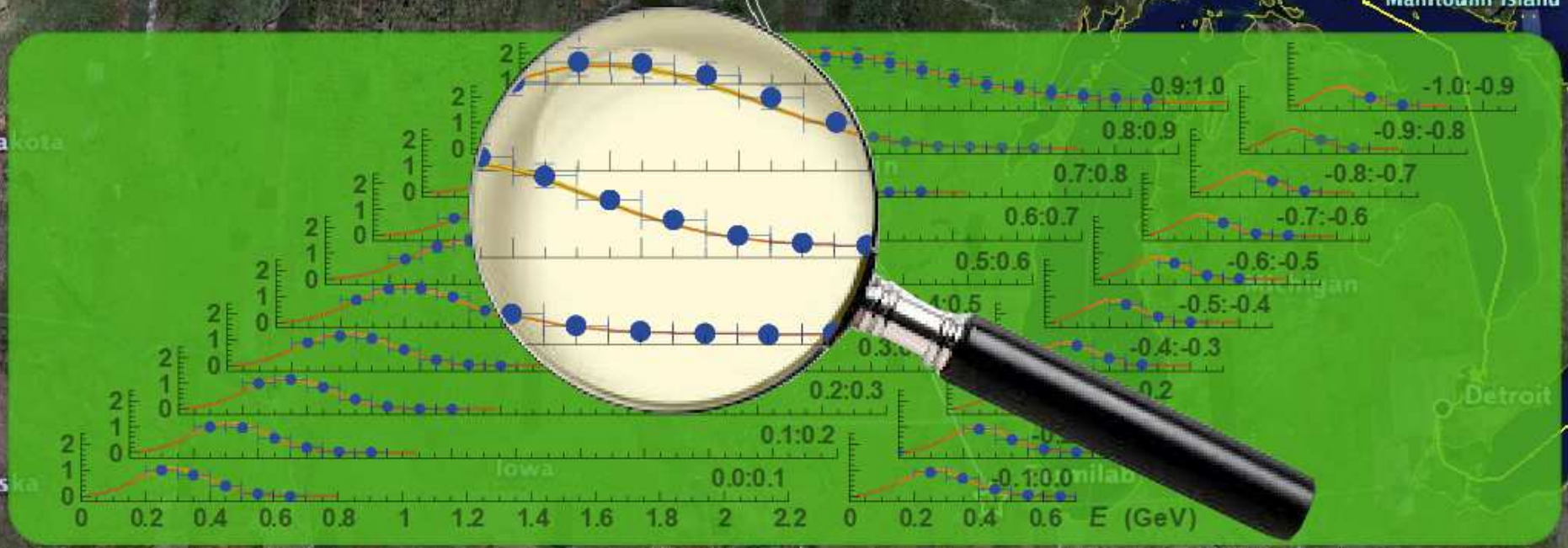
# Running axial mass for CCQE neutrino-nucleus scattering

I. Kakorin, K. Kuzmin, & V. Naumov

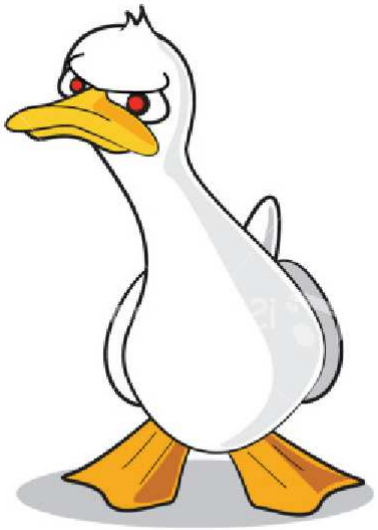
JINR NOvA Group (Dubna)




NOvA Collaboration Meeting at FNAL, October 20–23 , 2016

The aim of this report is to provide a simple "cookbook recipe" for a precise description of the CCQE neutrino and antineutrino interactions with nuclei using only two adjustable parameters.

*Cooking a duck*



*NO! Ducking a cook*

We do not intend discussing the physical meaning of the proposed recipe; instead we wish to demonstrate its usefulness as an empirical tool which could be used in the data processing of the neutrino oscillation experiments with accelerator and atmospheric (anti)neutrino beams.

# Outline

- Introduction
- Total Cross Sections – Recent data
- FNAL MiniBooNE 2010–2013
- FNAL MINERνA 2013
- T2K ND280 2016
- Parameters of the Smith–Moniz RFG Model
- Comparison with the early "golden" data
- Comparison with certain datasets unclaimed in the fits
- Implementation in GENIE
- Conclusions
- Backup slides (details of the fit, inputs, and all that)

# Introduction

One of the main sources of ambiguity in predicted neutrino event rates in the accelerator experiments at low and intermediate energies is caused by nuclear effects for the charged-current quasielastic (CCQE) $\nu/\overline{\nu}$ interactions with various detection targets.

Another closely-related problem arises from the experimental uncertainty of the nucleon axial mass parameter, $M_A$, in the dipole model of the nucleon axial form factor

$$F_A\left(Q^2\right) = F_A(0)\left(1+\frac{Q^2}{M_A^2}\right)^{-2}.$$

The most familiar Relativistic Fermi Gas (RFG) model cannot describe the up-to-date CCQE data (NOMAD, MiniBooNE, MINERνA, T2K, etc.) with a unique value of $M_A$.

In fact, there are a lot of different models based on the Fermi-gas approach. One of the most popular models is the model by Smith and Moniz (SM).[a] Below, the term "RFG" is used for the slightly-modified SM model,[b] with an updated set of the input parameters.

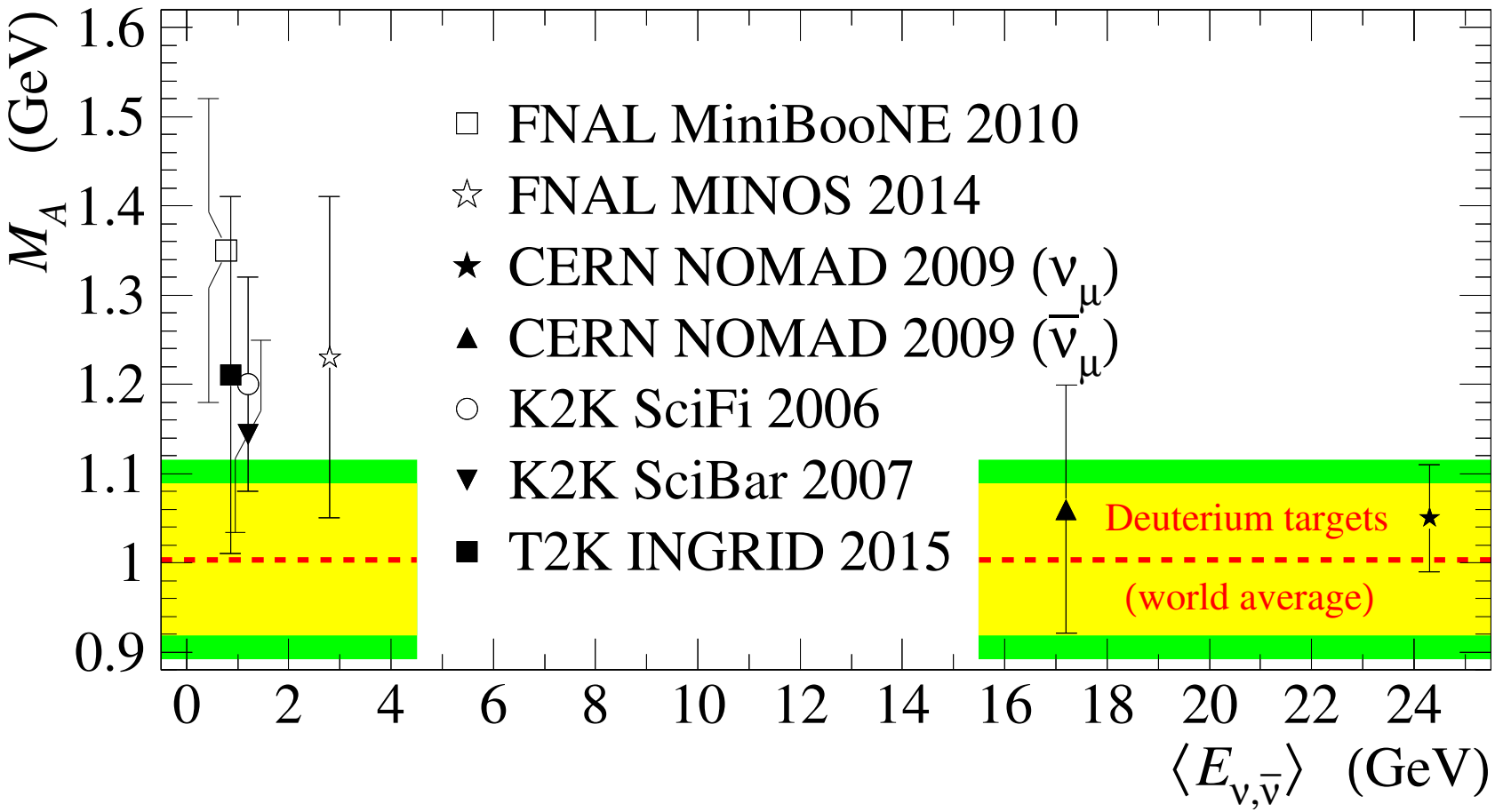
**NOTE:** The default RFG model in GENIE is the Bodek-Ritchie (BR) model rather than the SM.

[a] R.A. Smith and E.J. Moniz, *"Neutrino reactions on nuclear targets,"* Nucl. Phys. B **43**, 605 (1972); erratum – *ibid*. **101**, 547 (1975).

[b] K.S. Kuzmin, V.V. Lyubushkin and V.A. Naumov, *"Quasielastic axial-vector mass from experiments on neutrino-nucleus scattering,"* Eur. Phys. J. C **54**, 517 (2008) [arXiv:0712.4384 [hep-ph]].

## Some nuclear models currently on the market.

- **RFG Extensions:**
  - **Local Fermi Gas (LFG)** or **Local Density Approximation (LDA)**
    [Fermi momenta and separation energies are functions of the position in the nucleus]
  - **Spectral Function (SF)**
    [correlations between removal energies and initial state momenta of the nucleons, Gaussian spectrum of each shell]
- **Scaling and Superscaling (SuSA) models.**
- **Relativistic Mean Field (RMF) models.**
- **Relativistic Green's Function (RGF) models.**
- **Transverse Enhancement Model (TEM).**
- **Many-body Currents:**
  - **Short-range correlations (SRC),**
  - **Meson Exchange Currents (MEC, 2p2h),** $\Longleftarrow$ (most popular and successful)
  - **Random Phase Approximation (RPA)** and **Quasiparticle RPA (QRPA).**
- **Hotchpotchs** (e.g., SuSA+MEC, MEC+RPA, etc.).




The nucleon axial mass vs. neutrino energy, obtained in several recent experiments.
The dashed line and surrounding shaded double band represent the value of $M_A$ and its $1\sigma$ and $2\sigma$ uncertainties extracted from the deuterium experiments ($M_A^D$ hereafter).

This effect is now explained to an extent by the advanced nuclear models.

Besides, the much simpler RFG-based calculations can be somewhat fine-tuned by using a larger value of $M_A$ in the lower energy range.[a] This observation can be used for an empirical description of the experimental data within the RFG model. Namely, it is shown[b] that disadvantages of the RFG model can be effectively compensated by introducing the energy-dependent ("running") axial mass

$$M_A^{\mathsf{run}} = M_0 \left(1 + \frac{E_0}{E_\nu}\right)$$

with the parameters values

$$M_0 = 1.006 \pm 0.025\,(0.030)\ \mathsf{GeV} \quad \text{and} \quad E_0 = 334^{+58\,(70)}_{-54\,(64)}\ \mathsf{MeV}$$
$$\left(\frac{\chi^2}{\mathsf{NDF}} = \frac{300.4}{435 - 19} = 0.72\right)$$

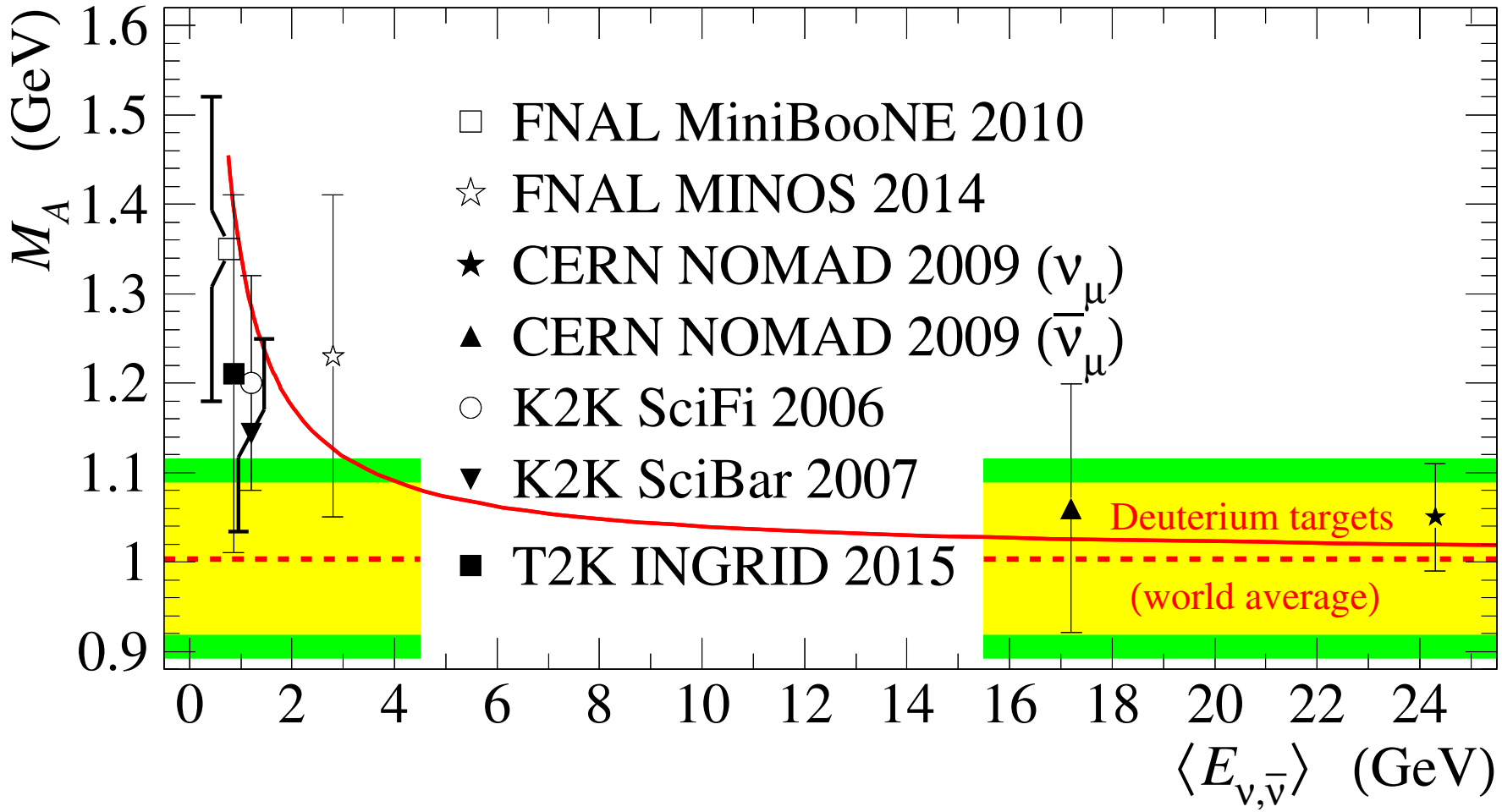
obtained from a global fit to all available data on neutrino-nucleus and antineutrino-nucleus CCQE scattering. This allows to phenomenologically account for the intricate nuclear effects beyond RFG, significant at low and medium (anti)neutrino energies, for the nuclei with $A \gtrsim 10$.

As a bonus, the parameter $M_0$ can be treated as the "current" dipole axial mass of the nucleon, that is the good old parameter $M_A$ appearing in the weak hadronic current.

[a] A lot of papers; see, e.g., A.M. Ankowski, O. Benhar, C. Mariani and E. Vagnoni, Phys. Rev. D **93**, 113004 (2016) [arXiv:1603.01072 [hep-ph]] and references therein.

[b] K. S. Kuzmin and V. A. Naumov, in preparation; see also Refs. [70–72] in backup slide 103.

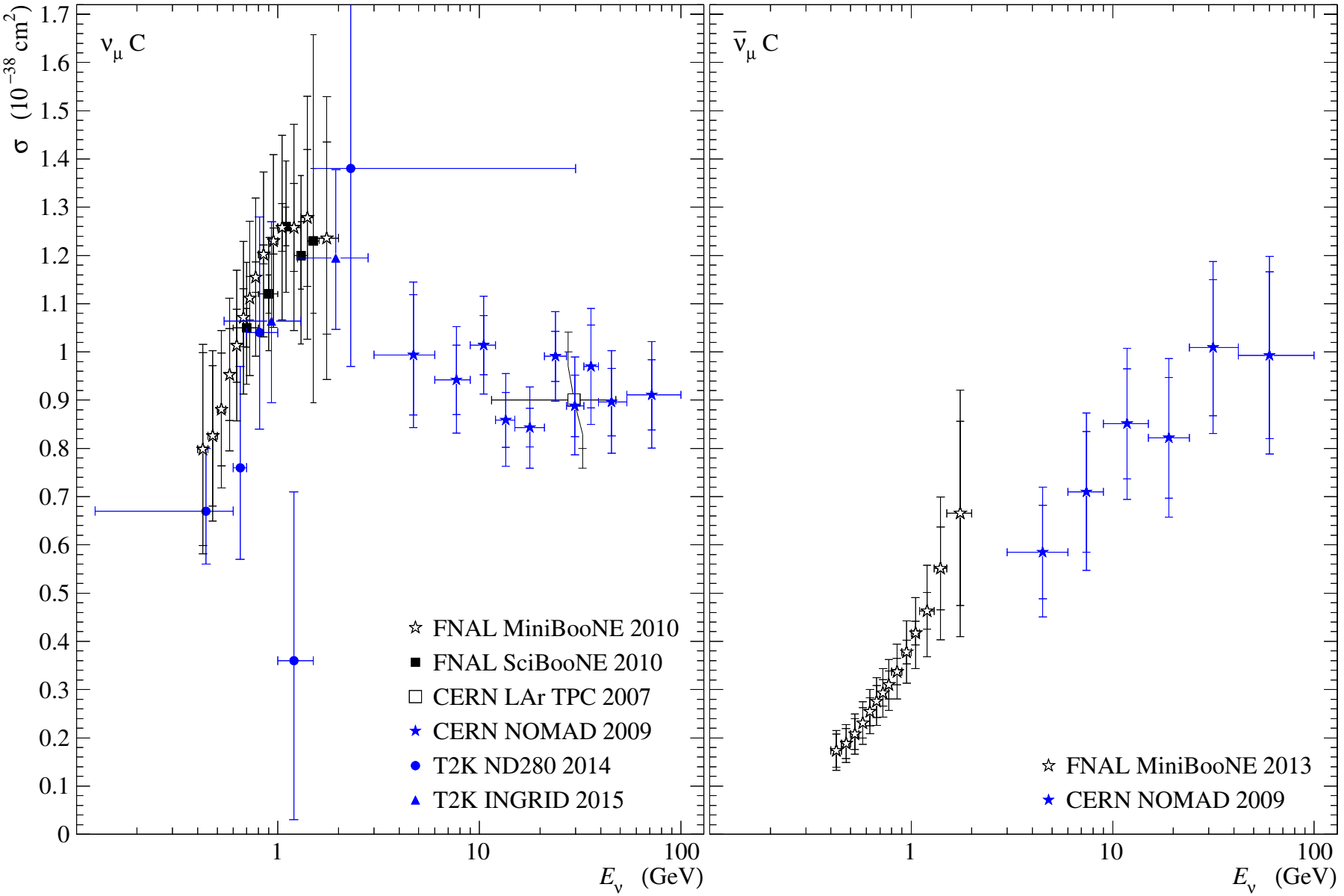
The red solid curve represents the behavior of $M_A^{\text{run}}$:

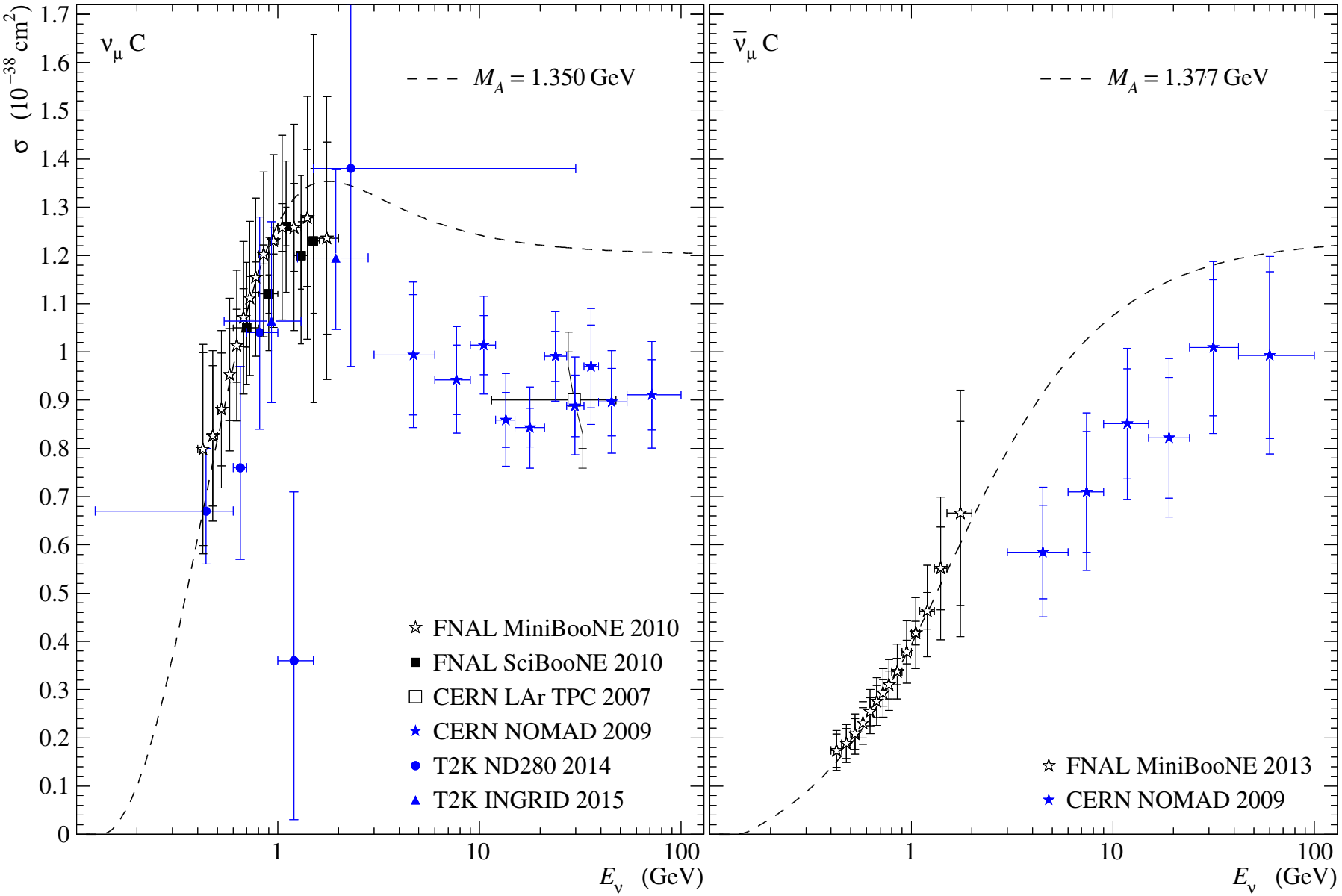


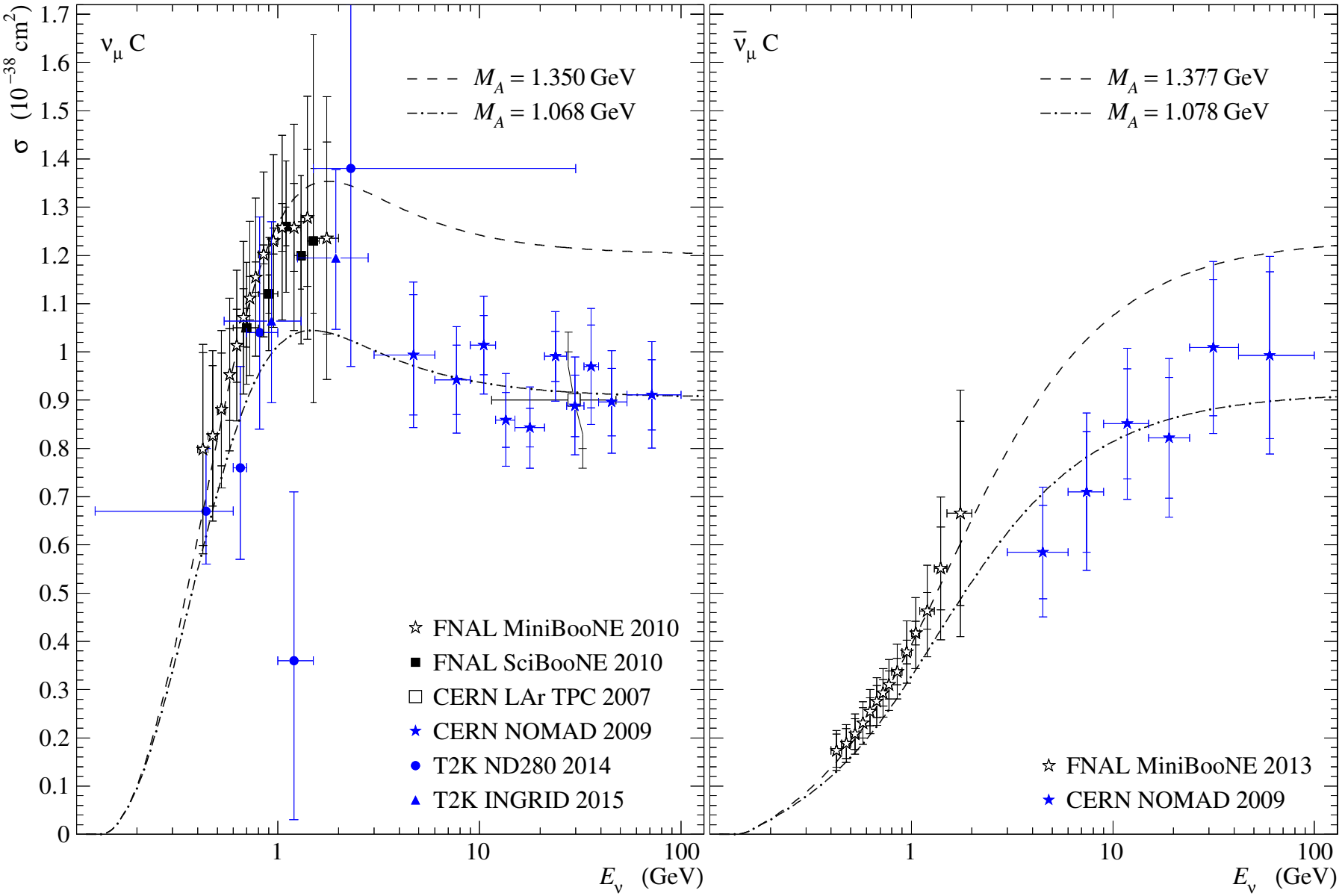

You may not be much impressed by that...

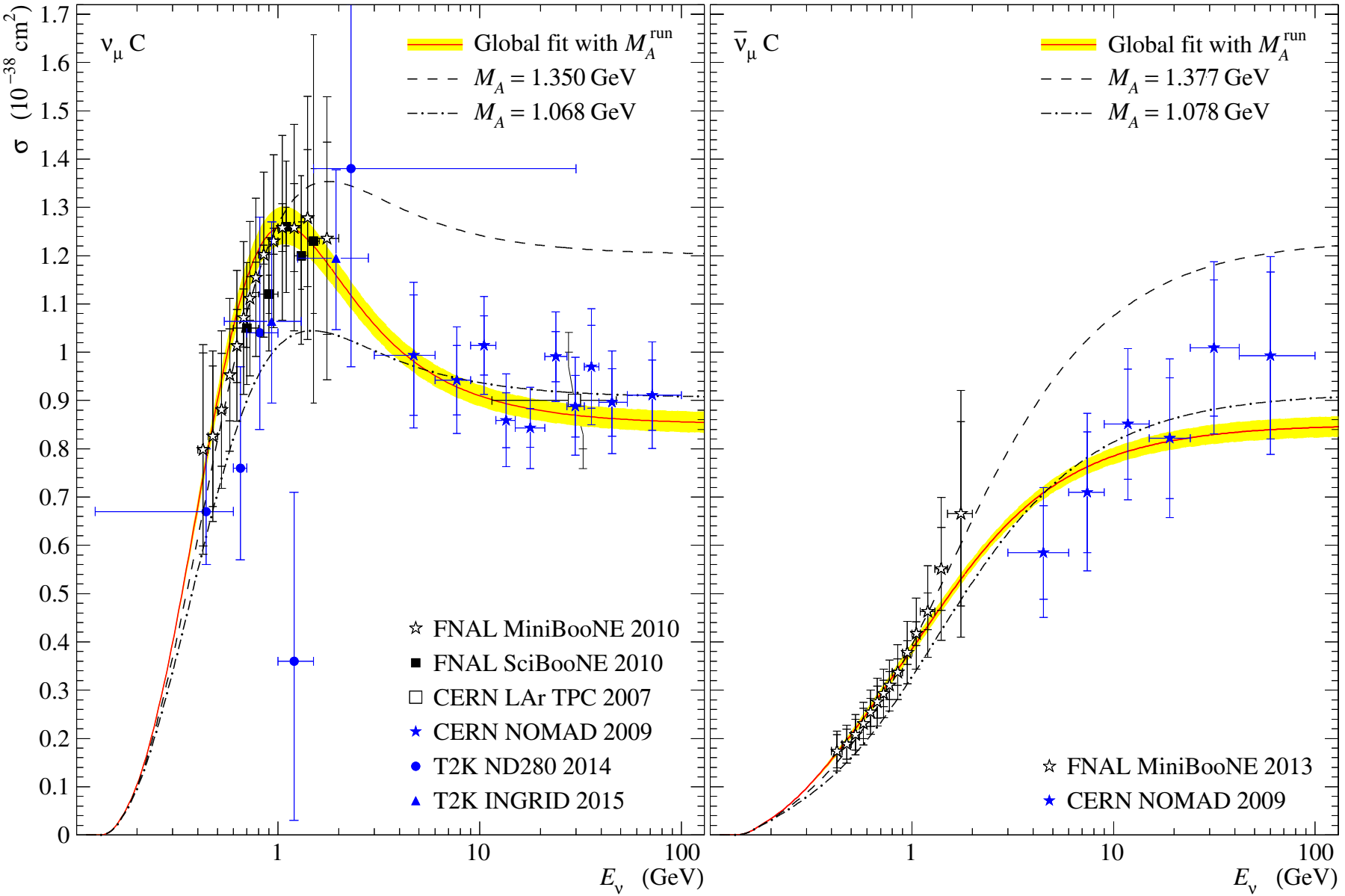
But let us now examine, how does the running axial mass describe the real data rather than the values of $M_A$ forcibly extracted from these data.

