented as follows:
  212 data points for the flux-averaged double differential cross sections $\langle d^2\sigma/dE_\mu d\cos\theta_\mu \rangle$ (49.3% of the full dataset, FNAL MiniBooNE),
  47 data points for the flux-averaged differential cross sections $\langle d\sigma/dQ^2 \rangle$ (10.9%),
  128 data points for the flux-averaged Q^2 distributions $\langle dN/dQ^2 \rangle$ (29.8%), and
  43 data points for the total cross sections (10.0%).
- The dataset for extracting the current axial mass M_A (deuterium and hydrogen targets) contains 85 data points (27.2% of the full dataset): ANL 1977 [5], ANL 1982 [7], BNL 1980 [10], BNL 1990 [14, 15], FNAL 1983 [16], CERN 1990 [44].

Criteria for exclusion of data from the global fit:

- Obsolete data (superseded or reconsidered due to increased statistics, revised normalization, etc. in the posterior reports of the same experimental groups).
- Data from the experiments with inactive targets (suggested by Arie Bodek).
- Data with χ^2/NDF > 5 and $|N-1| > 2\sigma_N$ (where N is the flux normalization).

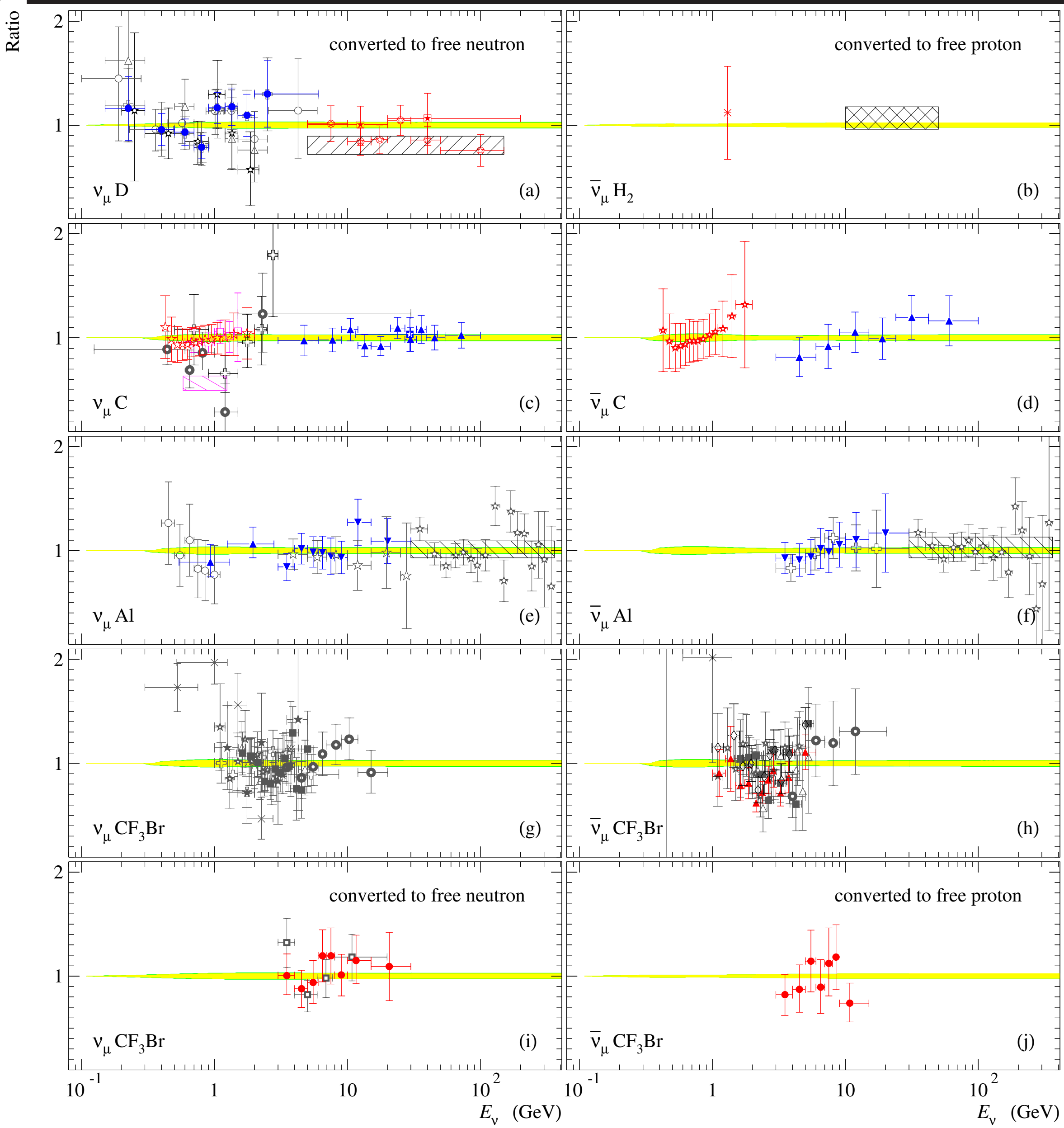
Convention: below, the **solid** symbols in the figures represent the data included into the global fit while **open** symbols represent the data excluded from the fit.

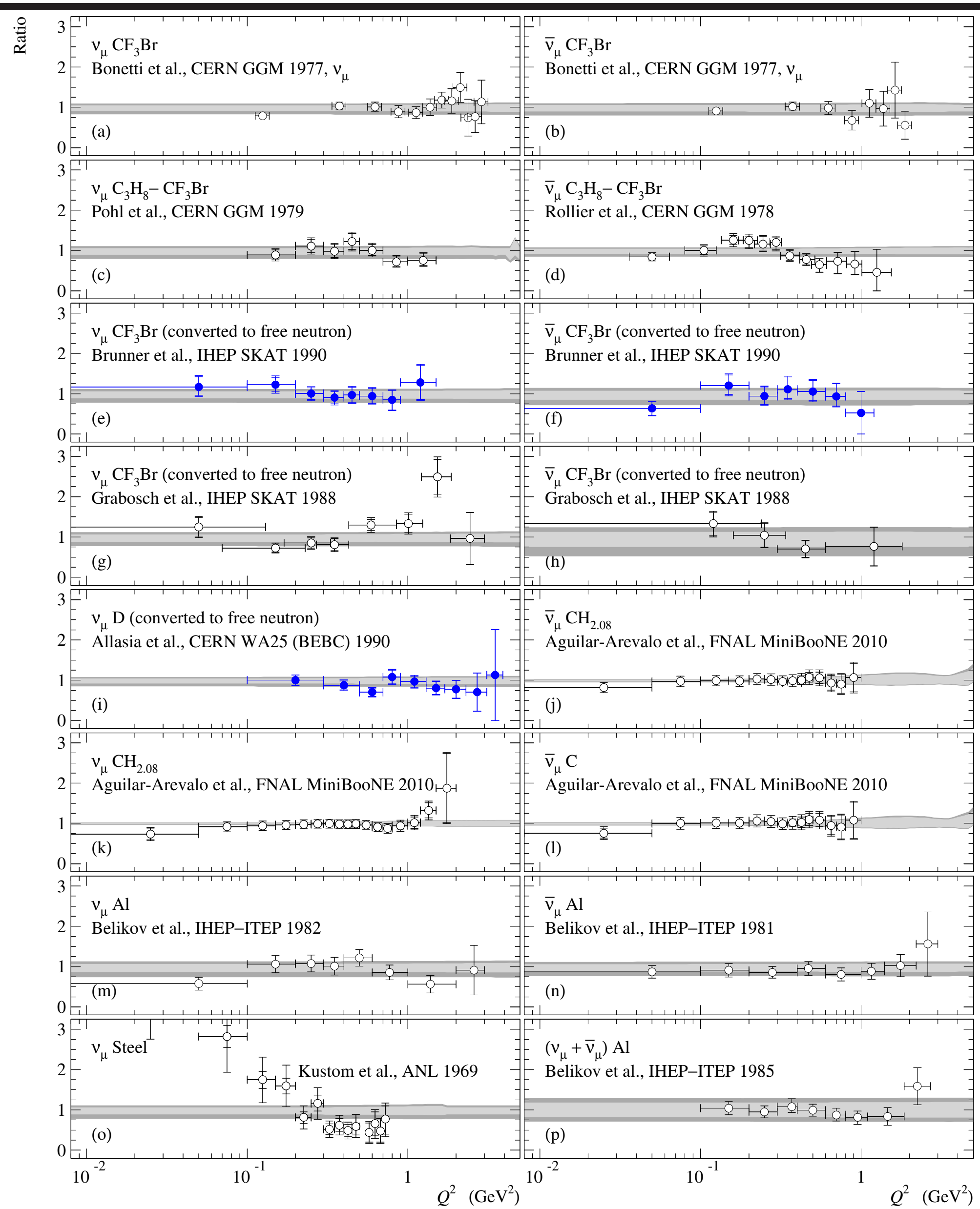
A few more details

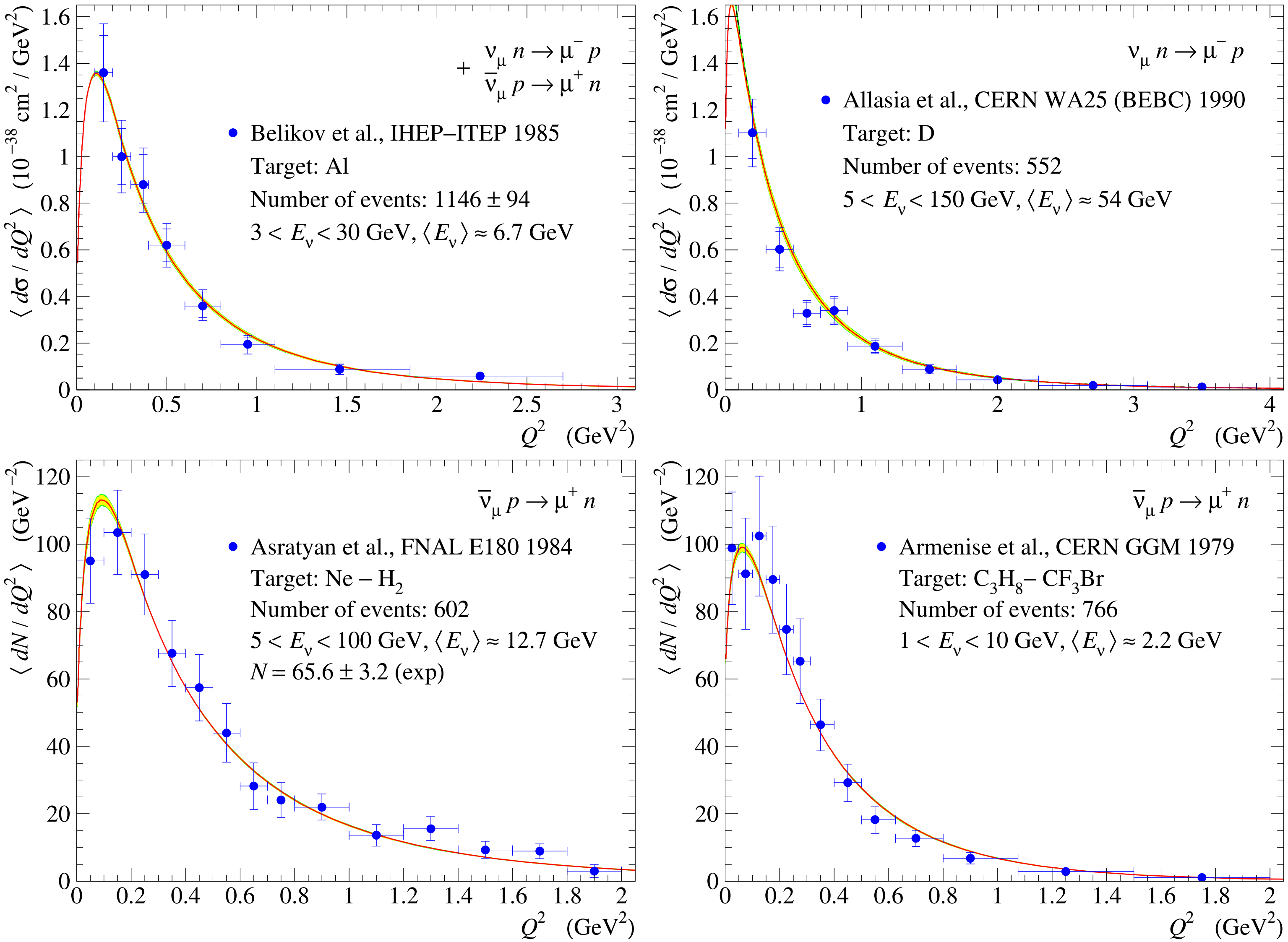
Experiment	Data type	N	χ^2/NP
Deuterium & hydrogen data and data for nuclear targets recalculated to free nucleons			
Barish et al., ANL 1977 [5]	σ_ν *	0.893 ± 0.034 (0.068)	15.97/8 = 2.00
Miller et al., ANL 1982 [7]	$\langle dN_\nu/dQ^2\rangle$ *	99.7 × 0.979	73.90/38 = 1.95
Fanourakis et al., BNL 1980 [10]	$\langle dN_{\overline{\nu}}/dQ^2\rangle$ *	2.4 × 0.901	1.37/4 = 0.34
	$\sigma_{\overline{\nu}}$	—	0.05/1 = 0.05
Kitagaki et al., BNL 1990 (2003 update) [14]	$\langle dN_\nu/dQ^2\rangle$ *	245.4 ×0.991	21.49/37 = 0.58
Kitagaki et al., FNAL 1983 [16]	$\langle dN_\nu/dQ^2\rangle$ *	41.5 × 0.945	8.53/20 = 0.43
	σ_ν	1.020 ± 0.077 (0.152)	0.18/2 = 0.09
Allasia et al., CERN BEBC 1990 [44]	$\langle d\sigma_\nu/dQ^2\rangle$ *	0.935 ± 0.035 (0.070)	4.77/8 = 0.68
	σ_ν	0.873 ± 0.032 (0.064)	15.11/6 = 3.02
Brunner et al., IHEP SKAT 1990 [55, 56]	$\langle d\sigma_\nu/dQ^2\rangle$ *	1.077 ± 0.059 (0.117)	3.89/8 = 0.49
	σ_ν	1.056 ± 0.058 (0.115)	3.14/8 = 0.39
	$\langle d\sigma_{\overline{\nu}}/dQ^2\rangle$ *	0.902 ± 0.064 (0.126)	7.31/7 = 1.04
	$\sigma_{\overline{\nu}}$	0.823 ± 0.056 (0.111)	8.10/7 = 1.15
Heavy targets data			
Asratyan et al., FNAL 1984 [17]	$\langle dN_{\overline{\nu}}/dQ^2\rangle$ *	68.7 × 0.993	7.93/14 = 0.57
	$\sigma_{\overline{\nu}}$	—	0.06/1 = 0.06
Aguilar-Arevalo et al., FNAL MiniBooNE 2010 [24]	$\langle d^2\sigma_\nu/dE_\mu d\cos\theta_\mu\rangle$ *	1.000 ± 0.008 (0.015)	35.76/137= 0.26
Aguilar-Arevalo et al., FNAL MiniBooNE 2013 [25]	$\langle d^2\sigma_{\overline{\nu}}/dE_\mu d\cos\theta_\mu\rangle$ *	1.017 ± 0.012 (0.023)	46.86/75 = 0.63
Fiorentini et al., FNAL MINERνA 2013 [27]	$\langle d\sigma_\nu/dQ^2\rangle$ *	—	8.04/8 = 1.01
Fields et al., FNAL MINERνA 2013 [28]	$\langle d\sigma_{\overline{\nu}}/dQ^2\rangle$ *	—	5.51/8 = 0.67
Armenise et al., CERN GGM 1979 [43, 45]	$\sigma_{\overline{\nu}}$	0.781 ± 0.031 (0.062)	20.57/11 = 1.87
Martinez de la Ossa Romero et al., CERN LAr-TPC 2007 [46]	σ_ν *	—	0.09/1 = 0.09
Lyubushkin et al., CERN NOMAD 2009 [47, 48]	σ_ν *	1.036 ± 0.026 (0.052)	5.91/10 = 0.59
	$\sigma_{\overline{\nu}}$ *	1.016 ± 0.058 (0.114)	3.09/6 = 0.52
Belikov et al., IHEP–ITEP 1985 [51]	$\langle d\sigma_{\nu+\overline{\nu}}/dQ^2\rangle$ *	0.975 ± 0.114 (0.184)	5.68/8 = 0.71
		0.988 ± 0.131 (0.212)	
	σ_ν *	0.873 ± 0.036 (0.071)	7.02/8 = 0.88
	$\sigma_{\overline{\nu}}$ *	0.897 ± 0.045 (0.089)	2.66/8 = 0.33
Abe et al., T2K ND280 2014 [61]	σ_ν *	—	9.57/5 = 2.39
Abe et al., T2K INGRID 2015 [63]	σ_ν *	0.977 ± 0.086 (0.170)	0.55/2 = 0.28

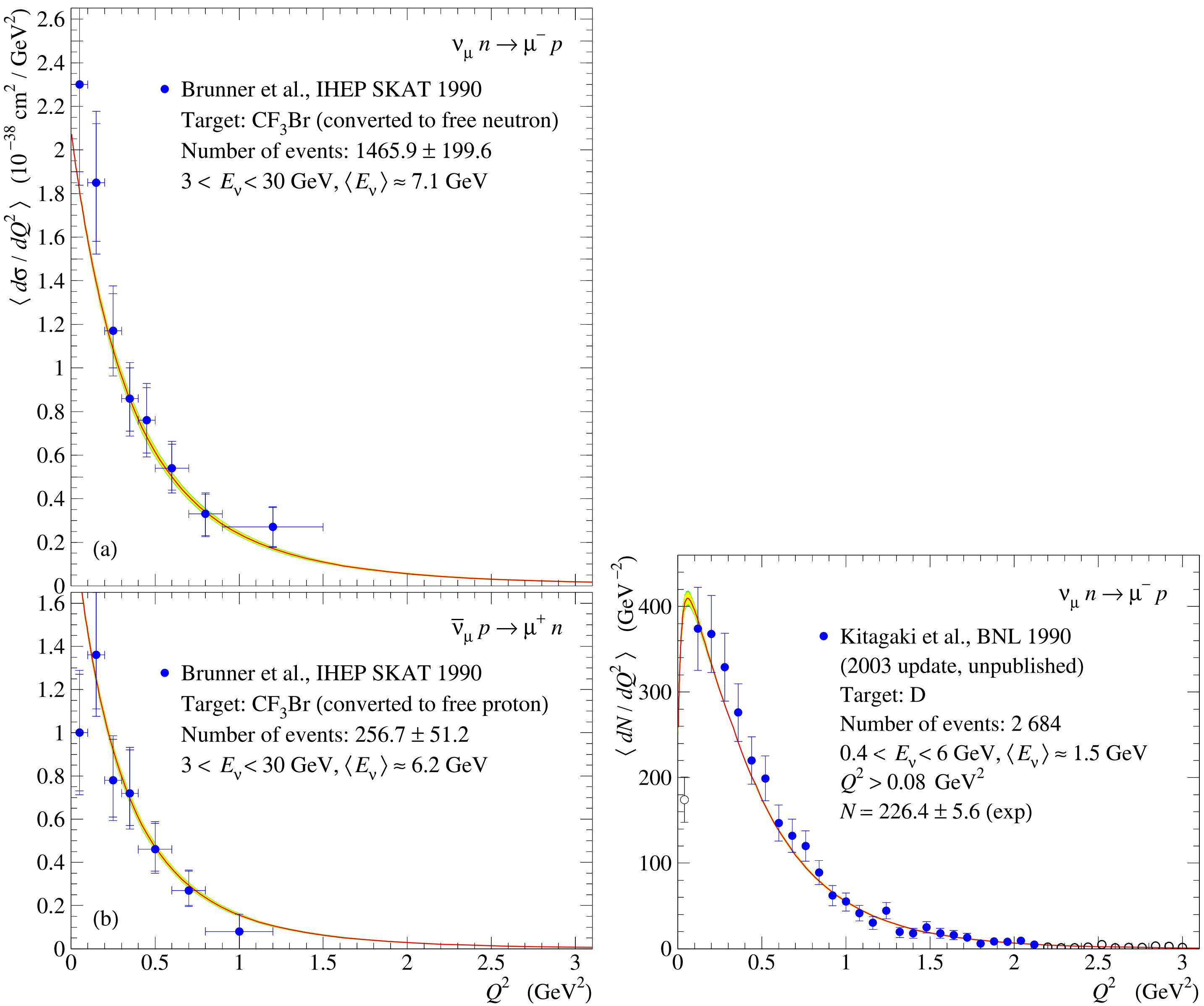
* The data included into the global fits.

Comparison with the early golden data

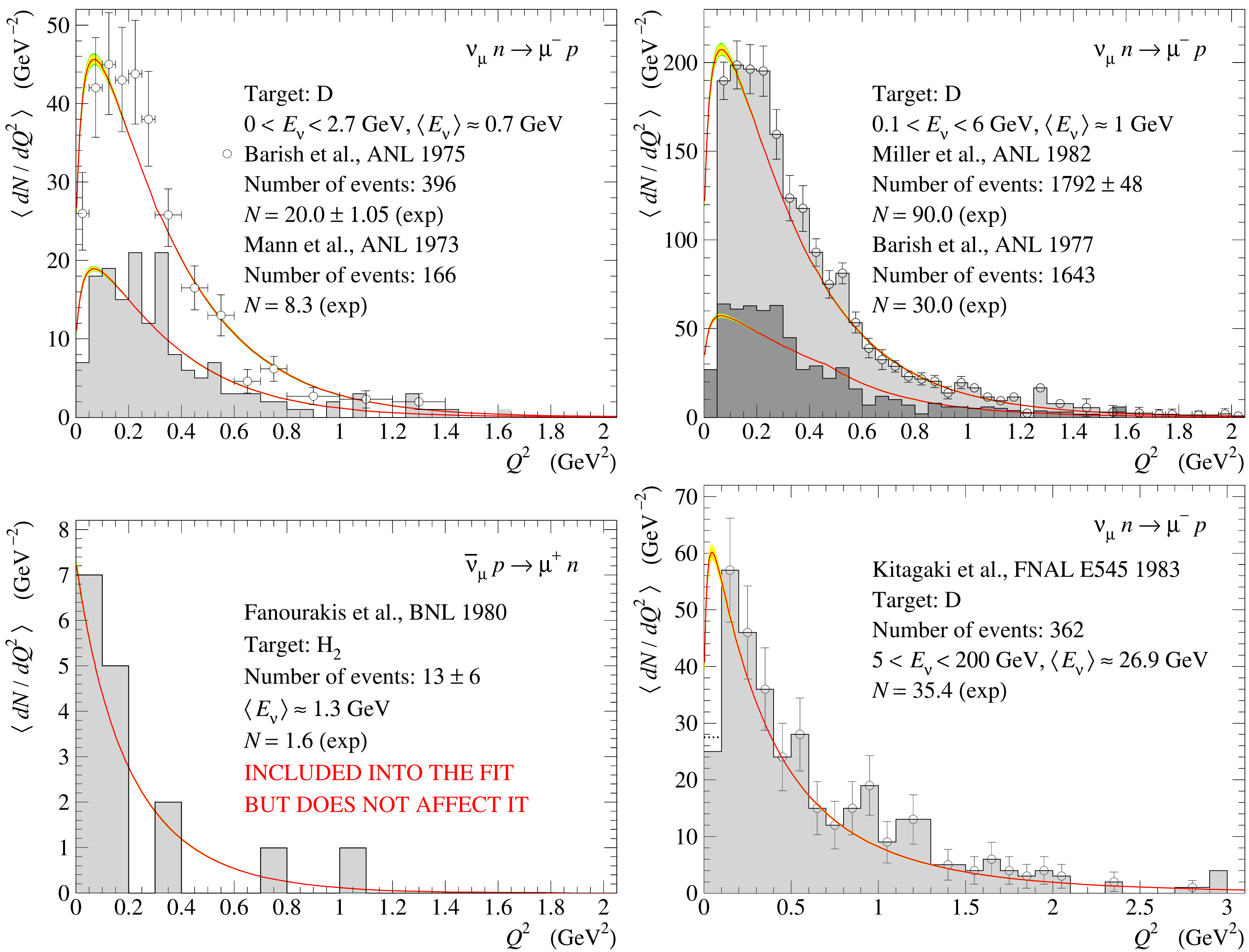

Ratio
converted to free neutron
converted to free proton
ν_μ D
(a)
$\bar{\nu}_\mu$ H_2
(b)
ν_μ C
(c)
$\bar{\nu}_\mu$ C
(d)
ν_μ Al
(e)
$\bar{\nu}_\mu$ Al
(f)
ν_μ CF_3Br
(g)
$\bar{\nu}_\mu$ CF_3Br
(h)
converted to free neutron
converted to free proton
ν_μ CF_3Br
(i)
$\bar{\nu}_\mu$ CF_3Br
(j)
E_ν (GeV)
E_ν (GeV)

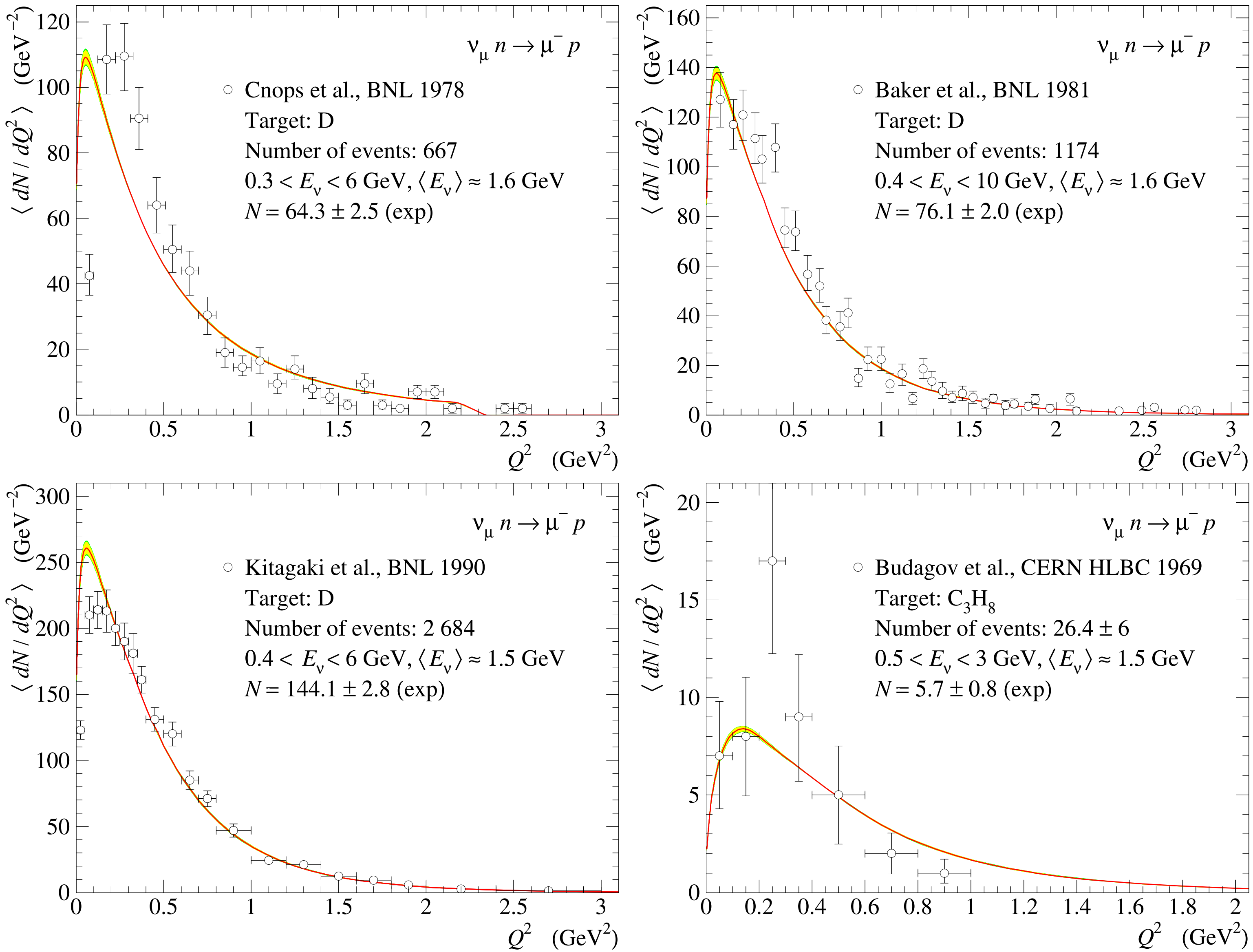

Ratio
νμ CF3Br
Bonetti et al., CERN GGM 1977, νμ
(a)
ν̄μ CF3Br
Bonetti et al., CERN GGM 1977, νμ
(b)
νμ C3H8– CF3Br
Pohl et al., CERN GGM 1979
(c)
ν̄μ C3H8– CF3Br
Rollier et al., CERN GGM 1978
(d)
νμ CF3Br (converted to free neutron)
Brunner et al., IHEP SKAT 1990
(e)
ν̄μ CF3Br (converted to free neutron)
Brunner et al., IHEP SKAT 1990
(f)
νμ CF3Br (converted to free neutron)
Grabosch et al., IHEP SKAT 1988
(g)
ν̄μ CF3Br (converted to free neutron)
Grabosch et al., IHEP SKAT 1988
(h)
νμ D (converted to free neutron)
Allasia et al., CERN WA25 (BEBC) 1990
(i)
ν̄μ CH2.08
Aguilar-Arevalo et al., FNAL MiniBooNE 2010
(j)
νμ CH2.08
Aguilar-Arevalo et al., FNAL MiniBooNE 2010
(k)
ν̄μ C
Aguilar-Arevalo et al., FNAL MiniBooNE 2010
(l)
νμ Al
Belikov et al., IHEP–ITEP 1982
(m)
ν̄μ Al
Belikov et al., IHEP–ITEP 1981
(n)
νμ Steel
Kustom et al., ANL 1969
(o)
(νμ + ν̄μ) Al
Belikov et al., IHEP–ITEP 1985
(p)
10^-2
10^-1
1
Q^2 (GeV^2)

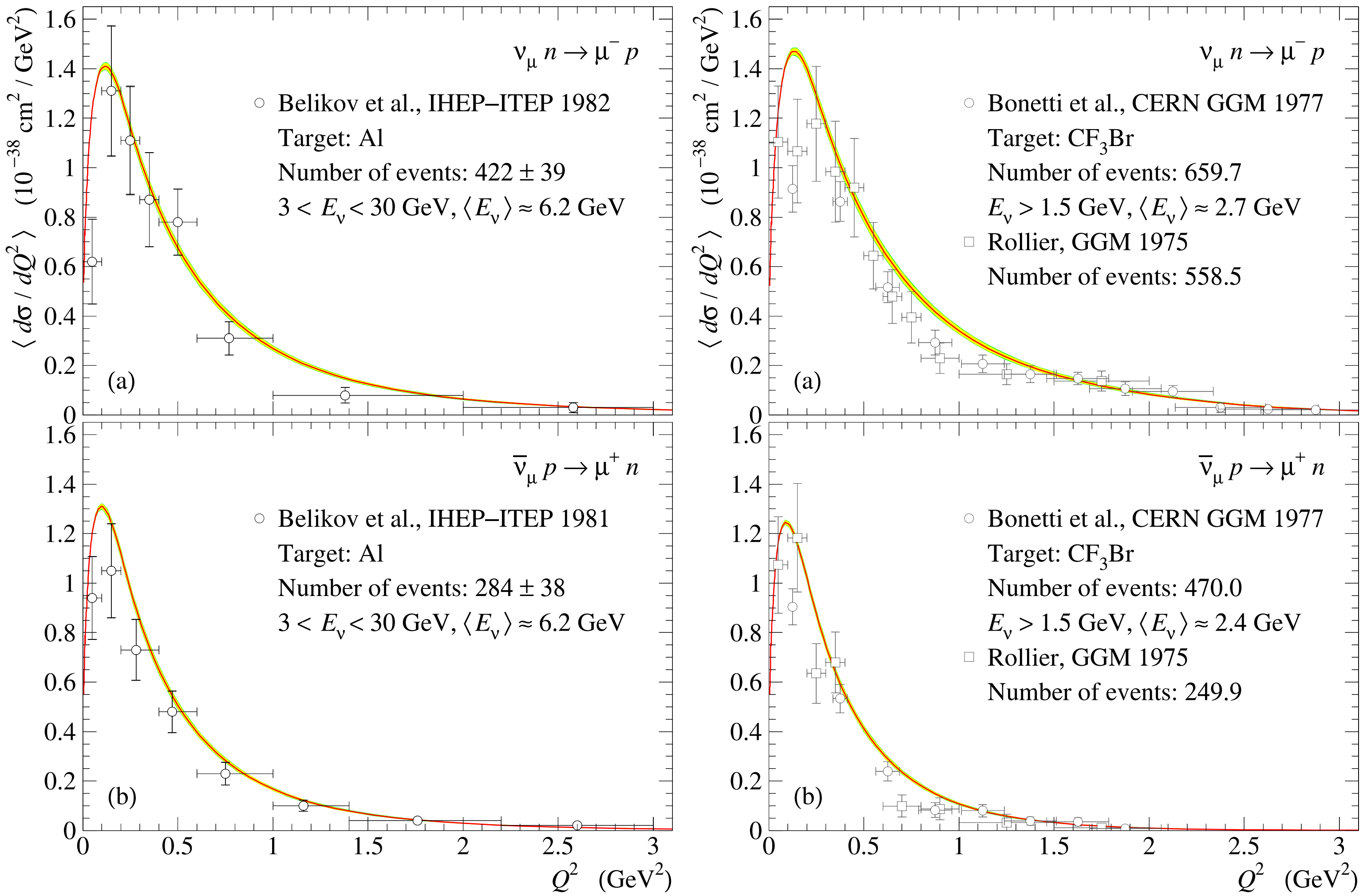

$\nu_\mu n \to \mu^- p$ + $\bar{\nu}_\mu p \to \mu^+ n$
Belikov et al., IHEP–ITEP 1985
Target: Al
Number of events: 1146 ± 94
$3 < E_\nu < 30$ GeV, $\langle E_\nu \rangle \approx 6.7$ GeV
$\langle d\sigma / dQ^2 \rangle$ (10^{-38} cm^2 / GeV^2)
Q^2 (GeV^2)
$\nu_\mu n \to \mu^- p$
Allasia et al., CERN WA25 (BEBC) 1990
Target: D
Number of events: 552
$5 < E_\nu < 150$ GeV, $\langle E_\nu \rangle \approx 54$ GeV
$\langle d\sigma / dQ^2 \rangle$ (10^{-38} cm^2 / GeV^2)
Q^2 (GeV^2)
$\bar{\nu}_\mu p \to \mu^+ n$
Asratyan et al., FNAL E180 1984
Target: Ne – H_2
Number of events: 602
$5 < E_\nu < 100$ GeV, $\langle E_\nu \rangle \approx 12.7$ GeV
$N = 65.6 \pm 3.2$ (exp)
$\langle dN / dQ^2 \rangle$ (GeV^{-2})
Q^2 (GeV^2)
$\bar{\nu}_\mu p \to \mu^+ n$
Armenise et al., CERN GGM 1979
Target: C_3H_8– CF_3Br
Number of events: 766
$1 < E_\nu < 10$ GeV, $\langle E_\nu \rangle \approx 2.2$ GeV
$\langle dN / dQ^2 \rangle$ (GeV^{-2})
Q^2 (GeV^2)