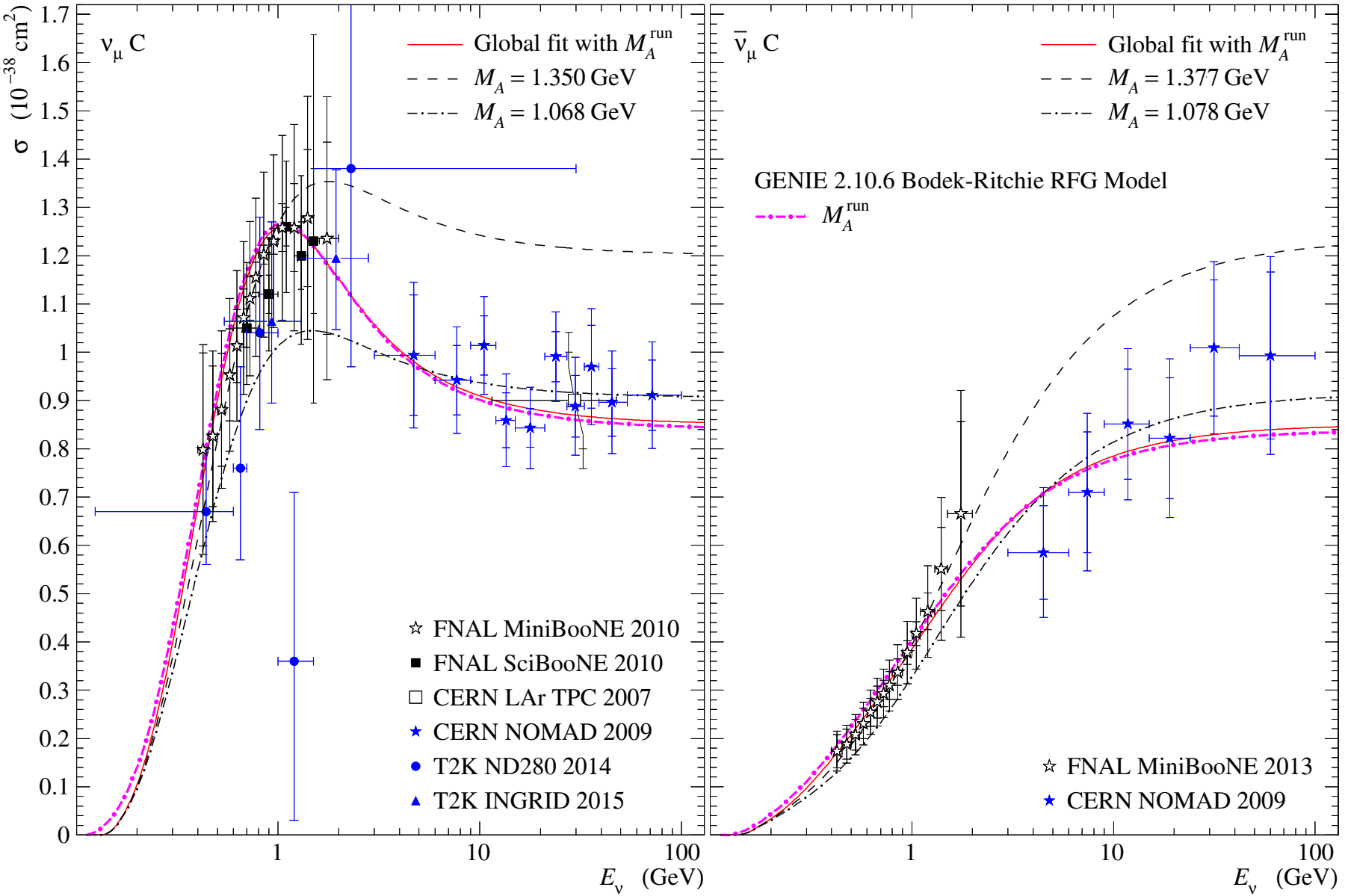
# Total Cross Sections
# Recent data

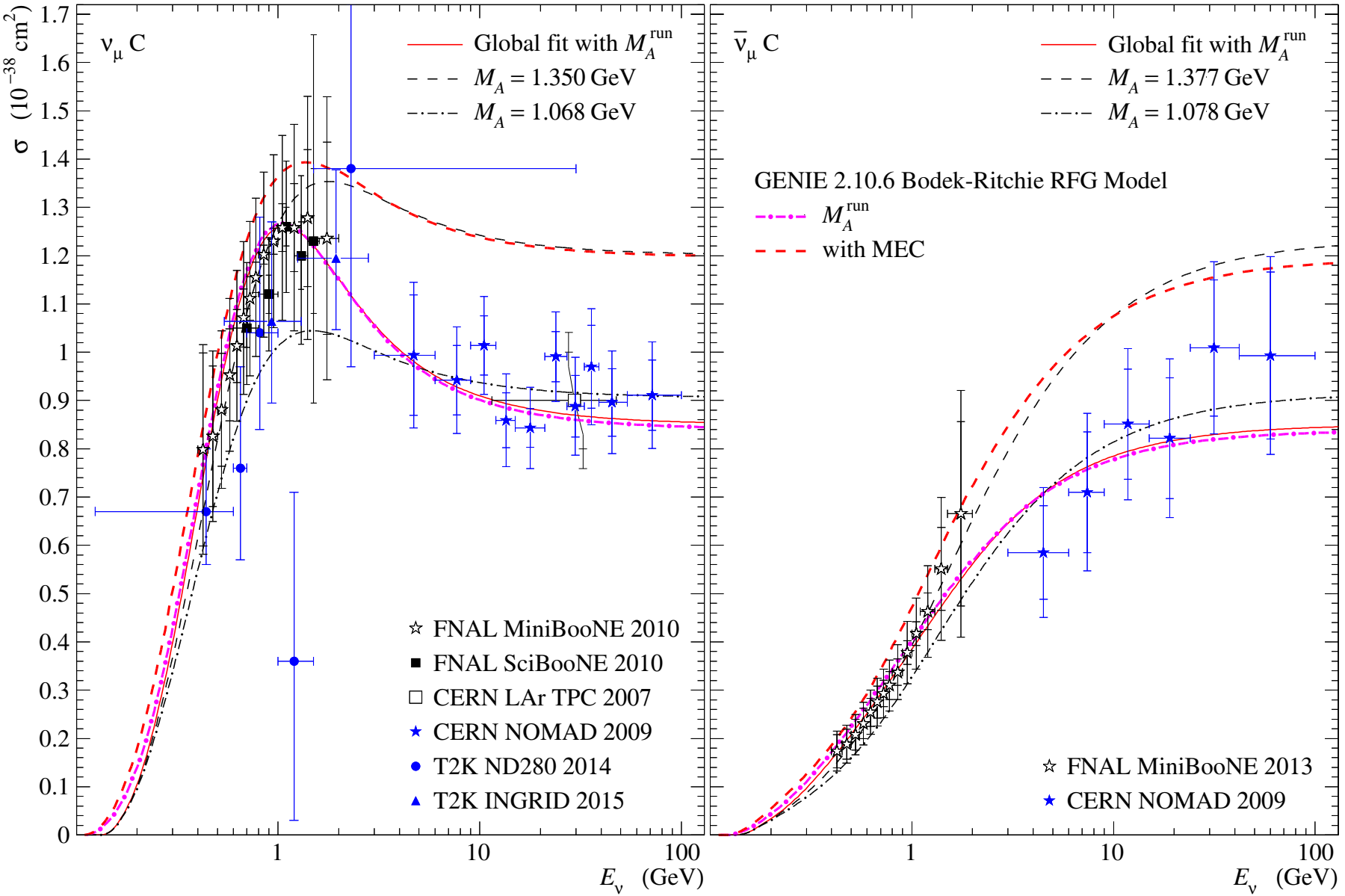

$\nu_\mu$ C
$\bar{\nu}_\mu$ C
$\sigma$ ($10^{-38}$ cm$^2$)
$E_\nu$ (GeV)
FNAL MiniBooNE 2010
FNAL SciBooNE 2010
CERN LAr TPC 2007
CERN NOMAD 2009
T2K ND280 2014
T2K INGRID 2015
FNAL MiniBooNE 2013
CERN NOMAD 2009

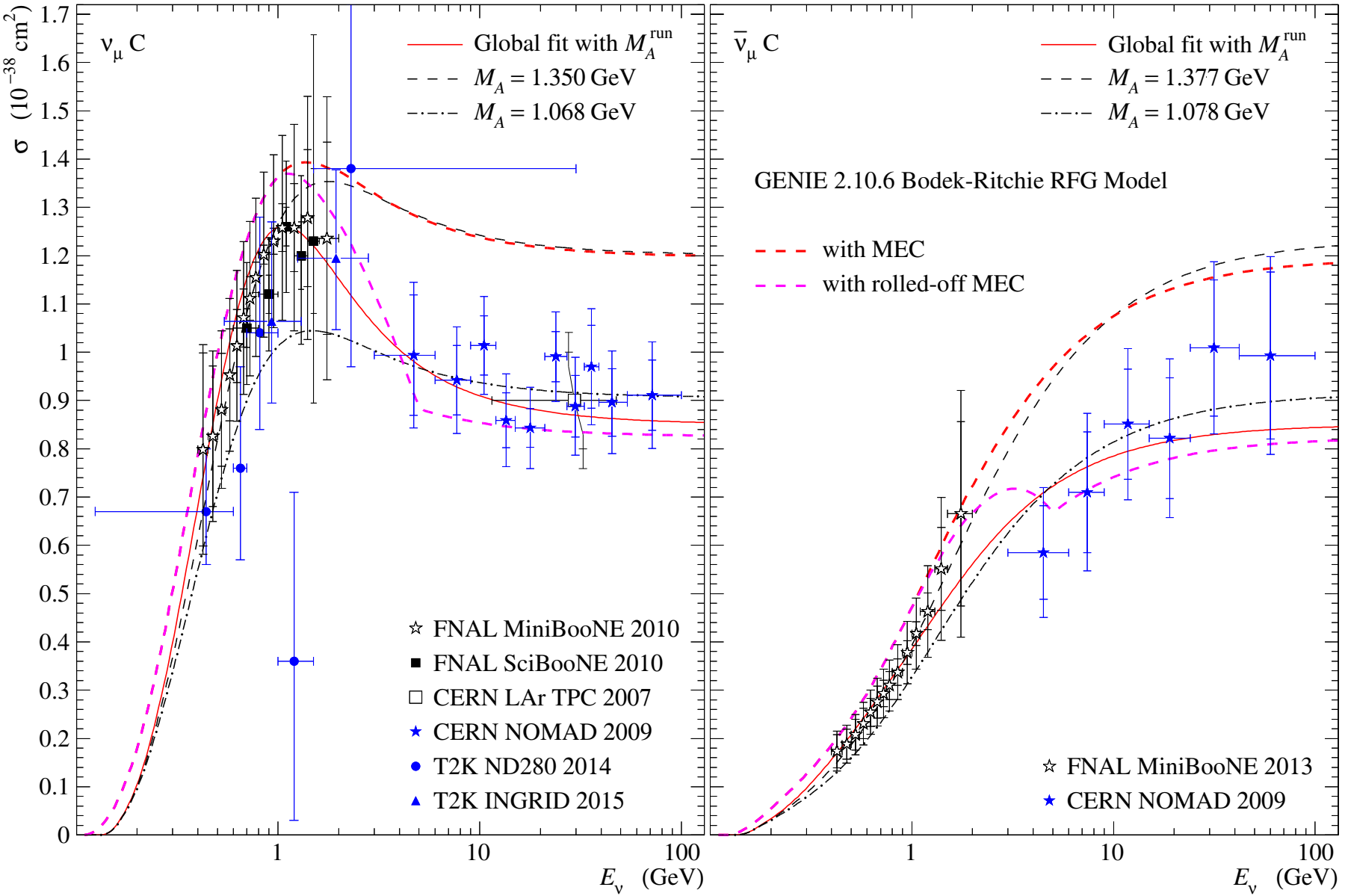

σ (10^−38 cm^2)
ν_μ C
M_A = 1.350 GeV
FNAL MiniBooNE 2010
FNAL SciBooNE 2010
CERN LAr TPC 2007
CERN NOMAD 2009
T2K ND280 2014
T2K INGRID 2015
E_ν (GeV)
ν̄_μ C
M_A = 1.377 GeV
FNAL MiniBooNE 2013
CERN NOMAD 2009
E_ν (GeV)


$\nu_\mu$ C
$M_A$ = 1.350 GeV
$M_A$ = 1.068 GeV
FNAL MiniBooNE 2010
FNAL SciBooNE 2010
CERN LAr TPC 2007
CERN NOMAD 2009
T2K ND280 2014
T2K INGRID 2015
$\sigma$ ($10^{-38}$ cm$^2$)
$E_\nu$ (GeV)
$\bar{\nu}_\mu$ C
$M_A$ = 1.377 GeV
$M_A$ = 1.078 GeV
FNAL MiniBooNE 2013
CERN NOMAD 2009
$E_\nu$ (GeV)

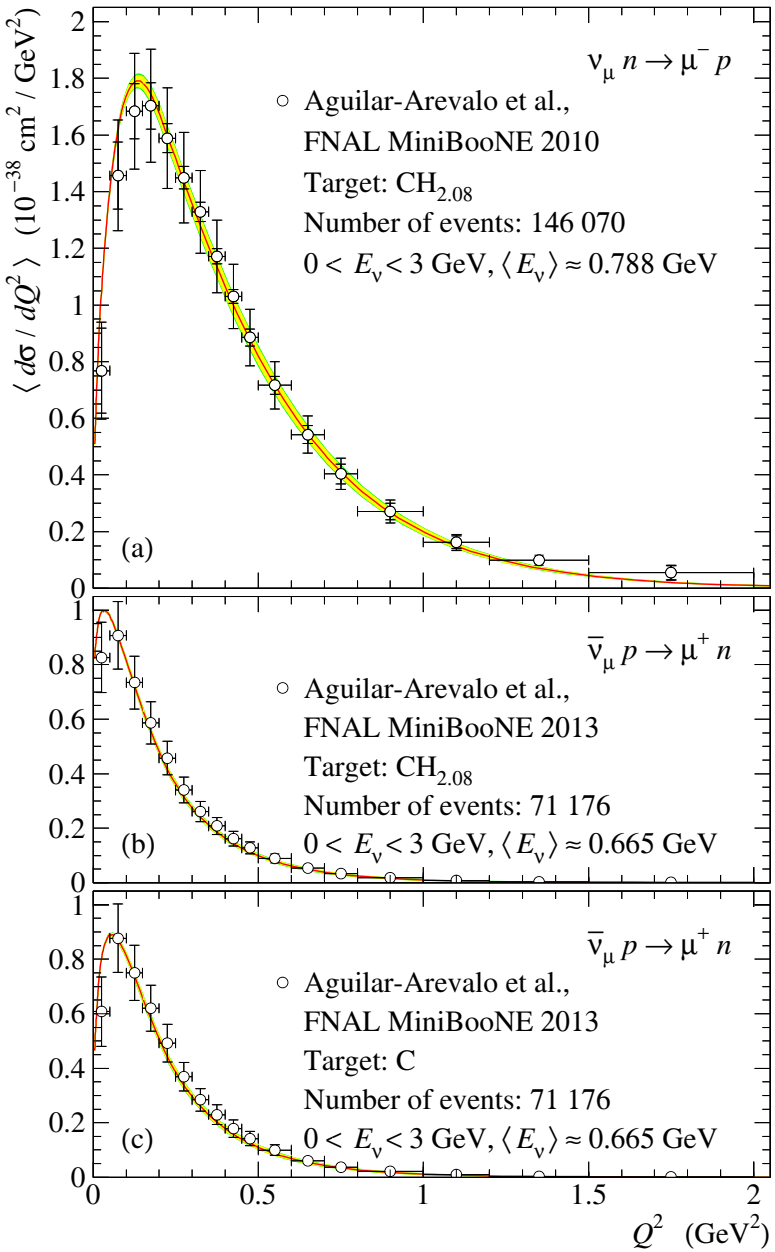

σ (10^−38 cm^2)
ν_μ C
Global fit with M_A^run
M_A = 1.350 GeV
M_A = 1.068 GeV
FNAL MiniBooNE 2010
FNAL SciBooNE 2010
CERN LAr TPC 2007
CERN NOMAD 2009
T2K ND280 2014
T2K INGRID 2015
E_ν (GeV)
ν̄_μ C
Global fit with M_A^run
M_A = 1.377 GeV
M_A = 1.078 GeV
FNAL MiniBooNE 2013
CERN NOMAD 2009
E_ν (GeV)


$\nu_\mu$ C
Global fit with $M_A^{run}$
$M_A$ = 1.350 GeV
$M_A$ = 1.068 GeV
FNAL MiniBooNE 2010
FNAL SciBooNE 2010
CERN LAr TPC 2007
CERN NOMAD 2009
T2K ND280 2014
T2K INGRID 2015
σ (10$^{-38}$ cm$^2$)
$E_\nu$ (GeV)
$\bar{\nu}_\mu$ C
Global fit with $M_A^{run}$
$M_A$ = 1.377 GeV
$M_A$ = 1.078 GeV
GENIE 2.10.6 Bodek-Ritchie RFG Model
$M_A^{run}$
FNAL MiniBooNE 2013
CERN NOMAD 2009
$E_\nu$ (GeV)

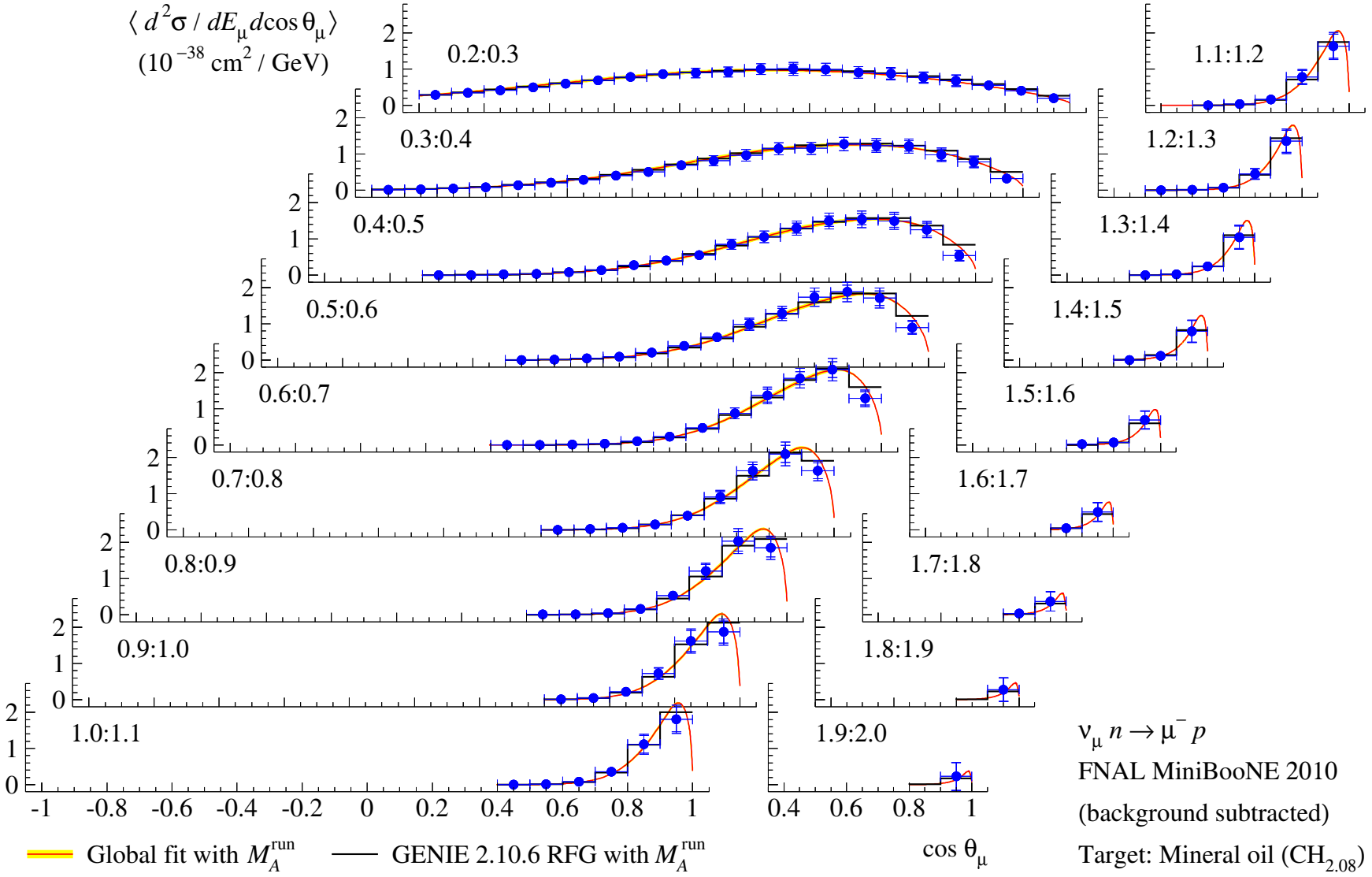

σ (10^−38 cm^2)
ν_μ C
Global fit with M_A^run
M_A = 1.350 GeV
M_A = 1.068 GeV
FNAL MiniBooNE 2010
FNAL SciBooNE 2010
CERN LAr TPC 2007
CERN NOMAD 2009
T2K ND280 2014
T2K INGRID 2015
E_ν (GeV)
ν̄_μ C
Global fit with M_A^run
M_A = 1.377 GeV
M_A = 1.078 GeV
GENIE 2.10.6 Bodek-Ritchie RFG Model
M_A^run
with MEC
FNAL MiniBooNE 2013
CERN NOMAD 2009
E_ν (GeV)

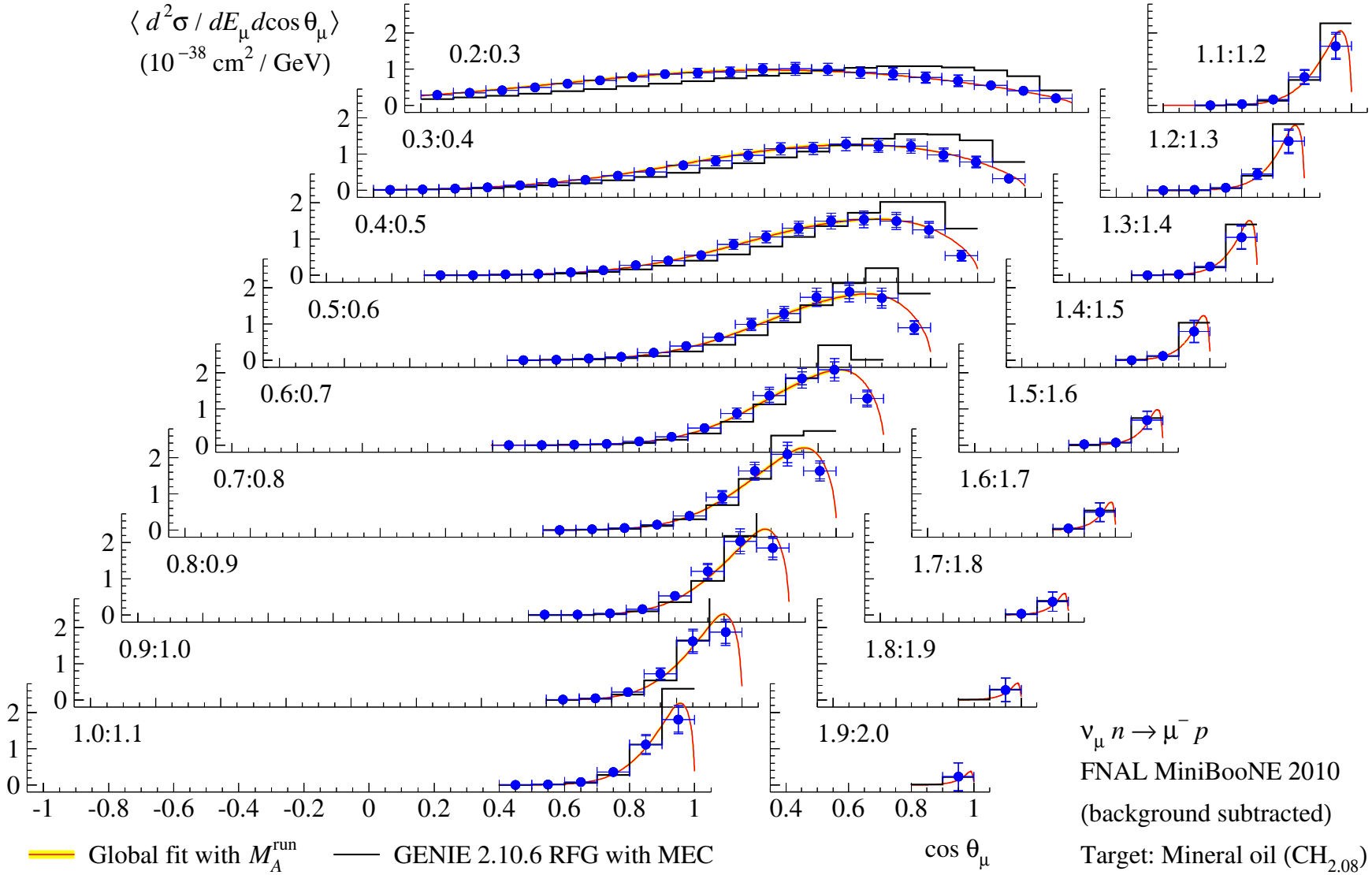

σ (10^−38 cm^2)
ν_μ C
Global fit with M_A^run
M_A = 1.350 GeV
M_A = 1.068 GeV
FNAL MiniBooNE 2010
FNAL SciBooNE 2010
CERN LAr TPC 2007
CERN NOMAD 2009
T2K ND280 2014
T2K INGRID 2015
E_ν (GeV)
ν̄_μ C
Global fit with M_A^run
M_A = 1.377 GeV
M_A = 1.078 GeV
GENIE 2.10.6 Bodek-Ritchie RFG Model
with MEC
with rolled-off MEC
FNAL MiniBooNE 2013
CERN NOMAD 2009
E_ν (GeV)

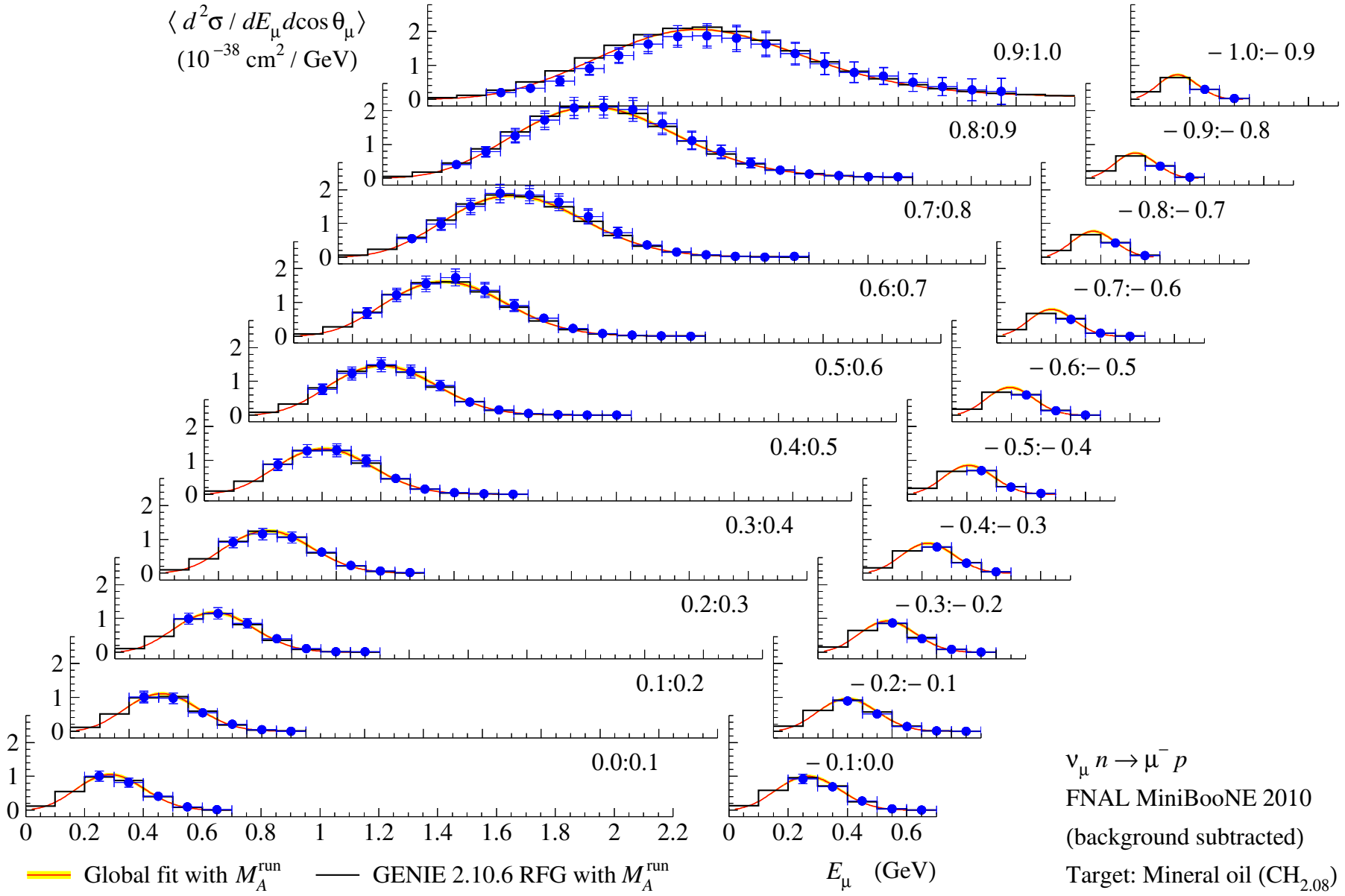
# FNAL MiniBooNE 2010–2013

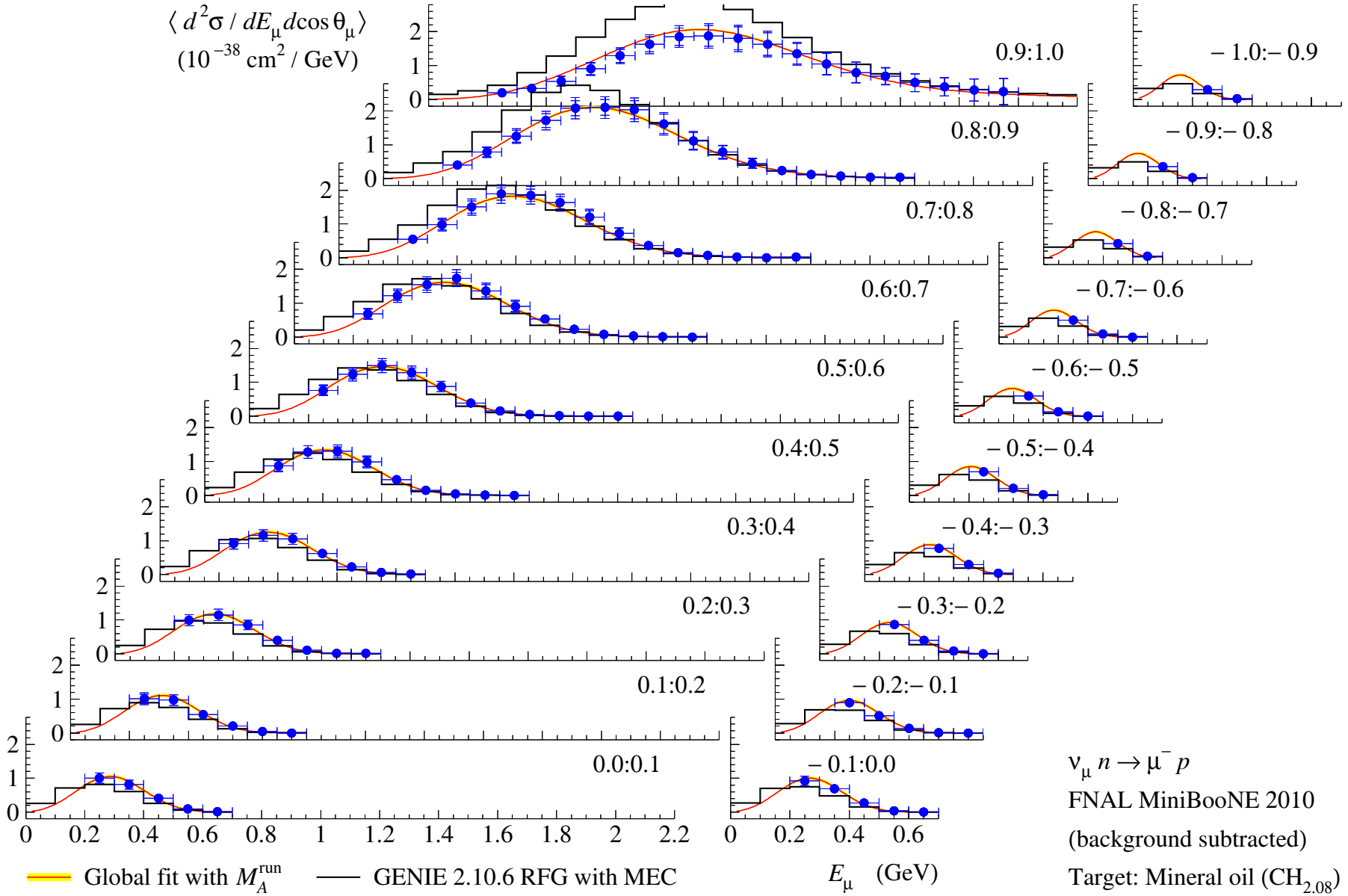


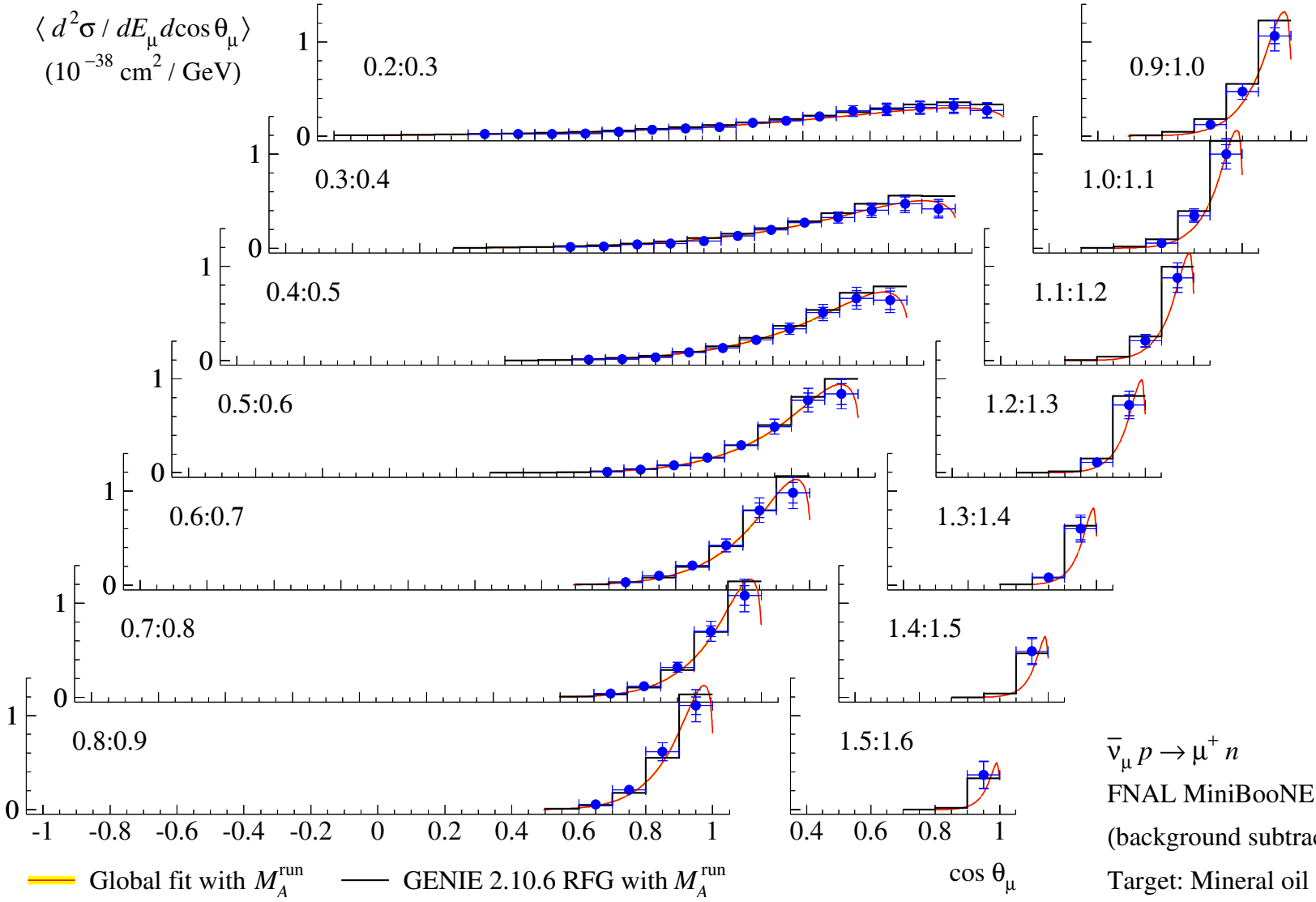

← Recalculated by the MiniBooNE Co. from the "raw" data on the $CH_{2.08}$ target. The agreement is a little bit worse and we know why...