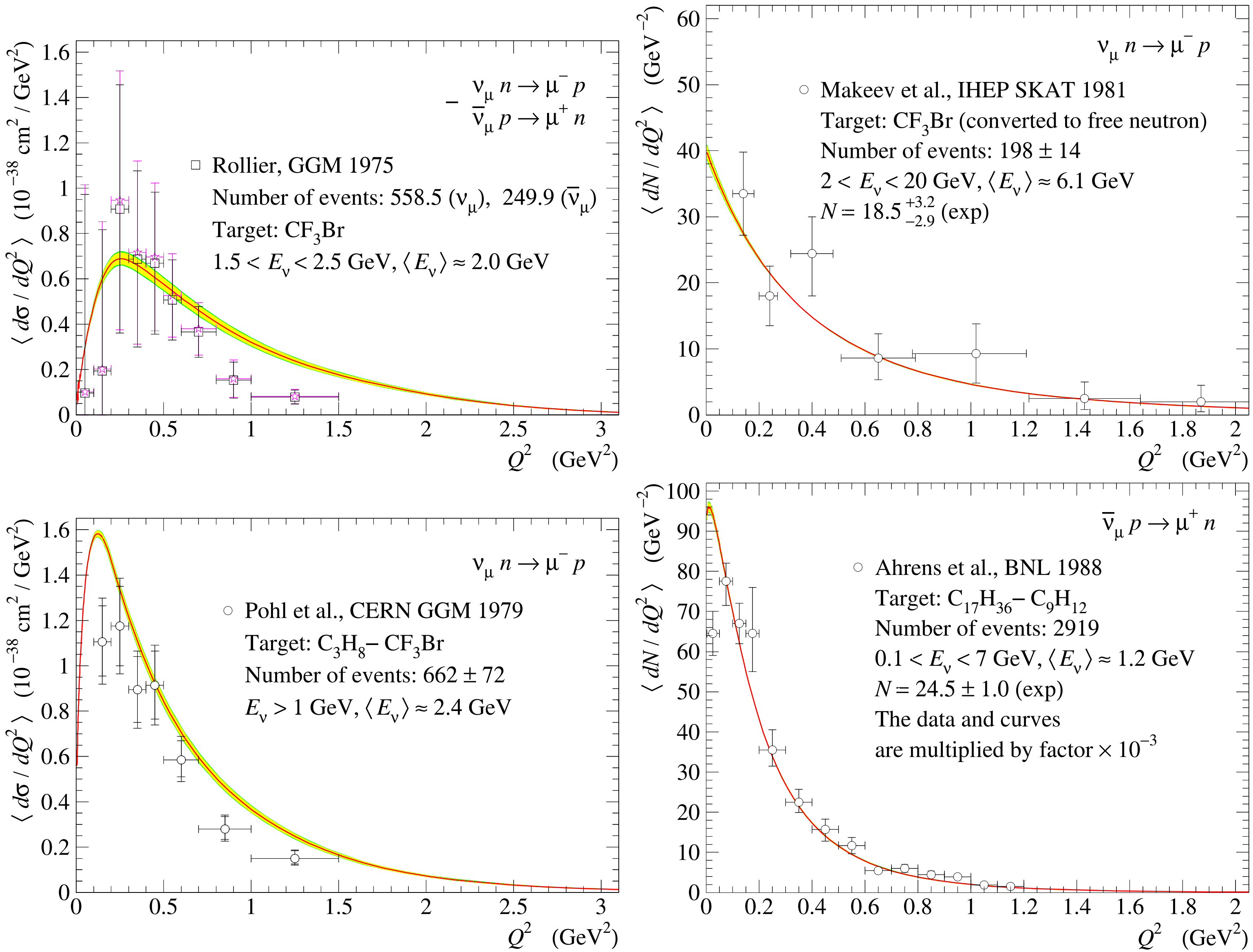

$\langle d\sigma / dQ^2 \rangle$ (10^{-38} cm^2 / GeV^2)
$\nu_\mu n \to \mu^- p$
Brunner et al., IHEP SKAT 1990
Target: CF_3Br (converted to free neutron)
Number of events: 1465.9 ± 199.6
$3 < E_\nu < 30$ GeV, $\langle E_\nu \rangle \approx 7.1$ GeV
(a)
$\bar{\nu}_\mu p \to \mu^+ n$
Brunner et al., IHEP SKAT 1990
Target: CF_3Br (converted to free proton)
Number of events: 256.7 ± 51.2
$3 < E_\nu < 30$ GeV, $\langle E_\nu \rangle \approx 6.2$ GeV
(b)
Q^2 (GeV^2)
$\langle dN / dQ^2 \rangle$ (GeV^{-2})
$\nu_\mu n \to \mu^- p$
Kitagaki et al., BNL 1990
(2003 update, unpublished)
Target: D
Number of events: 2 684
$0.4 < E_\nu < 6$ GeV, $\langle E_\nu \rangle \approx 1.5$ GeV
$Q^2 > 0.08$ GeV^2
$N = 226.4 \pm 5.6$ (exp)
Q^2 (GeV^2)

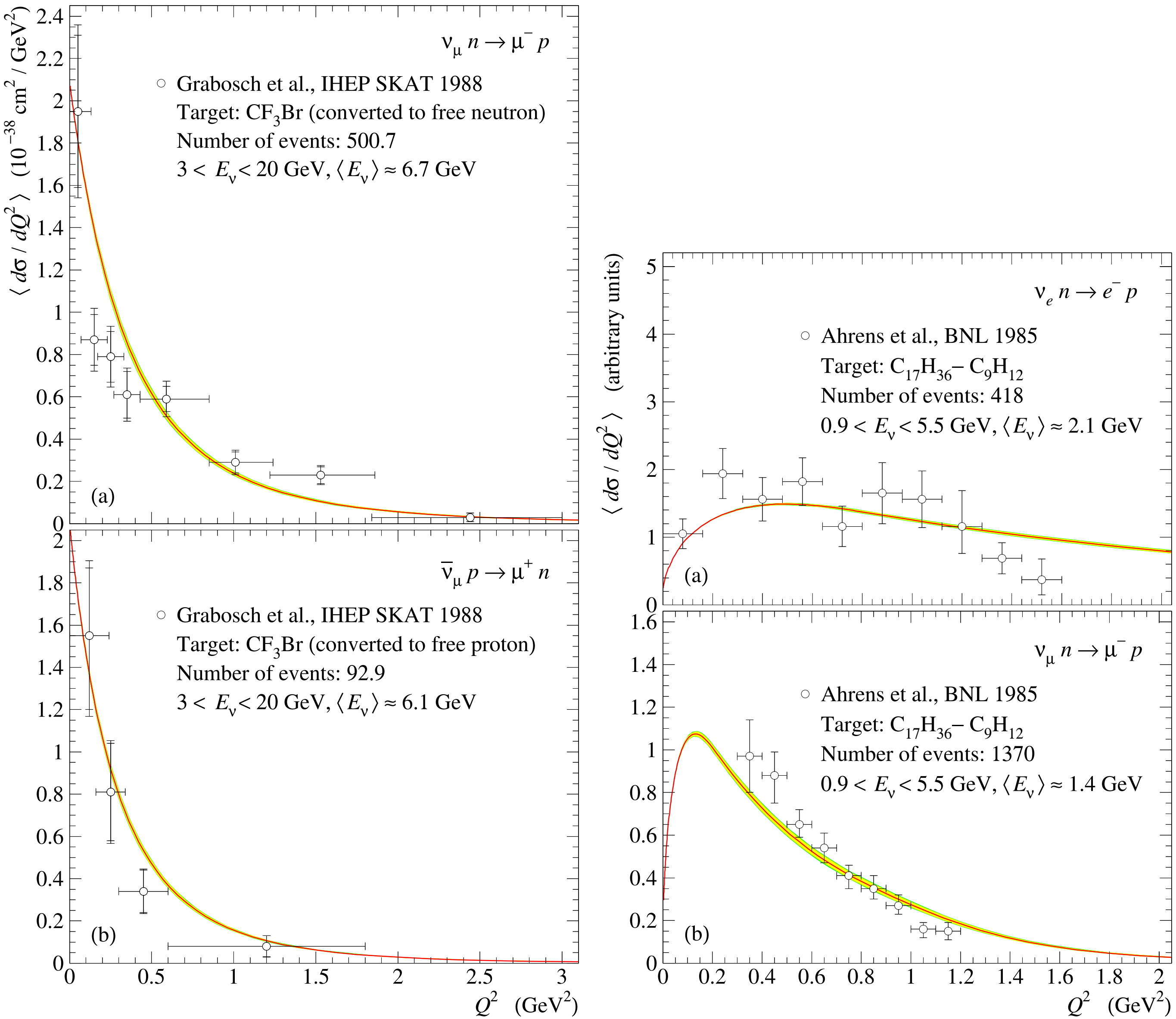
Comparison with certain datasets unclaimed in the global fit

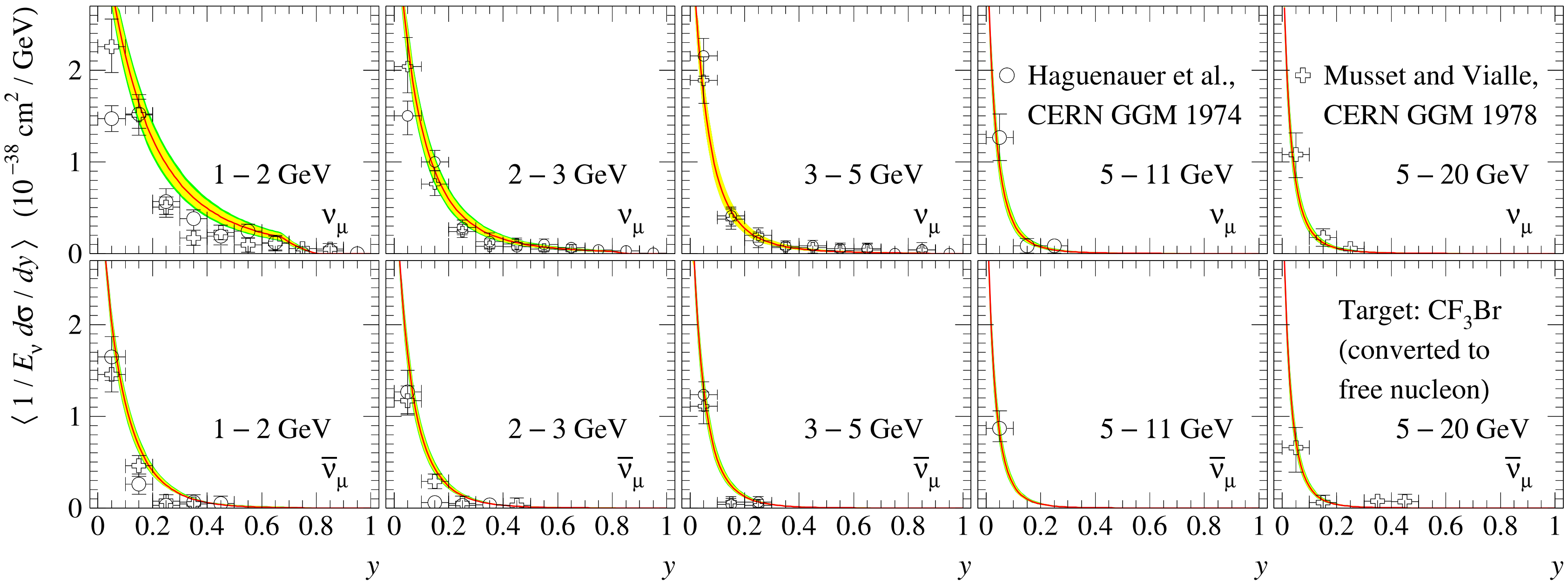

νμ n → μ⁻ p
Target: D
0 < Eν < 2.7 GeV, ⟨Eν⟩ ≈ 0.7 GeV
Barish et al., ANL 1975
Number of events: 396
N = 20.0 ± 1.05 (exp)
Mann et al., ANL 1973
Number of events: 166
N = 8.3 (exp)
⟨dN / dQ²⟩ (GeV⁻²)
Q² (GeV²)
νμ n → μ⁻ p
Target: D
0.1 < Eν < 6 GeV, ⟨Eν⟩ ≈ 1 GeV
Miller et al., ANL 1982
Number of events: 1792 ± 48
N = 90.0 (exp)
Barish et al., ANL 1977
Number of events: 1643
N = 30.0 (exp)
⟨dN / dQ²⟩ (GeV⁻²)
Q² (GeV²)
ν̄μ p → μ⁺ n
Fanourakis et al., BNL 1980
Target: H2
Number of events: 13 ± 6
⟨Eν⟩ ≈ 1.3 GeV
N = 1.6 (exp)
INCLUDED INTO THE FIT
BUT DOES NOT AFFECT IT
⟨dN / dQ²⟩ (GeV⁻²)
Q² (GeV²)
νμ n → μ⁻ p
Kitagaki et al., FNAL E545 1983
Target: D
Number of events: 362
5 < Eν < 200 GeV, ⟨Eν⟩ ≈ 26.9 GeV
N = 35.4 (exp)
⟨dN / dQ²⟩ (GeV⁻²)
Q² (GeV²)


νμ n → μ⁻ p
Cnops et al., BNL 1978
Target: D
Number of events: 667
0.3 < Eν < 6 GeV, ⟨Eν⟩ ≈ 1.6 GeV
N = 64.3 ± 2.5 (exp)
νμ n → μ⁻ p
Baker et al., BNL 1981
Target: D
Number of events: 1174
0.4 < Eν < 10 GeV, ⟨Eν⟩ ≈ 1.6 GeV
N = 76.1 ± 2.0 (exp)
νμ n → μ⁻ p
Kitagaki et al., BNL 1990
Target: D
Number of events: 2 684
0.4 < Eν < 6 GeV, ⟨Eν⟩ ≈ 1.5 GeV
N = 144.1 ± 2.8 (exp)
νμ n → μ⁻ p
Budagov et al., CERN HLBC 1969
Target: C3H8
Number of events: 26.4 ± 6
0.5 < Eν < 3 GeV, ⟨Eν⟩ ≈ 1.5 GeV
N = 5.7 ± 0.8 (exp)
⟨dN / dQ²⟩ (GeV⁻²)
Q² (GeV²)

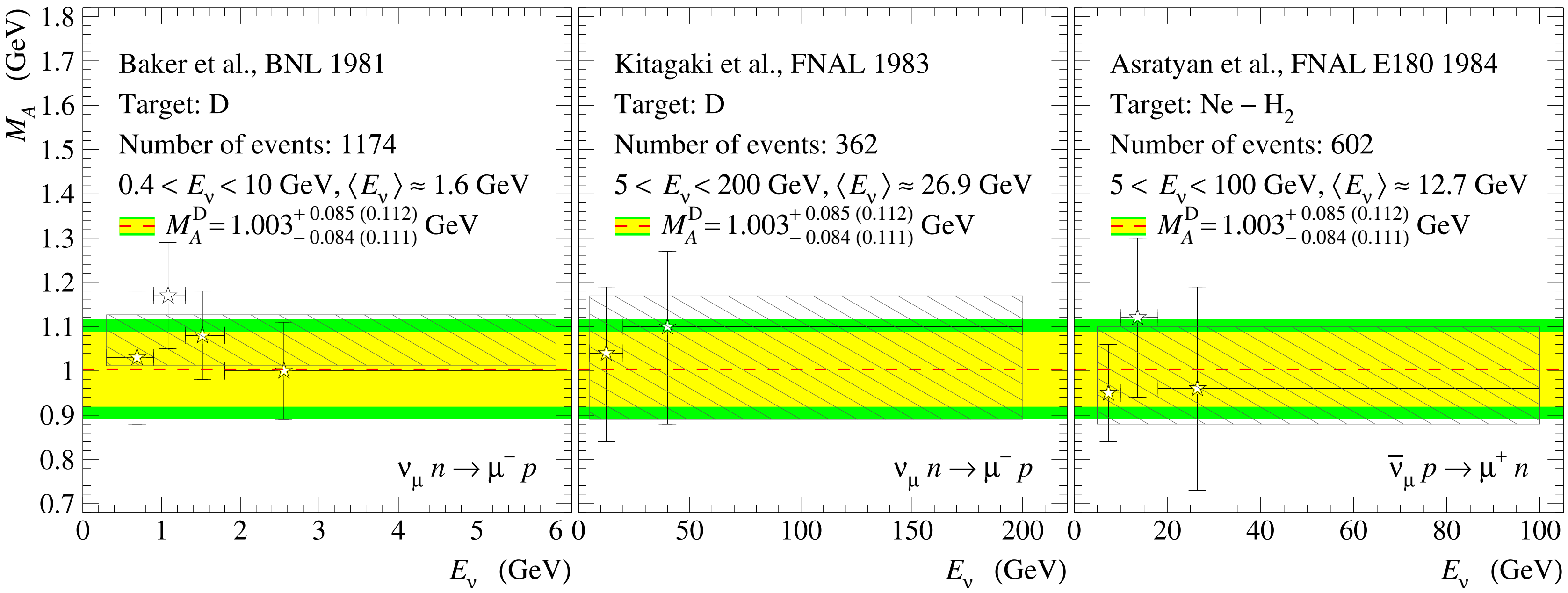

$\nu_\mu\, n \to \mu^-\, p$
Belikov et al., IHEP–ITEP 1982
Target: Al
Number of events: 422 ± 39
$3 < E_\nu < 30$ GeV, $\langle E_\nu \rangle \approx 6.2$ GeV
(a)
$\nu_\mu\, n \to \mu^-\, p$
Bonetti et al., CERN GGM 1977
Target: CF_3Br
Number of events: 659.7
$E_\nu > 1.5$ GeV, $\langle E_\nu \rangle \approx 2.7$ GeV
Rollier, GGM 1975
Number of events: 558.5
(a)
$\bar{\nu}_\mu\, p \to \mu^+\, n$
Belikov et al., IHEP–ITEP 1981
Target: Al
Number of events: 284 ± 38
$3 < E_\nu < 30$ GeV, $\langle E_\nu \rangle \approx 6.2$ GeV
(b)
$\bar{\nu}_\mu\, p \to \mu^+\, n$
Bonetti et al., CERN GGM 1977
Target: CF_3Br
Number of events: 470.0
$E_\nu > 1.5$ GeV, $\langle E_\nu \rangle \approx 2.4$ GeV
Rollier, GGM 1975
Number of events: 249.9
(b)
$\langle d\sigma / dQ^2 \rangle$ (10^{-38} cm^2 / GeV^2)
Q^2 (GeV^2)


$\nu_\mu n \to \mu^- p$
$\bar{\nu}_\mu p \to \mu^+ n$
Rollier, GGM 1975
Number of events: 558.5 (ν_μ), 249.9 ($\bar{\nu}_\mu$)
Target: CF_3Br
$1.5 < E_\nu < 2.5$ GeV, $\langle E_\nu \rangle \approx 2.0$ GeV
$\langle d\sigma / dQ^2 \rangle$ (10^{-38} cm^2 / GeV^2)
Q^2 (GeV^2)
$\nu_\mu n \to \mu^- p$
Makeev et al., IHEP SKAT 1981
Target: CF_3Br (converted to free neutron)
Number of events: 198 ± 14
$2 < E_\nu < 20$ GeV, $\langle E_\nu \rangle \approx 6.1$ GeV
$N = 18.5^{+3.2}_{-2.9}$ (exp)
$\langle dN / dQ^2 \rangle$ (GeV^{-2})
Q^2 (GeV^2)
$\nu_\mu n \to \mu^- p$
Pohl et al., CERN GGM 1979
Target: $C_3H_8 - CF_3Br$
Number of events: 662 ± 72
$E_\nu > 1$ GeV, $\langle E_\nu \rangle \approx 2.4$ GeV
$\langle d\sigma / dQ^2 \rangle$ (10^{-38} cm^2 / GeV^2)
Q^2 (GeV^2)
$\bar{\nu}_\mu p \to \mu^+ n$
Ahrens et al., BNL 1988
Target: $C_{17}H_{36} - C_9H_{12}$
Number of events: 2919
$0.1 < E_\nu < 7$ GeV, $\langle E_\nu \rangle \approx 1.2$ GeV
$N = 24.5 \pm 1.0$ (exp)
The data and curves
are multiplied by factor $\times 10^{-3}$
$\langle dN / dQ^2 \rangle$ (GeV^{-2})
Q^2 (GeV^2)


$\langle d\sigma / dQ^2 \rangle$ (10^{-38} cm^2 / GeV^2)
$\nu_\mu n \to \mu^- p$
Grabosch et al., IHEP SKAT 1988
Target: CF_3Br (converted to free neutron)
Number of events: 500.7
$3 < E_\nu < 20$ GeV, $\langle E_\nu \rangle \approx 6.7$ GeV
(a)
$\bar{\nu}_\mu p \to \mu^+ n$
Grabosch et al., IHEP SKAT 1988
Target: CF_3Br (converted to free proton)
Number of events: 92.9
$3 < E_\nu < 20$ GeV, $\langle E_\nu \rangle \approx 6.1$ GeV
(b)
Q^2 (GeV^2)
$\langle d\sigma / dQ^2 \rangle$ (arbitrary units)
$\nu_e n \to e^- p$
Ahrens et al., BNL 1985
Target: $C_{17}H_{36} - C_9H_{12}$
Number of events: 418
$0.9 < E_\nu < 5.5$ GeV, $\langle E_\nu \rangle \approx 2.1$ GeV
(a)
$\nu_\mu n \to \mu^- p$
Ahrens et al., BNL 1985
Target: $C_{17}H_{36} - C_9H_{12}$
Number of events: 1370
$0.9 < E_\nu < 5.5$ GeV, $\langle E_\nu \rangle \approx 1.4$ GeV
(b)
Q^2 (GeV^2)



Flux-weighted differential cross sections for $\nu_\mu n \to \mu^- p$ (top) and $\overline{\nu}_\mu p \to \mu^+ n$ (bottom). The cross sections were measured in the Gargamelle bubble chamber filled with heavy liquid freon and exposed to the CERN-PS ν_μ and $\overline{\nu}_\mu$ beams.
The shaded regions around the solid curves represent 1σ and 2σ deviations from the best-fitted values of M_A^{run} obtained in the global fit. Only the events with $E_{\nu,\overline{\nu}} > 1.5$ GeV were accepted.
The energy range for each experiment is shown in the legends.
Obsolete GGM 1975 data are also shown for completeness.

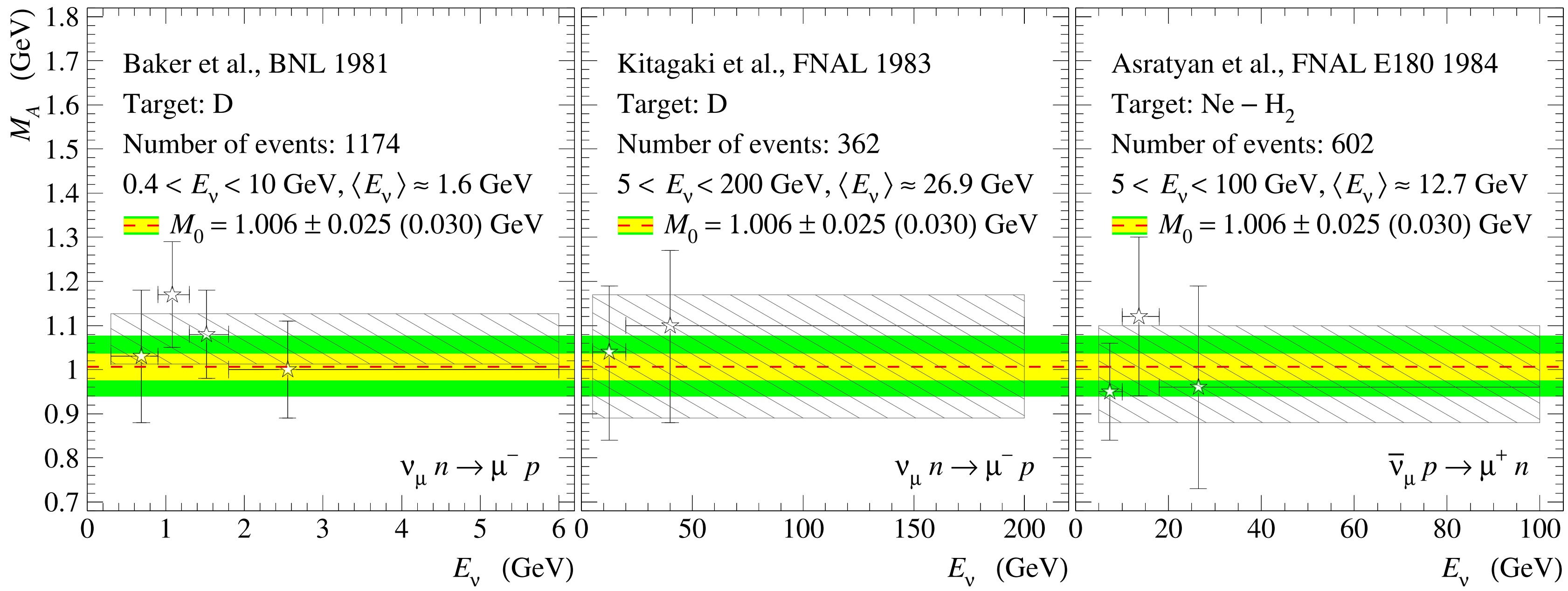


Current axial mass, M_A vs. neutrino energy measured for the $\nu_\mu n \to \mu^- p$ reaction with the deuterium filled detector at BNL [11]. Rectangle denotes the average value of M_A obtained by the authors of the experiment for the mean neutrino energy $\langle E_\nu \rangle = 1.48$ GeV.

Current axial mass, M_A vs. neutrino energy measured for the $\nu_\mu n \to \mu^- p$ reaction with the deuterium filled detector at BNL [16]. Rectangle denotes the average value of M_A obtained by the authors of the experiment for the whole neutrino energy range at the mean energy of $\langle E_\nu \rangle = 26.9$ GeV.

Current axial mass, M_A vs. antineutrino energy measured for the $\overline{\nu}_\mu p \to \mu^+ n$ reaction with the Ne-H_2 filled detector in the FNAL experiment E180 [17, 18]. Dashed rectangle denotes the average value of M_A obtained by the authors of the experiment for the whole energy range at the mean energy of $\langle E_\nu \rangle = 15.4$ GeV.

Dashed line and shaded bands in each panel represent the best-fit value of M_A^D and corresponding 1σ and 2σ uncertainties, obtained in our global fit to the deuterium and hydrogen QECC data.



The same as in the previous slide except the lines and bands which now represent the best-fit value of the parameter M_0 (which can be identified with the current axial mass of the nucleon, M_A) and corresponding 1σ and 2σ uncertainties, obtained in our global fit to all CCQE data described in slide 47.

In all our fits, the nuclear corrections for the $\nu - d$ CCQE scattering were accounted for within the impulse approximation and by using the closure over the final dinucleon states and meson exchange current correction at low Q^2; the Reid hard core potential and Hulthen wave function for the deuteron were applied.[a]

[a]See S.K. Singh and H. Arenhovel, *"Pion exchange current effects in $\nu_\mu + d \to \mu^- + p + p$,"* Z. Phys. A **324**, 347 (1986) and references therein.

Conclusions

- The Smith-Moniz RFG model supplemented by the energy-dependent (running) axial mass, M_A^{run}, defined by only two universal parameters M_0 and E_0, describes well the earlier and modern CCQE data on various nuclear targets.
- The best-fit values of M_0 and E_0 are sensitive to the details of the RFG model (including the input Fermi momenta and separation energies) and to the model of the nucleon electromagnetic form factors. However the fit can almost automatically be repeated with other versions of RFG (e.g., Bodek–Ritchie), or its extensions (SF, LFG, etc.), or with more advanced models and improved inputs.
- A more sophisticated parametrization of M_A^{run} is unreasonable for the present-day level of accuracy of the CCQE (and CCQE-like) data but may be needed in the future.
- Individual parametrizations for different nuclei or nucler groups are not warranted, but mainly because the currently available dataset for the inorganic heavy nuclei is not accurate enough and not quite self-consistent.
- There's no statistically significant difference between the M_A^{run} parameters extracted separately from the ν and $\overline{\nu}$ data, but there's a weak hint to a possible difference. Any case, the available $\overline{\nu}$ dataset is not yet sufficient for a more definite statement.

Finally, we suggest incorporating (as an option) the running axial mass into the neutrino MC generators used in the modern neutrino oscillation experiments.

Thanks for attention


BACKUP

χ^2 schematically

$$\chi^2 = \sum_{i\,(\text{uncorr})} \sum_{j\in G_i} \frac{\left[N_i T_{ij}(\boldsymbol{\lambda}) - E_{ij}\right]^2}{\sigma_{ij}^2} + \sum_{i\,(\text{uncorr})} \frac{(N_i-1)^2}{\sigma_i^2}$$
$$+ \sum_{i\,(\text{corr})} \left[\mathbf{T}_i(\boldsymbol{\lambda}) - \mathbf{E}_i\right] \mathbf{W}_i^{-1} \left[\mathbf{T}_i(\boldsymbol{\lambda}) - \mathbf{E}_i\right]^T .$$

Here

- index i enumerates the experiments (or data groups) G_i,
- index $j \in G_i$ enumerates the bin-averaged experimental data E_{ij} from the group G_i with the errors σ_{ij} which do not include the flux normalization uncertainty, σ_i. The data sets include the total CCQE cross sections, flux-averaged differential cross sections ($d\sigma_{\nu,\overline{\nu}}/dQ^2$, $d^2\sigma_{\nu,\overline{\nu}}/dE_\mu d\cos\theta_\mu$, etc.), and unnormalized Q^2 distributions ($dN_{\nu,\overline{\nu}}/dQ^2$).
- $T_{ij}(\boldsymbol{\lambda})$ represent the model predictions dependent on the set of fitting parameters $\boldsymbol{\lambda}$ (in our particular case, $\boldsymbol{\lambda} = (M_0, E_0)$ for heavy nuclear targets and $\boldsymbol{\lambda} = M_0$ for hydrogen and deuterium targets).
- $\mathbf{W}_i$ is the correlation matrix for the ith group.

Nuclear corrections for the CCQE cross sections on deuterium are, as a rule, already taken into account by the authors of the experiment and are represented as the cross sections or Q^2 distribution for a free nucleon target.
The formal analytic expressions for the normalization factors N_i can be found by solving the system of equations

$$\partial\chi^2/\partial N_i = 0.$$

The solution is of the form:

$$N_i = \frac{1 + \sigma_i^2 \sum\limits_{j\in G_i} \sigma_{ij}^{-2} T_{ij}(\boldsymbol{\lambda}) E_{ij}}{1 + \sigma_i^2 \sum\limits_{j\in G_i} \sigma_{ij}^{-2} \left[T_{ij}(\boldsymbol{\lambda})\right]^2}.$$

The penalty terms $(N_i - 1)^2/\sigma_i^2$ contain the flux uncertainties (σ_i) common to the data group G_i.
The χ^2 for the final fit to all data also includes another penalty term

$$\left(\frac{M_0 - M_A^D}{\Delta M_A^D}\right)^2,$$

which provides a "soft anchoring" of the parameter M_0 to the current axial mass $M_A^D \pm \Delta M_A^D$ obtained from the fitting the robust deuterium data only.

Including the "mixed" data into the χ^2 sum

The expression for χ^2 shown in the slide 67 becomes a bit more complicated when the data is a combination of the values obtained in the experiments with the neutrino and antineutrino beams.
Here we consider the case when we have to fit the *uncorrelated* data like E_1, E_2 and $E_3 = E_1' + E_2'$, where the pairs (E_1, E_1') and (E_2, E_2') have the common normalization uncertainty. Let's construct the sum for minimization for this case:

$$\begin{aligned}\chi^2_{\text{mix}} = &\sum_{j\in G_1} \frac{\left[N_1 T_{1j}(\boldsymbol{\lambda}) - E_{1j}\right]^2}{\sigma^2_{1j}} + \sum_{j\in G_2} \frac{\left[N_2 T_{2j}(\boldsymbol{\lambda}) - E_{1j}\right]^2}{\sigma^2_{2j}} \\ &+ \sum_{j\in G_3} \frac{\left[N_1 T_{1'j}(\boldsymbol{\lambda}) + N_2 T_{2'j}(\boldsymbol{\lambda}) - E_{3j}\right]^2}{\sigma^2_{3j}} \\ &+ \left(\frac{N_1 - 1}{\sigma_1}\right)^2 + \left(\frac{N_2 - 1}{\sigma_2}\right)^2\end{aligned}$$

(the notation is obvious). From the minimization conditions

$$\frac{\partial \chi^2_{\text{mix}}}{\partial N_1} = \frac{\partial \chi^2_{\text{mix}}}{\partial N_2} = 0$$

we obtain the expressions for the normalization factors:

$$N_1 = \frac{1}{d}\left(C_1 A_{22} - C_2 A_{12}\right), \quad N_2 = \frac{1}{d}\left(C_2 A_{11} - C_1 A_{12}\right),$$

where

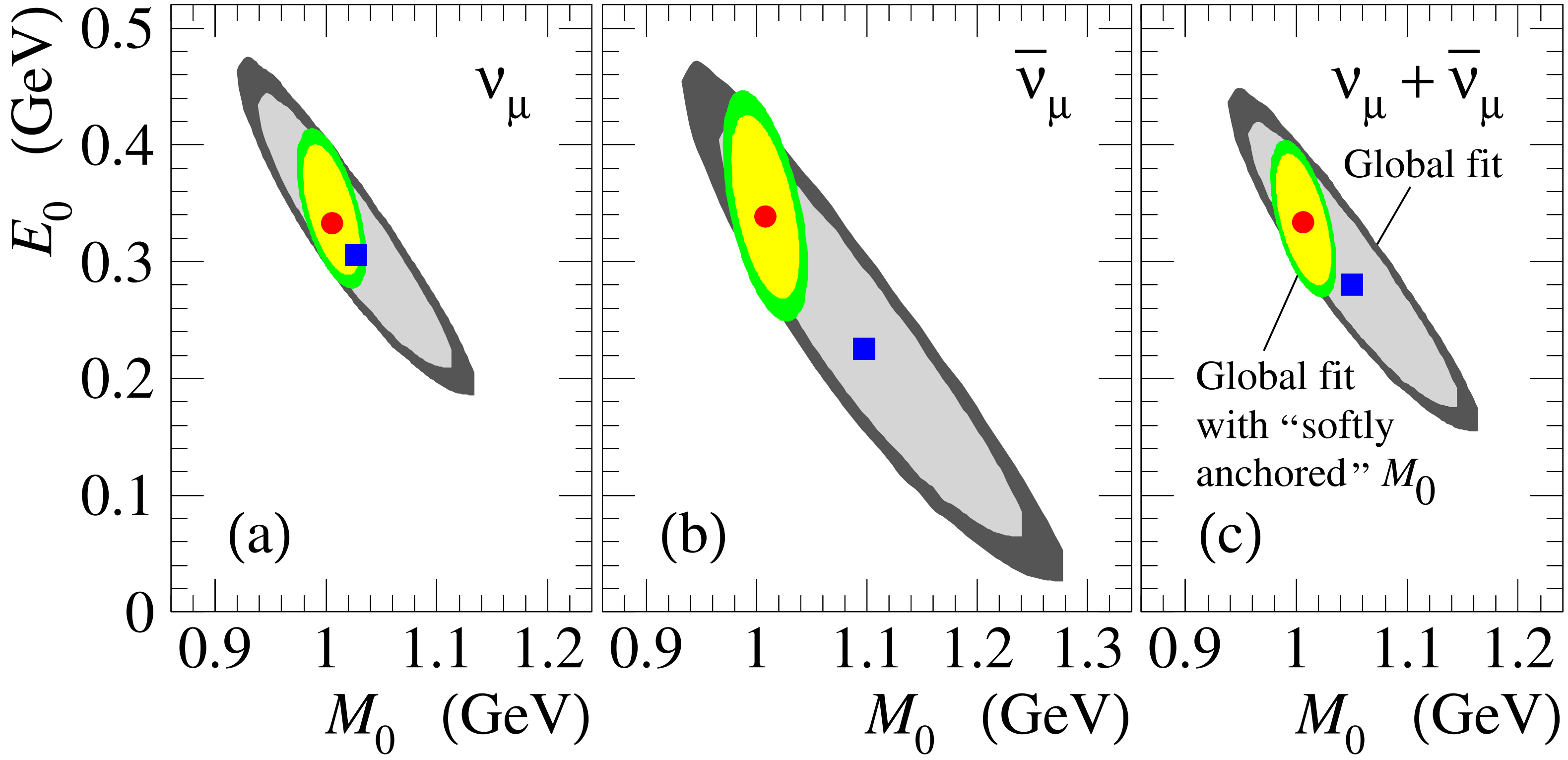
$$d = A_{11}A_{22} - A_{12}^2,$$

$$A_{11} = \sum_{j\in G_1} \frac{\left[T_{1j}(\boldsymbol{\lambda})\right]^2}{\sigma_{1j}^2} + \sum_{j\in G_3} \frac{\left[T_{1'j}(\boldsymbol{\lambda})\right]^2}{\sigma_{3j}^2} + \frac{1}{\sigma_1^2},$$