⟨ d²σ / dE_μ d cos θ_μ ⟩
(10⁻³⁸ cm² / GeV)
0.2:0.3
0.3:0.4
0.4:0.5
0.5:0.6
0.6:0.7
0.7:0.8
0.8:0.9
0.9:1.0
1.0:1.1
1.1:1.2
1.2:1.3
1.3:1.4
1.4:1.5
1.5:1.6
1.6:1.7
1.7:1.8
1.8:1.9
1.9:2.0
cos θ_μ
Global fit with M_A^run
GENIE 2.10.6 RFG with M_A^run
ν_μ n → μ⁻ p
FNAL MiniBooNE 2010
(background subtracted)
Target: Mineral oil (CH_2.08)


⟨ d²σ / dE_μ dcos θ_μ ⟩
(10⁻³⁸ cm² / GeV)
0.2:0.3
0.3:0.4
0.4:0.5
0.5:0.6
0.6:0.7
0.7:0.8
0.8:0.9
0.9:1.0
1.0:1.1
1.1:1.2
1.2:1.3
1.3:1.4
1.4:1.5
1.5:1.6
1.6:1.7
1.7:1.8
1.8:1.9
1.9:2.0
cos θ_μ
Global fit with M_A^run
GENIE 2.10.6 RFG with MEC
ν_μ n → μ⁻ p
FNAL MiniBooNE 2010
(background subtracted)
Target: Mineral oil (CH_2.08)


⟨ d²σ / dE_μ d cos θ_μ ⟩
(10⁻³⁸ cm² / GeV)
0.9:1.0
– 1.0:– 0.9
0.8:0.9
– 0.9:– 0.8
0.7:0.8
– 0.8:– 0.7
0.6:0.7
– 0.7:– 0.6
0.5:0.6
– 0.6:– 0.5
0.4:0.5
– 0.5:– 0.4
0.3:0.4
– 0.4:– 0.3
0.2:0.3
– 0.3:– 0.2
0.1:0.2
– 0.2:– 0.1
0.0:0.1
– 0.1:0.0
E_μ (GeV)
Global fit with M_A^run
GENIE 2.10.6 RFG with M_A^run
ν_μ n → μ⁻ p
FNAL MiniBooNE 2010
(background subtracted)
Target: Mineral oil (CH_2.08)


⟨ d²σ / dE_μ d cos θ_μ ⟩
(10⁻³⁸ cm² / GeV)
0.9:1.0
– 1.0:– 0.9
0.8:0.9
– 0.9:– 0.8
0.7:0.8
– 0.8:– 0.7
0.6:0.7
– 0.7:– 0.6
0.5:0.6
– 0.6:– 0.5
0.4:0.5
– 0.5:– 0.4
0.3:0.4
– 0.4:– 0.3
0.2:0.3
– 0.3:– 0.2
0.1:0.2
– 0.2:– 0.1
0.0:0.1
– 0.1:0.0
0 0.2 0.4 0.6 0.8 1 1.2 1.4 1.6 1.8 2 2.2
0 0.2 0.4 0.6
E_μ (GeV)
Global fit with M_A^run
GENIE 2.10.6 RFG with MEC
ν_μ n → μ⁻ p
FNAL MiniBooNE 2010
(background subtracted)
Target: Mineral oil (CH_2.08)

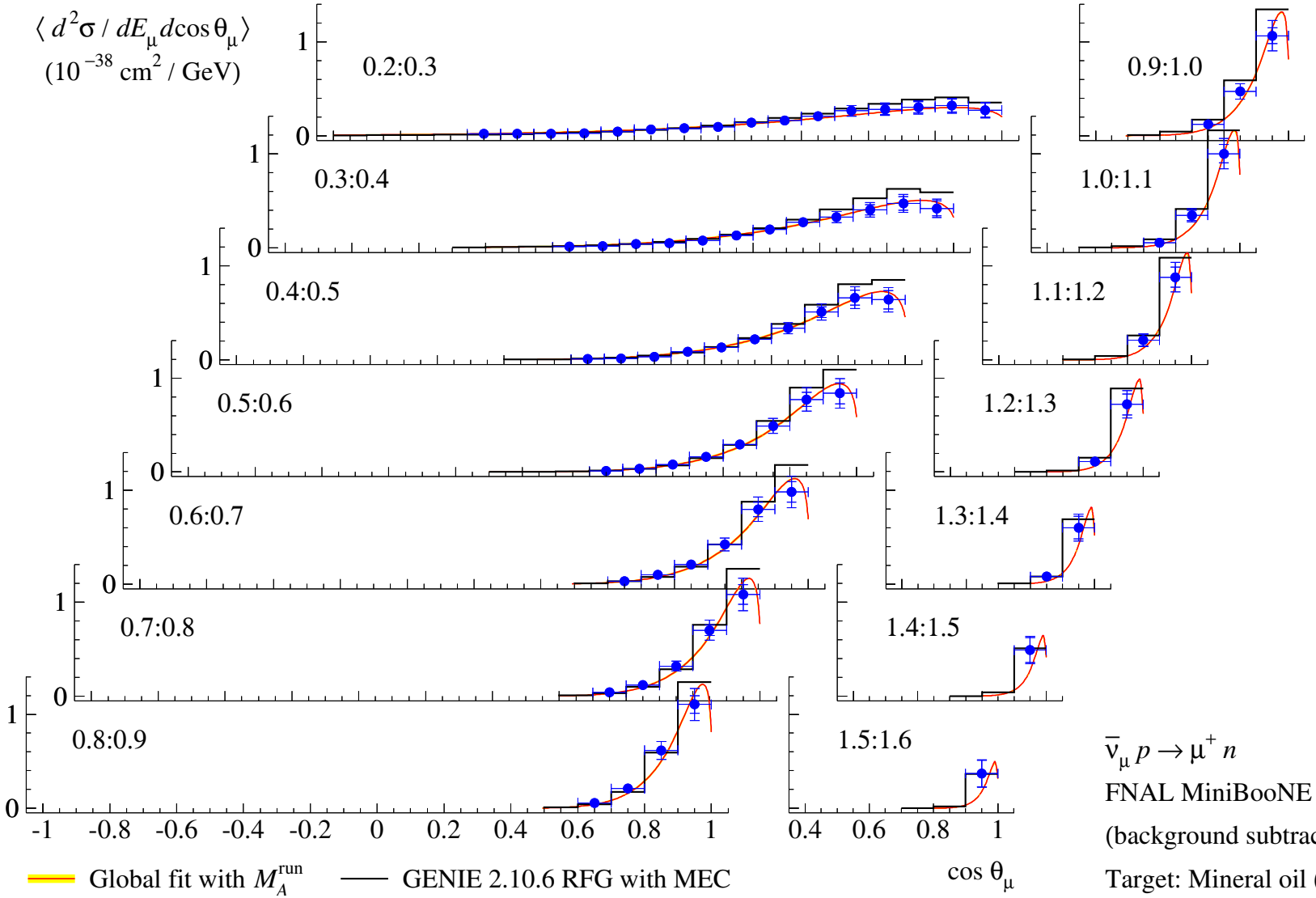



$\overline{\nu}_\mu\, p \to \mu^+\, n$

FNAL MiniBooNE 2013

(background subtracted)

Target: Mineral oil ($CH_{2.08}$)

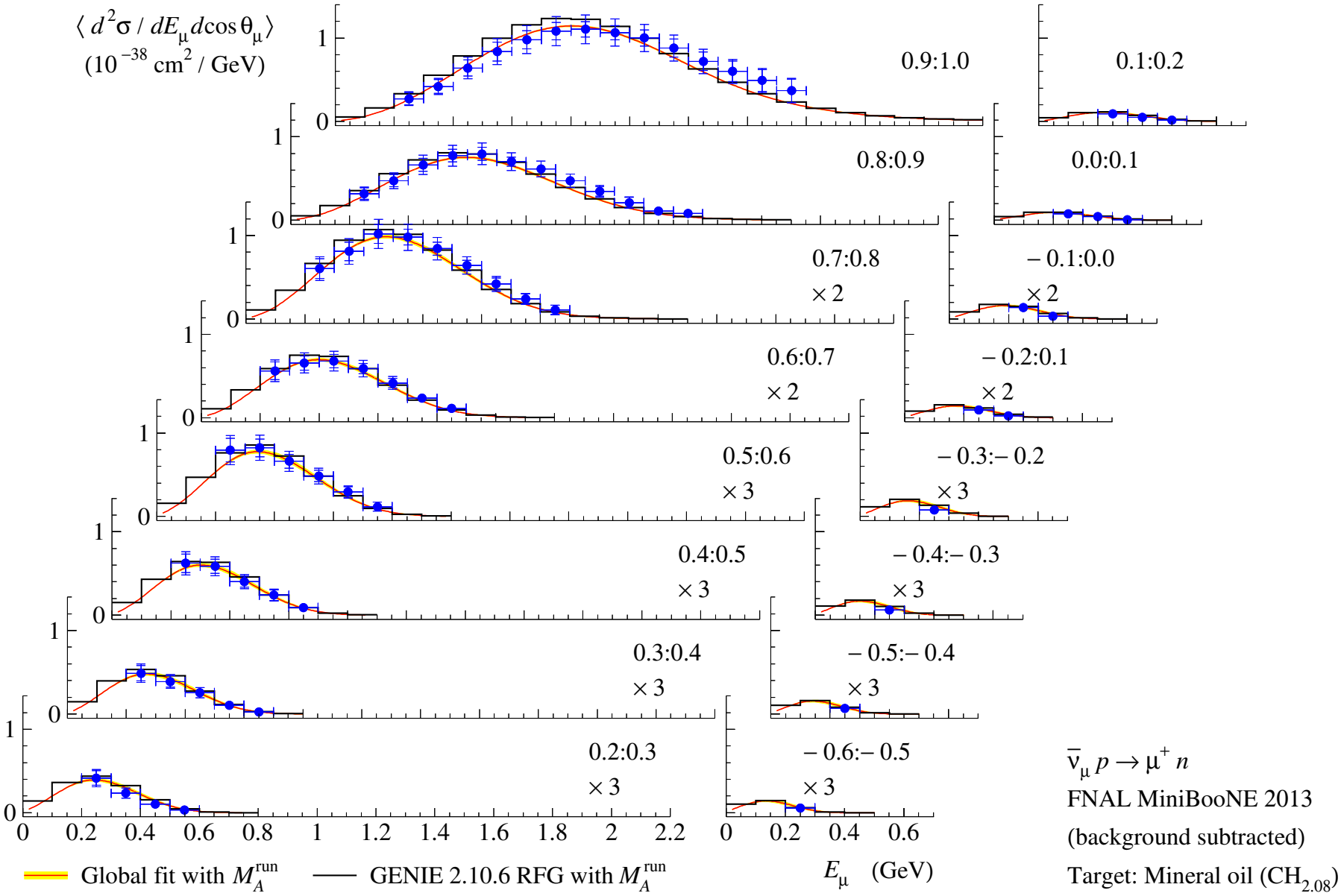



$\overline{\nu}_\mu\, p \to \mu^+\, n$

FNAL MiniBooNE 2013

(background subtracted)

Target: Mineral oil ($CH_{2.08}$)

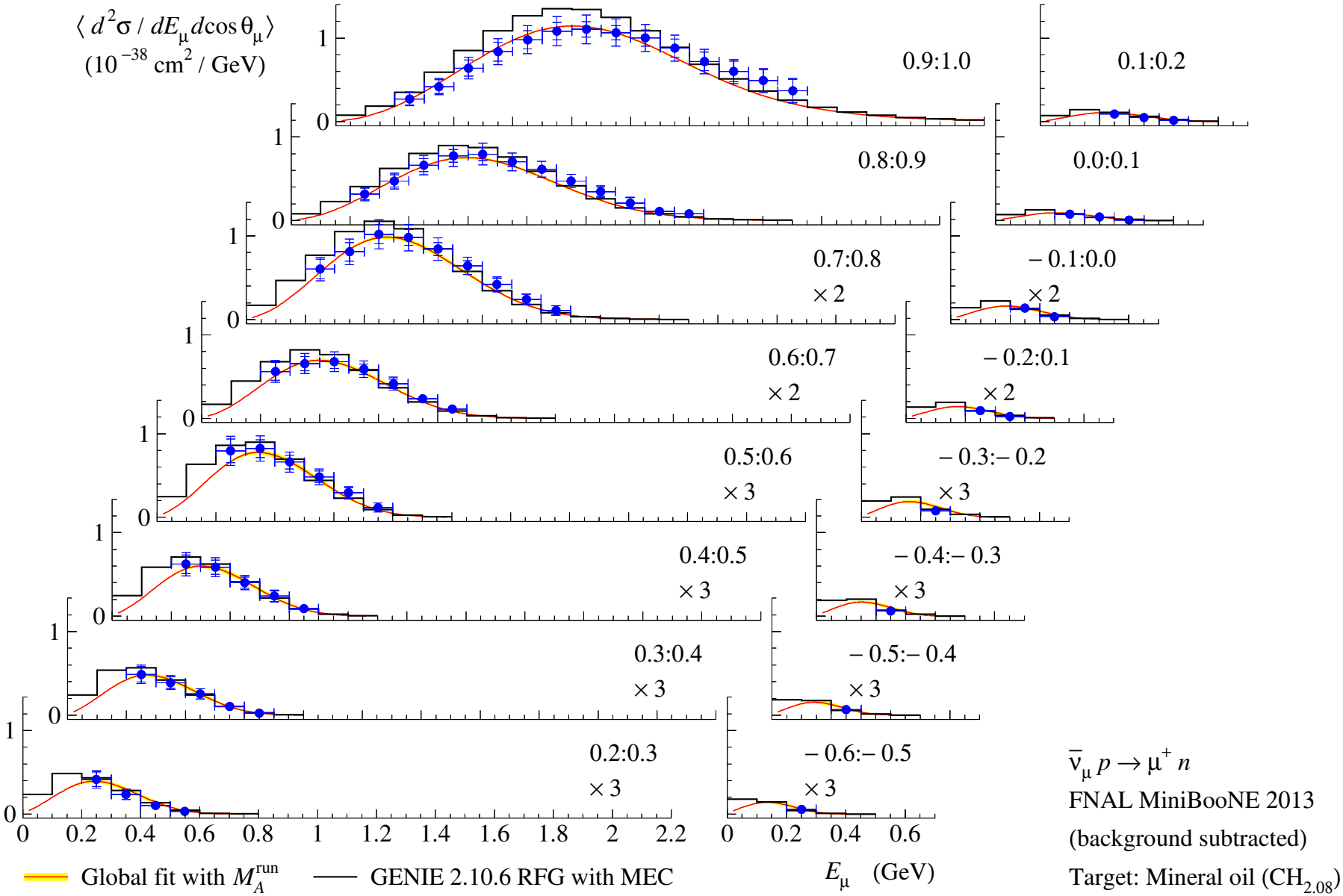



$\bar{\nu}_\mu\, p \to \mu^+\, n$

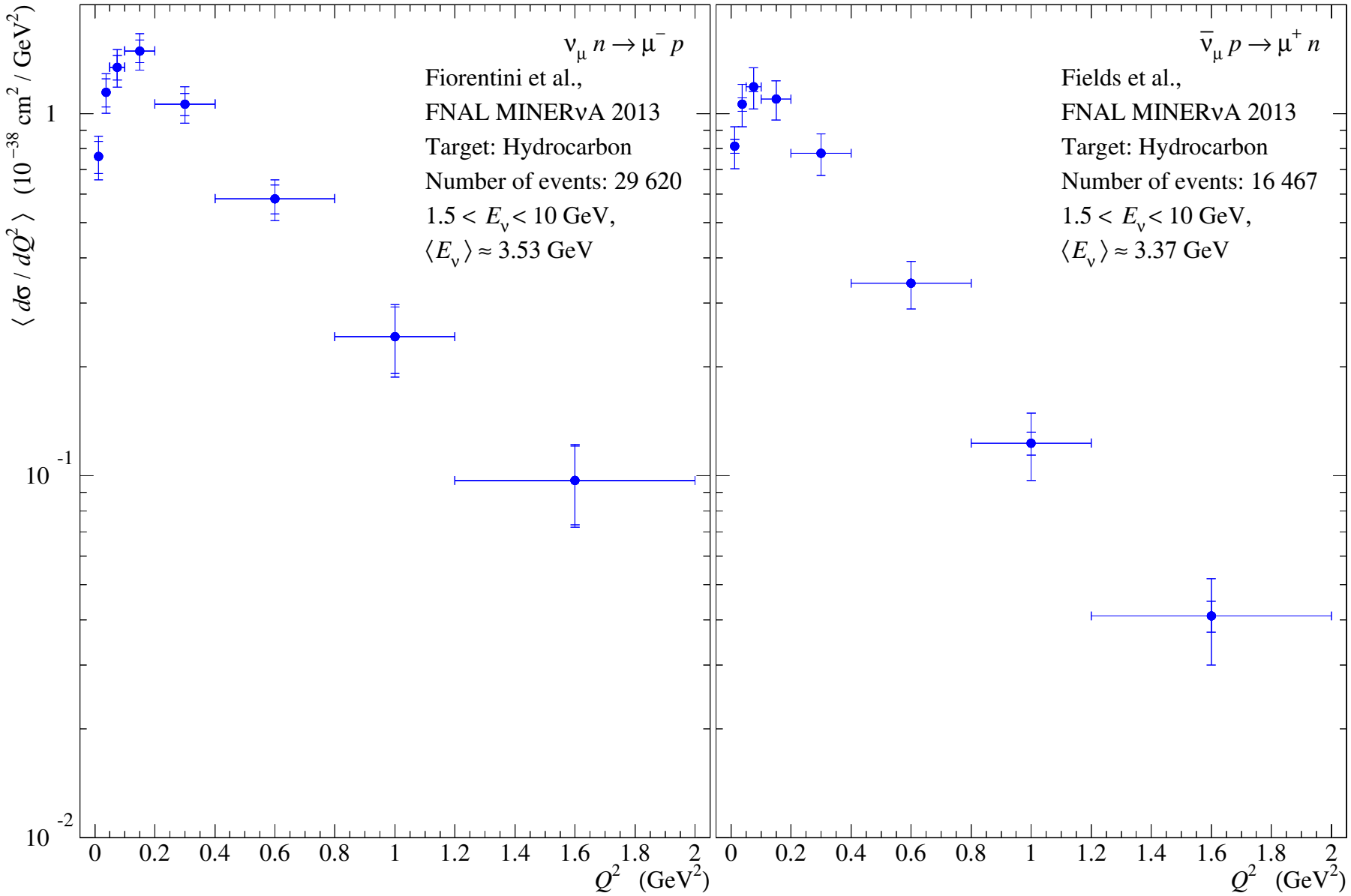
FNAL MiniBooNE 2013

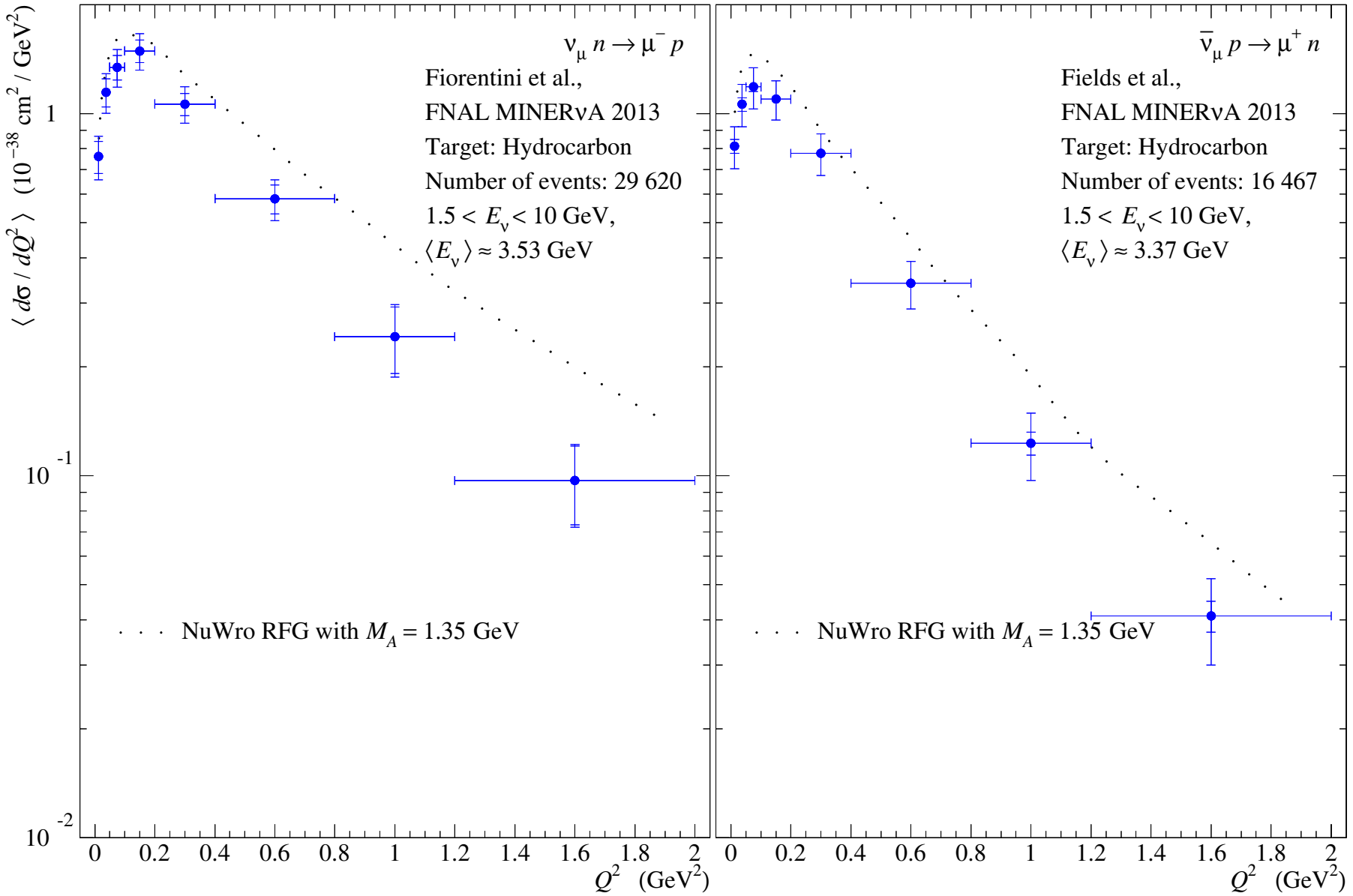
(background subtracted)

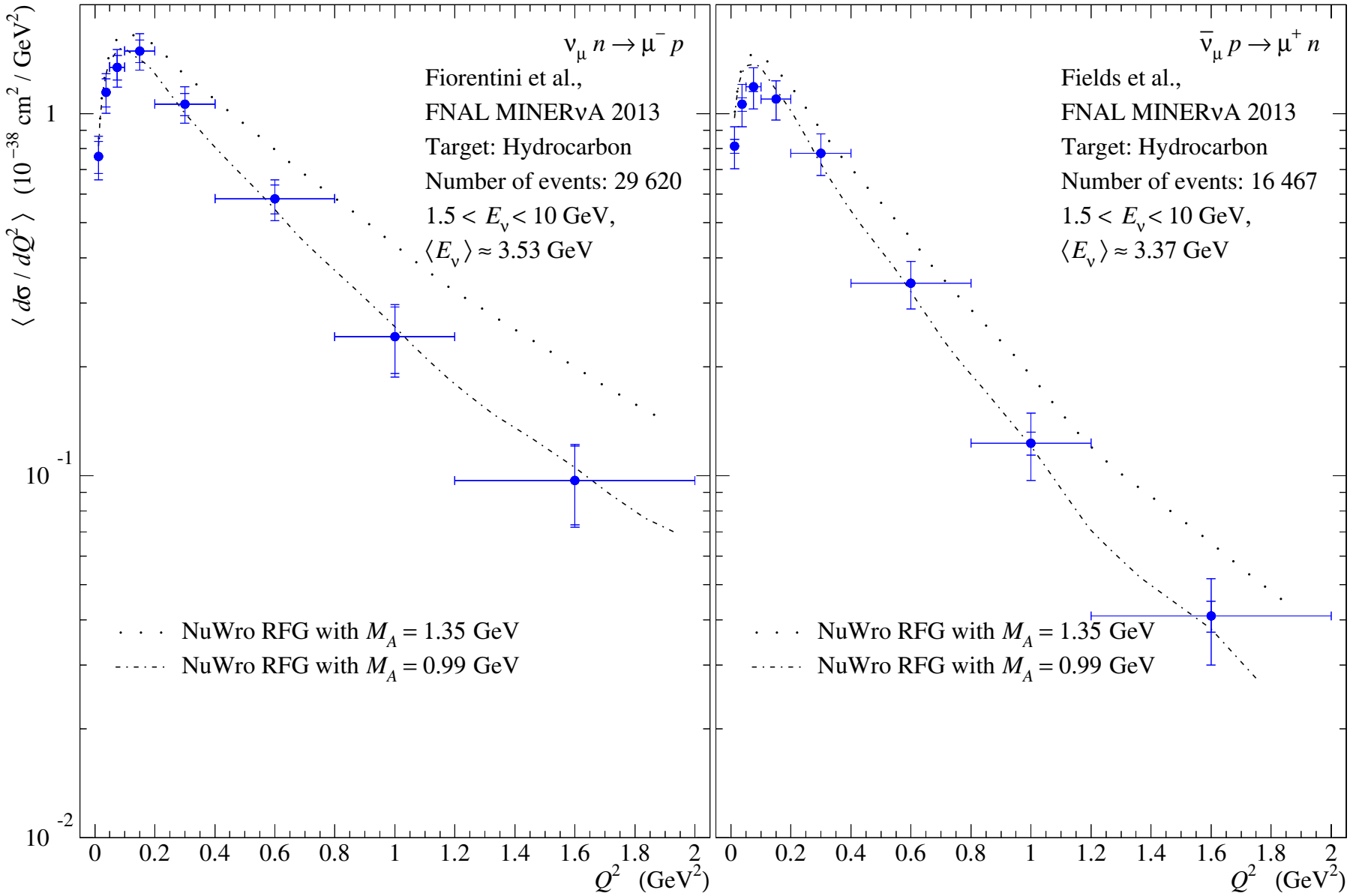
Target: Mineral oil ($CH_{2.08}$)

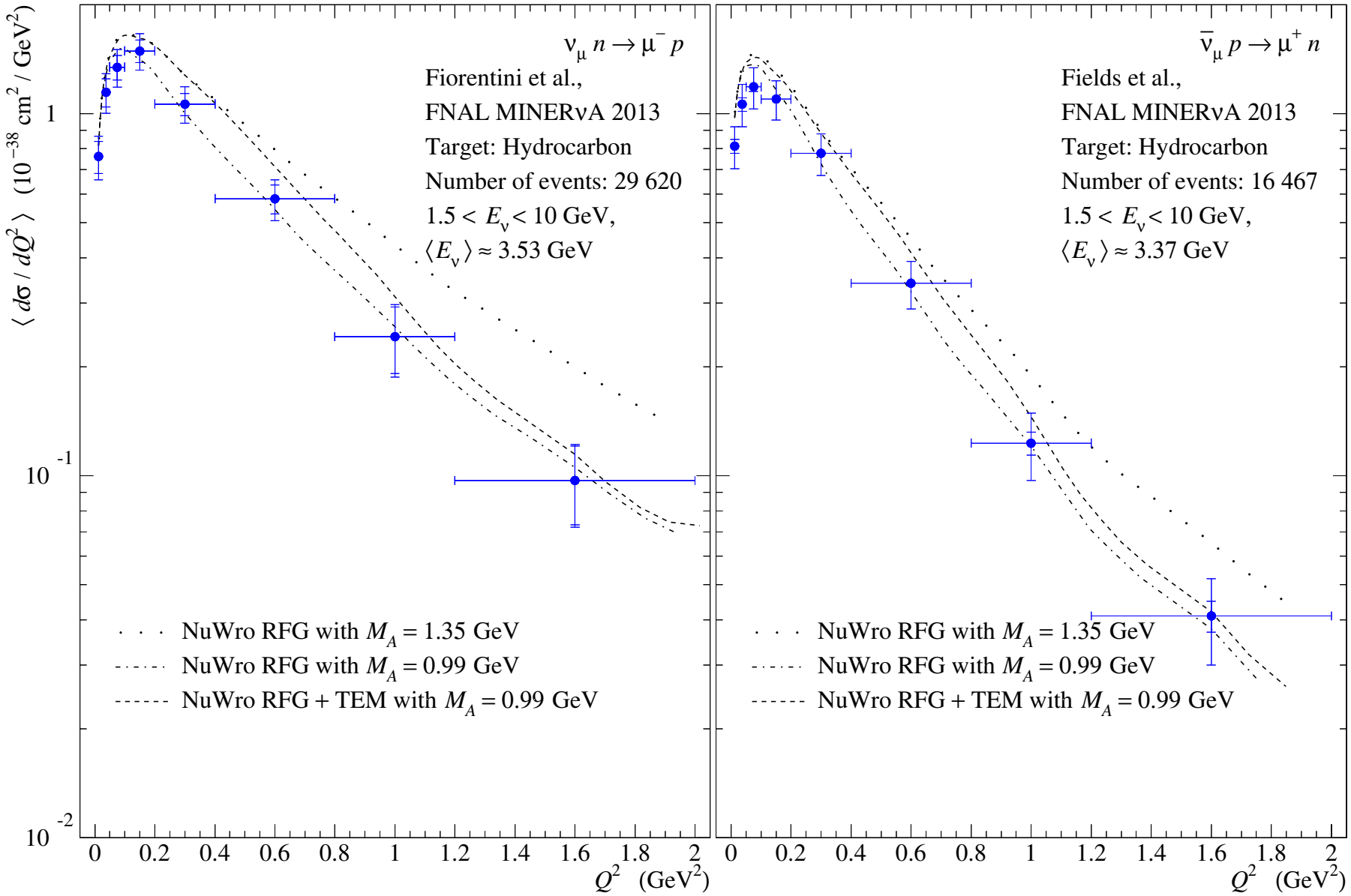

⟨ d²σ / dE_μ d cos θ_μ ⟩
(10^−38 cm² / GeV)
0.9:1.0
0.8:0.9
0.7:0.8
× 2
0.6:0.7
× 2
0.5:0.6
× 3
0.4:0.5
× 3
0.3:0.4
× 3
0.2:0.3
× 3
0.1:0.2
0.0:0.1
– 0.1:0.0
× 2
– 0.2:0.1
× 2
– 0.3:– 0.2
× 3
– 0.4:– 0.3
× 3
– 0.5:– 0.4
× 3
– 0.6:– 0.5
× 3
0 0.2 0.4 0.6 0.8 1 1.2 1.4 1.6 1.8 2 2.2
0 0.2 0.4 0.6
E_μ (GeV)
Global fit with M_A^run
GENIE 2.10.6 RFG with MEC
ν̄_μ p → μ⁺ n
FNAL MiniBooNE 2013
(background subtracted)
Target: Mineral oil (CH_2.08)

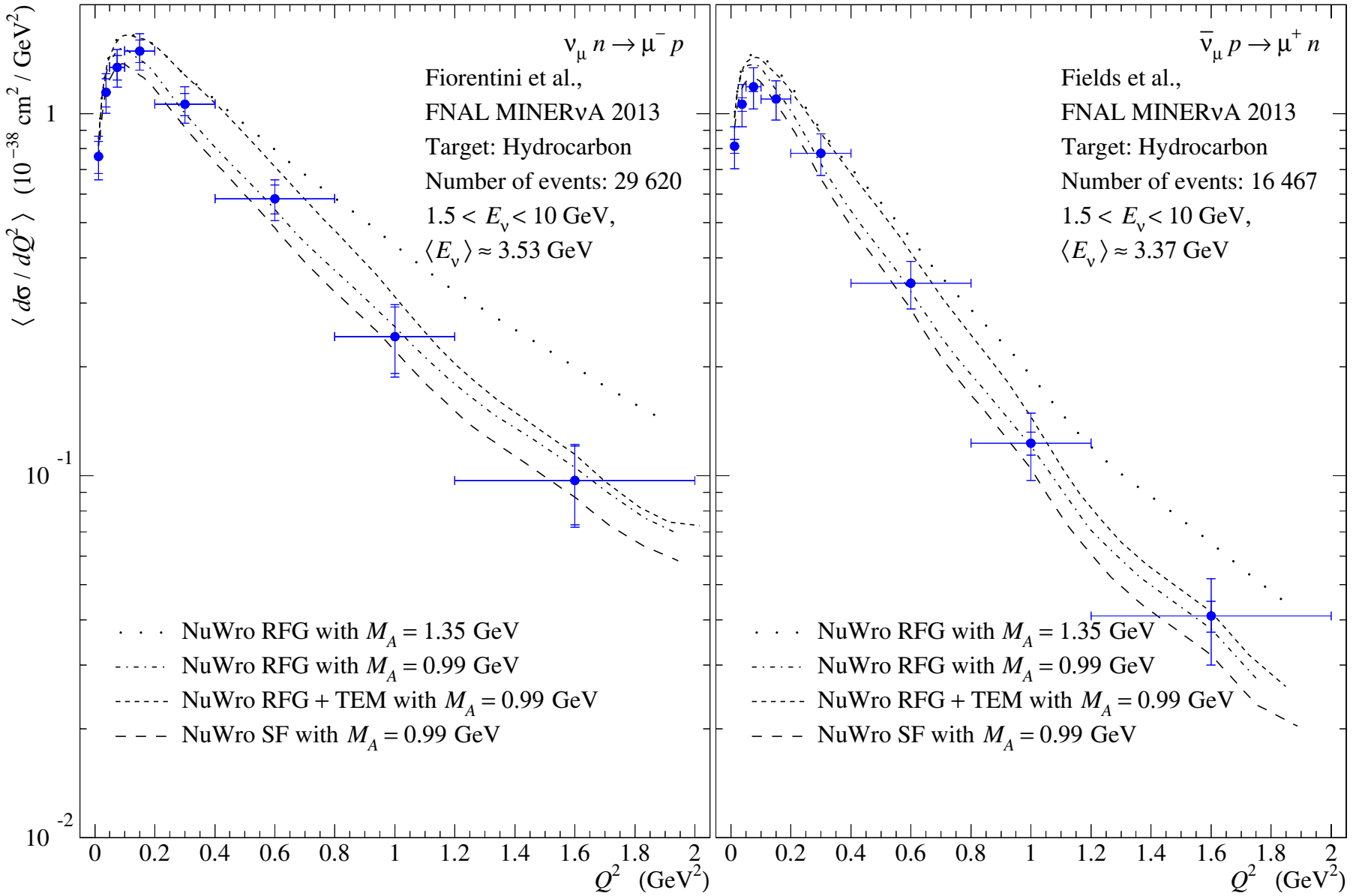
# FNAL MINER$\nu$A 2013

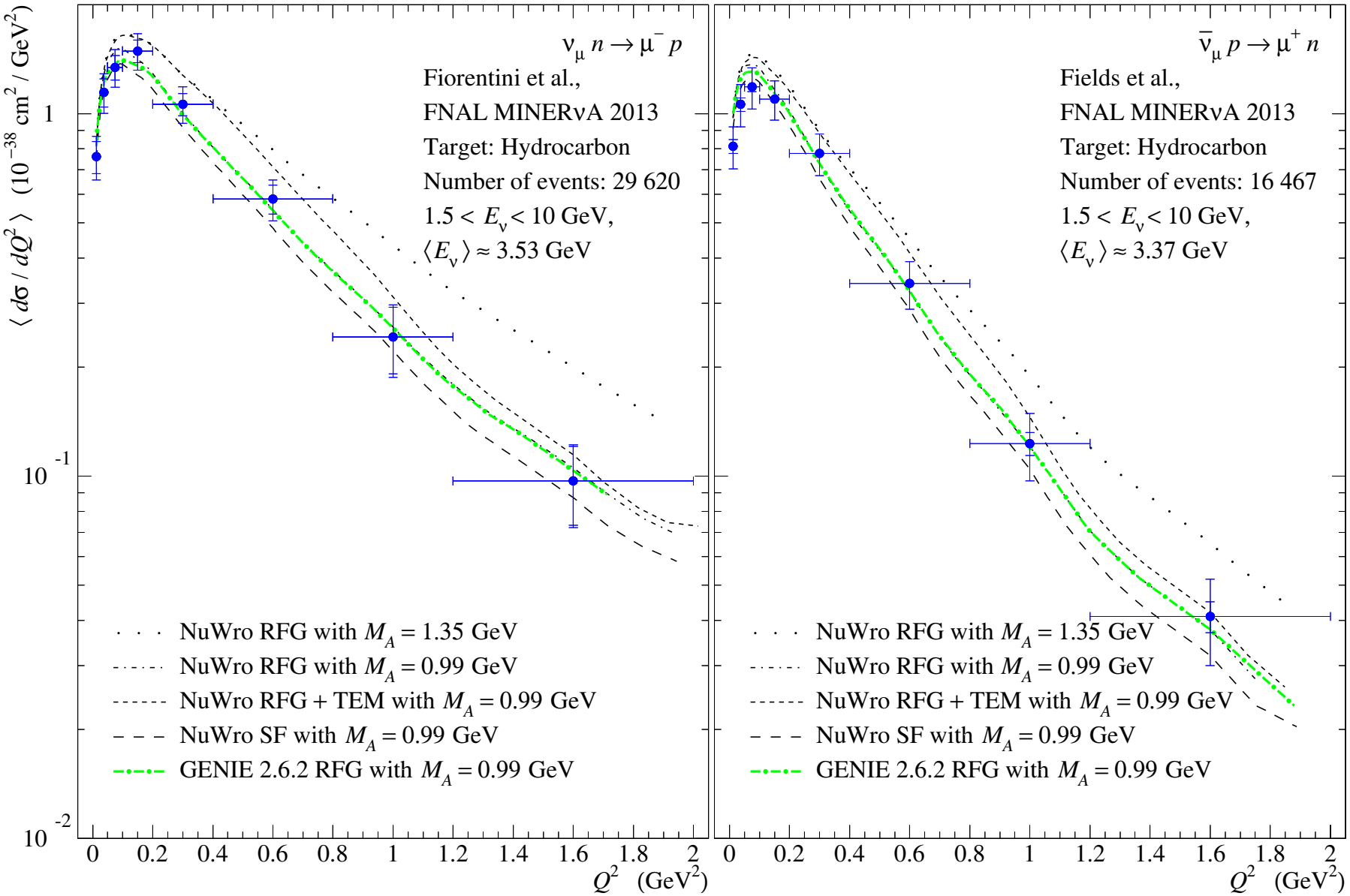

$\nu_\mu\, n \to \mu^-\, p$
Fiorentini et al.,
FNAL MINERνA 2013
Target: Hydrocarbon
Number of events: 29 620
$1.5 < E_\nu < 10$ GeV,
$\langle E_\nu \rangle \approx 3.53$ GeV
$\bar{\nu}_\mu\, p \to \mu^+\, n$
Fields et al.,
FNAL MINERνA 2013
Target: Hydrocarbon
Number of events: 16 467
$1.5 < E_\nu < 10$ GeV,
$\langle E_\nu \rangle \approx 3.37$ GeV
$\langle d\sigma / dQ^2 \rangle$ $(10^{-38}$ cm$^2$ / GeV$^2)$
$Q^2$ (GeV$^2$)

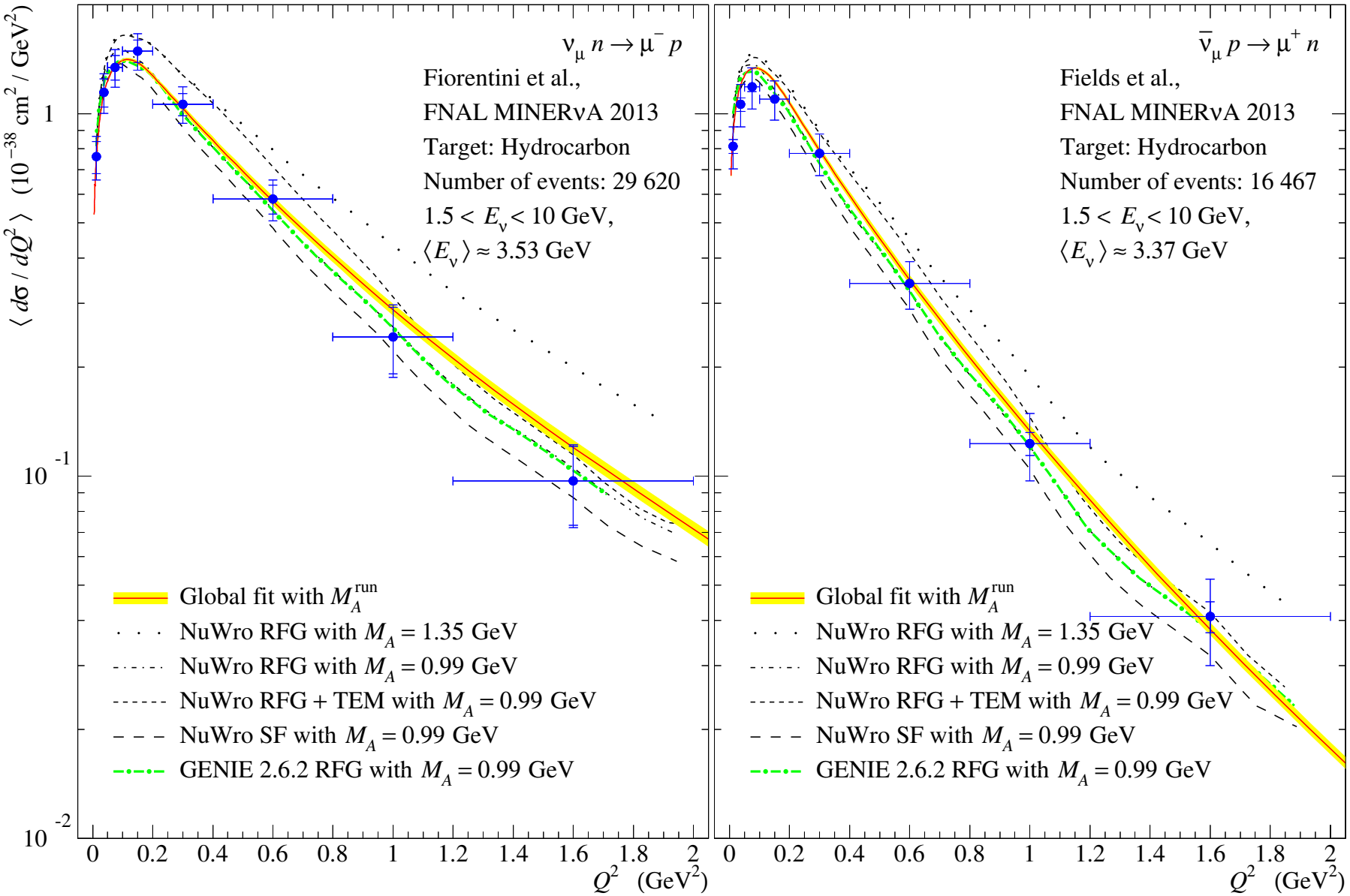

$\nu_\mu\, n \to \mu^-\, p$
Fiorentini et al.,
FNAL MINERνA 2013
Target: Hydrocarbon
Number of events: 29 620
$1.5 < E_\nu < 10$ GeV,
$\langle E_\nu \rangle \approx 3.53$ GeV
NuWro RFG with $M_A = 1.35$ GeV
$\bar{\nu}_\mu\, p \to \mu^+\, n$
Fields et al.,
FNAL MINERνA 2013
Target: Hydrocarbon
Number of events: 16 467
$1.5 < E_\nu < 10$ GeV,
$\langle E_\nu \rangle \approx 3.37$ GeV
NuWro RFG with $M_A = 1.35$ GeV
$\langle d\sigma / dQ^2 \rangle$ $(10^{-38}$ cm$^2$ / GeV$^2)$
$Q^2$ (GeV$^2$)

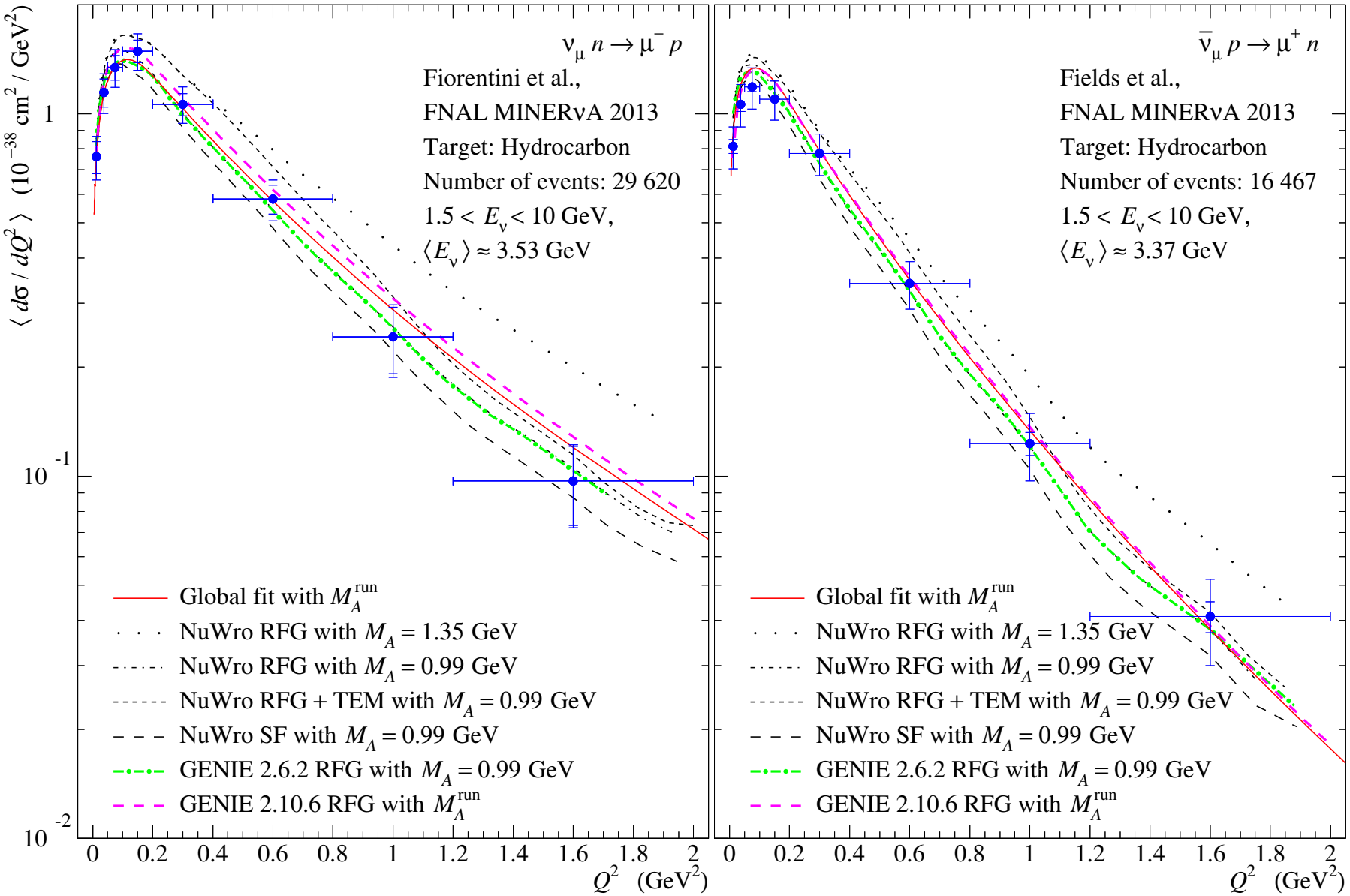

$\nu_\mu\, n \to \mu^-\, p$
Fiorentini et al.,
FNAL MINERνA 2013
Target: Hydrocarbon
Number of events: 29 620
$1.5 < E_\nu < 10$ GeV,
$\langle E_\nu \rangle \approx 3.53$ GeV
NuWro RFG with $M_A = 1.35$ GeV
NuWro RFG with $M_A = 0.99$ GeV
$\bar{\nu}_\mu\, p \to \mu^+\, n$
Fields et al.,
FNAL MINERνA 2013
Target: Hydrocarbon
Number of events: 16 467
$1.5 < E_\nu < 10$ GeV,
$\langle E_\nu \rangle \approx 3.37$ GeV
NuWro RFG with $M_A = 1.35$ GeV
NuWro RFG with $M_A = 0.99$ GeV
$\langle d\sigma / dQ^2 \rangle$ $(10^{-38}$ cm$^2$ / GeV$^2)$
$Q^2$ (GeV$^2$)


$\nu_\mu\, n \to \mu^-\, p$
Fiorentini et al.,
FNAL MINERνA 2013
Target: Hydrocarbon
Number of events: 29 620
$1.5 < E_\nu < 10$ GeV,
$\langle E_\nu \rangle \approx 3.53$ GeV
NuWro RFG with $M_A = 1.35$ GeV
NuWro RFG with $M_A = 0.99$ GeV
NuWro RFG + TEM with $M_A = 0.99$ GeV
$\langle d\sigma / dQ^2 \rangle$ ($10^{-38}$ cm$^2$ / GeV$^2$)
$Q^2$ (GeV$^2$)
$\bar{\nu}_\mu\, p \to \mu^+\, n$
Fields et al.,
FNAL MINERνA 2013
Target: Hydrocarbon
Number of events: 16 467
$1.5 < E_\nu < 10$ GeV,
$\langle E_\nu \rangle \approx 3.37$ GeV
NuWro RFG with $M_A = 1.35$ GeV
NuWro RFG with $M_A = 0.99$ GeV
NuWro RFG + TEM with $M_A = 0.99$ GeV
$Q^2$ (GeV$^2$)


$\nu_\mu\, n \to \mu^-\, p$
Fiorentini et al.,
FNAL MINERνA 2013
Target: Hydrocarbon
Number of events: 29 620
$1.5 < E_\nu < 10$ GeV,
$\langle E_\nu \rangle \approx 3.53$ GeV
NuWro RFG with $M_A = 1.35$ GeV
NuWro RFG with $M_A = 0.99$ GeV
NuWro RFG + TEM with $M_A = 0.99$ GeV
NuWro SF with $M_A = 0.99$ GeV
$\langle d\sigma / dQ^2 \rangle$ $(10^{-38}$ cm$^2$ / GeV$^2)$
$Q^2$ (GeV$^2$)
$\bar{\nu}_\mu\, p \to \mu^+\, n$
Fields et al.,
FNAL MINERνA 2013
Target: Hydrocarbon
Number of events: 16 467
$1.5 < E_\nu < 10$ GeV,
$\langle E_\nu \rangle \approx 3.37$ GeV
NuWro RFG with $M_A = 1.35$ GeV
NuWro RFG with $M_A = 0.99$ GeV
NuWro RFG + TEM with $M_A = 0.99$ GeV
NuWro SF with $M_A = 0.99$ GeV
$Q^2$ (GeV$^2$)

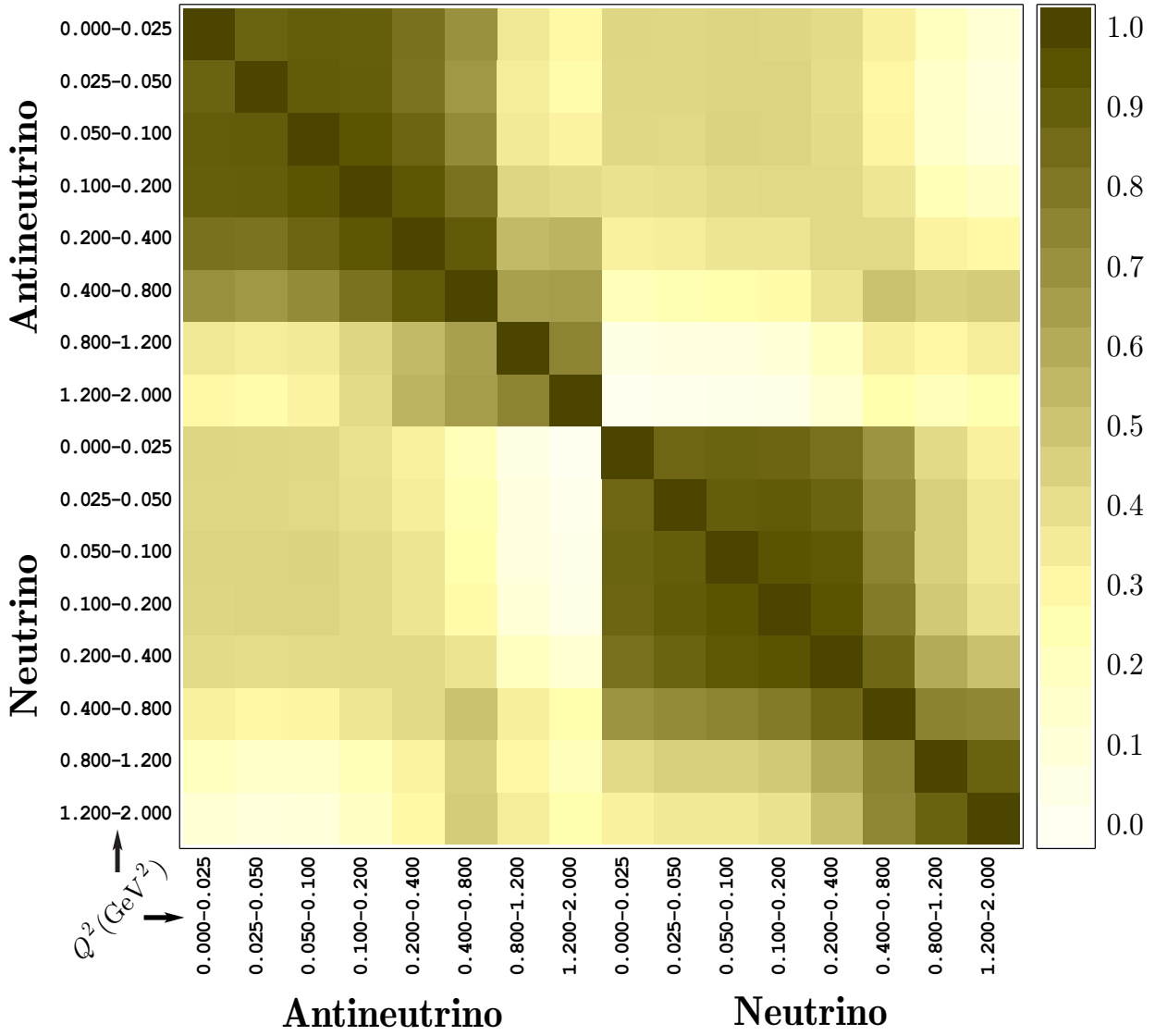

$\nu_\mu\, n \to \mu^-\, p$
Fiorentini et al.,
FNAL MINERνA 2013
Target: Hydrocarbon
Number of events: 29 620
$1.5 < E_\nu < 10$ GeV,
$\langle E_\nu \rangle \approx 3.53$ GeV
$\bar{\nu}_\mu\, p \to \mu^+\, n$
Fields et al.,
FNAL MINERνA 2013
Target: Hydrocarbon
Number of events: 16 467
$1.5 < E_\nu < 10$ GeV,
$\langle E_\nu \rangle \approx 3.37$ GeV
NuWro RFG with $M_A = 1.35$ GeV
NuWro RFG with $M_A = 0.99$ GeV
NuWro RFG + TEM with $M_A = 0.99$ GeV
NuWro SF with $M_A = 0.99$ GeV
GENIE 2.6.2 RFG with $M_A = 0.99$ GeV
$\langle d\sigma / dQ^2 \rangle$ $(10^{-38}$ cm$^2$ / GeV$^2)$
$Q^2$ (GeV$^2$)


$\nu_\mu\, n \to \mu^-\, p$
Fiorentini et al.,
FNAL MINERνA 2013
Target: Hydrocarbon
Number of events: 29 620
$1.5 < E_\nu < 10$ GeV,
$\langle E_\nu \rangle \approx 3.53$ GeV
$\bar{\nu}_\mu\, p \to \mu^+\, n$
Fields et al.,
FNAL MINERνA 2013
Target: Hydrocarbon
Number of events: 16 467
$1.5 < E_\nu < 10$ GeV,
$\langle E_\nu \rangle \approx 3.37$ GeV
Global fit with $M_A^{\rm run}$
NuWro RFG with $M_A = 1.35$ GeV
NuWro RFG with $M_A = 0.99$ GeV
NuWro RFG + TEM with $M_A = 0.99$ GeV
NuWro SF with $M_A = 0.99$ GeV
GENIE 2.6.2 RFG with $M_A = 0.99$ GeV
$\langle d\sigma / dQ^2 \rangle$ $(10^{-38}$ cm$^2$ / GeV$^2)$
$Q^2$ (GeV$^2$)


$\nu_\mu\, n \to \mu^-\, p$
Fiorentini et al.,
FNAL MINERνA 2013
Target: Hydrocarbon
Number of events: 29 620
$1.5 < E_\nu < 10$ GeV,
$\langle E_\nu \rangle \approx 3.53$ GeV
$\bar{\nu}_\mu\, p \to \mu^+\, n$
Fields et al.,
FNAL MINERνA 2013
Target: Hydrocarbon
Number of events: 16 467
$1.5 < E_\nu < 10$ GeV,
$\langle E_\nu \rangle \approx 3.37$ GeV
Global fit with $M_A^{\rm run}$
NuWro RFG with $M_A = 1.35$ GeV
NuWro RFG with $M_A = 0.99$ GeV
NuWro RFG + TEM with $M_A = 0.99$ GeV
NuWro SF with $M_A = 0.99$ GeV
GENIE 2.6.2 RFG with $M_A = 0.99$ GeV
GENIE 2.10.6 RFG with $M_A^{\rm run}$
$\langle d\sigma / dQ^2 \rangle$ $(10^{-38}$ cm$^2$ / GeV$^2)$
$Q^2$ (GeV$^2$)

As it is seen from the correlation matrix, the MINER$\nu$A data are highly correlated. Therefore the *by-eye* comparison between the data and the model predictions is not very conclusive.

The values of $\chi^2/N$ (where $N$ is the number of datapoints) listed in next slide allow us to conclude that our model provides the best description of the MINER$\nu$A results among the models shown in slides 30–36.



| Model | Author's estimation (by bin medians) | Our estimation (by bin medians) | Our estimation (by bin averages) |
|---|---|---|---|
| **$\nu_\mu$ FNAL MINER$\nu$A 2013 [27], $N = 8$** | | | |
| NuWro RFG with $M_A = 1.35$ | 3.7 | $29.26/8 = 3.66$ | $31.13/8 = 3.89$ |
| NuWro RFG with $M_A = 0.99$ + TEM | 2.4 | $18.40/8 = 2.30$ | $20.93/8 = 2.62$ |
| NuWro RFG with $M_A = 0.99$ | 3.5 | $28.19/8 = 3.52$ | $23.24/8 = 2.91$ |
| NuWro SF with $M_A = 0.99$ | 2.8 | $22.66/8 = 2.83$ | $18.69/8 = 2.34$ |
| GENIE 2.6.2 RFG with $M_A = 0.99$ | – | $16.04/8 = 2.00$ | $15.27/8 = 1.91$ |
| Global fit with $M_A^{\text{run}}$ | – | $7.19/8 = 0.90$ | $8.04/8 = 1.00$ |
| GENIE 2.10.6 with $M_A^{\text{run}}$ | – | $11.16/8 = 1.39$ | $10.77/8 = 1.35$ |
| **$\overline{\nu}_\mu$ FNAL MINER$\nu$A 2013 [28], $N = 8$** | | | |
| NuWro RFG with $M_A = 1.35$ | 2.90 | $23.50/8 = 2.94$ | $23.20/8 = 2.90$ |
| NuWro RFG with $M_A = 0.99$ + TEM | 1.06 | $8.35/8 = 1.04$ | $9.50/8 = 1.19$ |
| NuWro RFG with $M_A = 0.99$ | 2.64 | $21.39/8 = 2.67$ | $17.80/8 = 2.23$ |
| NuWro SF with $M_A = 0.99$ | 2.14 | $17.37/8 = 2.17$ | $14.59/8 = 1.82$ |
| GENIE 2.6.2 RFG with $M_A = 0.99$ | – | $19.96/8 = 2.50$ | $18.23/8 = 2.28$ |
| Global fit with $M_A^{\text{run}}$ | – | $7.29/8 = 0.91$ | $5.51/8 = 0.69$ |
| GENIE 2.10.6 with $M_A^{\text{run}}$ | – | $19.31/8 = 2.41$ | $15.92/8 = 1.99$ |
| **$\nu_\mu \oplus \overline{\nu}_\mu$ FNAL MINER$\nu$A 2013 [27, 28], $N = 16$** | | | |
| NuWro RFG with $M_A = 1.35$ | – | $59.15/16 = 3.70$ | $59.34/16 = 3.71$ |
| NuWro RFG with $M_A = 0.99$ + TEM | – | $28.33/16 = 1.77$ | $30.74/16 = 1.92$ |
| NuWro RFG with $M_A = 0.99$ | – | $48.11/16 = 3.01$ | $40.59/16 = 2.54$ |
| NuWro SF with $M_A = 0.99$ | – | $38.12/16 = 2.38$ | $32.39/16 = 2.02$ |
| GENIE 2.6.2 RFG with $M_A = 0.99$ | – | $34.87/16 = 2.18$ | $33.04/16 = 2.07$ |
| Global fit with $M_A^{\text{run}}$ | – | $19.93/16 = 1.25$ | $17.90/16 = 1.10$ |
| GENIE 2.10.6 with $M_A^{\text{run}}$ | – | $34.77/16 = 2.17$ | $30.02/16 = 1.88$ |

# T2K ND280 2016

# A caveat

We have not yet included the new (2016) T2K ND280 data [69] into the global fit due to the following reasons:

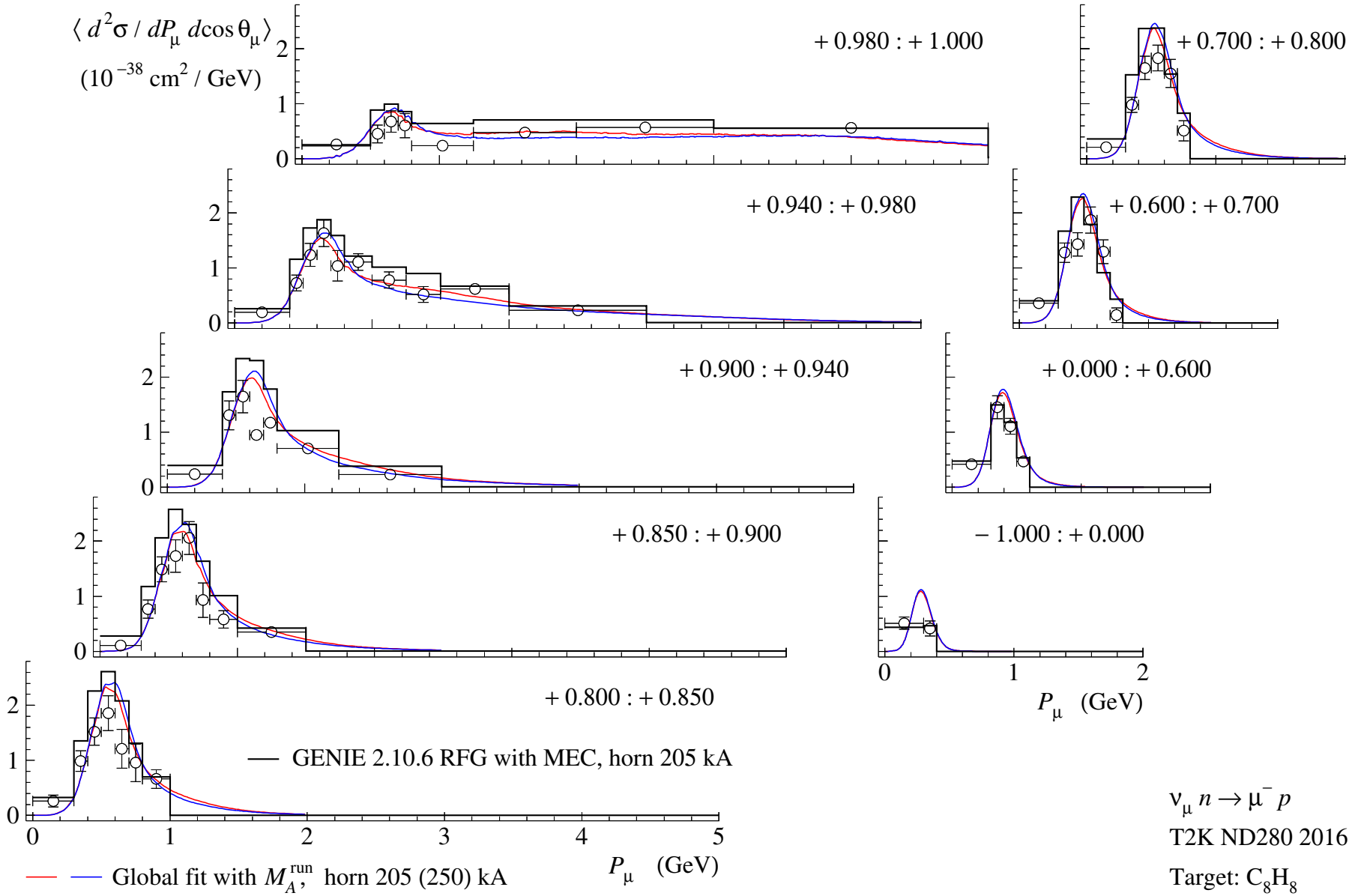
- We don't know the actual $\nu_\mu$ spectrum since in several runs of the experiment, the three magnetic horns were excited by different current pulses.
- We don't know which of two data analyses (or both with certain weights) should be used in the fit.
- There is some small additional uncertainty related to complicated elemental composition of the detector target (fine-grained detector FGD1).[a] The measurement has been effectively considered as if on the FGD1 scintillator ($C_8H_8$). But our experience suggests that such simplification is not quite harmless...

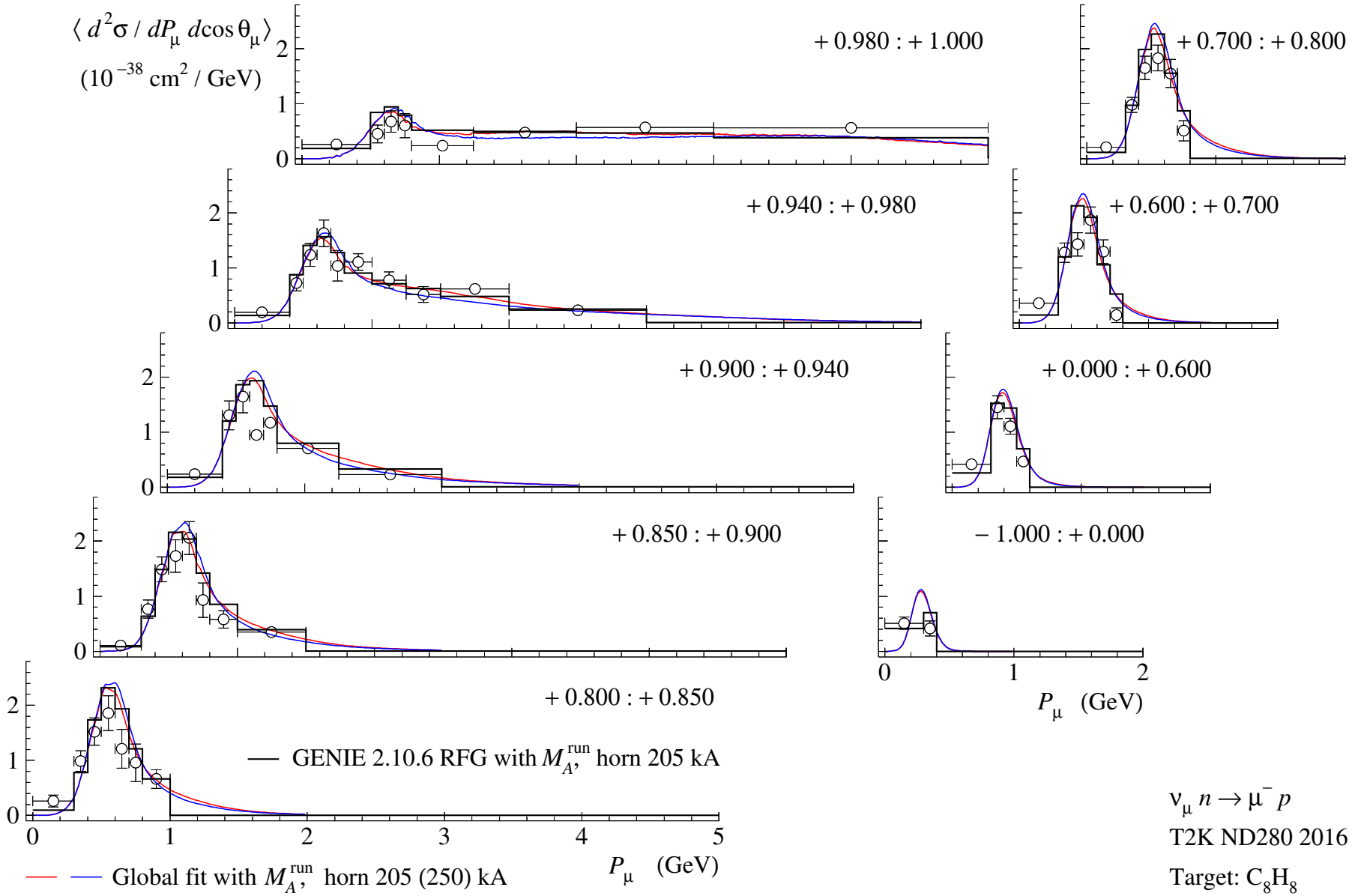
Here we only compare our calculations with the data from Analyses 1 & 2 [69] by using the two $\nu_\mu$ spectra corresponding to the horn currents of 250 and 205 kA.