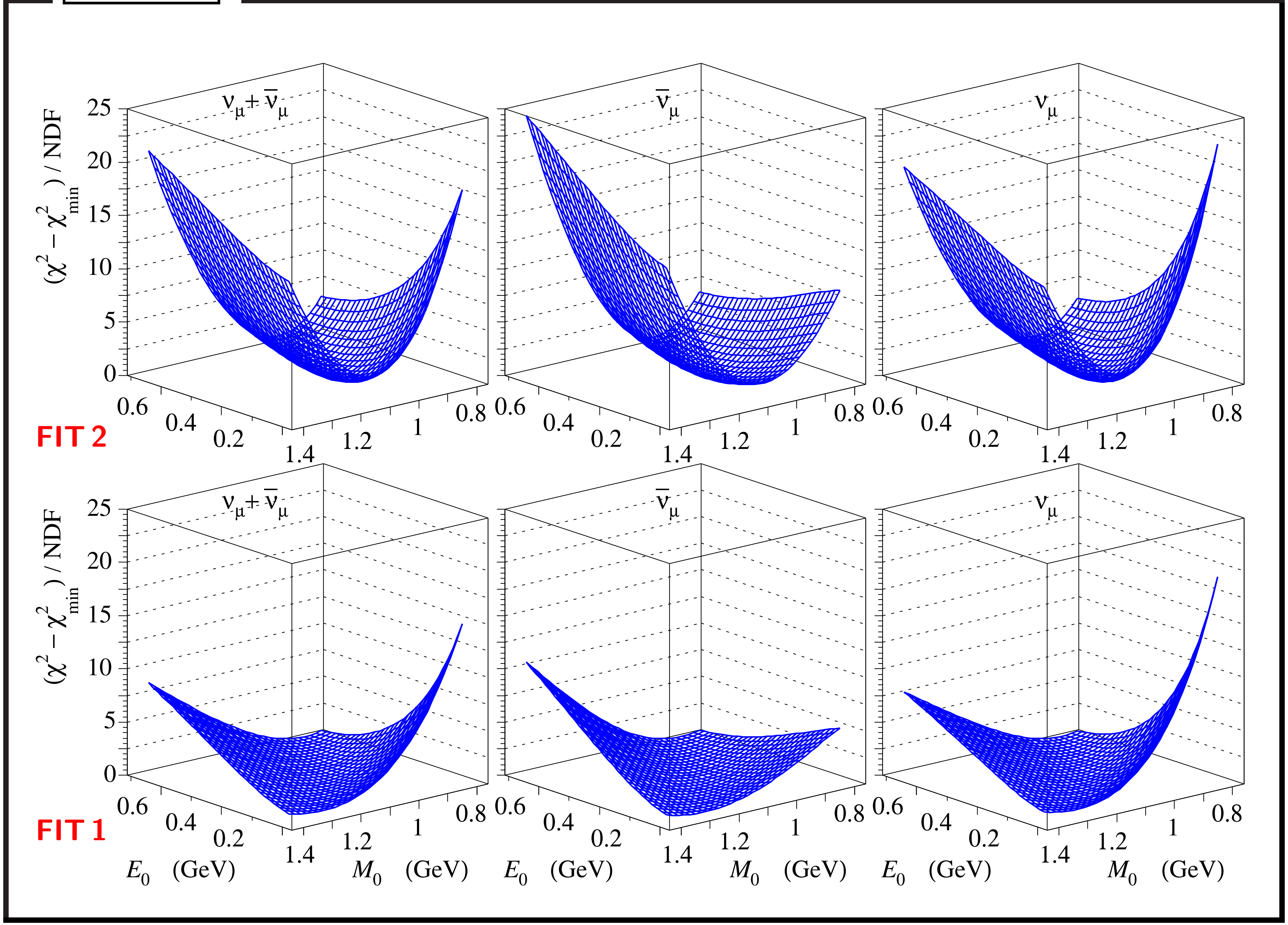
$$A_{22} = \sum_{j\in G_2} \frac{\left[T_{2j}(\boldsymbol{\lambda})\right]^2}{\sigma_{2j}^2} + \sum_{j\in G_3} \frac{\left[T_{2'j}(\boldsymbol{\lambda})\right]^2}{\sigma_{3j}^2} + \frac{1}{\sigma_2^2},$$

$$A_{12} = \sum_{j\in G_3} \frac{T_{1'j}(\boldsymbol{\lambda})T_{2'j}(\boldsymbol{\lambda})}{\sigma_{3j}^2},$$

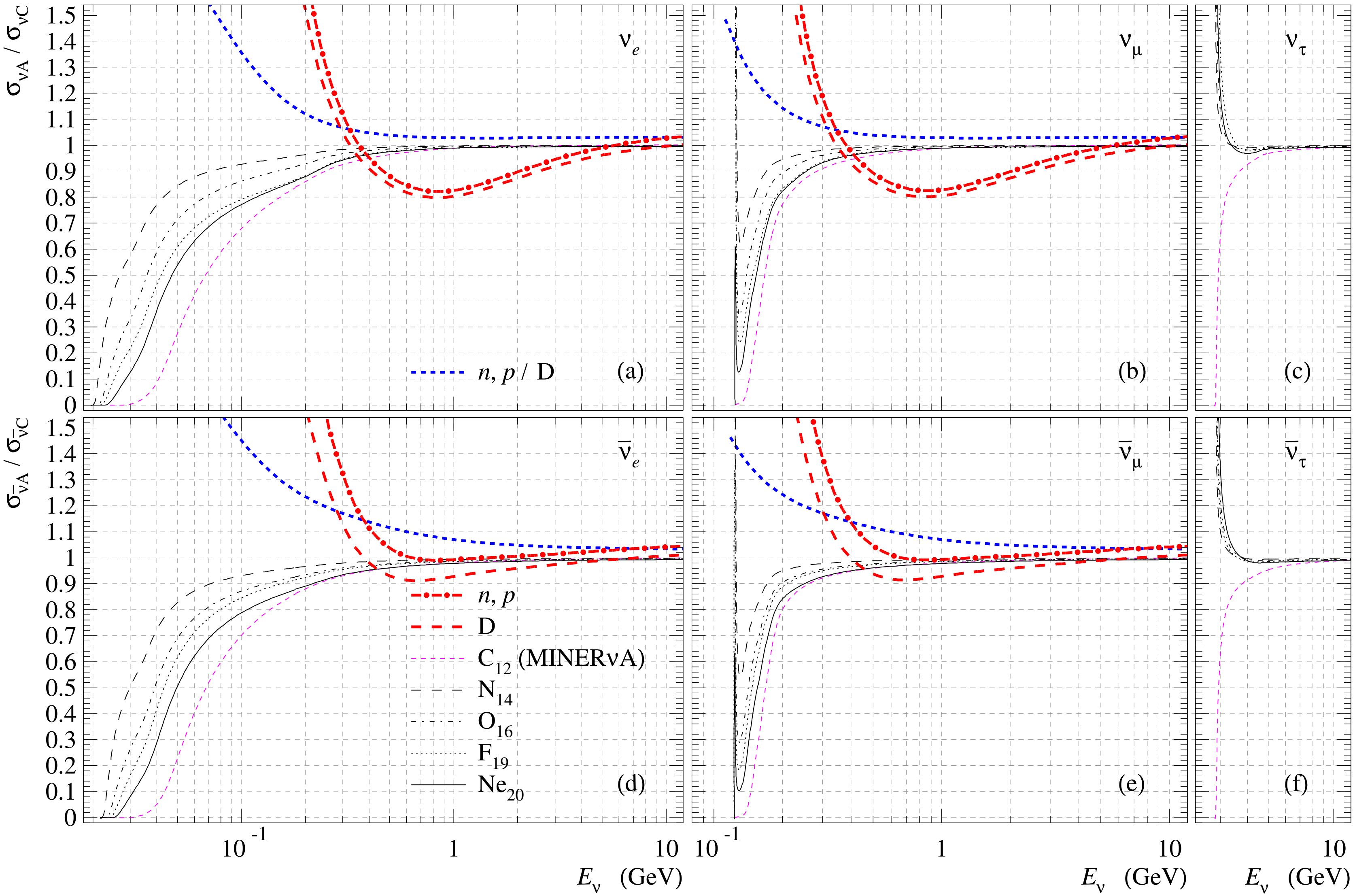
$$C_i = \sum_{j\in G_2} \frac{T_{ij}(\boldsymbol{\lambda})E_{ij}(\boldsymbol{\lambda})}{\sigma_{ij}^2} + \sum_{j\in G_3} \frac{T_{i'j}(\boldsymbol{\lambda})E_{3j}(\boldsymbol{\lambda})}{\sigma_{3j}^2} + \frac{1}{\sigma_2^2}.$$



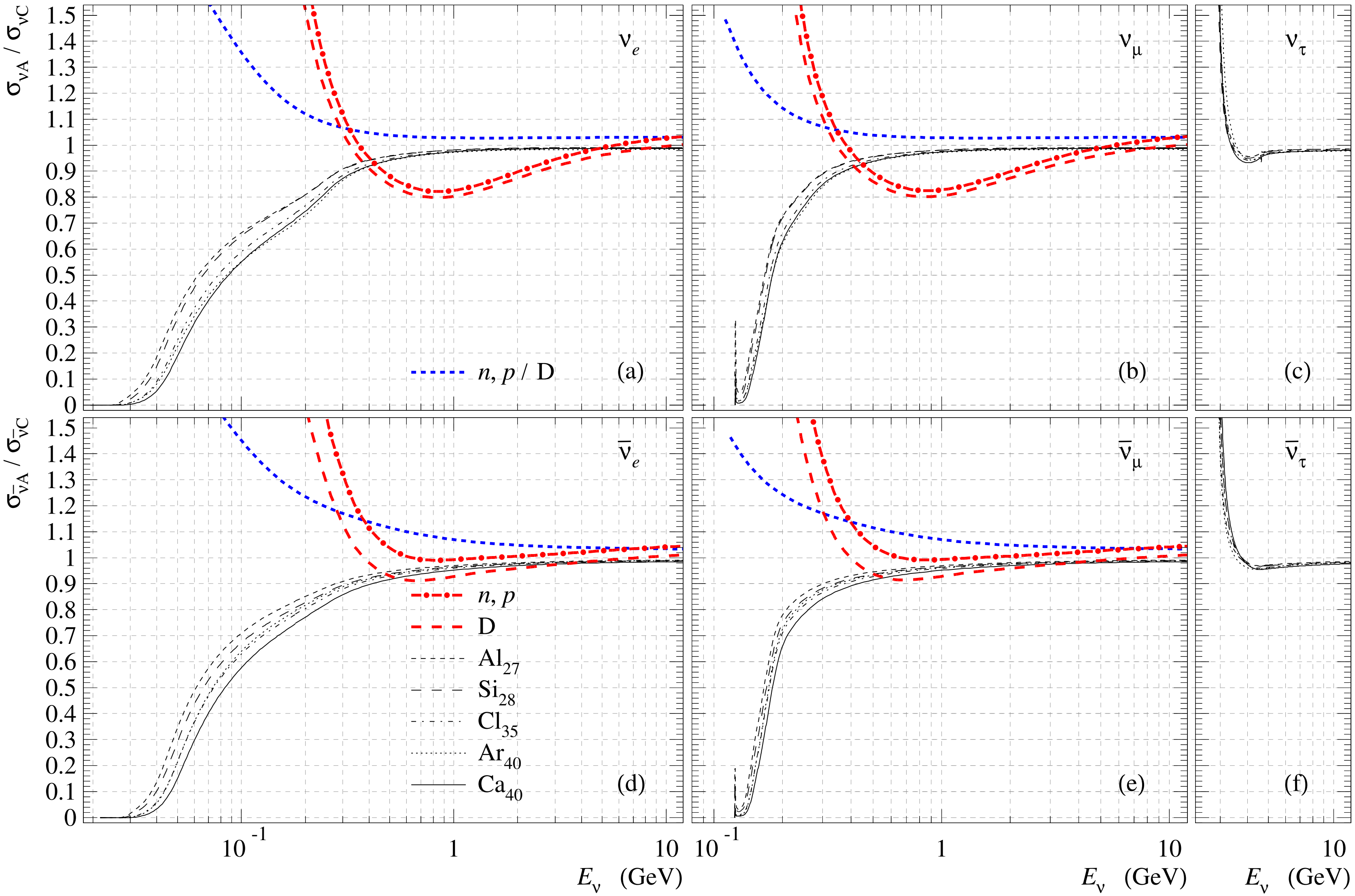
Error contours for the parameters M_0 and E_0 obtained from the global fits for ν (a), $\overline{\nu}$ (b), and $\nu + \overline{\nu}$ (c) CCQE data, performed *without* anchoring of M_0 (FIT 1, greater areas) and *with* anchoring of M_0 (FIT 2, smaller areas); see slide 68. The inner and outer contours for these fits indicate the 68% and 95% C.L. regions, respectively.
The shapes of $(\chi^2 - \chi^2_{\text{min}})$/NDF for FIT 1 and FIT 2 are shown in the next slide. ↓



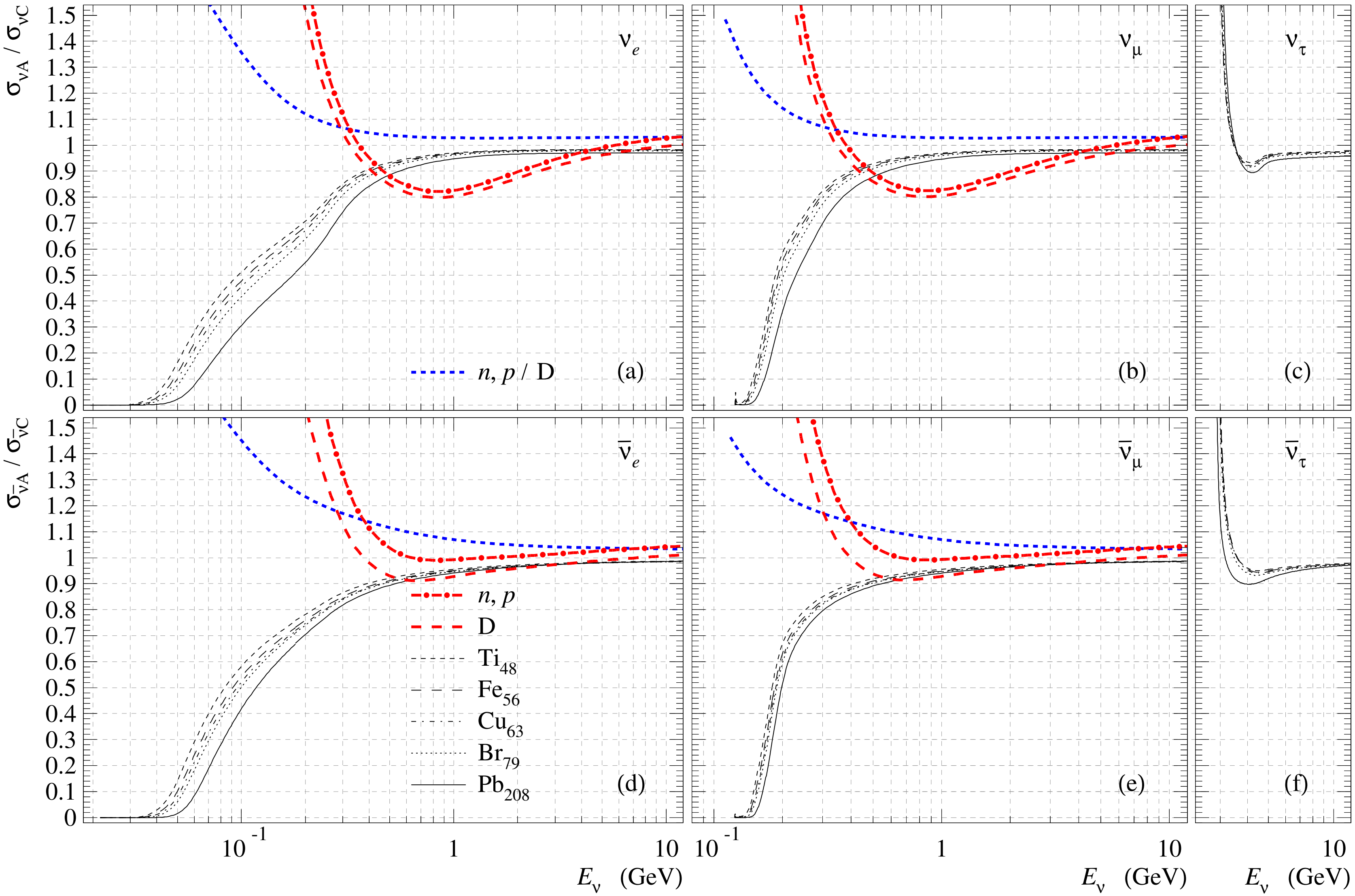
$(\chi^2 - \chi^2_{min})$ / NDF
$\nu_\mu + \bar{\nu}_\mu$
$\bar{\nu}_\mu$
ν_μ
FIT 2
FIT 1
E_0 (GeV)
M_0 (GeV)
0
5
10
15
20
25
0.6
0.4
0.2
1.4
1.2
1
0.8