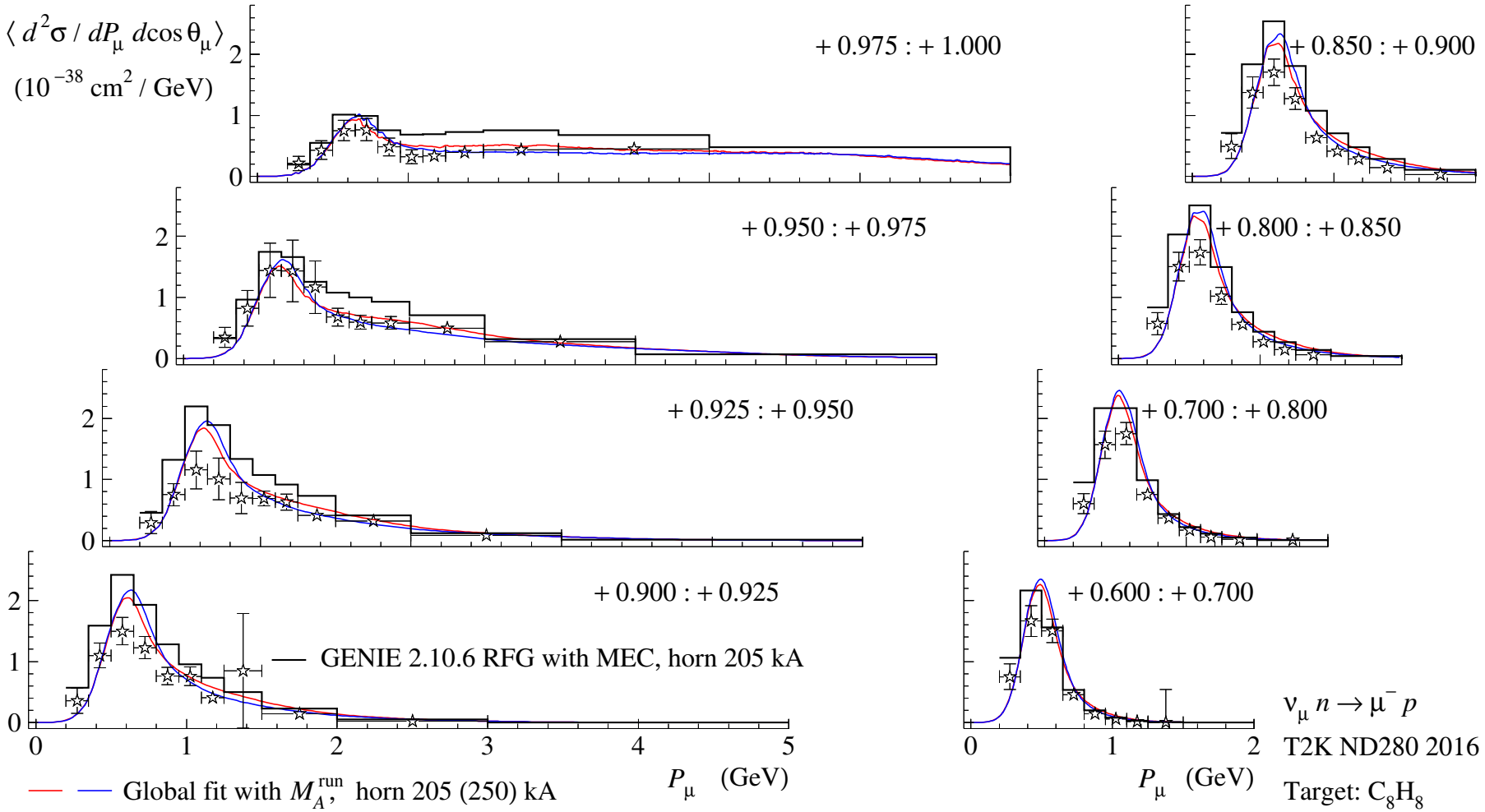
We find a reasonable (within the uncertainties) agreement between the ND280 results and those predicted by our model.

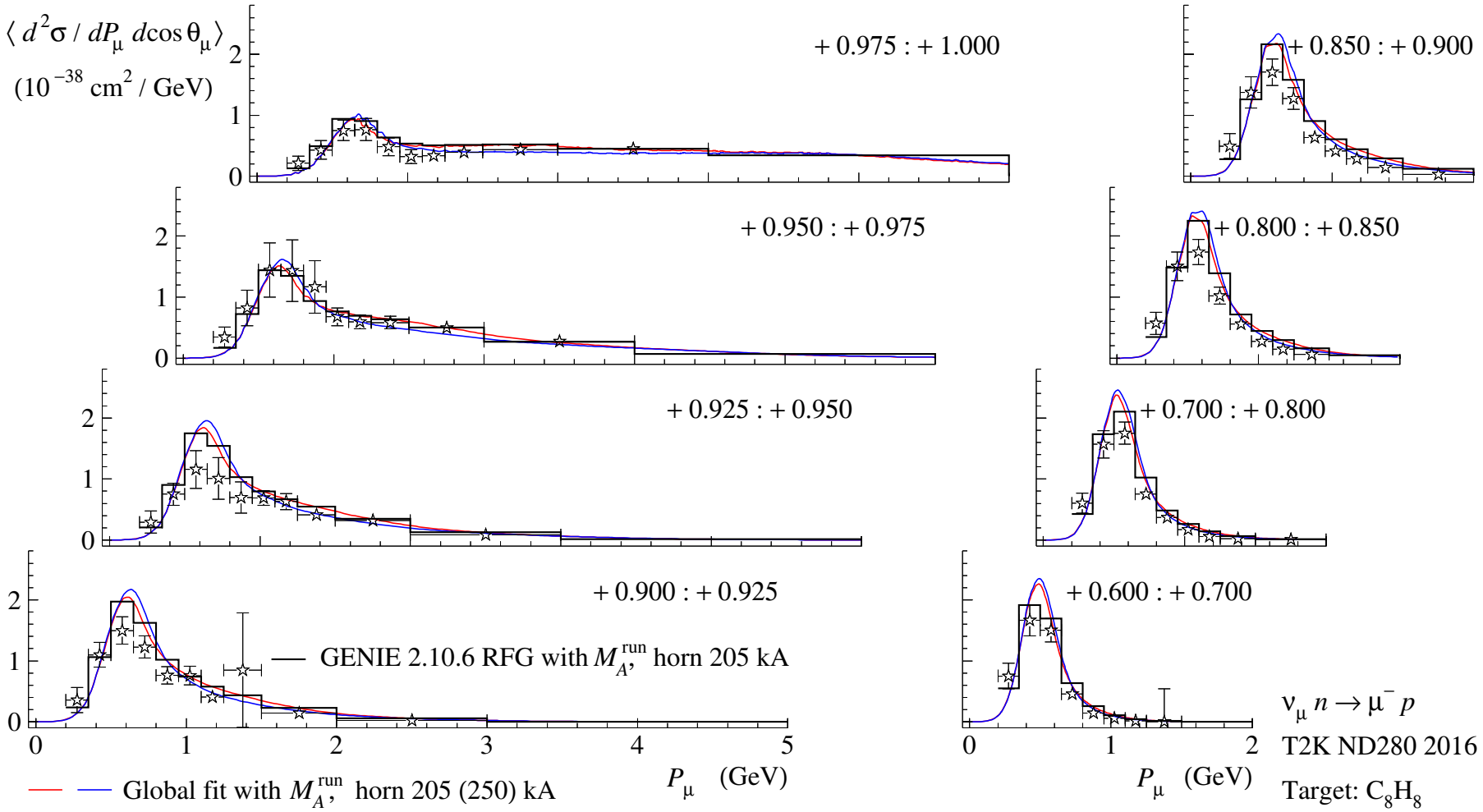
Therefore, it can be preliminary concluded that adding the T2K ND280 2016 data into the global fit would not drastically change the result.

[a] The FGD1 composition: carbon (86.1%), hydrogen (7.35%), oxygen (3.70%), and small quantities of other elements such as titanium, silicon, and nitrogen (2.85%) [69].


⟨ d²σ / dP_μ dcos θ_μ ⟩
(10⁻³⁸ cm² / GeV)
+ 0.980 : + 1.000
+ 0.700 : + 0.800
+ 0.940 : + 0.980
+ 0.600 : + 0.700
+ 0.900 : + 0.940
+ 0.000 : + 0.600
+ 0.850 : + 0.900
− 1.000 : + 0.000
+ 0.800 : + 0.850
P_μ (GeV)
GENIE 2.10.6 RFG with MEC, horn 205 kA
Global fit with M_A^run, horn 205 (250) kA
ν_μ n → μ⁻ p
T2K ND280 2016
Target: C8H8


⟨ d²σ / dP_μ dcos θ_μ ⟩
(10⁻³⁸ cm² / GeV)
+ 0.980 : + 1.000
+ 0.700 : + 0.800
+ 0.940 : + 0.980
+ 0.600 : + 0.700
+ 0.900 : + 0.940
+ 0.000 : + 0.600
+ 0.850 : + 0.900
− 1.000 : + 0.000
+ 0.800 : + 0.850
P_μ (GeV)
GENIE 2.10.6 RFG with M_A^run, horn 205 kA
Global fit with M_A^run, horn 205 (250) kA
P_μ (GeV)
ν_μ n → μ⁻ p
T2K ND280 2016
Target: C8H8


⟨ d²σ / dP_μ d cos θ_μ ⟩
(10⁻³⁸ cm² / GeV)
+ 0.975 : + 1.000
+ 0.950 : + 0.975
+ 0.925 : + 0.950
+ 0.900 : + 0.925
+ 0.850 : + 0.900
+ 0.800 : + 0.850
+ 0.700 : + 0.800
+ 0.600 : + 0.700
GENIE 2.10.6 RFG with MEC, horn 205 kA
Global fit with M_A^run, horn 205 (250) kA
P_μ (GeV)
ν_μ n → μ⁻ p
T2K ND280 2016
Target: C₈H₈


⟨ d²σ / dP_μ dcos θ_μ ⟩
(10⁻³⁸ cm² / GeV)
+ 0.975 : + 1.000
+ 0.850 : + 0.900
+ 0.950 : + 0.975
+ 0.800 : + 0.850
+ 0.925 : + 0.950
+ 0.700 : + 0.800
+ 0.900 : + 0.925
+ 0.600 : + 0.700
GENIE 2.10.6 RFG with M_A^run, horn 205 kA
Global fit with M_A^run, horn 205 (250) kA
P_μ (GeV)
P_μ (GeV)
ν_μ n → μ⁻ p
T2K ND280 2016
Target: C₈H₈

# Parameters of the SM RFG Model

# Interpolation formulas

The Fermi momenta for isoscalar nuclei and binding (separation) energies are calculated by using the following interpolation formulas (valid for $A \geq 6$):

$$p_F = 270[1 - 4.2/A + (6.0/A)^2 - (5.3/A)^3]$$

and

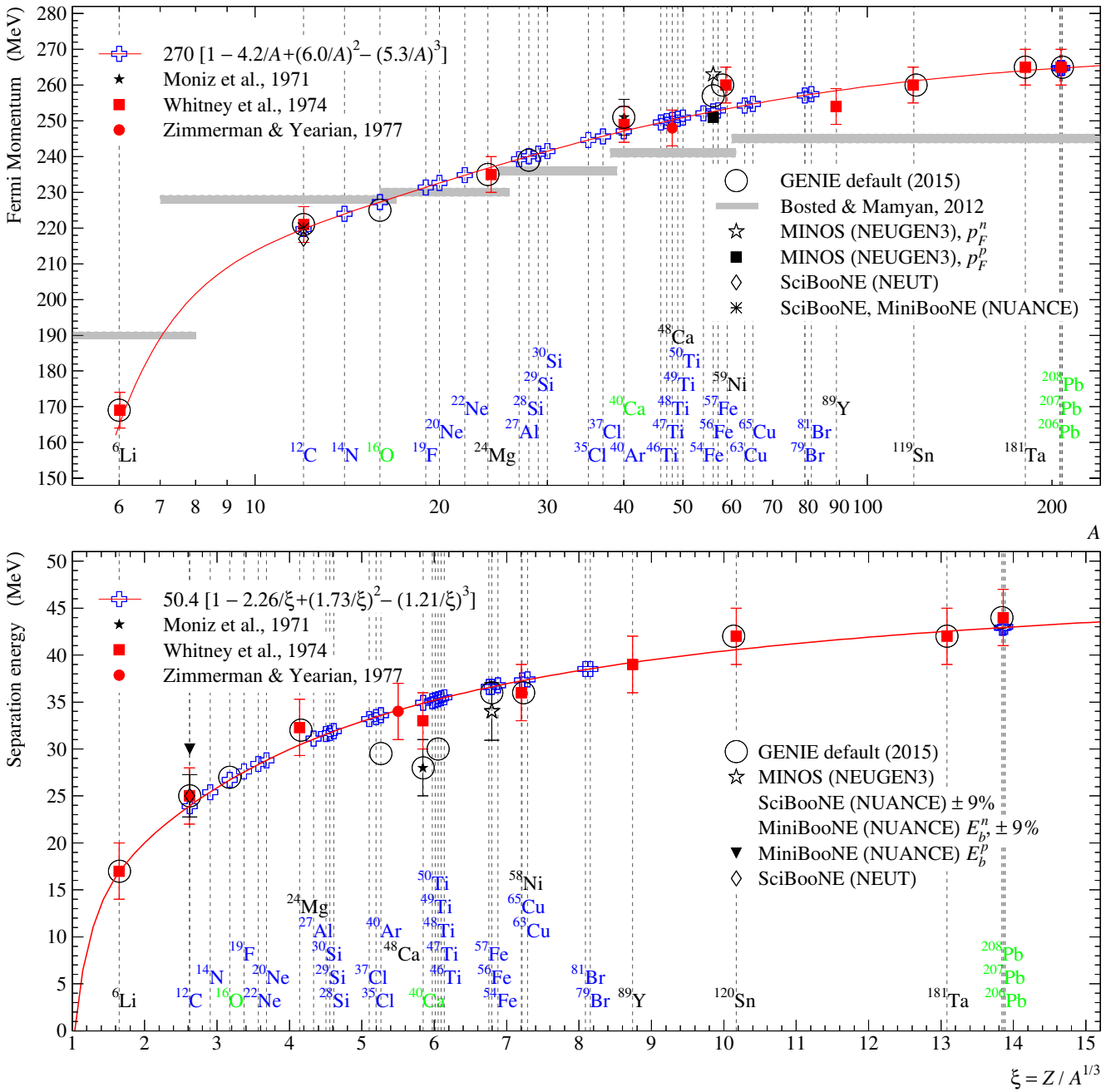
$$E_b = 50.4\left[1 - 2.26/\xi + (1.73/\xi)^2 - (1.21/\xi)^3\right], \quad \xi = Z/A^{1/3},$$

obtained from available (unfortunately, rather old) data on electron-nucleus scattering.[a]
The proton and neutron Fermi momenta ($p_F^p$ and $p_F^n$, respectively) are then calculated in the conventional way:[b]

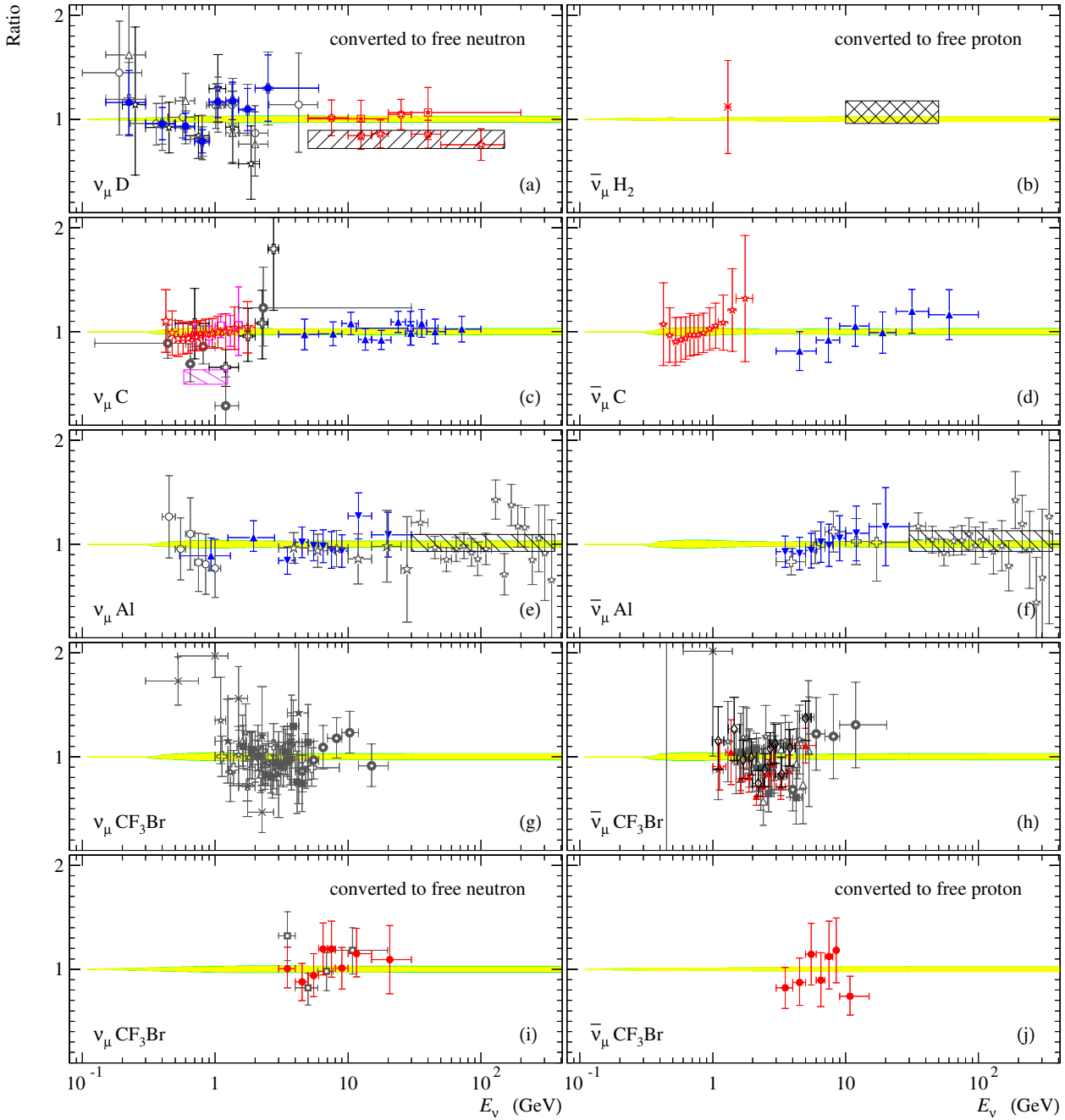
$$p_F^p = (2Z/A)^{1/3} p_F \quad \text{and} \quad p_F^n = (2N/A)^{1/3} p_F \qquad (N = A - Z).$$

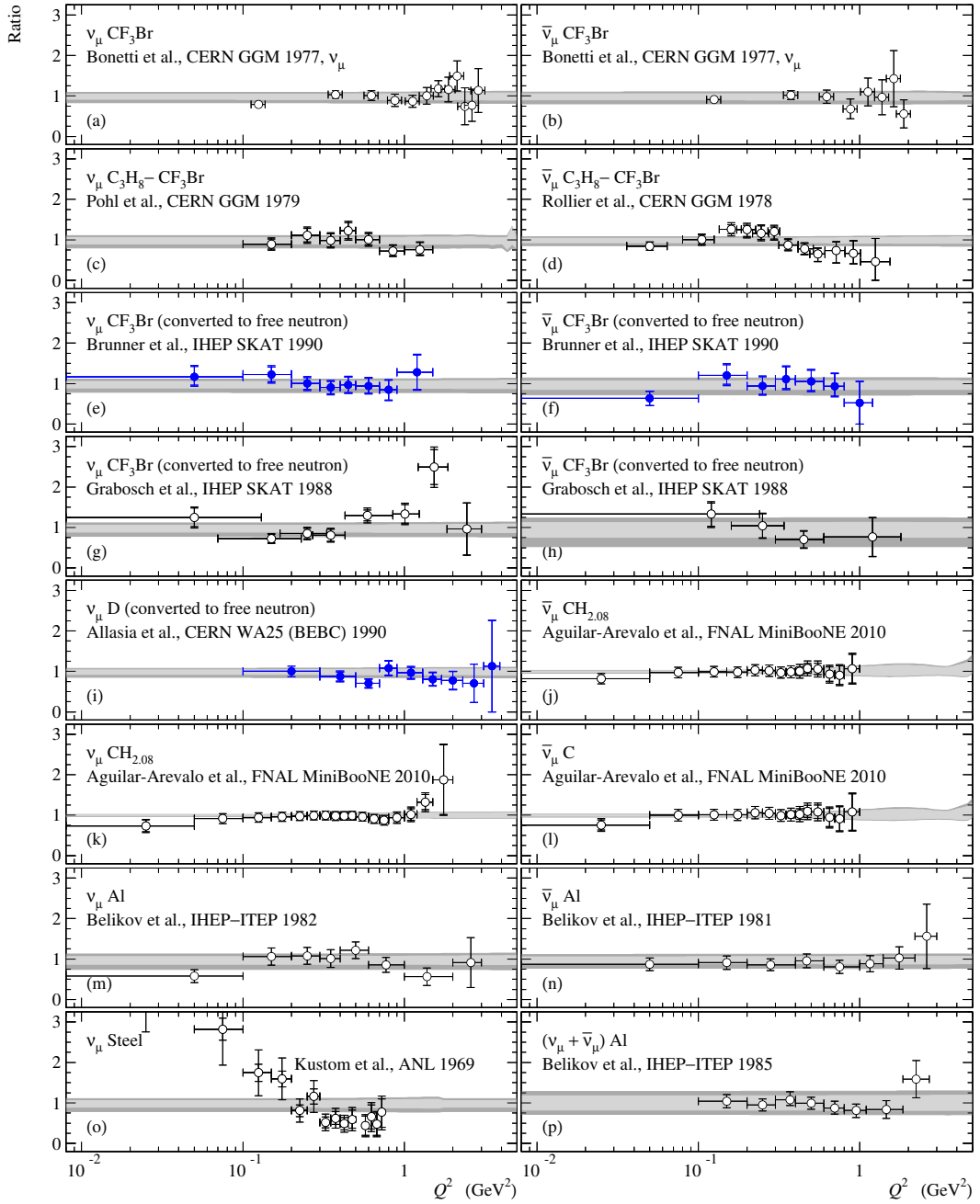
[a] E.J. Moniz, I. Sick, R.R. Whitney, J.R. Ficenec, R.D. Kephart, and W.P. Trower, *"Nuclear Fermi momenta from quasielastic electron scattering,"* Phys. Rev. Lett. **26**, 445 (1971); R.R. Whitney, I. Sick, J.R. Ficenec, R.D. Kephart and W.P. Trower, *"Quasielastic electron Scattering,"* Phys. Rev. C **9**, 2230 (1974); P.D. Zimmerman and M.R. Yearian, *"Fermi momenta and separation energies obtained from the quasi-elastic scattering of electrons from $^{48}$Ca and $^{40}$Ca,"* Z. Phys. A **278**, 291 (1976).

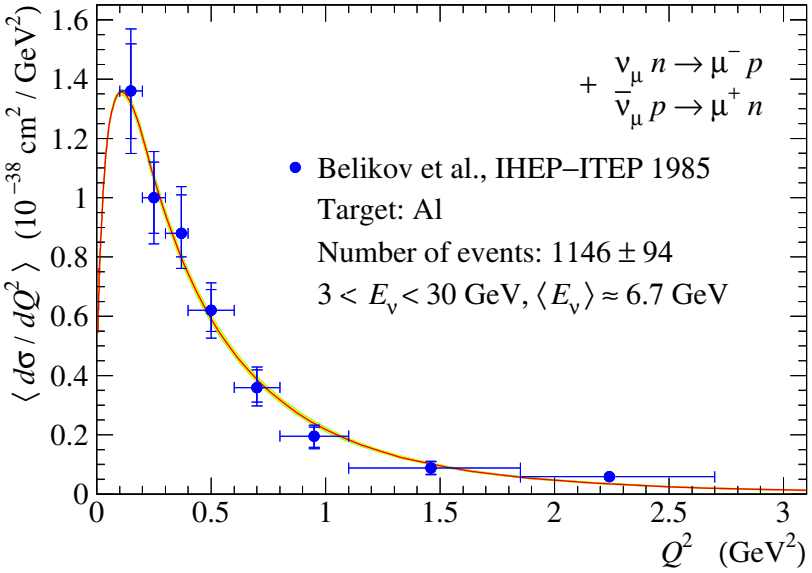
[b] These relations are based on the simplest assumption that the density of nuclear matter is approximately constant irrespective of the proton-to-neutron ratio $Z/N$.

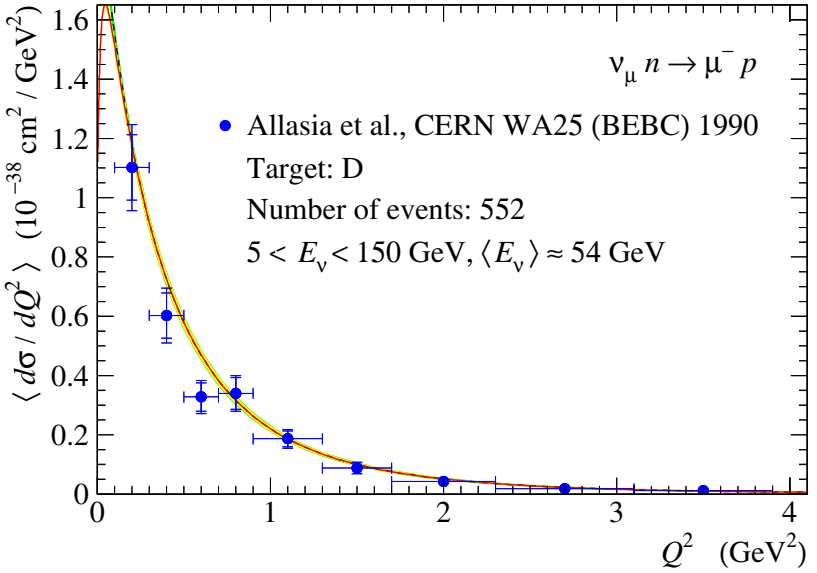

Fermi Momentum (MeV)
270 [1 – 4.2/A+(6.0/A)$^2$– (5.3/A)$^3$]
Moniz et al., 1971
Whitney et al., 1974
Zimmerman & Yearian, 1977
GENIE default (2015)
Bosted & Mamyan, 2012
MINOS (NEUGEN3), $p_F^n$
MINOS (NEUGEN3), $p_F^p$
SciBooNE (NEUT)
SciBooNE, MiniBooNE (NUANCE)
A
Separation energy (MeV)
50.4 [1 – 2.26/ξ+(1.73/ξ)$^2$– (1.21/ξ)$^3$]
Moniz et al., 1971
Whitney et al., 1974
Zimmerman & Yearian, 1977
GENIE default (2015)
MINOS (NEUGEN3)
SciBooNE (NUANCE) ± 9%
MiniBooNE (NUANCE) $E_b^n$, ± 9%
MiniBooNE (NUANCE) $E_b^p$
SciBooNE (NEUT)
ξ = Z / A$^{1/3}$

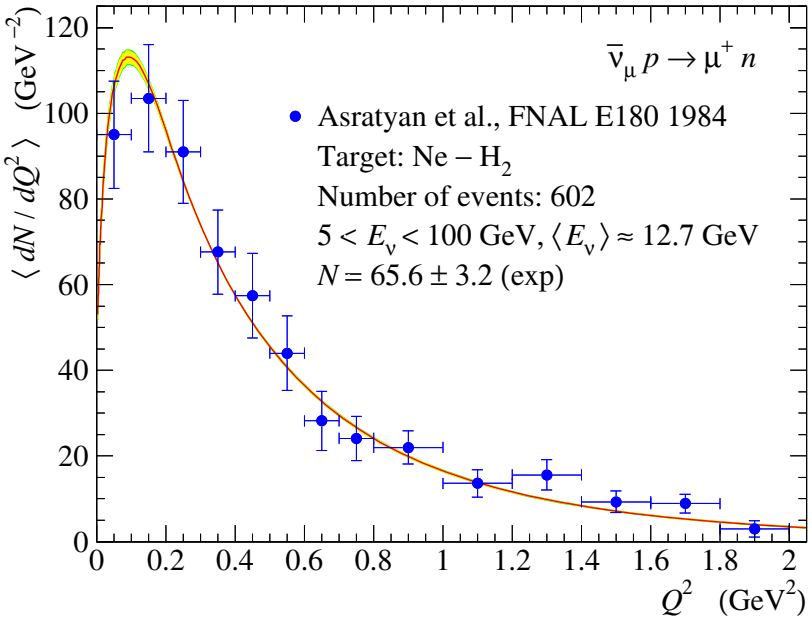
# Comparison with the early "golden" data

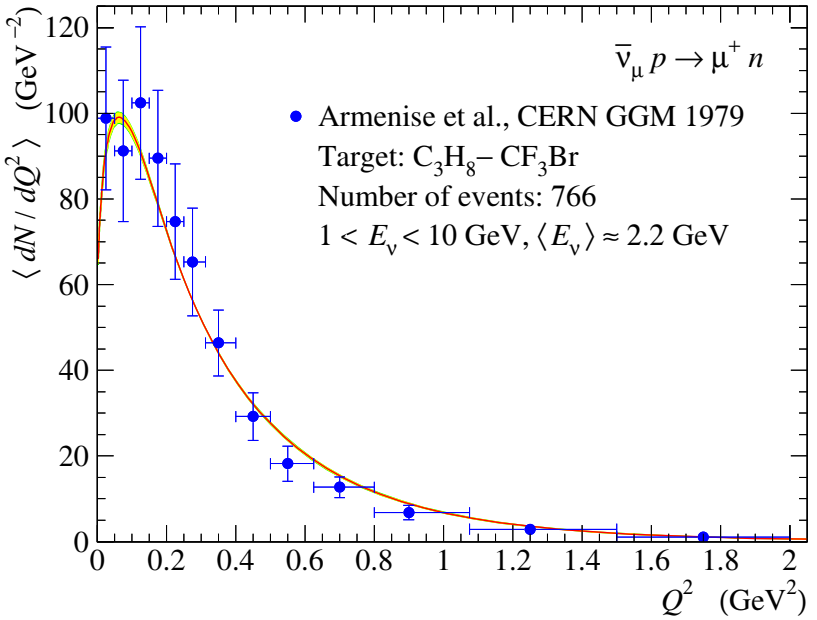

Ratio
converted to free neutron
converted to free proton
$\nu_\mu$ D
(a)
$\bar{\nu}_\mu$ $H_2$
(b)
$\nu_\mu$ C
(c)
$\bar{\nu}_\mu$ C
(d)
$\nu_\mu$ Al
(e)
$\bar{\nu}_\mu$ Al
(f)
$\nu_\mu$ $CF_3Br$
(g)
$\bar{\nu}_\mu$ $CF_3Br$
(h)
converted to free neutron
converted to free proton
$\nu_\mu$ $CF_3Br$
(i)
$\bar{\nu}_\mu$ $CF_3Br$
(j)
$E_\nu$ (GeV)
$E_\nu$ (GeV)

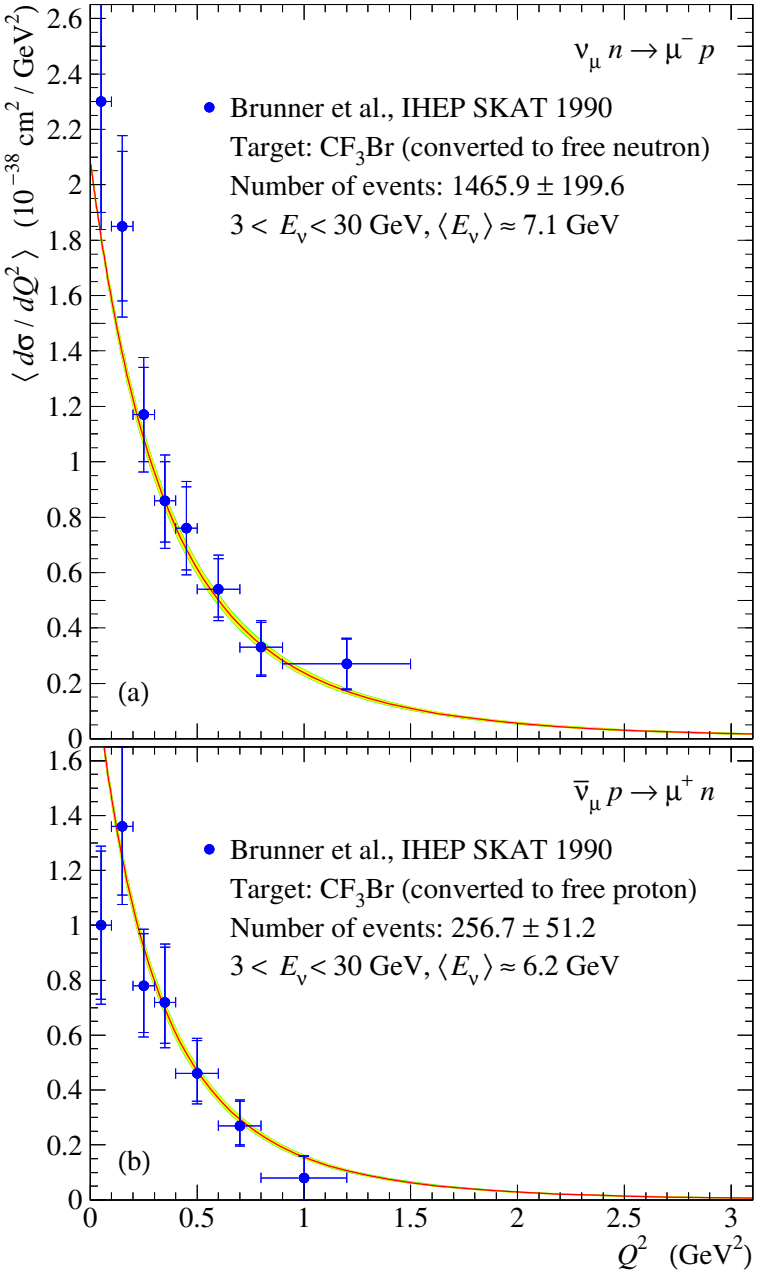

Ratio
$\nu_\mu$ $CF_3Br$
Bonetti et al., CERN GGM 1977, $\nu_\mu$
(a)
$\bar{\nu}_\mu$ $CF_3Br$
Bonetti et al., CERN GGM 1977, $\nu_\mu$
(b)
$\nu_\mu$ $C_3H_8$– $CF_3Br$
Pohl et al., CERN GGM 1979
(c)
$\bar{\nu}_\mu$ $C_3H_8$– $CF_3Br$
Rollier et al., CERN GGM 1978
(d)
$\nu_\mu$ $CF_3Br$ (converted to free neutron)
Brunner et al., IHEP SKAT 1990
(e)
$\bar{\nu}_\mu$ $CF_3Br$ (converted to free neutron)
Brunner et al., IHEP SKAT 1990
(f)
$\nu_\mu$ $CF_3Br$ (converted to free neutron)
Grabosch et al., IHEP SKAT 1988
(g)
$\bar{\nu}_\mu$ $CF_3Br$ (converted to free neutron)
Grabosch et al., IHEP SKAT 1988
(h)
$\nu_\mu$ D (converted to free neutron)
Allasia et al., CERN WA25 (BEBC) 1990
(i)
$\bar{\nu}_\mu$ $CH_{2.08}$
Aguilar-Arevalo et al., FNAL MiniBooNE 2010
(j)
$\nu_\mu$ $CH_{2.08}$
Aguilar-Arevalo et al., FNAL MiniBooNE 2010
(k)
$\bar{\nu}_\mu$ C
Aguilar-Arevalo et al., FNAL MiniBooNE 2010
(l)
$\nu_\mu$ Al
Belikov et al., IHEP–ITEP 1982
(m)
$\bar{\nu}_\mu$ Al
Belikov et al., IHEP–ITEP 1981
(n)
$\nu_\mu$ Steel
Kustom et al., ANL 1969
(o)
($\nu_\mu + \bar{\nu}_\mu$) Al
Belikov et al., IHEP–ITEP 1985
(p)
$10^{-2}$
$10^{-1}$
1
$Q^2$ (GeV$^2$)

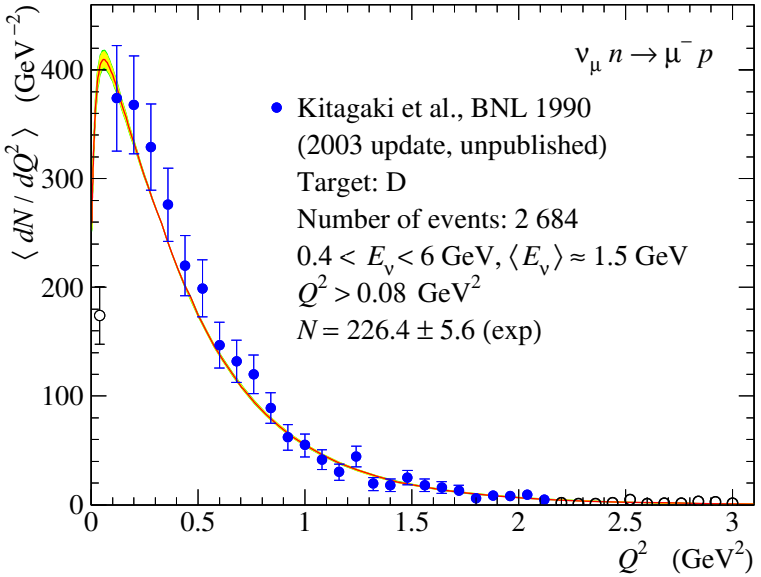

$\nu_\mu\, n \to \mu^- p$
$+\ \bar{\nu}_\mu\, p \to \mu^+ n$
Belikov et al., IHEP–ITEP 1985
Target: Al
Number of events: 1146 ± 94
$3 < E_\nu < 30$ GeV, $\langle E_\nu \rangle \approx 6.7$ GeV
$\langle d\sigma / dQ^2 \rangle$ ($10^{-38}$ cm$^2$ / GeV$^2$)
$Q^2$ (GeV$^2$)


$\nu_\mu\, n \to \mu^- p$
Allasia et al., CERN WA25 (BEBC) 1990
Target: D
Number of events: 552
$5 < E_\nu < 150$ GeV, $\langle E_\nu \rangle \approx 54$ GeV
$\langle d\sigma / dQ^2 \rangle$ ($10^{-38}$ cm$^2$ / GeV$^2$)
$Q^2$ (GeV$^2$)

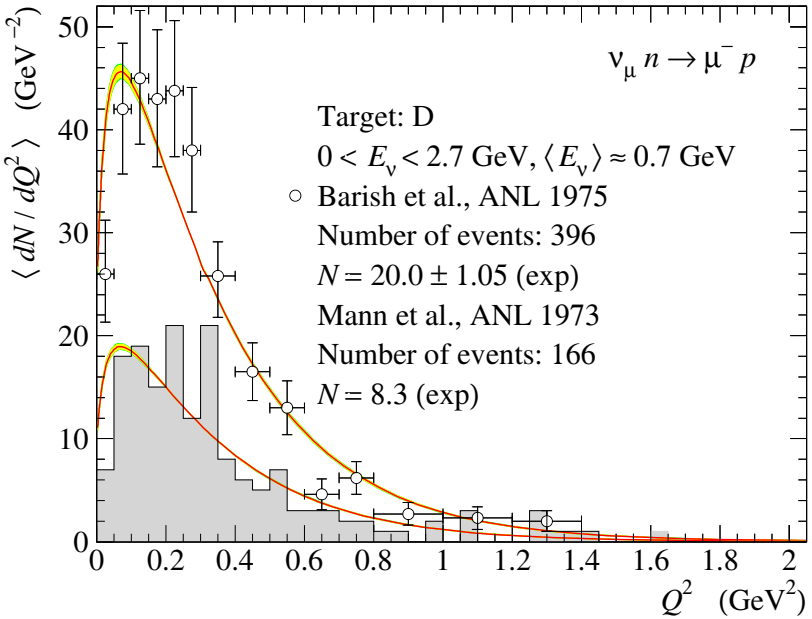

$\bar{\nu}_\mu\, p \to \mu^+ n$
Asratyan et al., FNAL E180 1984
Target: Ne – $H_2$
Number of events: 602
$5 < E_\nu < 100$ GeV, $\langle E_\nu \rangle \approx 12.7$ GeV
$N = 65.6 \pm 3.2$ (exp)
$\langle dN / dQ^2 \rangle$ (GeV$^{-2}$)
$Q^2$ (GeV$^2$)

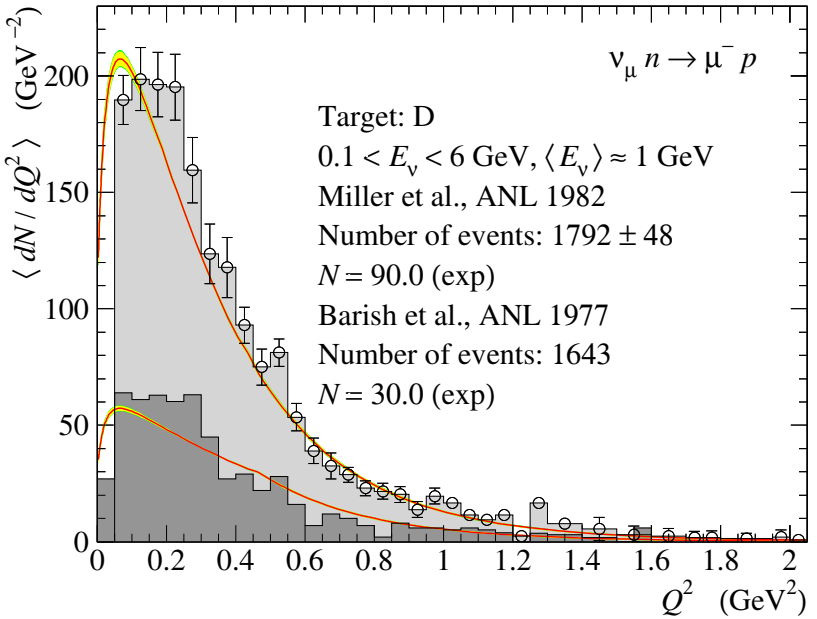

$\bar{\nu}_\mu\, p \to \mu^+ n$
Armenise et al., CERN GGM 1979
Target: $C_3H_8$– $CF_3Br$
Number of events: 766
$1 < E_\nu < 10$ GeV, $\langle E_\nu \rangle \approx 2.2$ GeV
$\langle dN / dQ^2 \rangle$ (GeV$^{-2}$)
$Q^2$ (GeV$^2$)

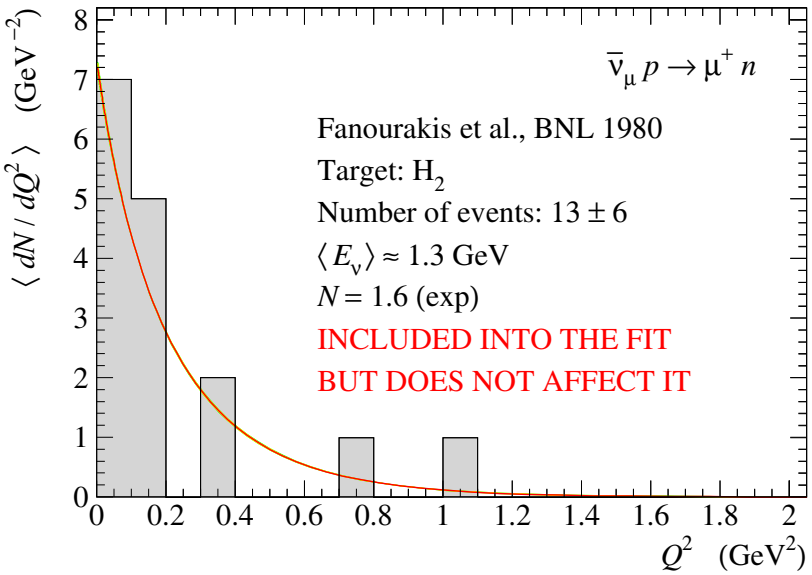

$\nu_\mu\, n \to \mu^-\, p$
Brunner et al., IHEP SKAT 1990
Target: $CF_3Br$ (converted to free neutron)
Number of events: 1465.9 ± 199.6
$3 < E_\nu < 30$ GeV, $\langle E_\nu \rangle \approx 7.1$ GeV
(a)
$\langle d\sigma / dQ^2 \rangle$ ($10^{-38}$ cm$^2$ / GeV$^2$)
$\bar{\nu}_\mu\, p \to \mu^+\, n$
Brunner et al., IHEP SKAT 1990
Target: $CF_3Br$ (converted to free proton)
Number of events: 256.7 ± 51.2
$3 < E_\nu < 30$ GeV, $\langle E_\nu \rangle \approx 6.2$ GeV
(b)
$Q^2$ (GeV$^2$)

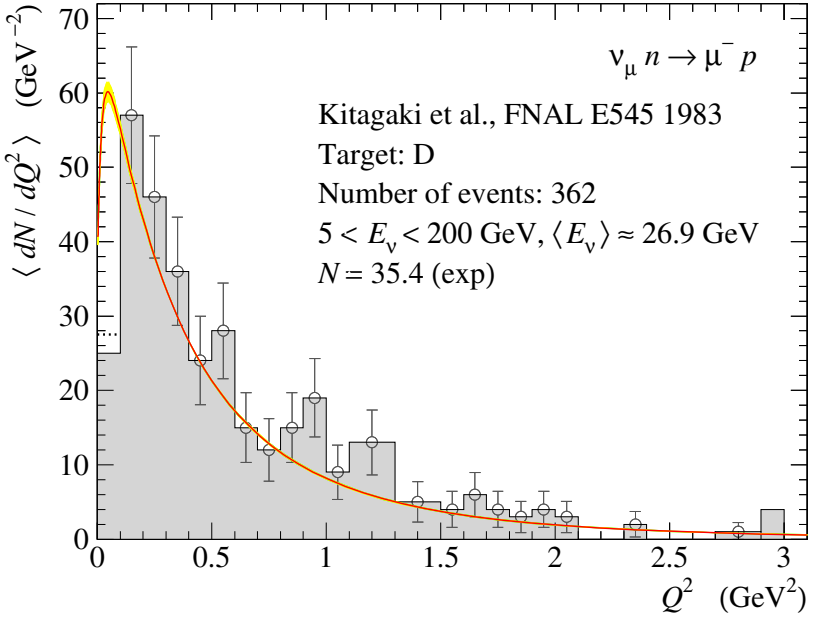

$\nu_\mu\, n \to \mu^-\, p$
Kitagaki et al., BNL 1990
(2003 update, unpublished)
Target: D
Number of events: 2 684
$0.4 < E_\nu < 6$ GeV, $\langle E_\nu \rangle \approx 1.5$ GeV
$Q^2 > 0.08$ GeV$^2$
$N = 226.4 \pm 5.6$ (exp)
$\langle dN / dQ^2 \rangle$ (GeV$^{-2}$)
$Q^2$ (GeV$^2$)

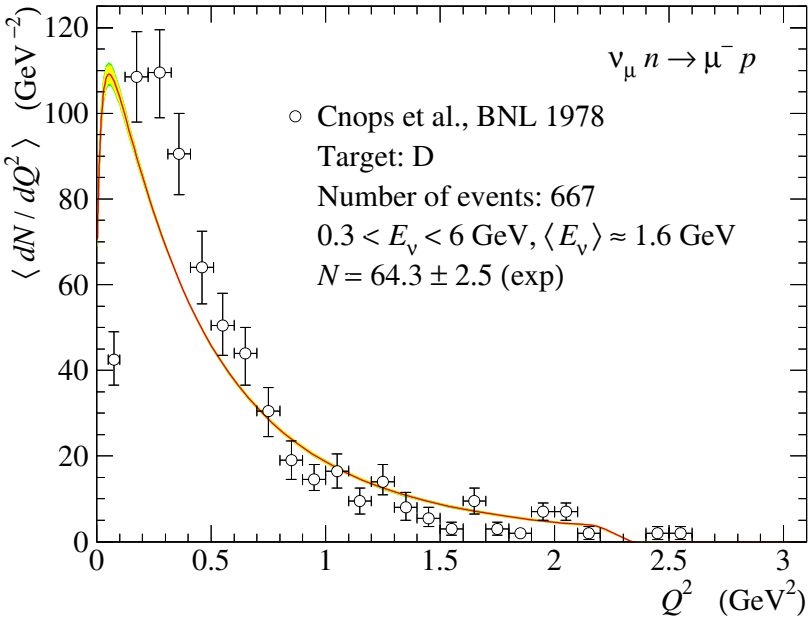
# Comparison with certain datasets unclaimed in the global fit

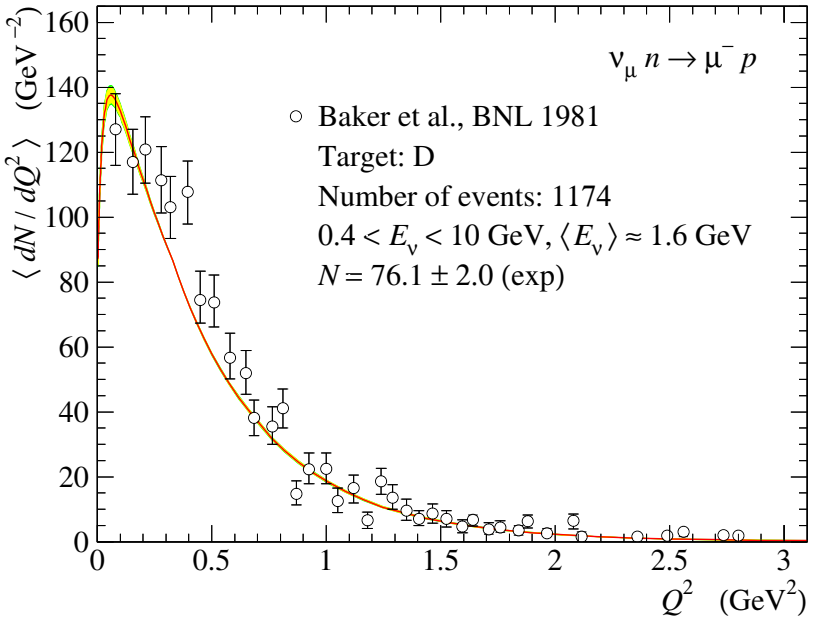

$\nu_\mu\, n \to \mu^-\, p$
Target: D
$0 < E_\nu < 2.7$ GeV, $\langle E_\nu \rangle \approx 0.7$ GeV
Barish et al., ANL 1975
Number of events: 396
$N = 20.0 \pm 1.05$ (exp)
Mann et al., ANL 1973
Number of events: 166
$N = 8.3$ (exp)
$\langle dN/dQ^2 \rangle$ (GeV$^{-2}$)
$Q^2$ (GeV$^2$)

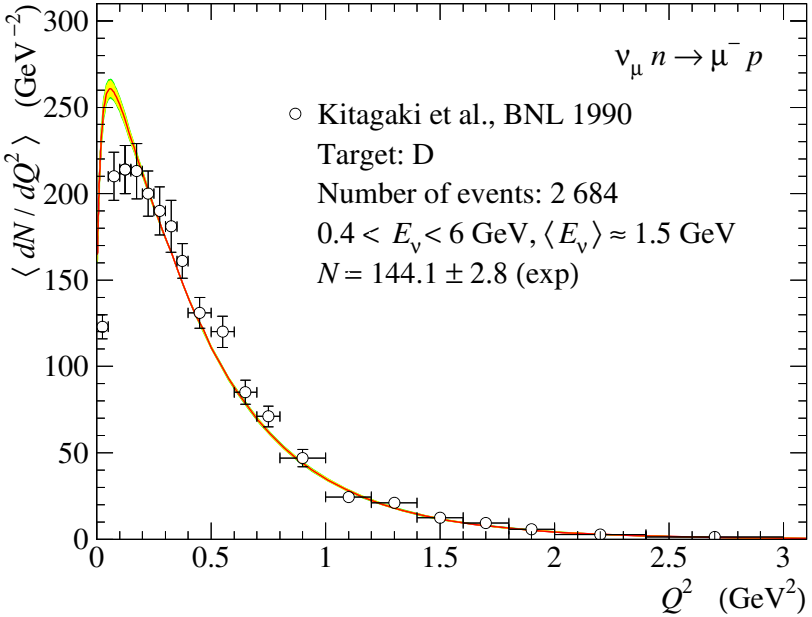

$\nu_\mu\, n \to \mu^-\, p$
Target: D
$0.1 < E_\nu < 6$ GeV, $\langle E_\nu \rangle \approx 1$ GeV
Miller et al., ANL 1982
Number of events: $1792 \pm 48$
$N = 90.0$ (exp)
Barish et al., ANL 1977
Number of events: 1643
$N = 30.0$ (exp)
$\langle dN/dQ^2 \rangle$ (GeV$^{-2}$)
$Q^2$ (GeV$^2$)

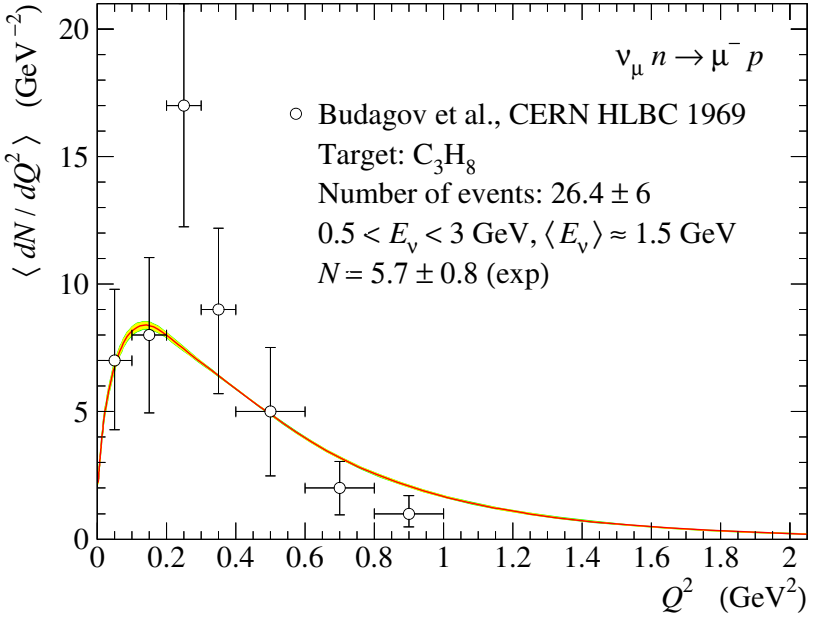

$\bar{\nu}_\mu\, p \to \mu^+\, n$
Fanourakis et al., BNL 1980
Target: $H_2$
Number of events: $13 \pm 6$
$\langle E_\nu \rangle \approx 1.3$ GeV
$N = 1.6$ (exp)
INCLUDED INTO THE FIT
BUT DOES NOT AFFECT IT
$\langle dN/dQ^2 \rangle$ (GeV$^{-2}$)
$Q^2$ (GeV$^2$)

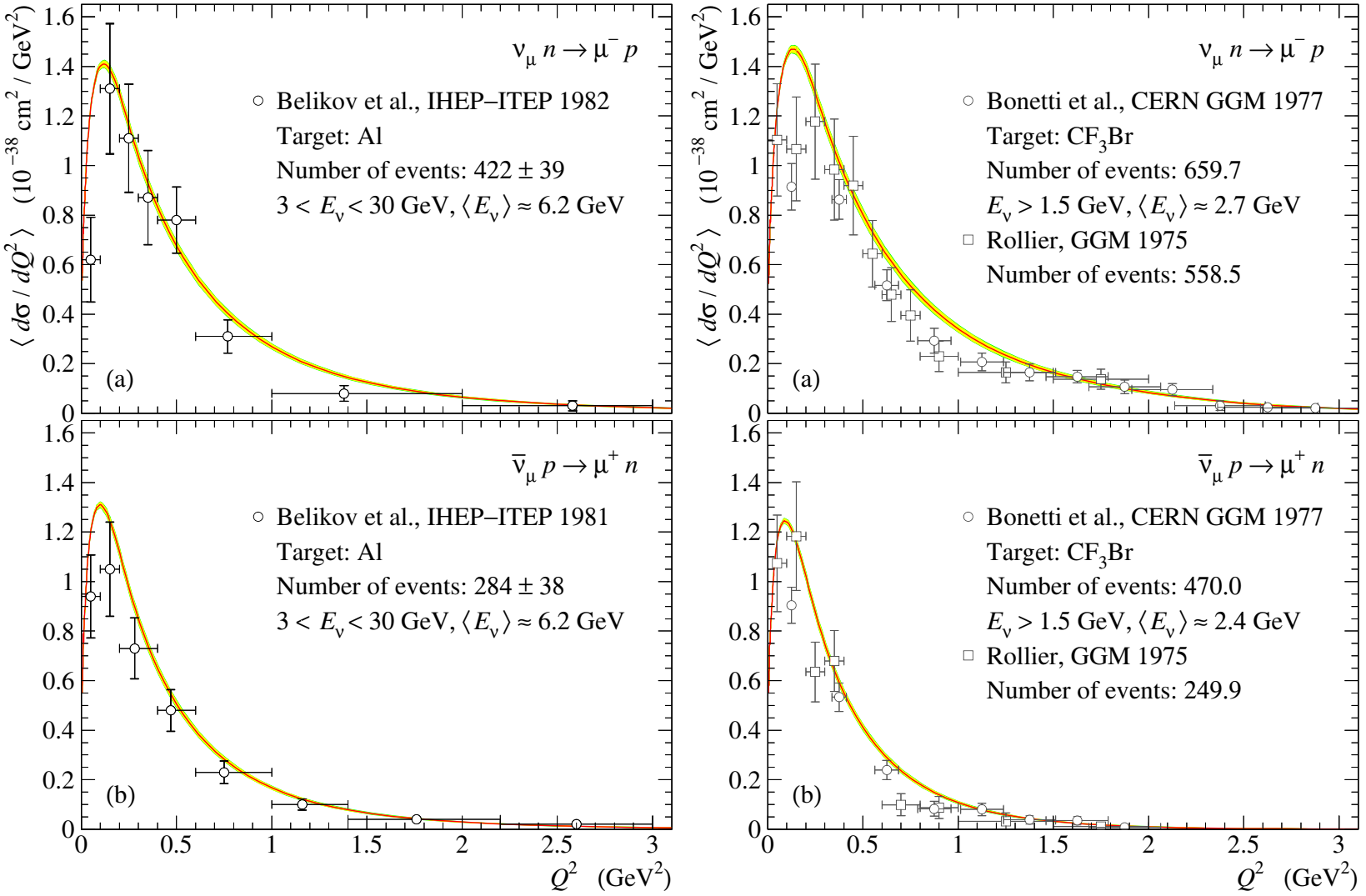

$\nu_\mu\, n \to \mu^-\, p$
Kitagaki et al., FNAL E545 1983
Target: D
Number of events: 362
$5 < E_\nu < 200$ GeV, $\langle E_\nu \rangle \approx 26.9$ GeV
$N = 35.4$ (exp)
$\langle dN/dQ^2 \rangle$ (GeV$^{-2}$)
$Q^2$ (GeV$^2$)

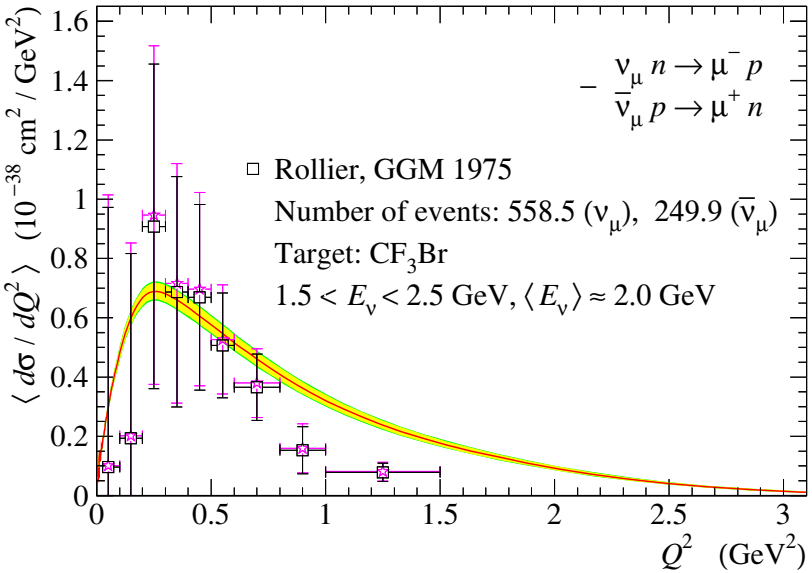

$\nu_\mu\, n \to \mu^-\, p$
Cnops et al., BNL 1978
Target: D
Number of events: 667
$0.3 < E_\nu < 6$ GeV, $\langle E_\nu \rangle \approx 1.6$ GeV
$N = 64.3 \pm 2.5$ (exp)
$\langle dN/dQ^2 \rangle$ (GeV$^{-2}$)
$Q^2$ (GeV$^2$)

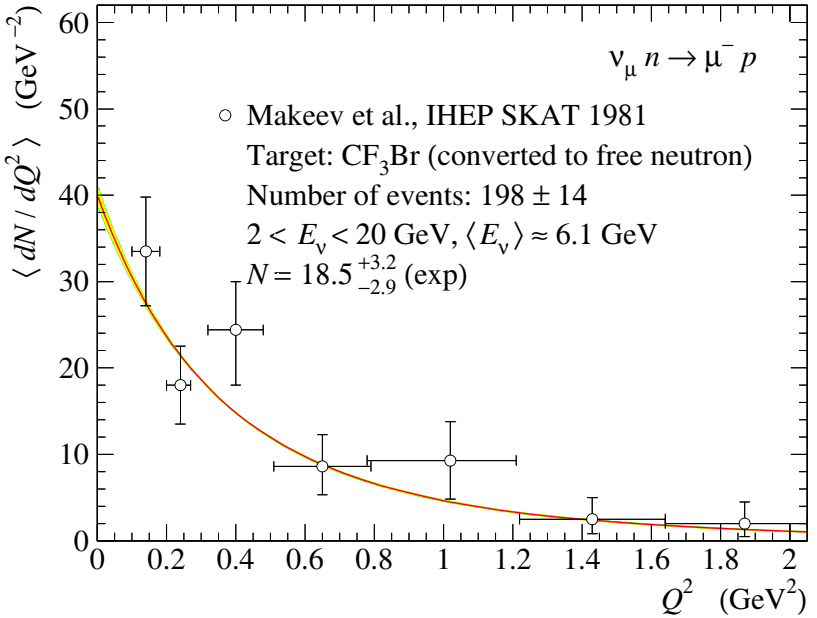

$\nu_\mu\, n \to \mu^-\, p$
Baker et al., BNL 1981
Target: D
Number of events: 1174
$0.4 < E_\nu < 10$ GeV, $\langle E_\nu \rangle \approx 1.6$ GeV
$N = 76.1 \pm 2.0$ (exp)
$\langle dN/dQ^2 \rangle$ (GeV$^{-2}$)
$Q^2$ (GeV$^2$)

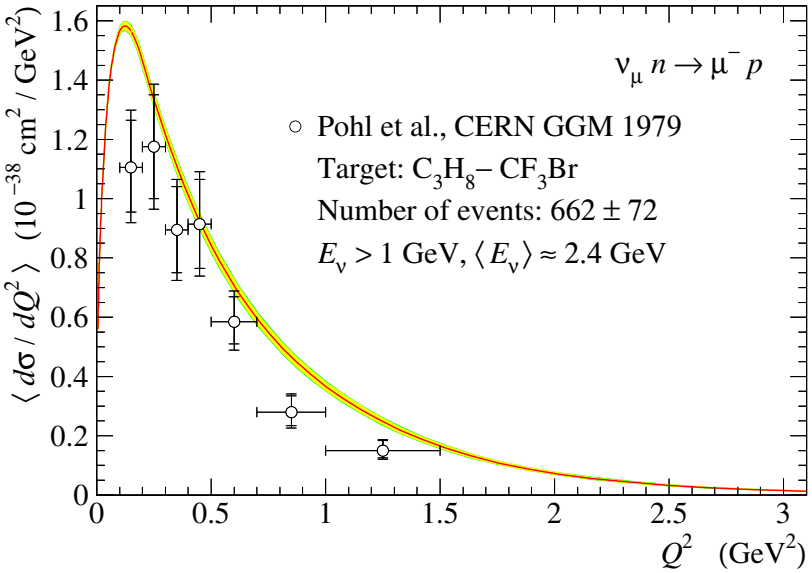

$\nu_\mu\, n \to \mu^-\, p$
Kitagaki et al., BNL 1990
Target: D
Number of events: 2 684
$0.4 < E_\nu < 6$ GeV, $\langle E_\nu \rangle \approx 1.5$ GeV
$N = 144.1 \pm 2.8$ (exp)
$\langle dN/dQ^2 \rangle$ (GeV$^{-2}$)
$Q^2$ (GeV$^2$)

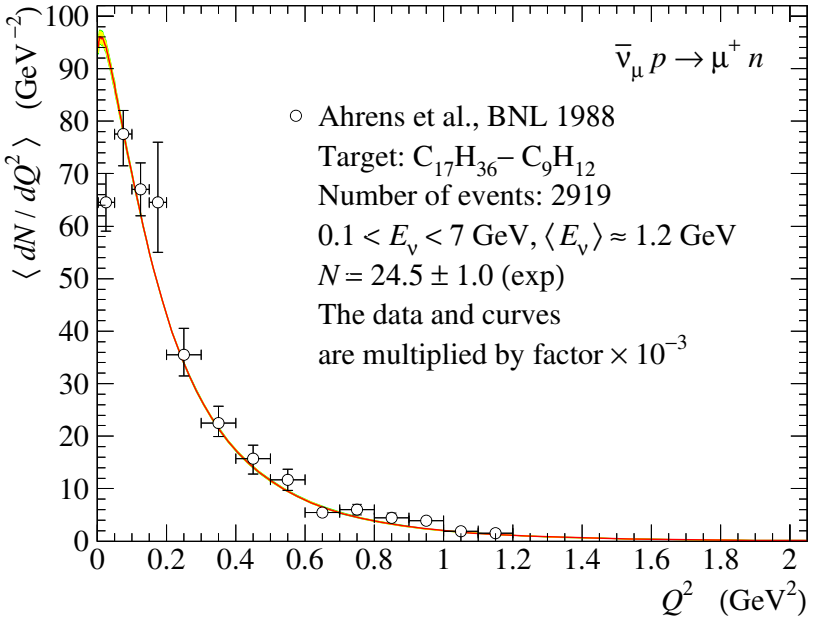

$\nu_\mu\, n \to \mu^-\, p$
Budagov et al., CERN HLBC 1969
Target: $C_3H_8$
Number of events: $26.4 \pm 6$
$0.5 < E_\nu < 3$ GeV, $\langle E_\nu \rangle \approx 1.5$ GeV
$N = 5.7 \pm 0.8$ (exp)
$\langle dN/dQ^2 \rangle$ (GeV$^{-2}$)
$Q^2$ (GeV$^2$)

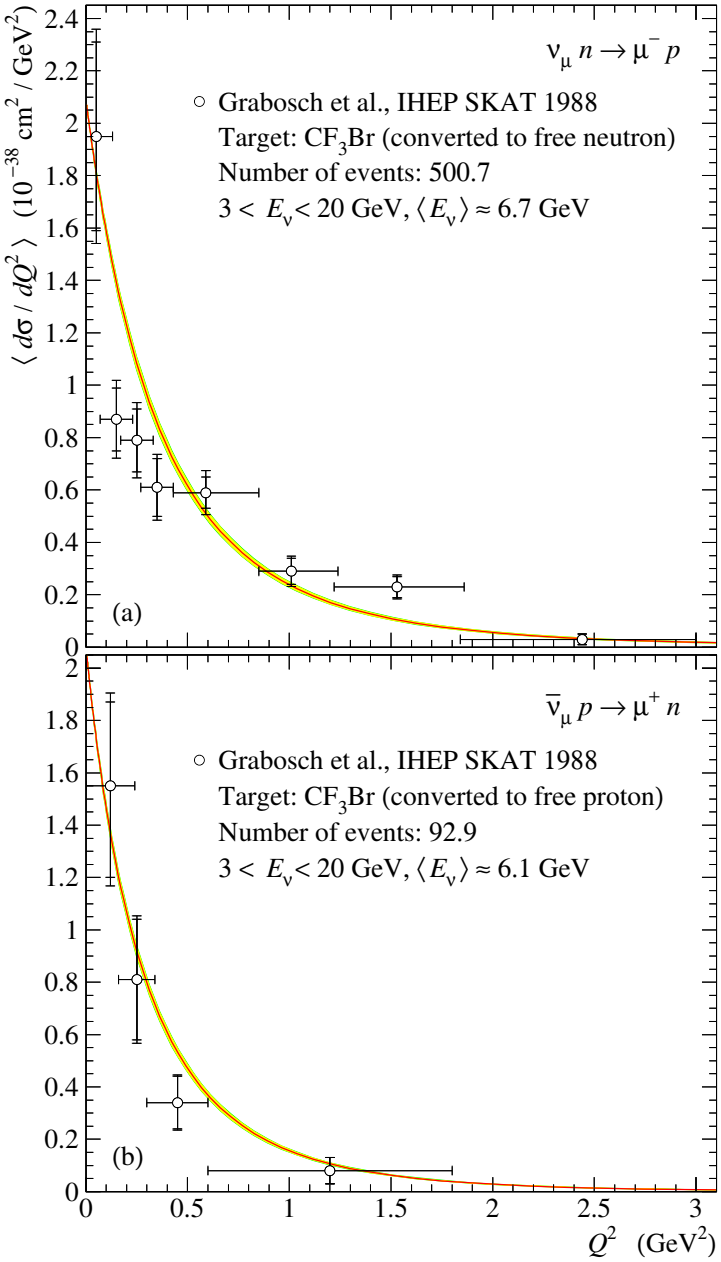

$\nu_\mu\, n \to \mu^-\, p$
Belikov et al., IHEP–ITEP 1982
Target: Al
Number of events: 422 ± 39
3 < $E_\nu$ < 30 GeV, $\langle E_\nu \rangle \approx$ 6.2 GeV
(a)
$\bar{\nu}_\mu\, p \to \mu^+\, n$
Belikov et al., IHEP–ITEP 1981
Target: Al
Number of events: 284 ± 38
3 < $E_\nu$ < 30 GeV, $\langle E_\nu \rangle \approx$ 6.2 GeV
(b)
$\nu_\mu\, n \to \mu^-\, p$
Bonetti et al., CERN GGM 1977
Target: $CF_3Br$
Number of events: 659.7
$E_\nu$ > 1.5 GeV, $\langle E_\nu \rangle \approx$ 2.7 GeV
Rollier, GGM 1975
Number of events: 558.5
(a)
$\bar{\nu}_\mu\, p \to \mu^+\, n$
Bonetti et al., CERN GGM 1977
Target: $CF_3Br$
Number of events: 470.0
$E_\nu$ > 1.5 GeV, $\langle E_\nu \rangle \approx$ 2.4 GeV
Rollier, GGM 1975
Number of events: 249.9
(b)
$\langle d\sigma / dQ^2 \rangle$ ($10^{-38}$ cm$^2$ / GeV$^2$)
$Q^2$ (GeV$^2$)