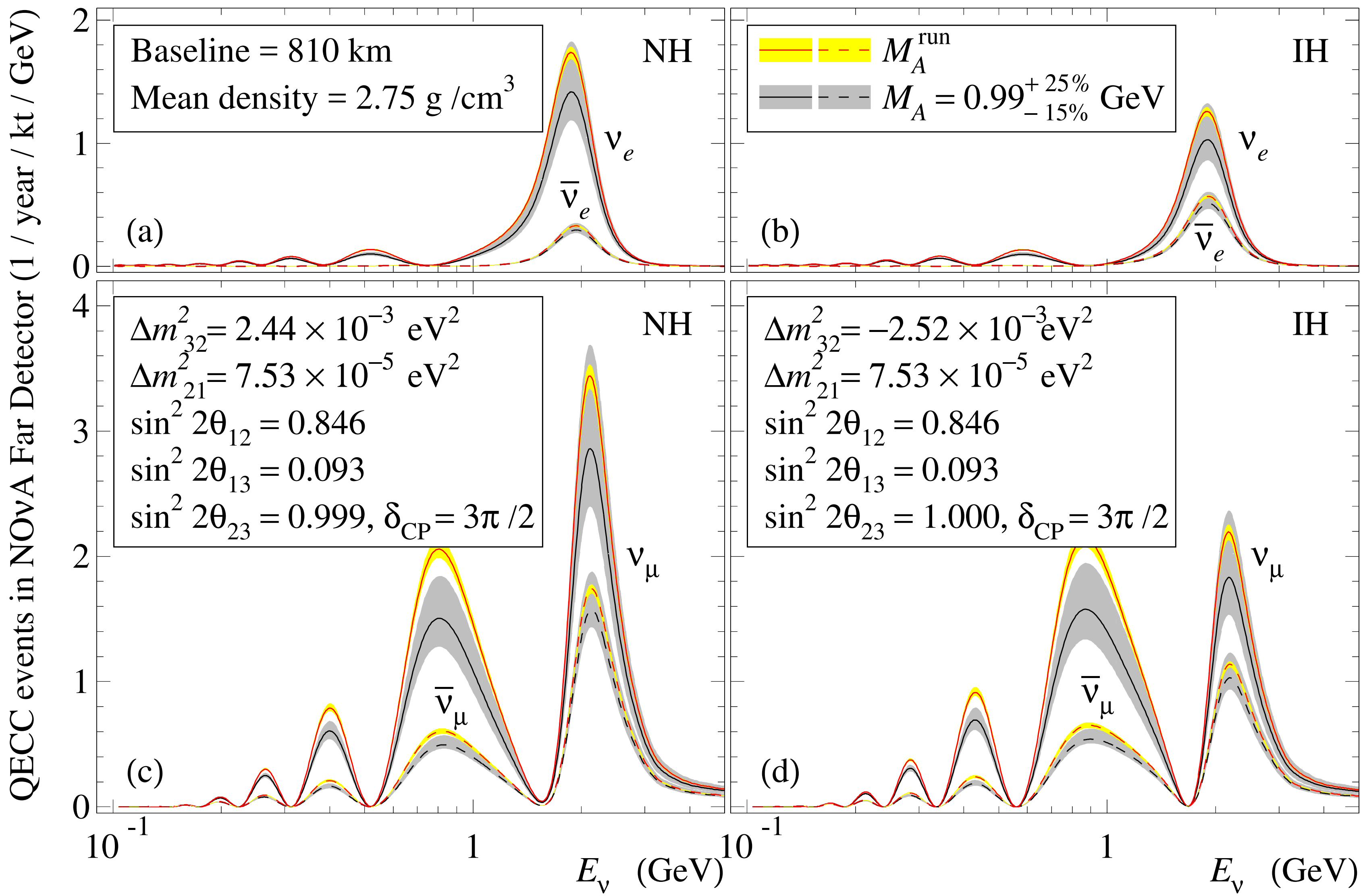
$\sigma_{\nu A} / \sigma_{\nu C}$
$\sigma_{\bar{\nu} A} / \sigma_{\bar{\nu} C}$
ν_e
ν_μ
ν_τ
$\bar{\nu}_e$
$\bar{\nu}_\mu$
$\bar{\nu}_\tau$
(a)
(b)
(c)
(d)
(e)
(f)
n, p / D
n, p
D
C_{12} (MINERνA)
N_{14}
O_{16}
F_{19}
Ne_{20}
E_ν (GeV)



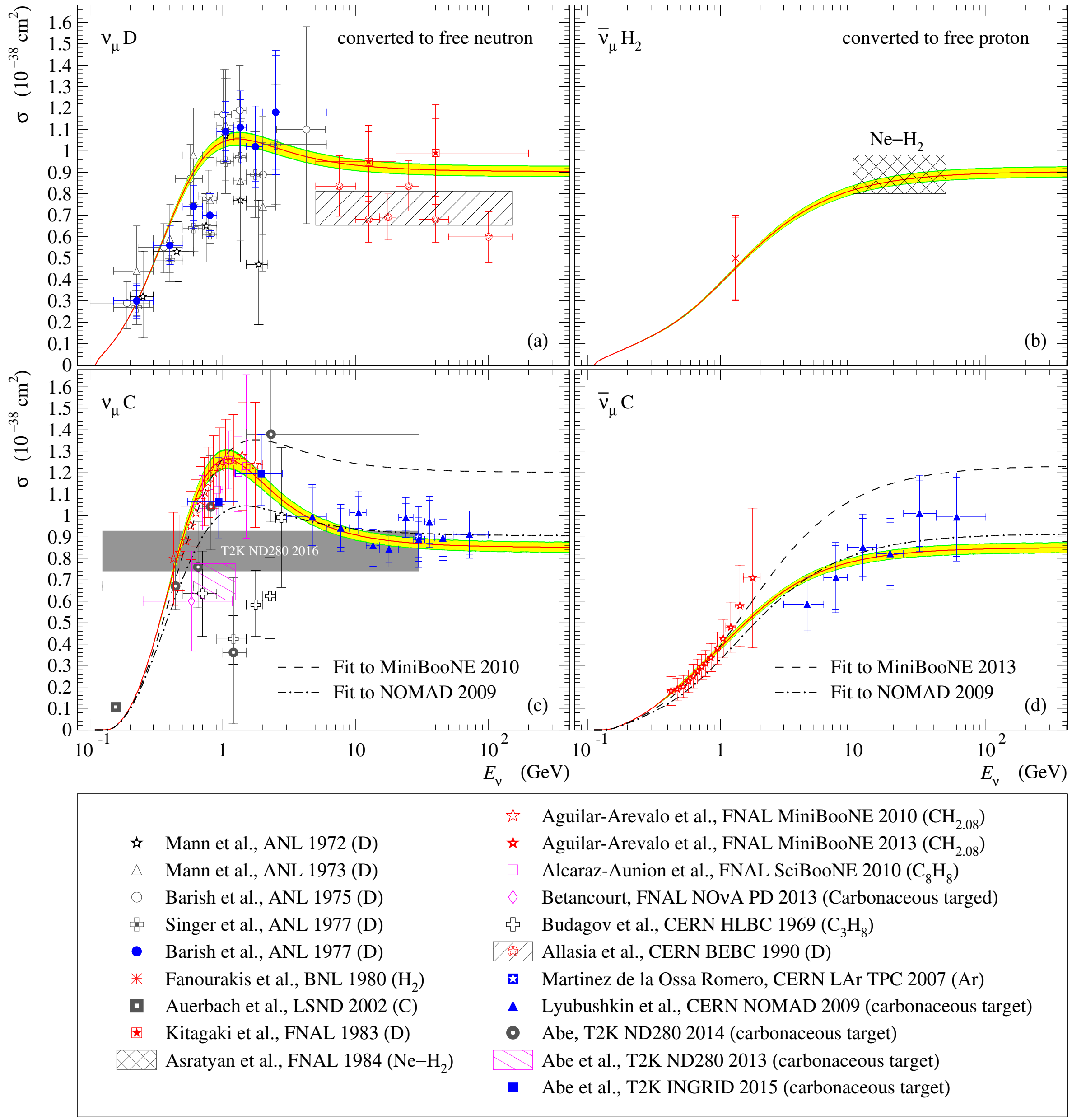
$\sigma_{\nu A} / \sigma_{\nu C}$
ν_e
ν_μ
ν_τ
n, p / D
(a)
(b)
(c)
$\sigma_{\bar{\nu} A} / \sigma_{\bar{\nu} C}$
$\bar{\nu}_e$
$\bar{\nu}_\mu$
$\bar{\nu}_\tau$
n, p
D
Al_{27}
Si_{28}
Cl_{35}
Ar_{40}
Ca_{40}
(d)
(e)
(f)
E_ν (GeV)

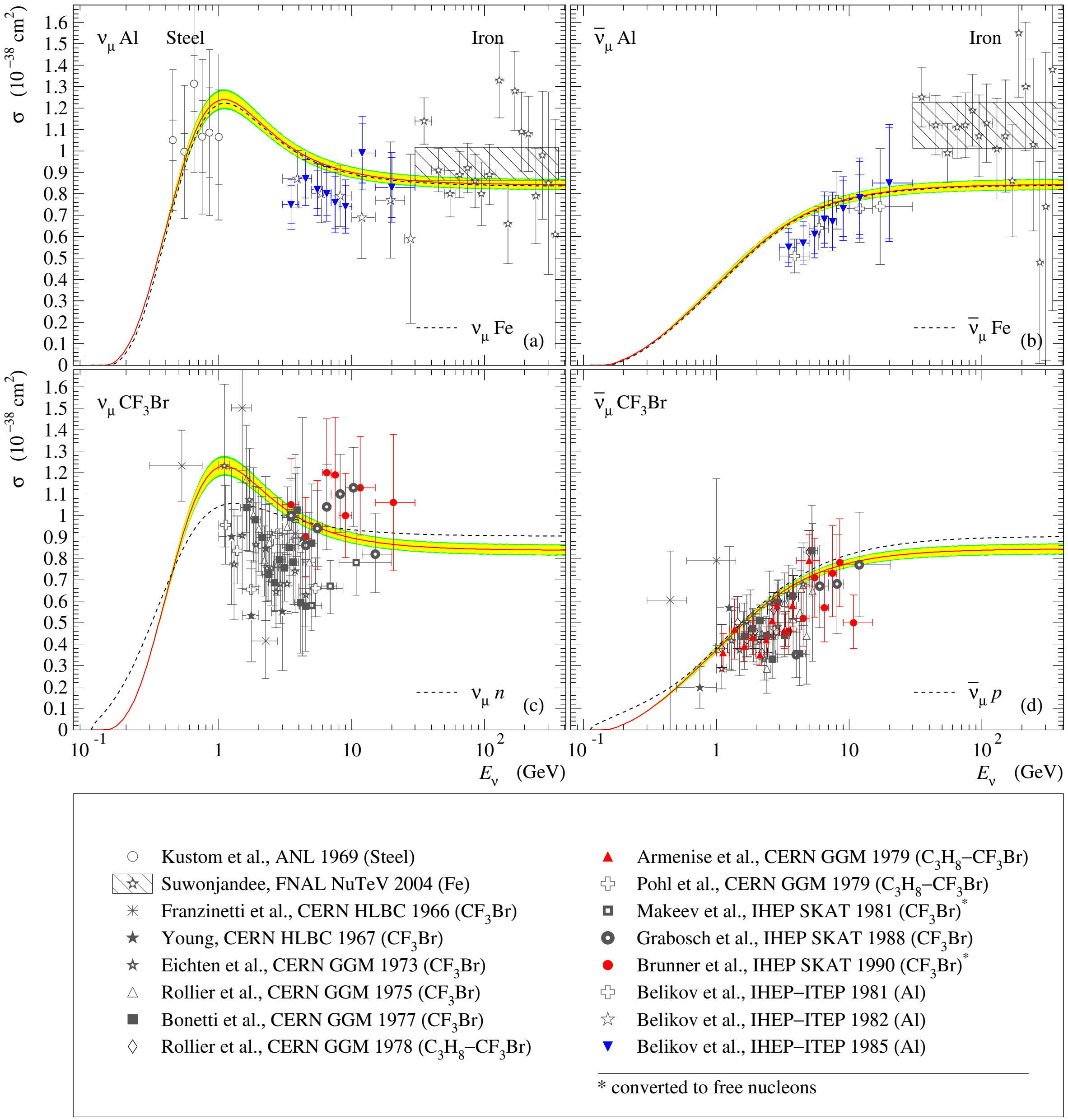


$\sigma_{\nu A} / \sigma_{\nu C}$
$\sigma_{\bar{\nu}A} / \sigma_{\bar{\nu}C}$
ν_e
ν_μ
ν_τ
$\bar{\nu}_e$
$\bar{\nu}_\mu$
$\bar{\nu}_\tau$
n, p / D
n, p
D
Ti_{48}
Fe_{56}
Cu_{63}
Br_{79}
Pb_{208}
(a)
(b)
(c)
(d)
(e)
(f)
E_ν (GeV)

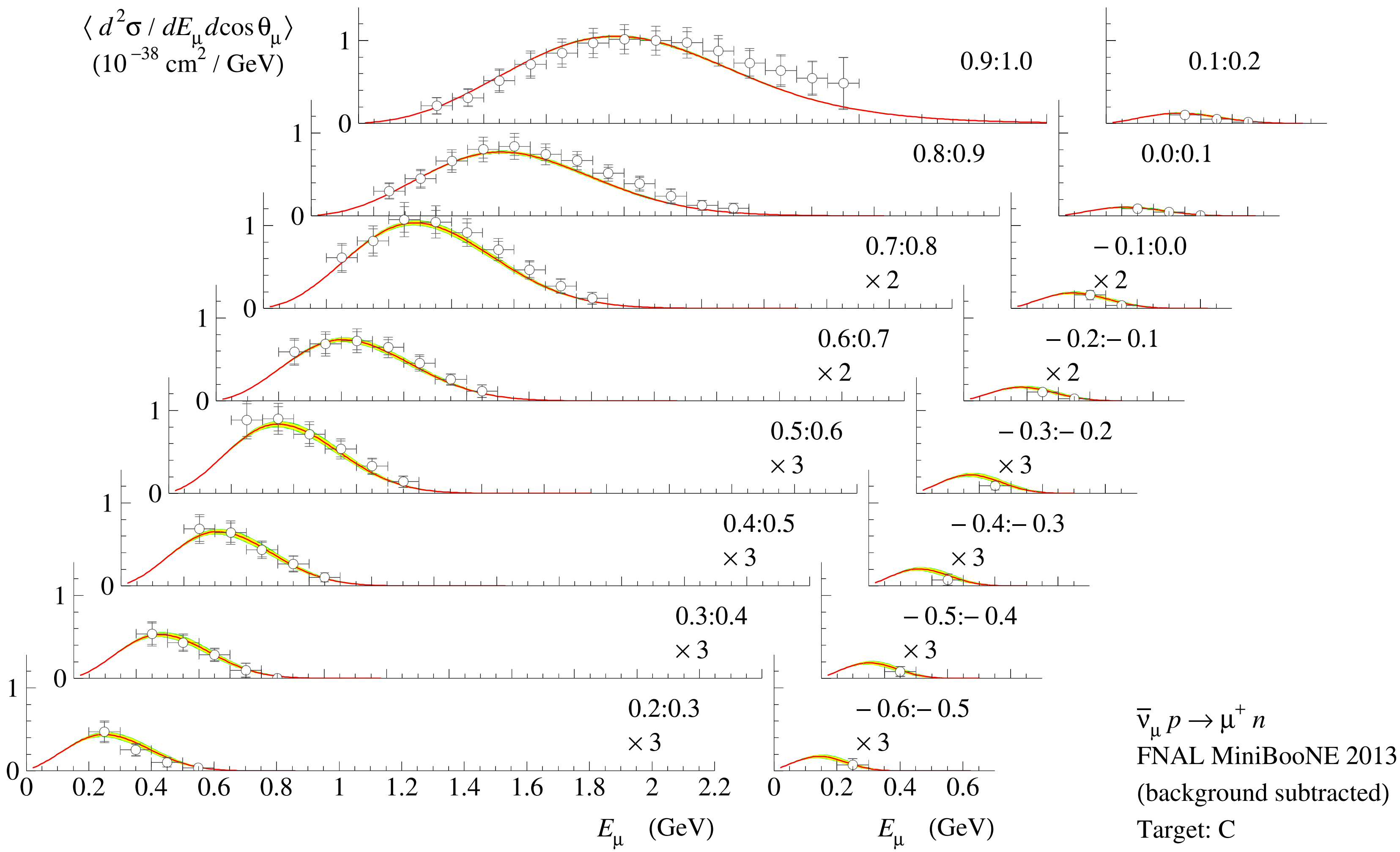


An application to NOνA experiment (see Refs. [70–72] for details).

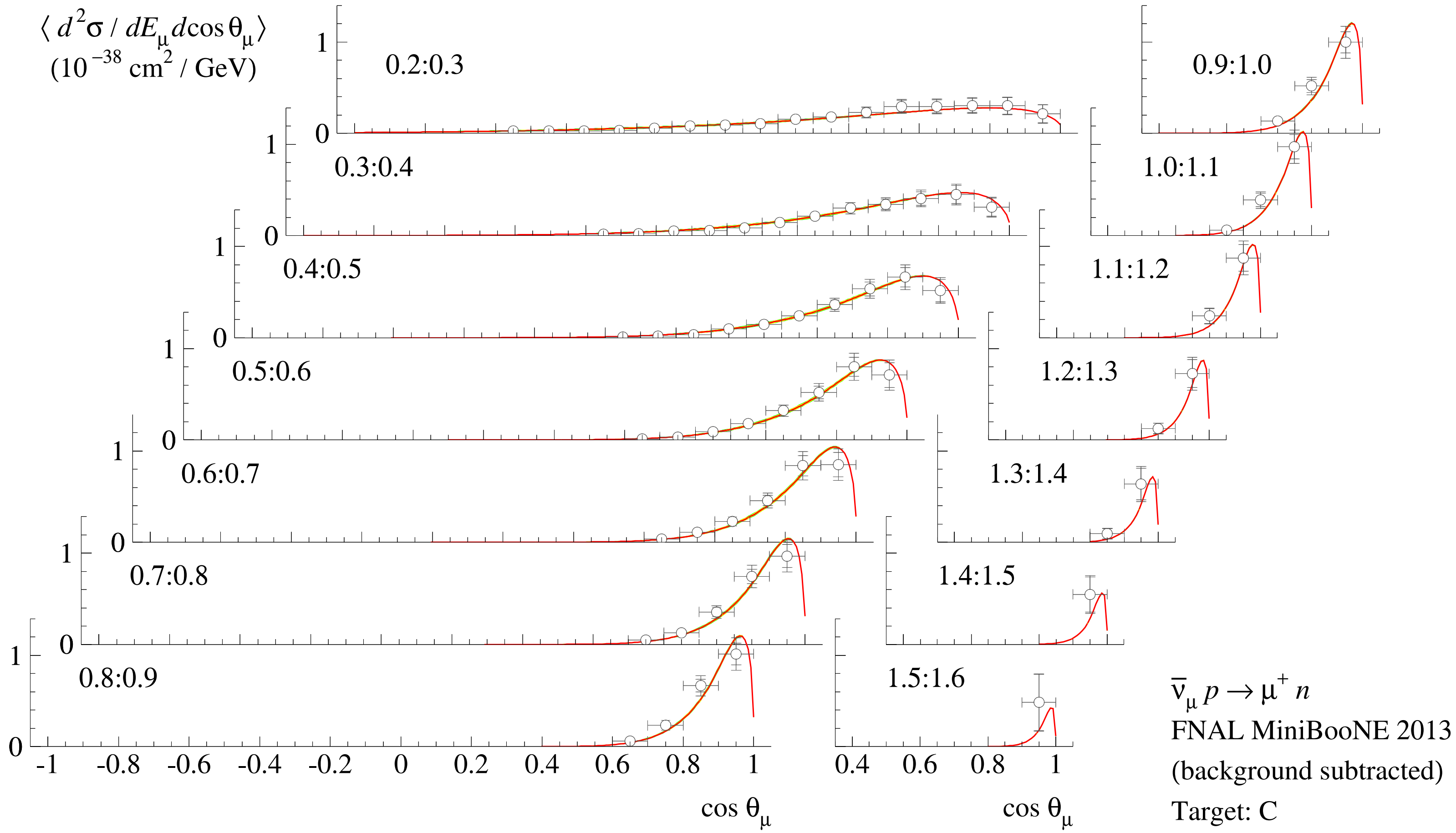

σ (10^−38 cm^2)
ν_μ D
converted to free neutron
(a)
$\bar{\nu}_\mu$ H_2
converted to free proton
Ne–H_2
(b)
ν_μ C
T2K ND280 2016
Fit to MiniBooNE 2010
Fit to NOMAD 2009
(c)
$\bar{\nu}_\mu$ C
Fit to MiniBooNE 2013
Fit to NOMAD 2009
(d)
E_ν (GeV)
Mann et al., ANL 1972 (D)
Mann et al., ANL 1973 (D)
Barish et al., ANL 1975 (D)
Singer et al., ANL 1977 (D)
Barish et al., ANL 1977 (D)
Fanourakis et al., BNL 1980 (H_2)
Auerbach et al., LSND 2002 (C)
Kitagaki et al., FNAL 1983 (D)
Asratyan et al., FNAL 1984 (Ne–H_2)
Aguilar-Arevalo et al., FNAL MiniBooNE 2010 ($CH_{2.08}$)
Aguilar-Arevalo et al., FNAL MiniBooNE 2013 ($CH_{2.08}$)
Alcaraz-Aunion et al., FNAL SciBooNE 2010 (C_8H_8)
Betancourt, FNAL NOvA PD 2013 (Carbonaceous targed)
Budagov et al., CERN HLBC 1969 (C_3H_8)
Allasia et al., CERN BEBC 1990 (D)
Martinez de la Ossa Romero, CERN LAr TPC 2007 (Ar)
Lyubushkin et al., CERN NOMAD 2009 (carbonaceous target)
Abe, T2K ND280 2014 (carbonaceous target)
Abe et al., T2K ND280 2013 (carbonaceous target)
Abe et al., T2K INGRID 2015 (carbonaceous target)



σ (10^−38 cm^2)
ν_μ Al
Steel
Iron
ν_μ Fe
(a)
$\bar{\nu}_\mu$ Al
Iron
$\bar{\nu}_\mu$ Fe
(b)
ν_μ CF$_3$Br
ν_μ n
(c)
$\bar{\nu}_\mu$ CF$_3$Br
$\bar{\nu}_\mu$ p
(d)
E_ν (GeV)
Kustom et al., ANL 1969 (Steel)
Suwonjandee, FNAL NuTeV 2004 (Fe)
Franzinetti et al., CERN HLBC 1966 (CF$_3$Br)
Young, CERN HLBC 1967 (CF$_3$Br)
Eichten et al., CERN GGM 1973 (CF$_3$Br)
Rollier et al., CERN GGM 1975 (CF$_3$Br)
Bonetti et al., CERN GGM 1977 (CF$_3$Br)
Rollier et al., CERN GGM 1978 (C$_3$H$_8$–CF$_3$Br)
Armenise et al., CERN GGM 1979 (C$_3$H$_8$–CF$_3$Br)
Pohl et al., CERN GGM 1979 (C$_3$H$_8$–CF$_3$Br)
Makeev et al., IHEP SKAT 1981 (CF$_3$Br)*
Grabosch et al., IHEP SKAT 1988 (CF$_3$Br)
Brunner et al., IHEP SKAT 1990 (CF$_3$Br)*
Belikov et al., IHEP–ITEP 1981 (Al)
Belikov et al., IHEP–ITEP 1982 (Al)
Belikov et al., IHEP–ITEP 1985 (Al)
* converted to free nucleons

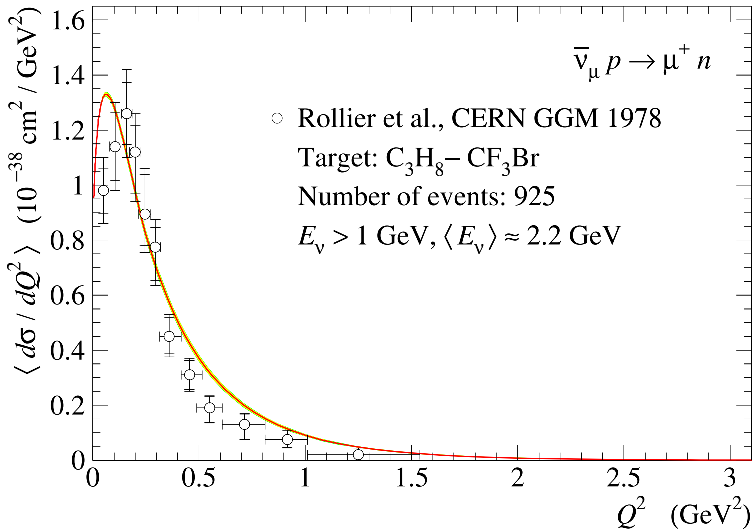


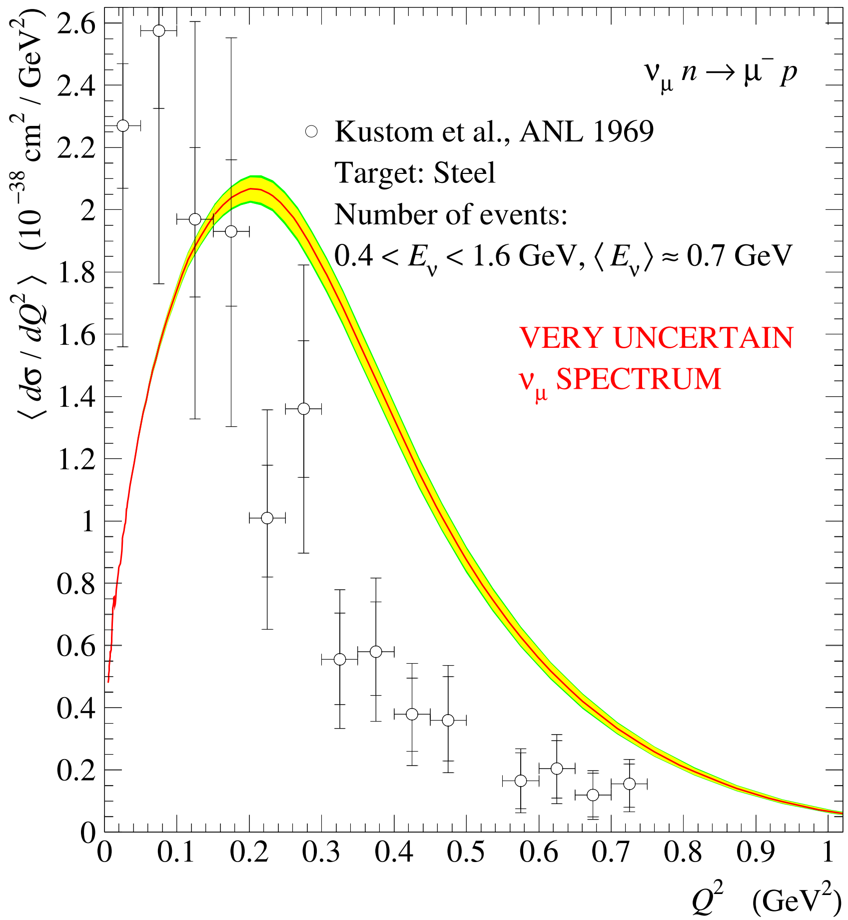
See note in slide 18.

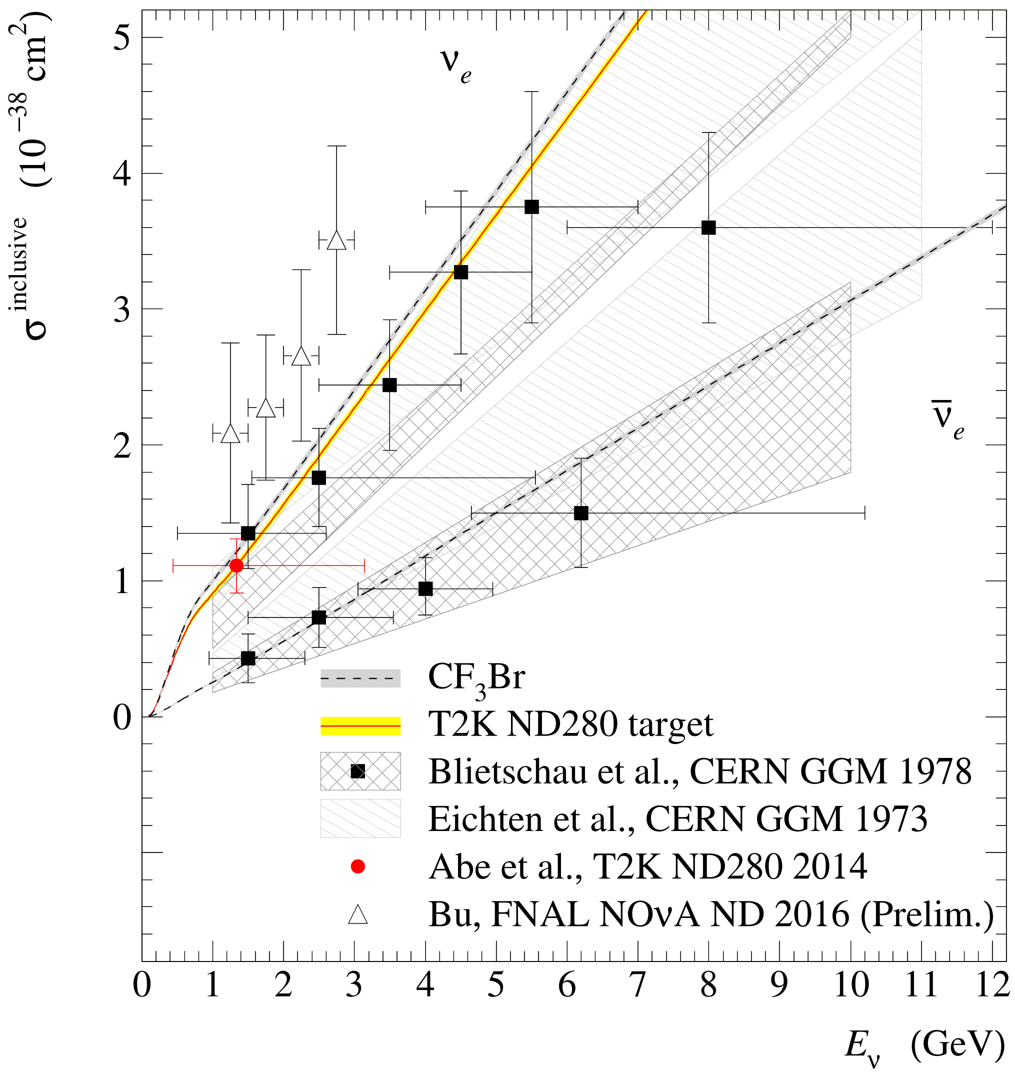


See note in slide 18.

A fly in the ointment (examples of doubtful agreement)

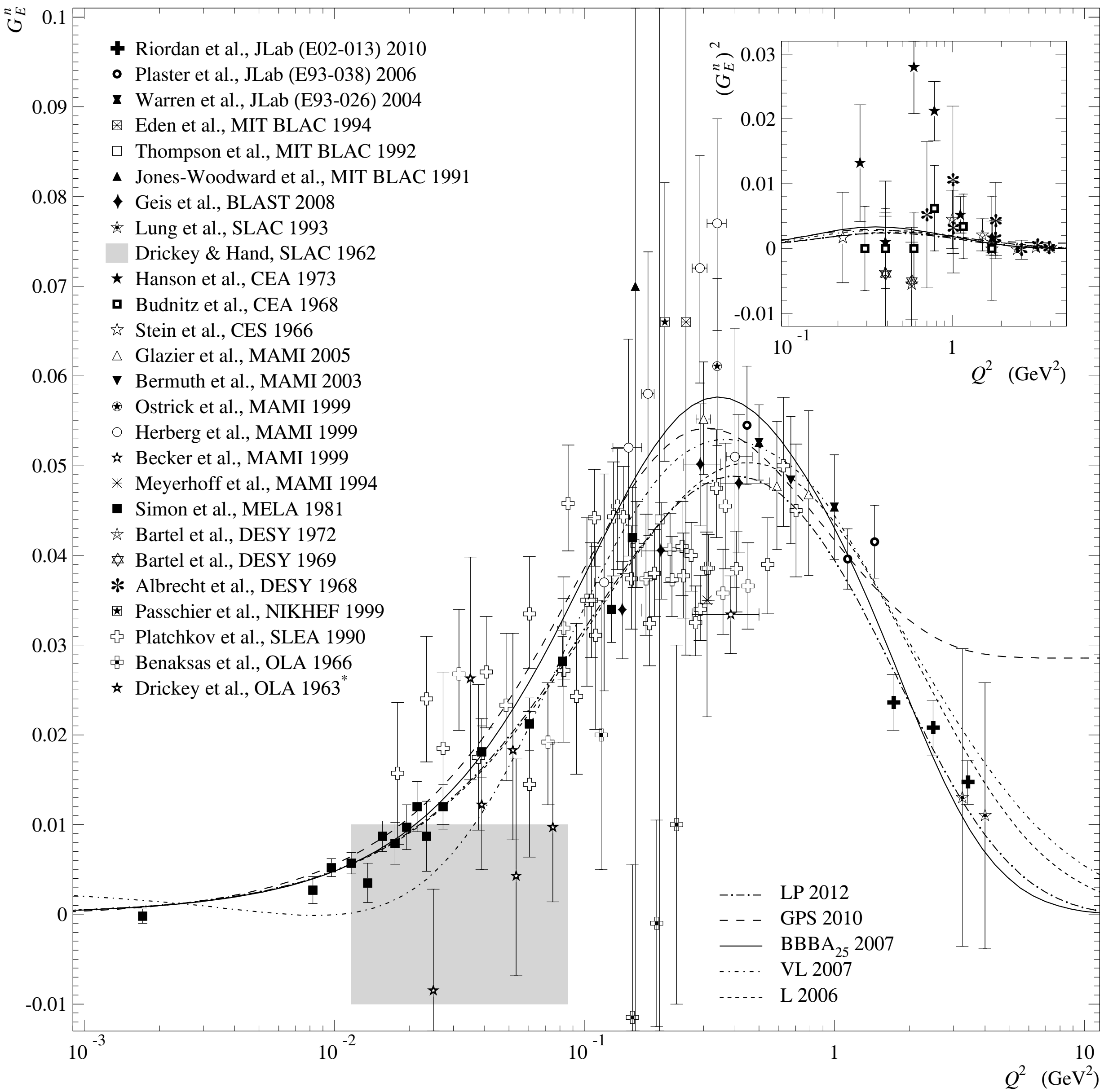




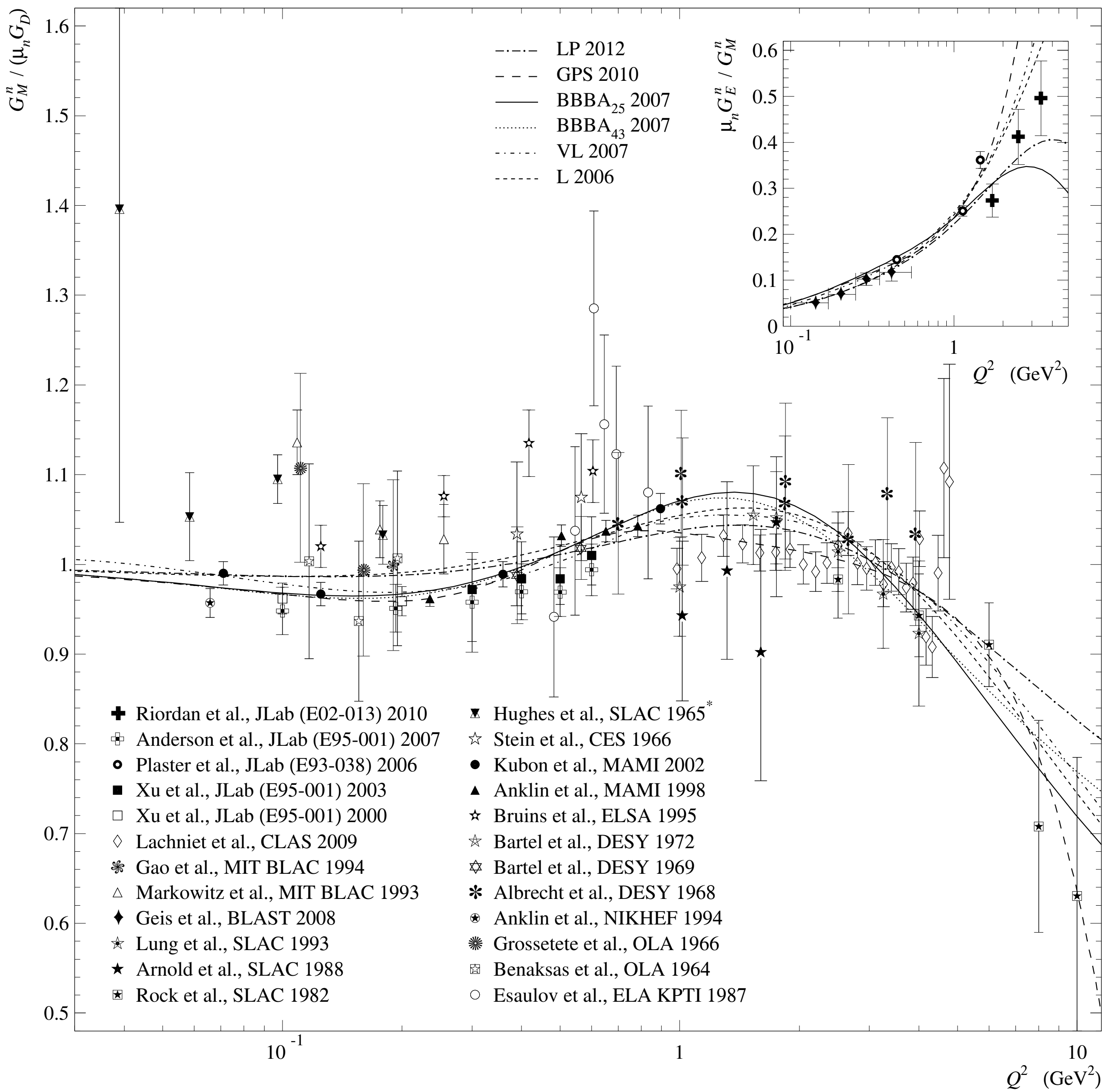


Total inclusive CC cross sections for ν_e and $\overline{\nu}_e$ at low energies with a bit surprising (*preliminary*) data from NOνA Near Detector (X. Bu, arXiv:1601.01213 [hep-ex]).