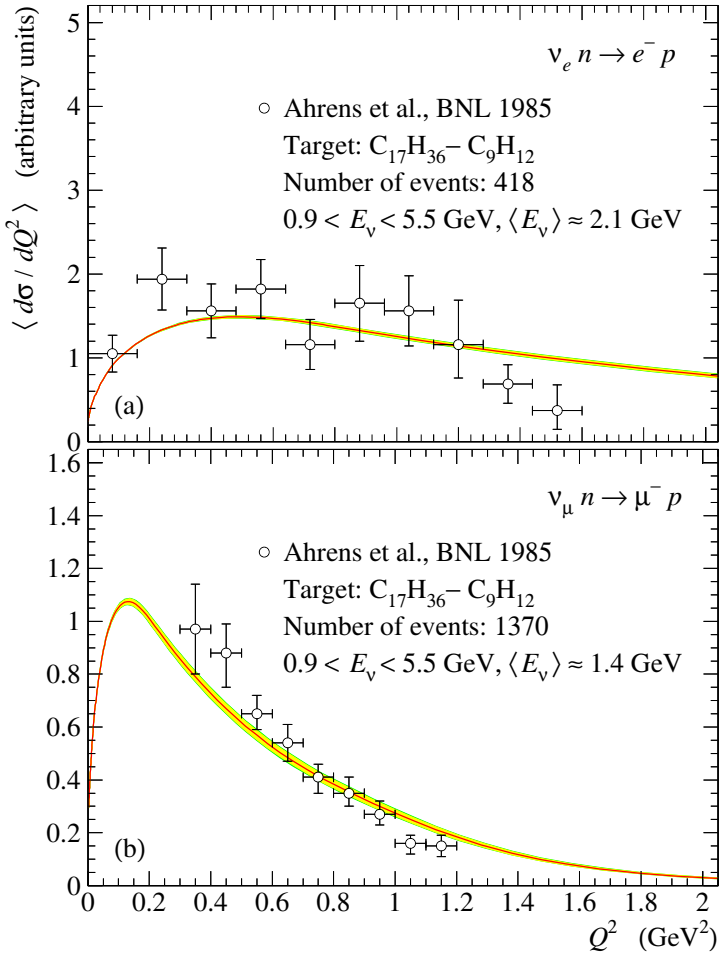

$\nu_\mu\, n \to \mu^-\, p$
$\bar{\nu}_\mu\, p \to \mu^+\, n$
Rollier, GGM 1975
Number of events: 558.5 ($\nu_\mu$), 249.9 ($\bar{\nu}_\mu$)
Target: $CF_3Br$
$1.5 < E_\nu < 2.5$ GeV, $\langle E_\nu \rangle \approx 2.0$ GeV
$\langle d\sigma / dQ^2 \rangle$ ($10^{-38}$ cm$^2$ / GeV$^2$)
$Q^2$ (GeV$^2$)

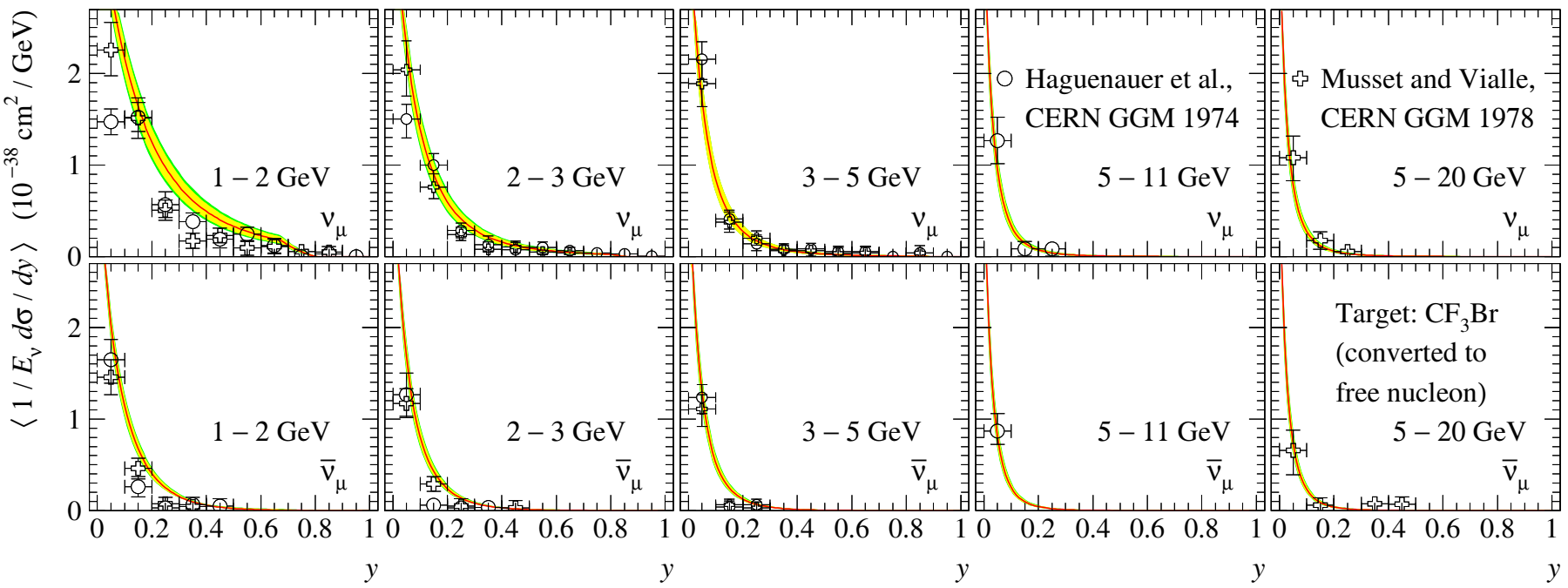

$\nu_\mu\, n \to \mu^-\, p$
Makeev et al., IHEP SKAT 1981
Target: $CF_3Br$ (converted to free neutron)
Number of events: 198 ± 14
$2 < E_\nu < 20$ GeV, $\langle E_\nu \rangle \approx 6.1$ GeV
$N = 18.5\,^{+3.2}_{-2.9}$ (exp)
$\langle dN / dQ^2 \rangle$ (GeV$^{-2}$)
$Q^2$ (GeV$^2$)


$\nu_\mu\, n \to \mu^-\, p$
Pohl et al., CERN GGM 1979
Target: $C_3H_8 - CF_3Br$
Number of events: 662 ± 72
$E_\nu > 1$ GeV, $\langle E_\nu \rangle \approx 2.4$ GeV
$\langle d\sigma / dQ^2 \rangle$ ($10^{-38}$ cm$^2$ / GeV$^2$)
$Q^2$ (GeV$^2$)


$\bar{\nu}_\mu\, p \to \mu^+\, n$
Ahrens et al., BNL 1988
Target: $C_{17}H_{36} - C_9H_{12}$
Number of events: 2919
$0.1 < E_\nu < 7$ GeV, $\langle E_\nu \rangle \approx 1.2$ GeV
$N = 24.5 \pm 1.0$ (exp)
The data and curves
are multiplied by factor $\times 10^{-3}$
$\langle dN / dQ^2 \rangle$ (GeV$^{-2}$)
$Q^2$ (GeV$^2$)


$\nu_\mu\, n \to \mu^-\, p$
Grabosch et al., IHEP SKAT 1988
Target: $CF_3Br$ (converted to free neutron)
Number of events: 500.7
$3 < E_\nu < 20$ GeV, $\langle E_\nu \rangle \approx 6.7$ GeV
(a)
$\bar{\nu}_\mu\, p \to \mu^+\, n$
Grabosch et al., IHEP SKAT 1988
Target: $CF_3Br$ (converted to free proton)
Number of events: 92.9
$3 < E_\nu < 20$ GeV, $\langle E_\nu \rangle \approx 6.1$ GeV
(b)
$\langle d\sigma / dQ^2 \rangle$ $(10^{-38}$ cm$^2$ / GeV$^2)$
$Q^2$ (GeV$^2$)


$\nu_e\, n \to e^-\, p$
Ahrens et al., BNL 1985
Target: $C_{17}H_{36}$– $C_9H_{12}$
Number of events: 418
$0.9 < E_\nu < 5.5$ GeV, $\langle E_\nu \rangle \approx 2.1$ GeV
(a)
$\nu_\mu\, n \to \mu^-\, p$
Ahrens et al., BNL 1985
Target: $C_{17}H_{36}$– $C_9H_{12}$
Number of events: 1370
$0.9 < E_\nu < 5.5$ GeV, $\langle E_\nu \rangle \approx 1.4$ GeV
(b)
$\langle d\sigma / dQ^2 \rangle$ (arbitrary units)
$Q^2$ (GeV$^2$)




Flux-weighted differential cross sections for $\nu_\mu n \to \mu^- p$ (top) and $\overline{\nu}_\mu p \to \mu^+ n$ (bottom). The cross sections were measured in the Gargamelle bubble chamber filled with heavy liquid freon and exposed to the CERN-PS $\nu_\mu$ and $\overline{\nu}_\mu$ beams.
The shaded regions around the solid curves represent $1\sigma$ and $2\sigma$ deviations from the best-fitted values of $M_A^{\text{run}}$ obtained in the global fit. Only the events with $E_{\nu,\overline{\nu}} > 1.5$ GeV were accepted.
The energy range for each experiment is shown in the legends.
Obsolete GGM 1975 data are also shown for completeness.

# Implementation in GENIE

The running axial mass model is implemented into release 2.11.0 of the GENIE Neutrino Event MC generator and is called genie::KuzminNaumov2016AxialFormFactorModel.

It works with the default CCQE cross sections model genie::LwlynSmithQELCCPXSec and there is no need to make any changes in the 'cross section model' part of the GENIE configuration file *$GENIE/config/UserPhysicsOptions.xml*:

```
<!--
 ~~~
```

To start working with the running axial mass, you need to set the corresponding parameter to genie::KuzminNaumov2016AxialFormFactorModel in the configuration file *$GENIE/config/UserPhysicsOptions.xml*:

```
<!--
  ~~~
```

The values of $M_A$ and $F_A(0)$ in the configuration file
*$GENIE/config/KuzminNaumov2016AxialFormFactorModel.xml*
should be set to our best-fit values (instead of the GENIE default ones).

```
<alg_conf>

<!--
Configuration sets for the Kuzmin-Naumov running axial form factors model.

Algorithm Configurable Parameters:
.....................................................................................
Name        Type    Optional   Comment                    Default
.....................................................................................
Ma          double  Yes        Axial Mass                 GPL value: QEL-Ma
FA0         double  Yes        FA(q2 = 0)                 GPL value: QEL-FA0
E0          double  Yes        E0                         GPL value: CCQE-E0
-->
-------------------------------------------------------------------------------------
  <param_set name="Default">
  </param_set>

</alg_conf>
```

```
-------------------------------------------------------------------------------------
  <param_set name="Default">
      <param type="double" name="Ma">  1.006   </param>
      <param type="double" name="FA0"> -1.2695 </param>
  </param_set>

</alg_conf>
```

There is not need to change the GENIE default value of the parameter $E_0$.

# A few more comments:

- The best fit parameters $M_0$ and $E_0$ were obtained within the SM RFG model but GENIE (in all releases) uses the BR RFG model. **Most important**

- The set of RFG input parameters ($p_F$ and $E_b$) in GENIE 2.11.0 is in general different from that used in our analysis. **Important for some targets**

- There is still uncorrected bug in GENIE 2.11.0 which affects the calculation accuracy for the elastic and quasielastic cross sections for **non-isoscalar** nuclei. **Generally important**

- The electromagnetic form factor models used in GENIE 2.11.0 and in our analysis are slightly different. **Almost negligible**

- Several general physics constants (e.g. masses of nuclei) used in our analysis are slightly different from those in GENIE 2.11.0. **Negligible**

We plan:

- Incorporation of the SM RFG into the GENIE (at least its local version).
- Adjustment of the $M_A^{\text{run}}$ parameters for the BR RFG model.

# Conclusions

- The Smith-Moniz RFG model supplemented by the energy-dependent (running) axial mass, $M_A^{\text{run}}$, defined by only two universal parameters $M_0$ and $E_0$, describes well the earlier and modern CCQE data on various nuclear targets.
- The best-fit values of $M_0$ and $E_0$ are sensitive to the details of the RFG model (including the input Fermi momenta and separation energies) and to the model of the nucleon electromagnetic form factors. However the fit can almost automatically be repeated with other versions of RFG (e.g., Bodek–Ritchie), or its extensions (SF, LFG, etc.), or with more advanced models and improved inputs.
- A more sophisticated parametrization of $M_A^{\text{run}}$ is unreasonable for the present-day level of accuracy of the CCQE (and CCQE-like) data but may be needed in the future.
- Individual parametrizations for different nuclei or nucler groups are not warranted, but mainly because the currently available dataset for the inorganic heavy nuclear targets is not accurate enough and is not fully self-consistent.
- There's no statistically significant difference between the $M_A^{\text{run}}$ parameters extracted separately from the $\nu$ and $\overline{\nu}$ data, but there is a weak hint to a possible difference. Any case, the available $\overline{\nu}$ dataset is not yet sufficient for a more definite statement.

Finally, we suggest to use the running axial mass option in the NO$\nu$A analyses.

# Thanks for attention

BACKUP

# "Golden" dataset for the global fits

# Accountancy & taxonomy

**Experiments with deuterium and hydrogen targets:**
ANL 1977 [5] ($\sigma_\nu$, 8 data points),
ANL 1982 [7] ($\langle dN_\nu/dQ^2\rangle$, 39 data points),
BNL 1980 [10] ($\sigma_{\overline{\nu}}$, 5 data point),
BNL 1990 [14, 15] ($\langle dN_\nu/dQ^2\rangle$, 37 data points),
FNAL 1983 [16] ($\langle dN_\nu/dQ^2\rangle$, 20 data points), and
CERN BEBC 1990 [44] ($\langle d\sigma_\nu/dQ^2\rangle$, 8 data points).

**Experiment with Ne-$H_2$ mixture**
FNAL 1984 [17] ($dN_{\overline{\nu}}/dQ^2$, 14 data points).

**Experiment with aluminium target**
IHEP–ITEP 1985 [51] ($\sigma_\nu$, $\sigma_{\overline{\nu}}$, $\langle d\sigma_{\nu+\overline{\nu}}/dQ^2\rangle$, 8 data points in each set).

**Experiments with carbonaceous targets**
FNAL MiniBooNE 2010 [24] ($\langle d^2\sigma_\nu/dE_\mu d\cos\theta_\mu\rangle$, 137 data points),
FNAL MiniBooNE 2013 [25] ($\langle d^2\sigma_{\overline{\nu}}/dE_\mu d\cos\theta_\mu\rangle$, 75 data points),
CERN NOMAD 2009 [47, 48] ($\sigma_\nu$ and $\sigma_{\overline{\nu}}$, 10 and 6 data points, respectively), and
FNAL MINERνA 2013 [27, 28] ($\langle d\sigma_\nu/dQ^2\rangle$, $\langle d\sigma_{\overline{\nu}}/dQ^2\rangle$, 8 data points in each set).

**Experiments with carbon-containing mixtures**
CERN GGM 1979 [43, 45] ($\langle dN_{\overline{\nu}}/dQ^2\rangle$, 13 data points),
CERN LAr-TPC 2007 [46] ($\sigma_\nu$, 1 data point),
IHEP SKAT 1990 [56] ($\langle d\sigma_\nu/dQ^2\rangle$ and $\langle d\sigma_{\overline{\nu}}/dQ^2\rangle$, 8 and 7 data points, respectively).

**Experiment with iron target** T2K INGRID 2015 [63] ($\sigma_\nu$, 2 data points).

- Thus the fitted dataset consists of 430 data points with
  286 data points for the $\nu_\mu$ cross sections (66.5% of the full dataset),
  136 data points for the $\overline{\nu}_\mu$ cross sections (31.6%), and
  8 data points for the summarized cross sections of $\nu_\mu + \overline{\nu}_\mu$ (1.9%).
- The experimental data are presented as follows:
  212 data points for the flux-averaged double differential cross sections $\langle d^2\sigma/dE_\mu d\cos\theta_\mu\rangle$ (49.3% of the full dataset, FNAL MiniBooNE),
  47 data points for the flux-averaged differential cross sections $\langle d\sigma/dQ^2\rangle$ (10.9%),
  128 data points for the flux-averaged $Q^2$ distributions $\langle dN/dQ^2\rangle$ (29.8%), and
  43 data points for the total cross sections (10.0%).
- The dataset for extracting the current axial mass $M_A$ (deuterium and hydrogen targets) contains 85 data points (27.2% of the full dataset): ANL 1977 [5], ANL 1982 [7], BNL 1980 [10], BNL 1990 [14, 15], FNAL 1983 [16], CERN 1990 [44].

## Criteria for exclusion of data from the global fit:

- Obsolete data (superseded or reconsidered due to increased statistics, revised normalization, etc. in the posterior reports of the same experimental groups).
- Data from the experiments with inactive targets (suggested by Arie Bodek).
- Data with $\chi^2/\text{NDF} > 5$ and $|N-1| > 2\sigma_N$ (where $N$ is the flux normalization).

**Convention:** below, the **solid** symbols in the figures represent the data included into the global fit while **open** symbols represent the data excluded from the fit.

# The set of experimental data used in the global fit.

| Experiment | Data type | $N$ | $\chi^2$/NP |
|---|---|---|---|
| Deuterium & hydrogen data and data for nuclear targets recalculated to free nucleons | | | |
| Barish *et al.*, ANL 1977 [5] | $\sigma_\nu$ * | 0.893 ± 0.034 (0.068) | 15.97/8 = 2.00 |
| Miller *et al.*, ANL 1982 [7] | $\langle dN_\nu/dQ^2\rangle$ * | 99.7 × 0.979 | 73.90/38 = 1.95 |
| Fanourakis *et al.*, BNL 1980 [10] | $\langle dN_{\overline{\nu}}/dQ^2\rangle$ * | 2.4 × 0.901 | 1.37/4 = 0.34 |
| | $\sigma_{\overline{\nu}}$ | – | 0.05/1 = 0.05 |
| Kitagaki *et al.*, BNL 1990 (2003 update) [14] | $\langle dN_\nu/dQ^2\rangle$ * | 245.4 ×0.991 | 21.49/37 = 0.58 |
| Kitagaki *et al.*, FNAL 1983 [16] | $\langle dN_\nu/dQ^2\rangle$ * | 41.5 × 0.945 | 8.53/20 = 0.43 |
| | $\sigma_\nu$ | 1.020 ± 0.077 (0.152) | 0.18/2 = 0.09 |
| Allasia *et al.*, CERN BEBC 1990 [44] | $\langle d\sigma_\nu/dQ^2\rangle$ * | 0.935 ± 0.035 (0.070) | 4.77/8 = 0.68 |
| | $\sigma_\nu$ | 0.873 ± 0.032 (0.064) | 15.11/6 = 3.02 |
| Brunner *et al.*, IHEP SKAT 1990 [55, 56] | $\langle d\sigma_\nu/dQ^2\rangle$ * | 1.077 ± 0.059 (0.117) | 3.89/8 = 0.49 |
| | $\sigma_\nu$ | 1.056 ± 0.058 (0.115) | 3.14/8 = 0.39 |
| | $\langle d\sigma_{\overline{\nu}}/dQ^2\rangle$ * | 0.902 ± 0.064 (0.126) | 7.31/7 = 1.04 |
| | $\sigma_{\overline{\nu}}$ | 0.823 ± 0.056 (0.111) | 8.10/7 = 1.15 |
| Heavy targets data | | | |
| Asratyan *et al.*, FNAL 1984 [17] | $\langle dN_{\overline{\nu}}/dQ^2\rangle$ * | 68.7 × 0.993 | 7.93/14 = 0.57 |
| | $\sigma_{\overline{\nu}}$ | – | 0.06/1 = 0.06 |
| Aguilar-Arevalo *et al.*, FNAL MiniBooNE 2010 [24] | $\langle d^2\sigma_\nu/dE_\mu d\cos\theta_\mu\rangle$ * | 1.000 ± 0.008 (0.015) | 35.76/137= 0.26 |
| Aguilar-Arevalo *et al.*, FNAL MiniBooNE 2013 [25] | $\langle d^2\sigma_{\overline{\nu}}/dE_\mu d\cos\theta_\mu\rangle$ * | 1.017 ± 0.012 (0.023) | 46.86/75 = 0.63 |
| Fiorentini *et al.*, FNAL MINERνA 2013 [27] | $\langle d\sigma_\nu/dQ^2\rangle$ * | – | 8.04/8 = 1.01 |
| Fields *et al.*, FNAL MINERνA 2013 [28] | $\langle d\sigma_{\overline{\nu}}/dQ^2\rangle$ * | – | 5.51/8 = 0.67 |
| Armenise *et al.*, CERN GGM 1979 [43, 45] | $\sigma_{\overline{\nu}}$ | 0.781 ± 0.031 (0.062) | 20.57/11 = 1.87 |
| Martinez de la Ossa Romero *et al.*, CERN LAr-TPC 2007 [46] | $\sigma_\nu$ * | – | 0.09/1 = 0.09 |
| Lyubushkin *et al.*, CERN NOMAD 2009 [47, 48] | $\sigma_\nu$ * | 1.036 ± 0.026 (0.052) | 5.91/10 = 0.59 |
| | $\sigma_{\overline{\nu}}$ * | 1.016 ± 0.058 (0.114) | 3.09/6 = 0.52 |
| Belikov *et al.*, IHEP–ITEP 1985 [51] | $\langle d\sigma_{\nu+\overline{\nu}}/dQ^2\rangle$ * | 0.975 ± 0.114 (0.184) | 5.68/8 = 0.71 |
| | | 0.988 ± 0.131 (0.212) | |
| | $\sigma_\nu$ * | 0.873 ± 0.036 (0.071) | 7.02/8 = 0.88 |
| | $\sigma_{\overline{\nu}}$ * | 0.897 ± 0.045 (0.089) | 2.66/8 = 0.33 |
| Abe *et al.*, T2K ND280 2014 [61] | $\sigma_\nu$ * | – | 9.57/5 = 2.39 |
| Abe *et al.*, T2K INGRID 2015 [63] | $\sigma_\nu$ * | 0.977 ± 0.086 (0.170) | 0.55/2 = 0.28 |

* The data included into the global fits.

# The fitting method

# $\chi^2$ schematically

$$\chi^2 = \sum_{i\,(\text{uncorr})} \sum_{j\in G_i} \frac{\left[N_i T_{ij}(\boldsymbol{\lambda}) - E_{ij}\right]^2}{\sigma_{ij}^2} + \sum_{i\,(\text{uncorr})} \frac{\left(N_i - 1\right)^2}{\sigma_i^2} + \sum_{i\,(\text{corr})} \left[\mathbf{T}_i(\boldsymbol{\lambda}) - \mathbf{E}_i\right] \mathbf{W}_i^{-1} \left[\mathbf{T}_i(\boldsymbol{\lambda}) - \mathbf{E}_i\right]^T .$$

Here

- index $i$ enumerates the experiments (or data groups) $G_i$,
- index $j \in G_i$ enumerates the bin-averaged experimental data $E_{ij}$ from the group $G_i$ with the errors $\sigma_{ij}$ which do not include the flux normalization uncertainty, $\sigma_i$. The data sets include the total CCQE cross sections, flux-averaged differential cross sections ($d\sigma_{\nu,\overline{\nu}}/dQ^2$, $d^2\sigma_{\nu,\overline{\nu}}/dE_\mu d\cos\theta_\mu$, etc.), and unnormalized $Q^2$ distributions ($dN_{\nu,\overline{\nu}}/dQ^2$).
- $T_{ij}(\boldsymbol{\lambda})$ represent the model predictions dependent on the set of fitting parameters $\boldsymbol{\lambda}$ (in our particular case, $\boldsymbol{\lambda} = (M_0, E_0)$ for heavy nuclear targets and $\boldsymbol{\lambda} = M_0$ for hydrogen and deuterium targets).
- $\mathbf{W}_i$ is the correlation matrix for the $i$th group.

Nuclear corrections for the CCQE cross sections on deuterium are, as a rule, already taken into account by the authors of the experiment and are represented as the cross sections or $Q^2$ distribution for a free nucleon target.
The formal analytic expressions for the normalization factors $N_i$ can be found by solving the system of equations

$$\partial\chi^2/\partial N_i = 0.$$

The solution is of the form:

$$N_i = \frac{1 + \sigma_i^2 \sum\limits_{j \in G_i} \sigma_{ij}^{-2} T_{ij}(\boldsymbol{\lambda}) E_{ij}}{1 + \sigma_i^2 \sum\limits_{j \in G_i} \sigma_{ij}^{-2} \left[T_{ij}(\boldsymbol{\lambda})\right]^2}.$$

The penalty terms $\left(N_i - 1\right)^2 / \sigma_i^2$ contain the flux uncertainties $(\sigma_i)$ common to the data group $G_i$.
The $\chi^2$ for the final fit to all data also includes another penalty term

$$\left(\frac{M_0 - M_A^D}{\Delta M_A^D}\right)^2,$$

which provides a "soft anchoring" of the parameter $M_0$ to the current axial mass $M_A^D \pm \Delta M_A^D$ obtained from the fitting the robust deuterium data only.

## Including the "mixed" data into the $\chi^2$ sum

The expression for $\chi^2$ shown in the slide 73 becomes a bit more complicated when the data is a combination of the values obtained in the experiments with the neutrino and antineutrino beams.

Here we consider the case when we have to fit the *uncorrelated* data like $E_1$, $E_2$ and $E_3 = E_1' + E_2'$, where the pairs $(E_1, E_1')$ and $(E_2, E_2')$ have the common normalization uncertainty. Let's construct the sum for minimization for this case:

$$\chi^2_{\text{mix}} = \sum_{j\in G_1} \frac{\left[N_1 T_{1j}(\boldsymbol{\lambda}) - E_{1j}\right]^2}{\sigma_{1j}^2} + \sum_{j\in G_2} \frac{\left[N_2 T_{2j}(\boldsymbol{\lambda}) - E_{1j}\right]^2}{\sigma_{2j}^2}$$
$$+ \sum_{j\in G_3} \frac{\left[N_1 T_{1'j}(\boldsymbol{\lambda}) + N_2 T_{2'j}(\boldsymbol{\lambda}) - E_{3j}\right]^2}{\sigma_{3j}^2}$$
$$+ \left(\frac{N_1 - 1}{\sigma_1}\right)^2 + \left(\frac{N_2 - 1}{\sigma_2}\right)^2$$

(the notation is obvious). From the minimization conditions

$$\frac{\partial \chi^2_{\text{mix}}}{\partial N_1} = \frac{\partial \chi^2_{\text{mix}}}{\partial N_2} = 0$$

we obtain the expressions for the normalization factors:

$$N_1 = \frac{1}{d}\left(C_1 A_{22} - C_2 A_{12}\right), \quad N_2 = \frac{1}{d}\left(C_2 A_{11} - C_1 A_{12}\right),$$

where

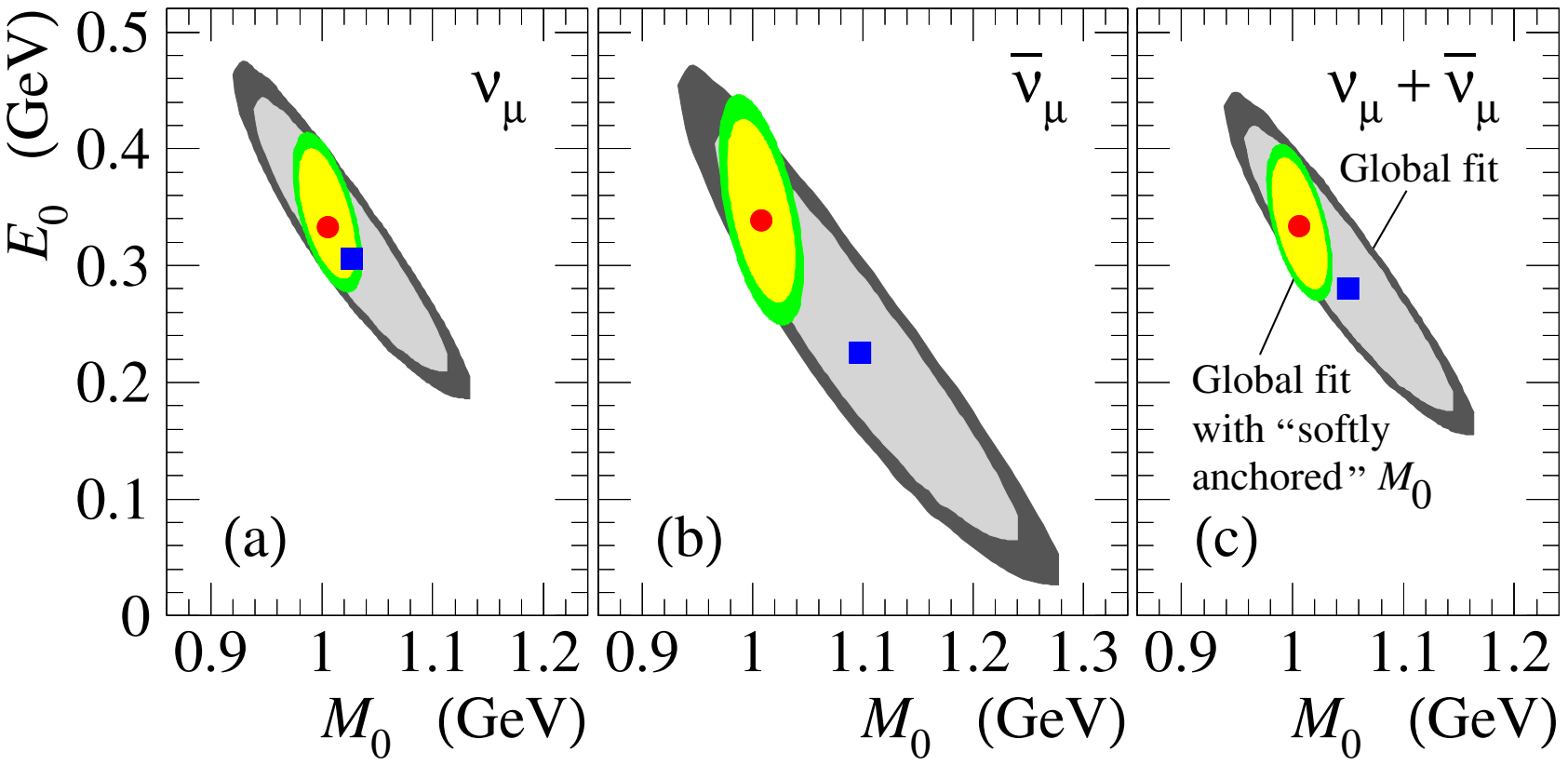
$$d = A_{11}A_{22} - A_{12}^2,$$

$$A_{11} = \sum_{j\in G_1} \frac{\left[T_{1j}(\boldsymbol{\lambda})\right]^2}{\sigma_{1j}^2} + \sum_{j\in G_3} \frac{\left[T_{1'j}(\boldsymbol{\lambda})\right]^2}{\sigma_{3j}^2} + \frac{1}{\sigma_1^2},$$

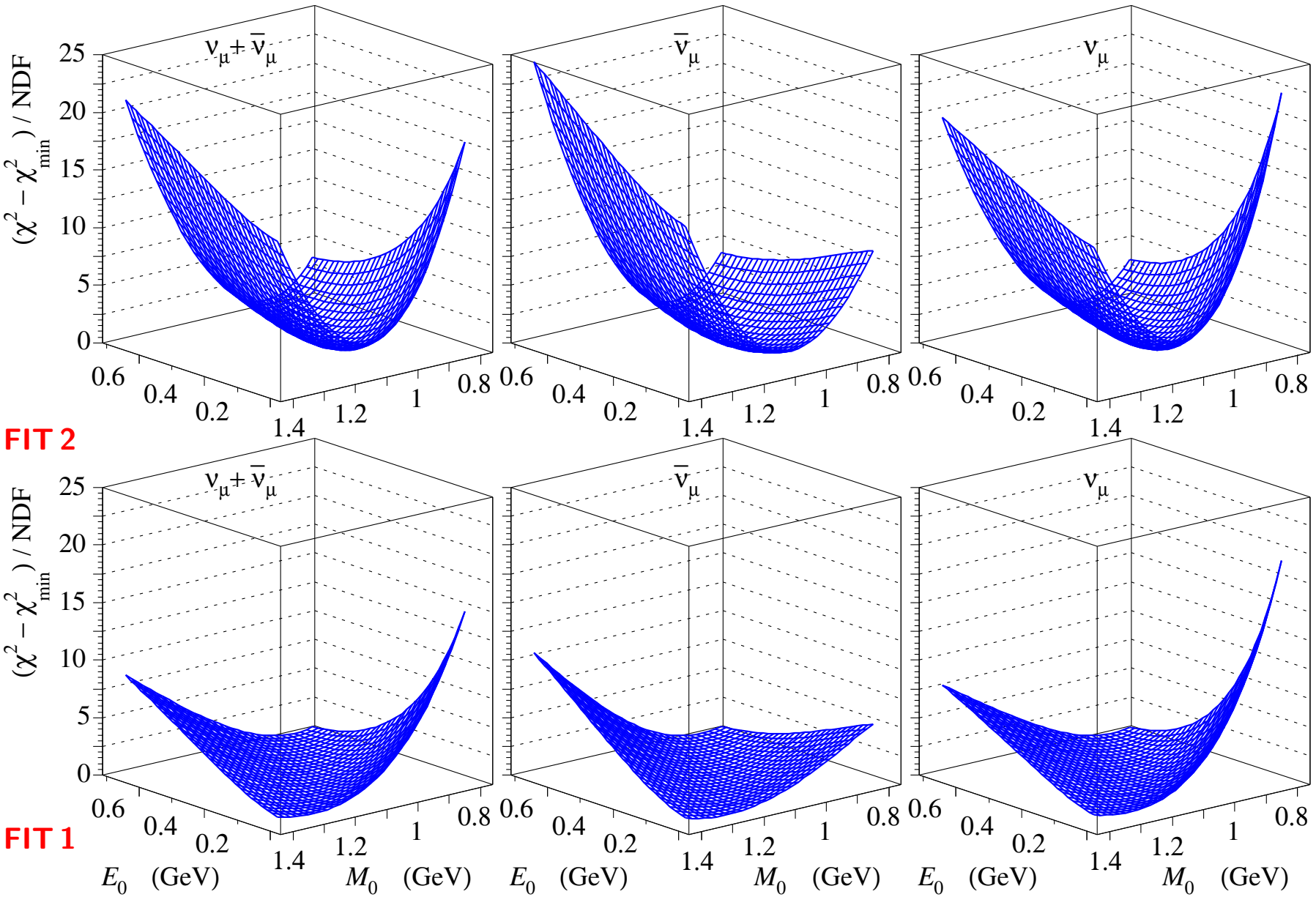
$$A_{22} = \sum_{j\in G_2} \frac{\left[T_{2j}(\boldsymbol{\lambda})\right]^2}{\sigma_{2j}^2} + \sum_{j\in G_3} \frac{\left[T_{2'j}(\boldsymbol{\lambda})\right]^2}{\sigma_{3j}^2} + \frac{1}{\sigma_2^2},$$

$$A_{12} = \sum_{j\in G_3} \frac{T_{1'j}(\boldsymbol{\lambda}) T_{2'j}(\boldsymbol{\lambda})}{\sigma_{3j}^2},$$