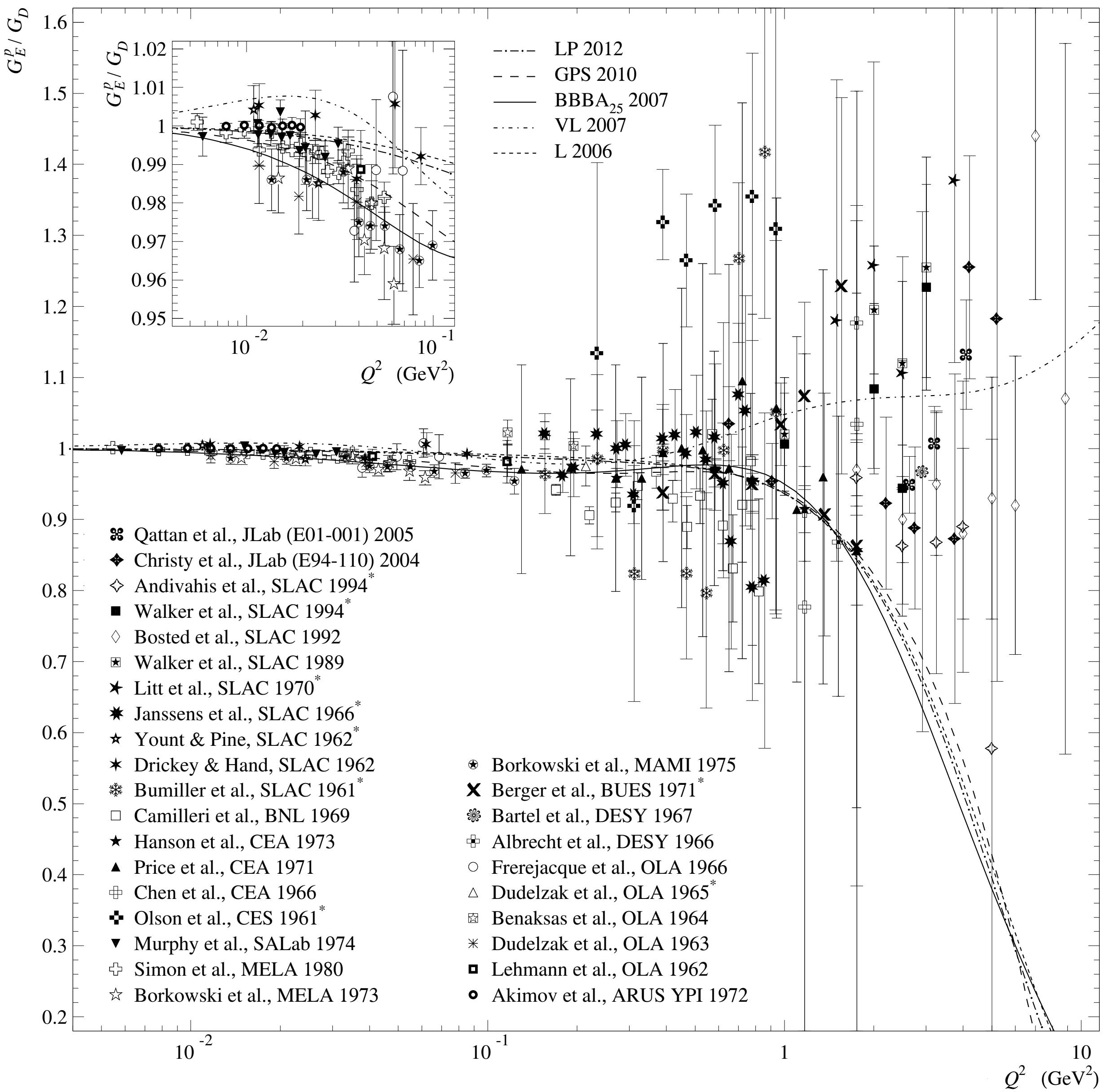
Comparison of several models for the nucleon electromagnetic form factors with data



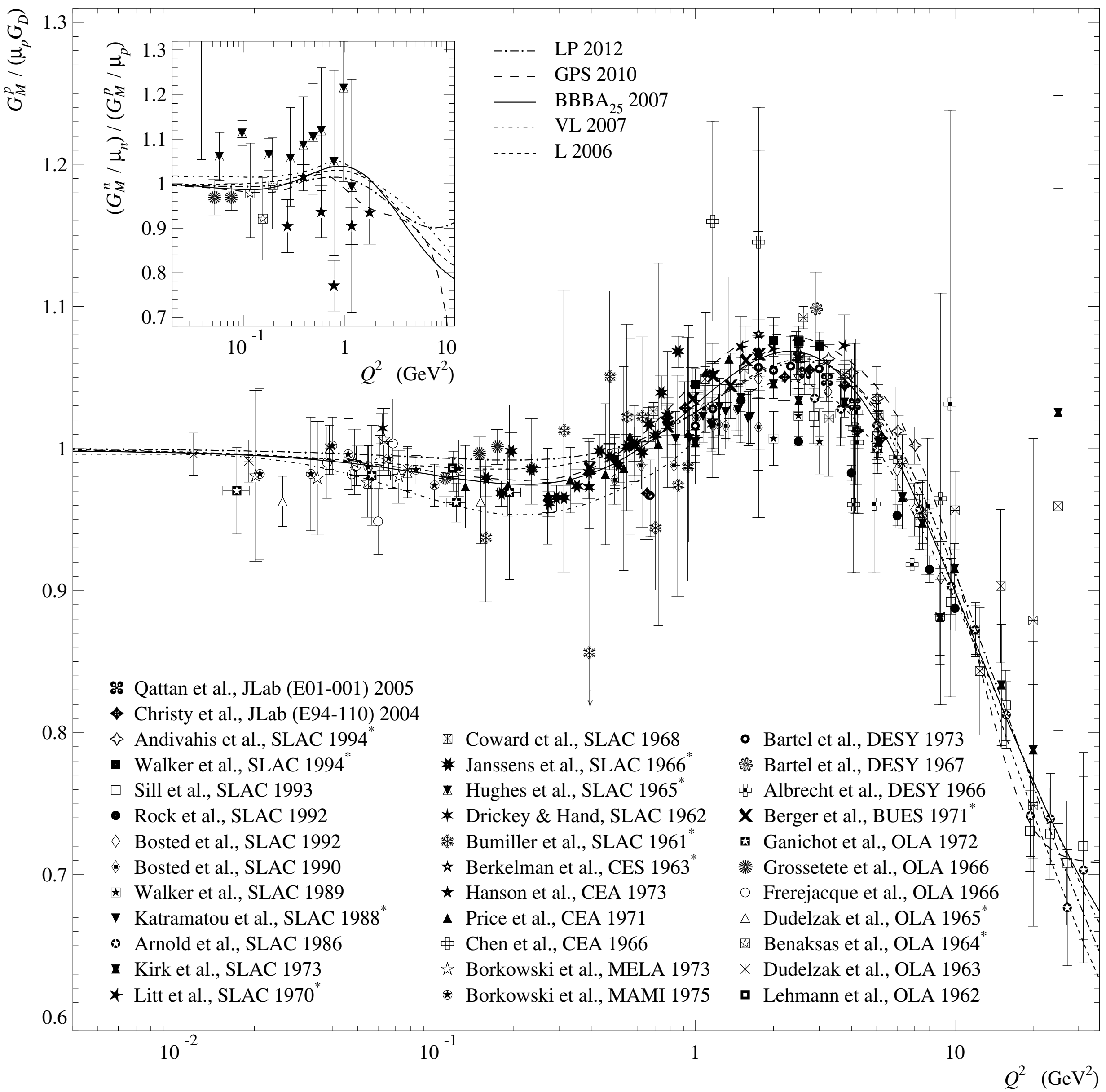
G_E^n
Riordan et al., JLab (E02-013) 2010
Plaster et al., JLab (E93-038) 2006
Warren et al., JLab (E93-026) 2004
Eden et al., MIT BLAC 1994
Thompson et al., MIT BLAC 1992
Jones-Woodward et al., MIT BLAC 1991
Geis et al., BLAST 2008
Lung et al., SLAC 1993
Drickey & Hand, SLAC 1962
Hanson et al., CEA 1973
Budnitz et al., CEA 1968
Stein et al., CES 1966
Glazier et al., MAMI 2005
Bermuth et al., MAMI 2003
Ostrick et al., MAMI 1999
Herberg et al., MAMI 1999
Becker et al., MAMI 1999
Meyerhoff et al., MAMI 1994
Simon et al., MELA 1981
Bartel et al., DESY 1972
Bartel et al., DESY 1969
Albrecht et al., DESY 1968
Passchier et al., NIKHEF 1999
Platchkov et al., SLEA 1990
Benaksas et al., OLA 1966
Drickey et al., OLA 1963*
LP 2012
GPS 2010
$BBBA_{25}$ 2007
VL 2007
L 2006
$(G_E^n)^2$
Q^2 (GeV^2)



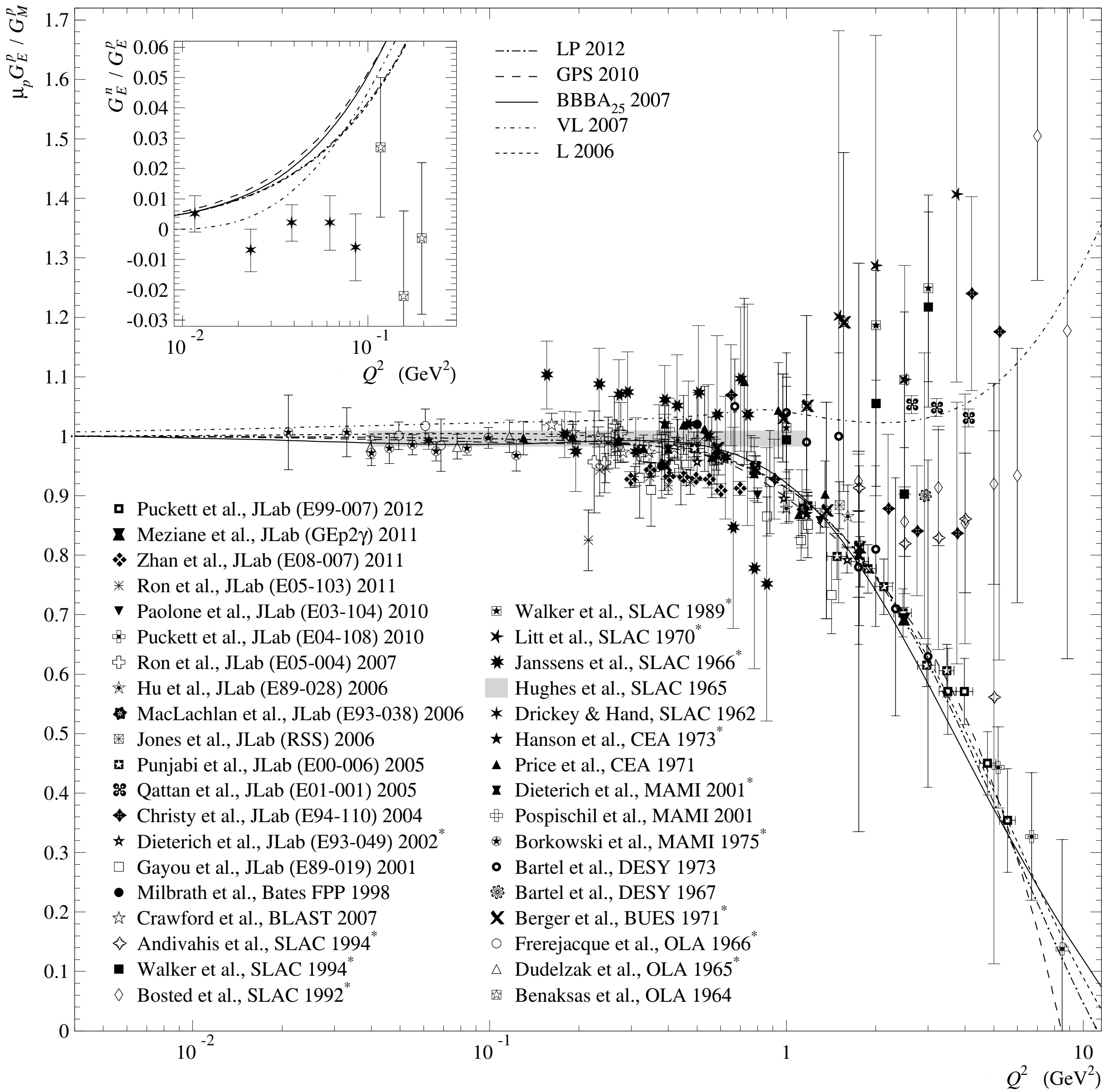
$G_M^n / (\mu_n G_D)$
LP 2012
GPS 2010
BBBA$_{25}$ 2007
BBBA$_{43}$ 2007
VL 2007
L 2006
$\mu_n G_E^n / G_M^n$
Q^2 (GeV^2)
Riordan et al., JLab (E02-013) 2010
Anderson et al., JLab (E95-001) 2007
Plaster et al., JLab (E93-038) 2006
Xu et al., JLab (E95-001) 2003
Xu et al., JLab (E95-001) 2000
Lachniet et al., CLAS 2009
Gao et al., MIT BLAC 1994
Markowitz et al., MIT BLAC 1993
Geis et al., BLAST 2008
Lung et al., SLAC 1993
Arnold et al., SLAC 1988
Rock et al., SLAC 1982
Hughes et al., SLAC 1965*
Stein et al., CES 1966
Kubon et al., MAMI 2002
Anklin et al., MAMI 1998
Bruins et al., ELSA 1995
Bartel et al., DESY 1972
Bartel et al., DESY 1969
Albrecht et al., DESY 1968
Anklin et al., NIKHEF 1994
Grossetete et al., OLA 1966
Benaksas et al., OLA 1964
Esaulov et al., ELA KPTI 1987
Q^2 (GeV^2)



G_E^p / G_D
LP 2012
GPS 2010
BBBA$_{25}$ 2007
VL 2007
L 2006
Q^2 (GeV^2)
Qattan et al., JLab (E01-001) 2005
Christy et al., JLab (E94-110) 2004
Andivahis et al., SLAC 1994*
Walker et al., SLAC 1994*
Bosted et al., SLAC 1992
Walker et al., SLAC 1989
Litt et al., SLAC 1970*
Janssens et al., SLAC 1966*
Yount & Pine, SLAC 1962*
Drickey & Hand, SLAC 1962
Bumiller et al., SLAC 1961*
Camilleri et al., BNL 1969
Hanson et al., CEA 1973
Price et al., CEA 1971
Chen et al., CEA 1966
Olson et al., CES 1961*
Murphy et al., SALab 1974
Simon et al., MELA 1980
Borkowski et al., MELA 1973
Borkowski et al., MAMI 1975
Berger et al., BUES 1971*
Bartel et al., DESY 1967
Albrecht et al., DESY 1966
Frerejacque et al., OLA 1966
Dudelzak et al., OLA 1965*
Benaksas et al., OLA 1964
Dudelzak et al., OLA 1963
Lehmann et al., OLA 1962
Akimov et al., ARUS YPI 1972



$G_M^p / (\mu_p G_D)$
$(G_M^n / \mu_n) / (G_M^p / \mu_p)$
Q^2 (GeV^2)
LP 2012
GPS 2010
BBBA$_{25}$ 2007
VL 2007
L 2006
Qattan et al., JLab (E01-001) 2005
Christy et al., JLab (E94-110) 2004
Andivahis et al., SLAC 1994*
Walker et al., SLAC 1994*
Sill et al., SLAC 1993
Rock et al., SLAC 1992
Bosted et al., SLAC 1992
Bosted et al., SLAC 1990
Walker et al., SLAC 1989
Katramatou et al., SLAC 1988*
Arnold et al., SLAC 1986
Kirk et al., SLAC 1973
Litt et al., SLAC 1970*
Coward et al., SLAC 1968
Janssens et al., SLAC 1966*
Hughes et al., SLAC 1965*
Drickey & Hand, SLAC 1962
Bumiller et al., SLAC 1961*
Berkelman et al., CES 1963*
Hanson et al., CEA 1973
Price et al., CEA 1971
Chen et al., CEA 1966
Borkowski et al., MELA 1973
Borkowski et al., MAMI 1975
Bartel et al., DESY 1973
Bartel et al., DESY 1967
Albrecht et al., DESY 1966
Berger et al., BUES 1971*
Ganichot et al., OLA 1972
Grossetete et al., OLA 1966
Frerejacque et al., OLA 1966
Dudelzak et al., OLA 1965*
Benaksas et al., OLA 1964*
Dudelzak et al., OLA 1963
Lehmann et al., OLA 1962



$\mu_p G_E^p / G_M^p$
G_E^n / G_E^p
Q^2 (GeV^2)
LP 2012
GPS 2010
BBBA$_{25}$ 2007
VL 2007
L 2006
Puckett et al., JLab (E99-007) 2012
Meziane et al., JLab (GEp2γ) 2011
Zhan et al., JLab (E08-007) 2011
Ron et al., JLab (E05-103) 2011
Paolone et al., JLab (E03-104) 2010
Puckett et al., JLab (E04-108) 2010
Ron et al., JLab (E05-004) 2007
Hu et al., JLab (E89-028) 2006
MacLachlan et al., JLab (E93-038) 2006
Jones et al., JLab (RSS) 2006
Punjabi et al., JLab (E00-006) 2005
Qattan et al., JLab (E01-001) 2005
Christy et al., JLab (E94-110) 2004
Dieterich et al., JLab (E93-049) 2002*
Gayou et al., JLab (E89-019) 2001
Milbrath et al., Bates FPP 1998
Crawford et al., BLAST 2007
Andivahis et al., SLAC 1994*
Walker et al., SLAC 1994*
Bosted et al., SLAC 1992*
Walker et al., SLAC 1989*
Litt et al., SLAC 1970*
Janssens et al., SLAC 1966*
Hughes et al., SLAC 1965
Drickey & Hand, SLAC 1962
Hanson et al., CEA 1973*
Price et al., CEA 1971
Dieterich et al., MAMI 2001*
Pospischil et al., MAMI 2001
Borkowski et al., MAMI 1975*
Bartel et al., DESY 1973
Bartel et al., DESY 1967
Berger et al., BUES 1971*
Frerejacque et al., OLA 1966*
Dudelzak et al., OLA 1965*
Benaksas et al., OLA 1964

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