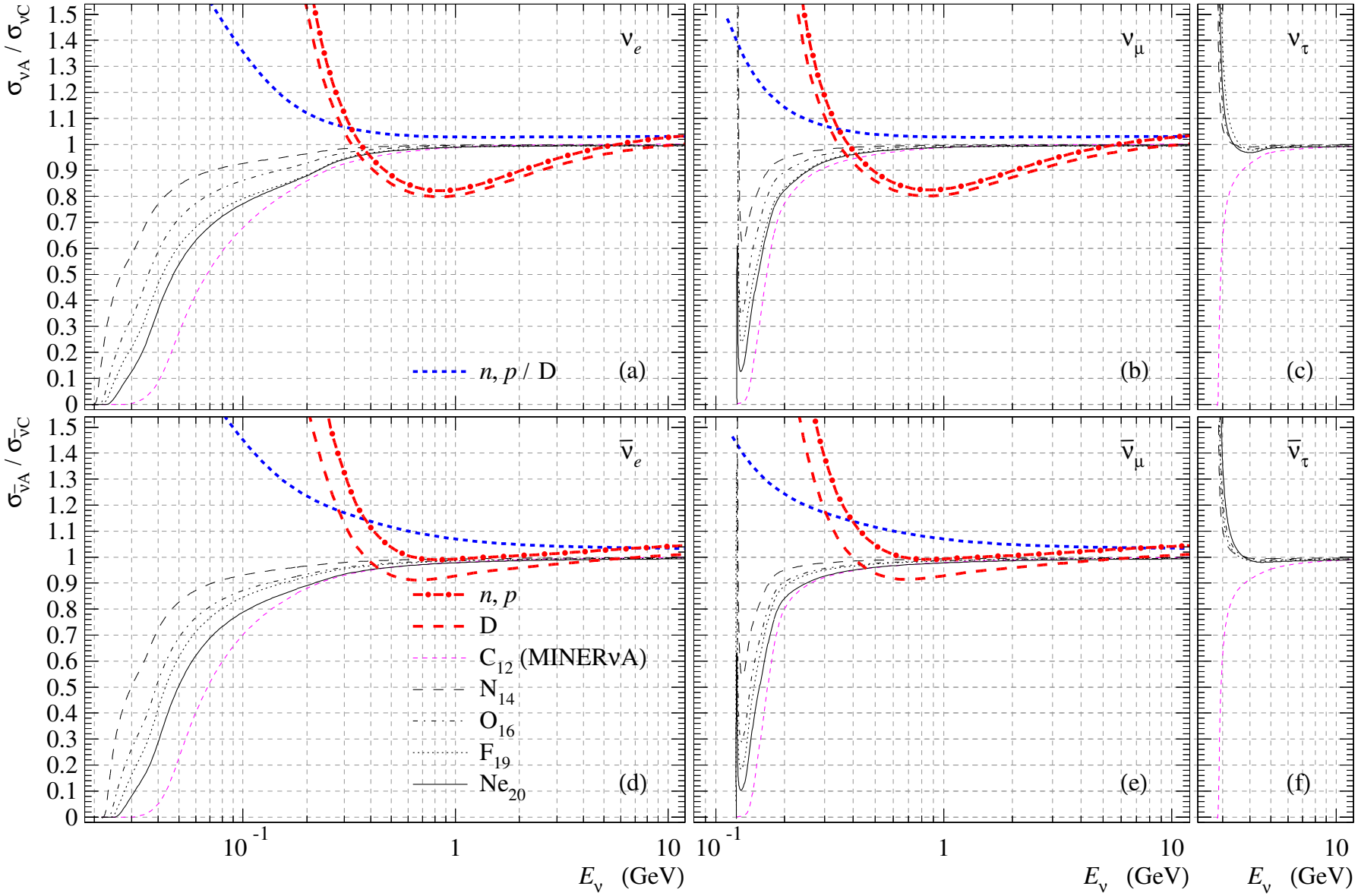
$$C_i = \sum_{j\in G_2} \frac{T_{ij}(\boldsymbol{\lambda}) E_{ij}(\boldsymbol{\lambda})}{\sigma_{ij}^2} + \sum_{j\in G_3} \frac{T_{i'j}(\boldsymbol{\lambda}) E_{3j}(\boldsymbol{\lambda})}{\sigma_{3j}^2} + \frac{1}{\sigma_2^2}.$$

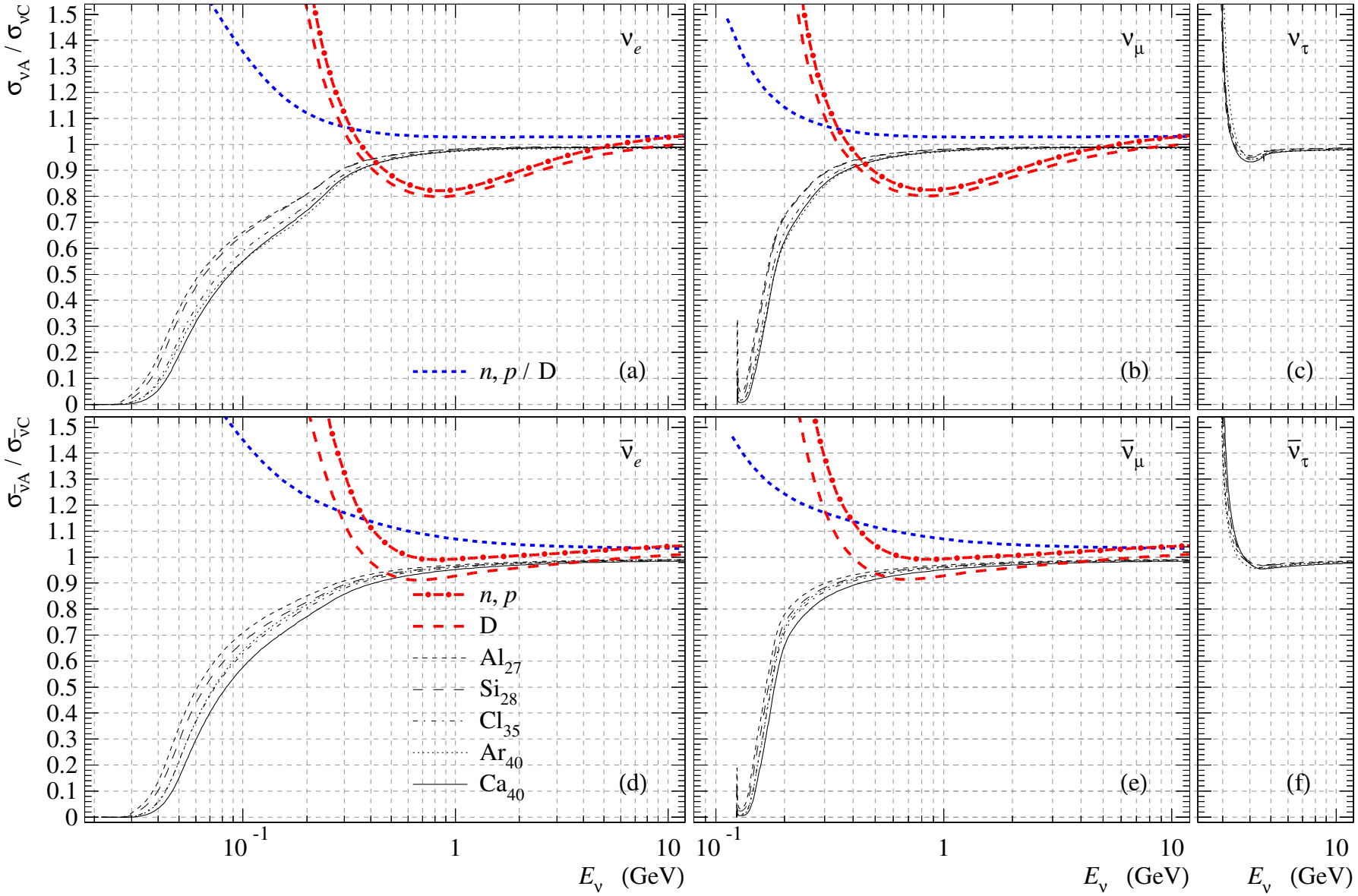


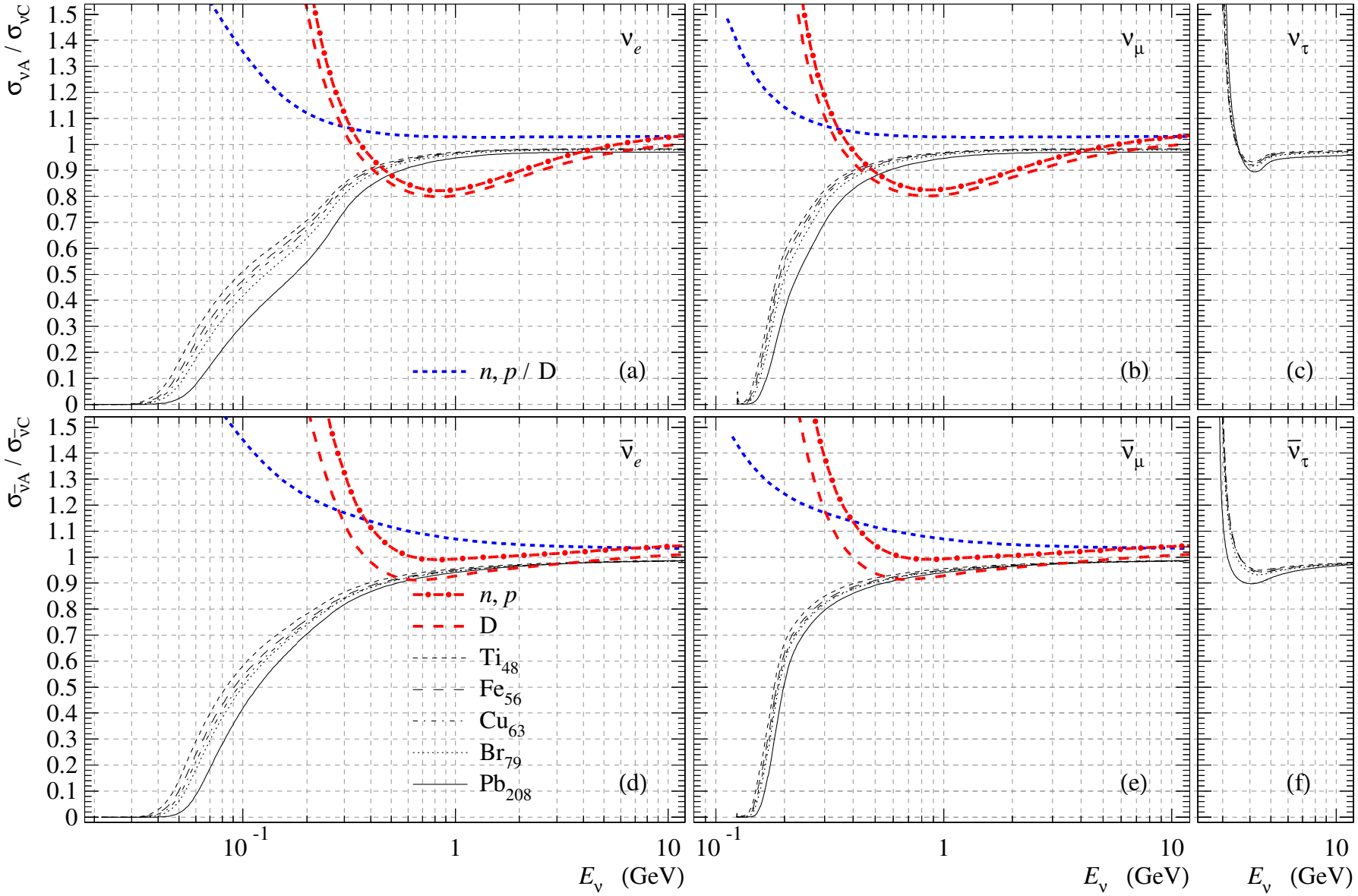

Error contours for the parameters $M_0$ and $E_0$ obtained from the global fits for $\nu$ (a), $\overline{\nu}$ (b), and $\nu + \overline{\nu}$ (c) CCQE data, performed *without* anchoring of $M_0$ (FIT 1, greater areas) and *with* anchoring of $M_0$ (FIT 2, smaller areas); see slide 74. The inner and outer contours for these fits indicate the $68\%$ and $95\%$ C.L. regions, respectively.
The shapes of $(\chi^2 - \chi^2_{\text{min}})/\text{NDF}$ for FIT 1 and FIT 2 are shown in the next slide. ↓


FIT 2
FIT 1
$\nu_\mu + \bar{\nu}_\mu$
$\bar{\nu}_\mu$
$\nu_\mu$
$(\chi^2 - \chi^2_{min})$ / NDF
$E_0$ (GeV)
$M_0$ (GeV)

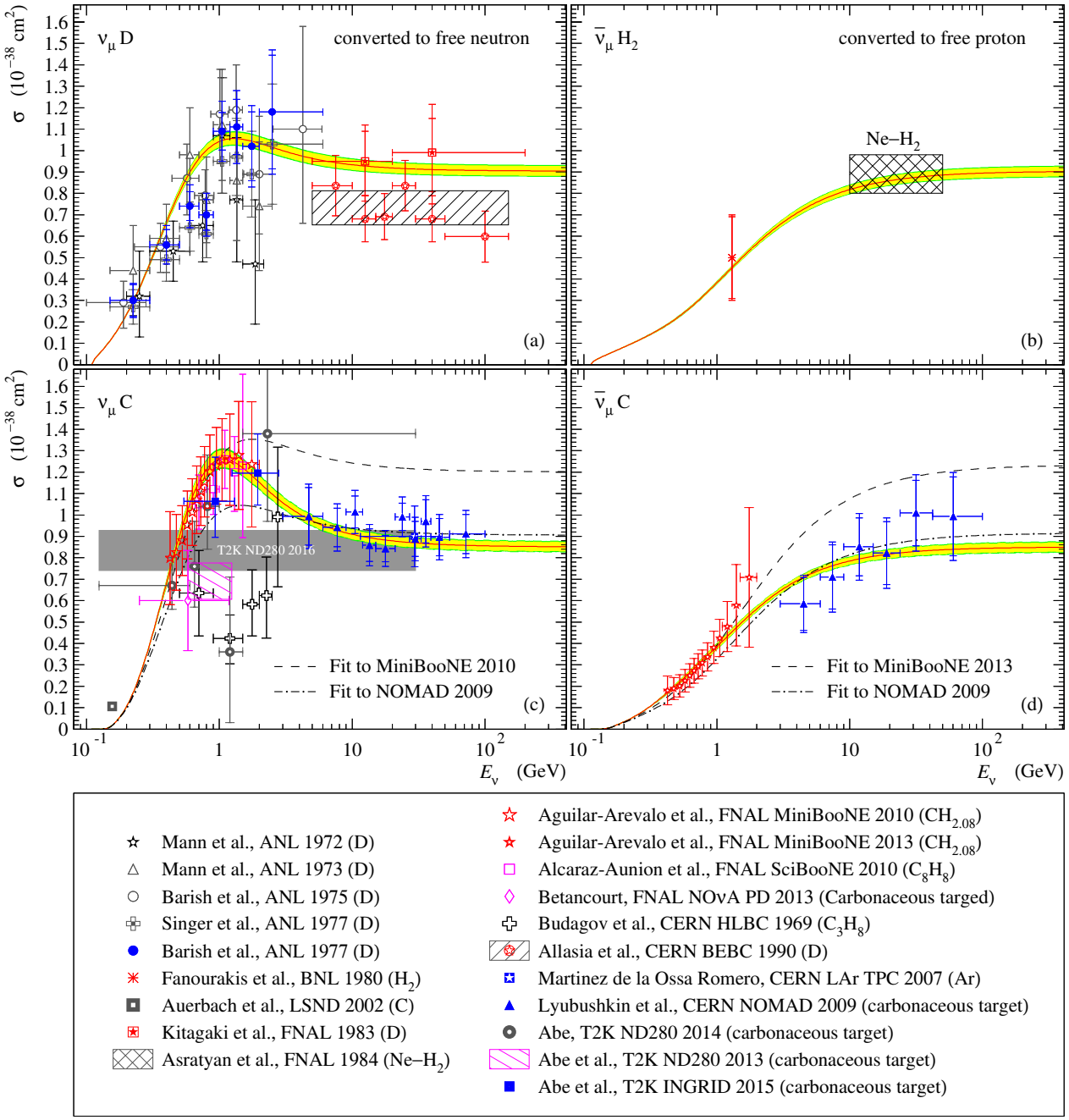
# Nuclear effects

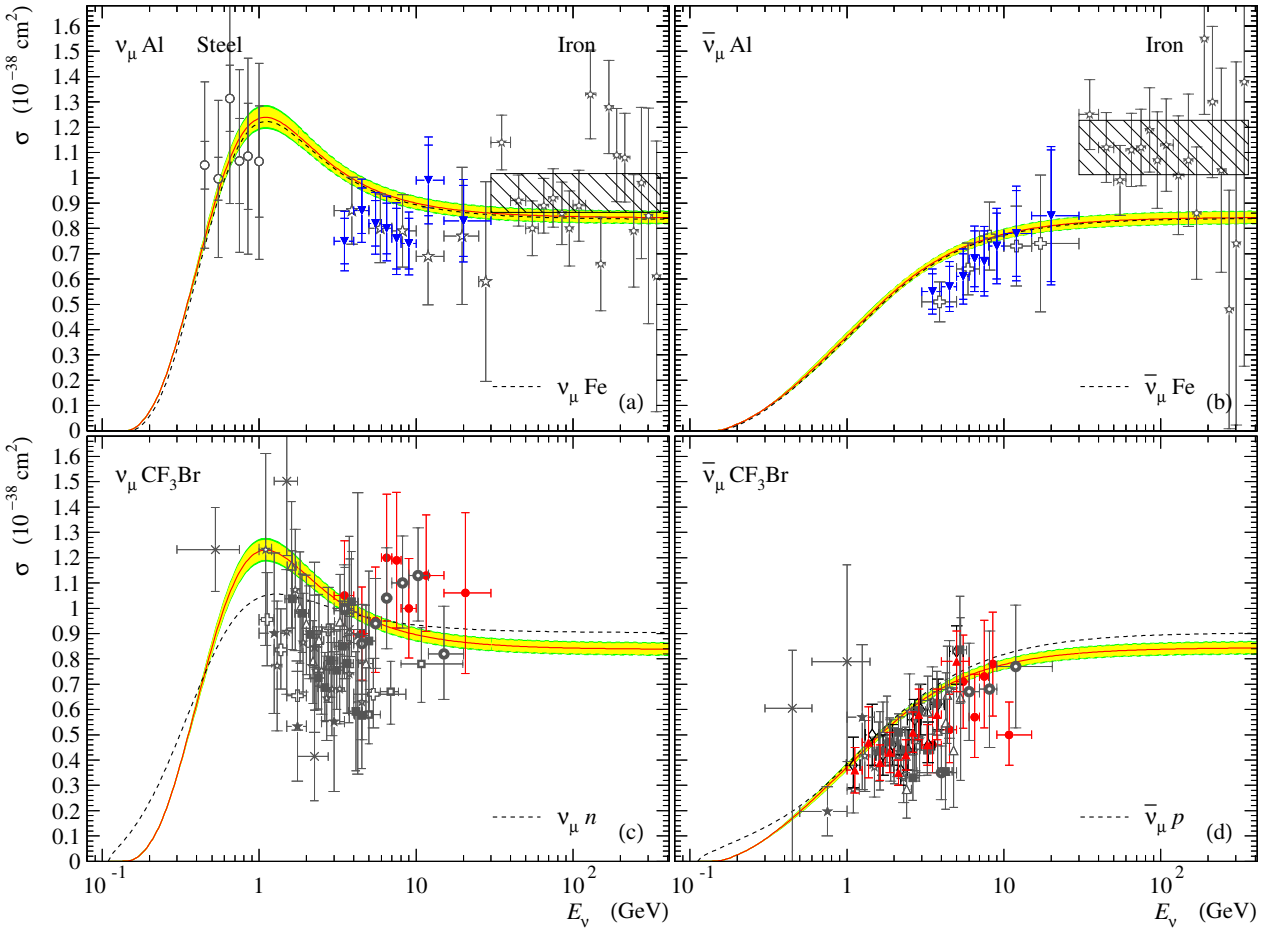

σνA / σνC
σν̄A / σν̄C
νe
νμ
ντ
ν̄e
ν̄μ
ν̄τ
n, p / D
n, p
D
C12 (MINERνA)
N14
O16
F19
Ne20
(a)
(b)
(c)
(d)
(e)
(f)
Eν (GeV)

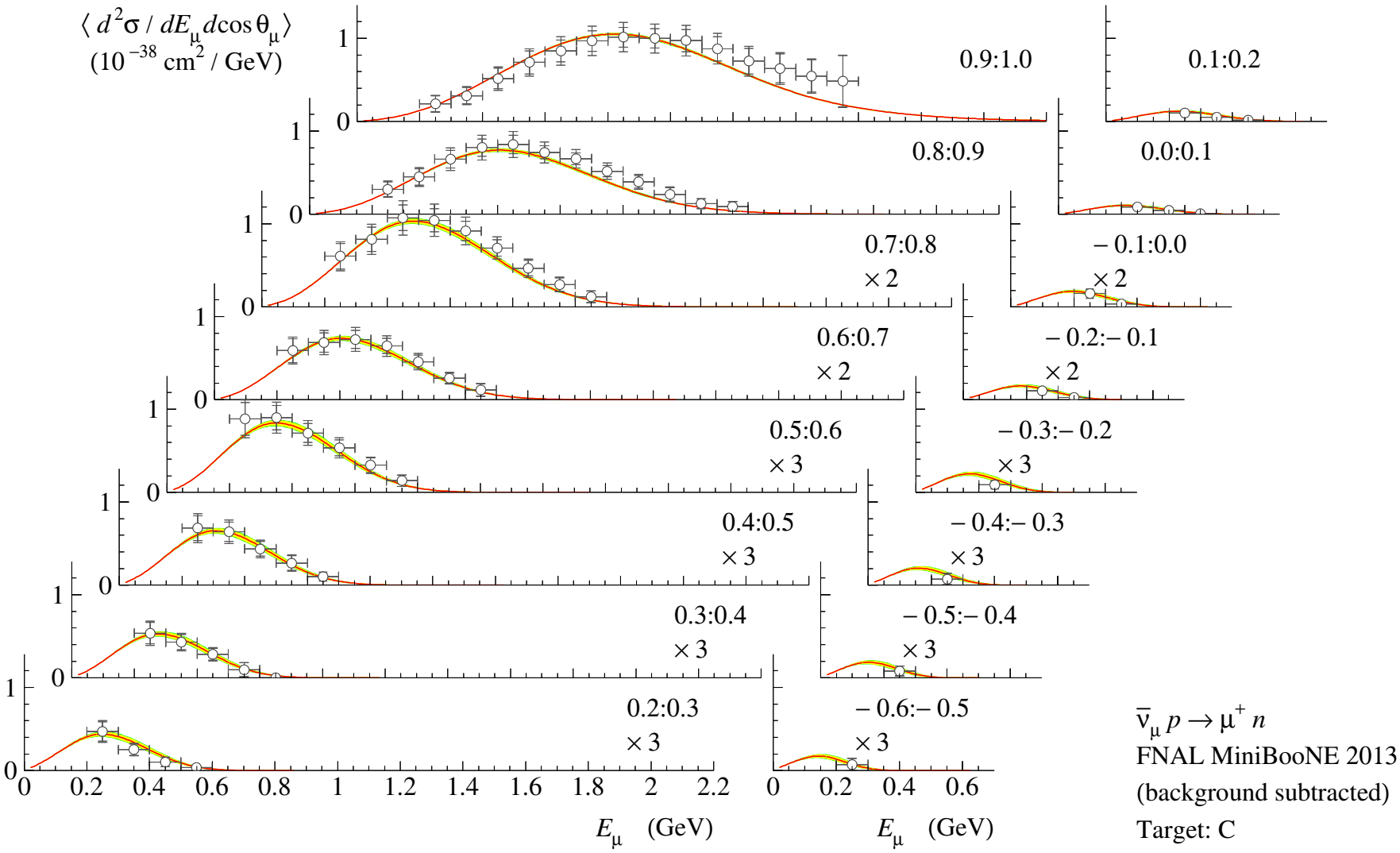

σνA / σνC
σν̄A / σν̄C
νe
νμ
ντ
ν̄e
ν̄μ
ν̄τ
n, p / D
n, p
D
Al27
Si28
Cl35
Ar40
Ca40
(a)
(b)
(c)
(d)
(e)
(f)
Eν (GeV)


σνA / σνC
σν̄A / σν̄C
νe
νμ
ντ
ν̄e
ν̄μ
ν̄τ
n, p / D
(a)
(b)
(c)
n, p
D
Ti48
Fe56
Cu63
Br79
Pb208
(d)
(e)
(f)
10 -1
1
10
Eν (GeV)

# Comparison with data
# (a few more examples)

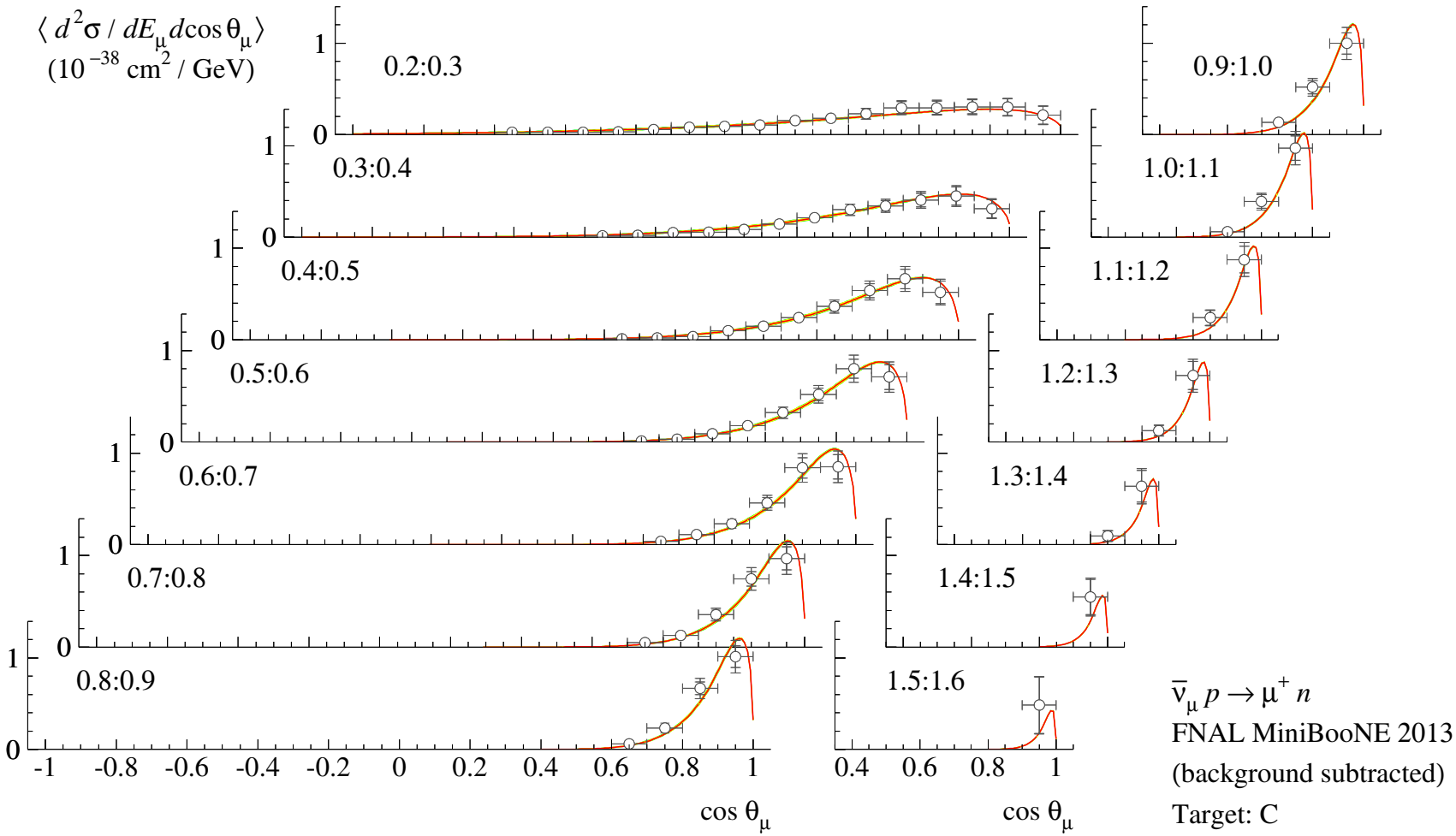

νμ D
converted to free neutron
(a)
ν̄μ H2
converted to free proton
Ne–H2
(b)
νμ C
T2K ND280 2016
Fit to MiniBooNE 2010
Fit to NOMAD 2009
(c)
ν̄μ C
Fit to MiniBooNE 2013
Fit to NOMAD 2009
(d)
σ (10^-38 cm^2)
Eν (GeV)
Mann et al., ANL 1972 (D)
Mann et al., ANL 1973 (D)
Barish et al., ANL 1975 (D)
Singer et al., ANL 1977 (D)
Barish et al., ANL 1977 (D)
Fanourakis et al., BNL 1980 (H2)
Auerbach et al., LSND 2002 (C)
Kitagaki et al., FNAL 1983 (D)
Asratyan et al., FNAL 1984 (Ne–H2)
Aguilar-Arevalo et al., FNAL MiniBooNE 2010 (CH2.08)
Aguilar-Arevalo et al., FNAL MiniBooNE 2013 (CH2.08)
Alcaraz-Aunion et al., FNAL SciBooNE 2010 (C8H8)
Betancourt, FNAL NOvA PD 2013 (Carbonaceous targed)
Budagov et al., CERN HLBC 1969 (C3H8)
Allasia et al., CERN BEBC 1990 (D)
Martinez de la Ossa Romero, CERN LAr TPC 2007 (Ar)
Lyubushkin et al., CERN NOMAD 2009 (carbonaceous target)
Abe, T2K ND280 2014 (carbonaceous target)
Abe et al., T2K ND280 2013 (carbonaceous target)
Abe et al., T2K INGRID 2015 (carbonaceous target)


σ (10^-38 cm^2)
νμ Al
Steel
Iron
νμ Fe
(a)
ν̄μ Al
Iron
ν̄μ Fe
(b)
νμ CF3Br
νμ n
(c)
ν̄μ CF3Br
ν̄μ p
(d)
Eν (GeV)
Kustom et al., ANL 1969 (Steel)
Suwonjandee, FNAL NuTeV 2004 (Fe)
Franzinetti et al., CERN HLBC 1966 (CF3Br)
Young, CERN HLBC 1967 (CF3Br)
Eichten et al., CERN GGM 1973 (CF3Br)
Rollier et al., CERN GGM 1975 (CF3Br)
Bonetti et al., CERN GGM 1977 (CF3Br)
Rollier et al., CERN GGM 1978 (C3H8–CF3Br)
Armenise et al., CERN GGM 1979 (C3H8–CF3Br)
Pohl et al., CERN GGM 1979 (C3H8–CF3Br)
Makeev et al., IHEP SKAT 1981 (CF3Br)*
Grabosch et al., IHEP SKAT 1988 (CF3Br)
Brunner et al., IHEP SKAT 1990 (CF3Br)*
Belikov et al., IHEP–ITEP 1981 (Al)
Belikov et al., IHEP–ITEP 1982 (Al)
Belikov et al., IHEP–ITEP 1985 (Al)
* converted to free nucleons

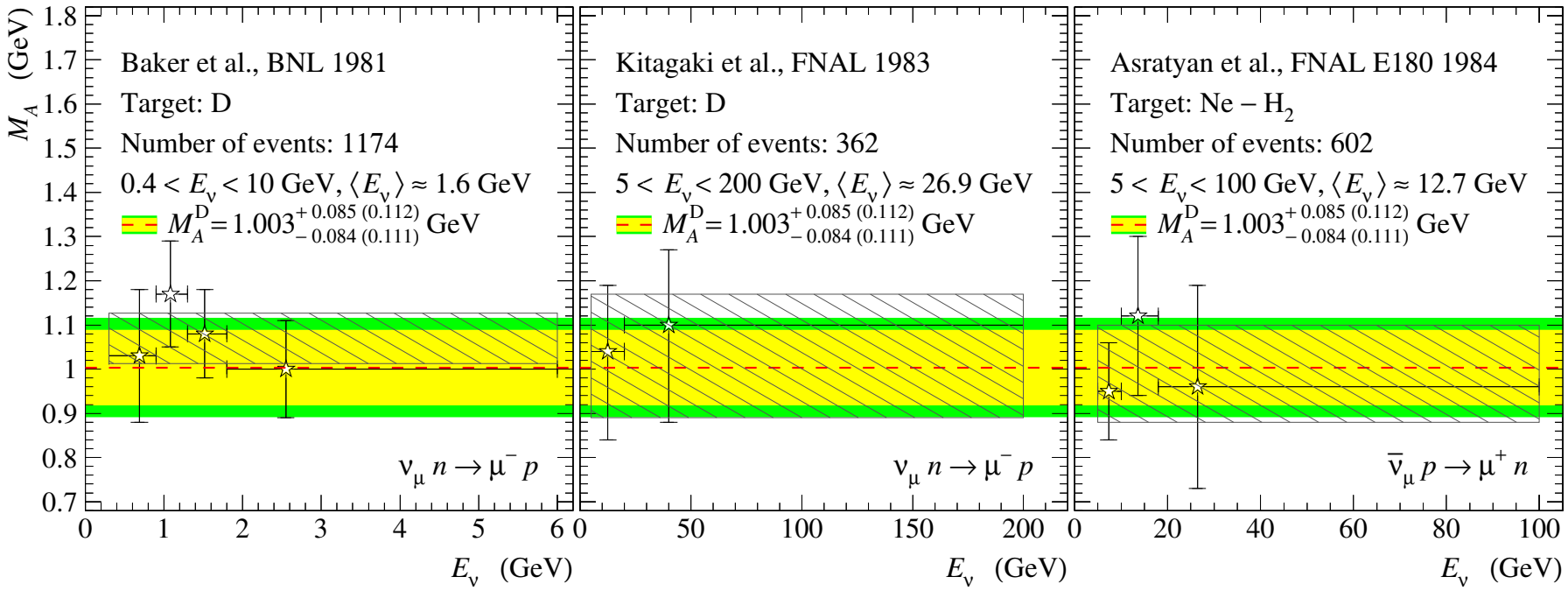



$\bar{\nu}_\mu\, p \to \mu^+\, n$
FNAL MiniBooNE 2013
(background subtracted)
Target: C

See note in slide 19.




See note in slide 19.

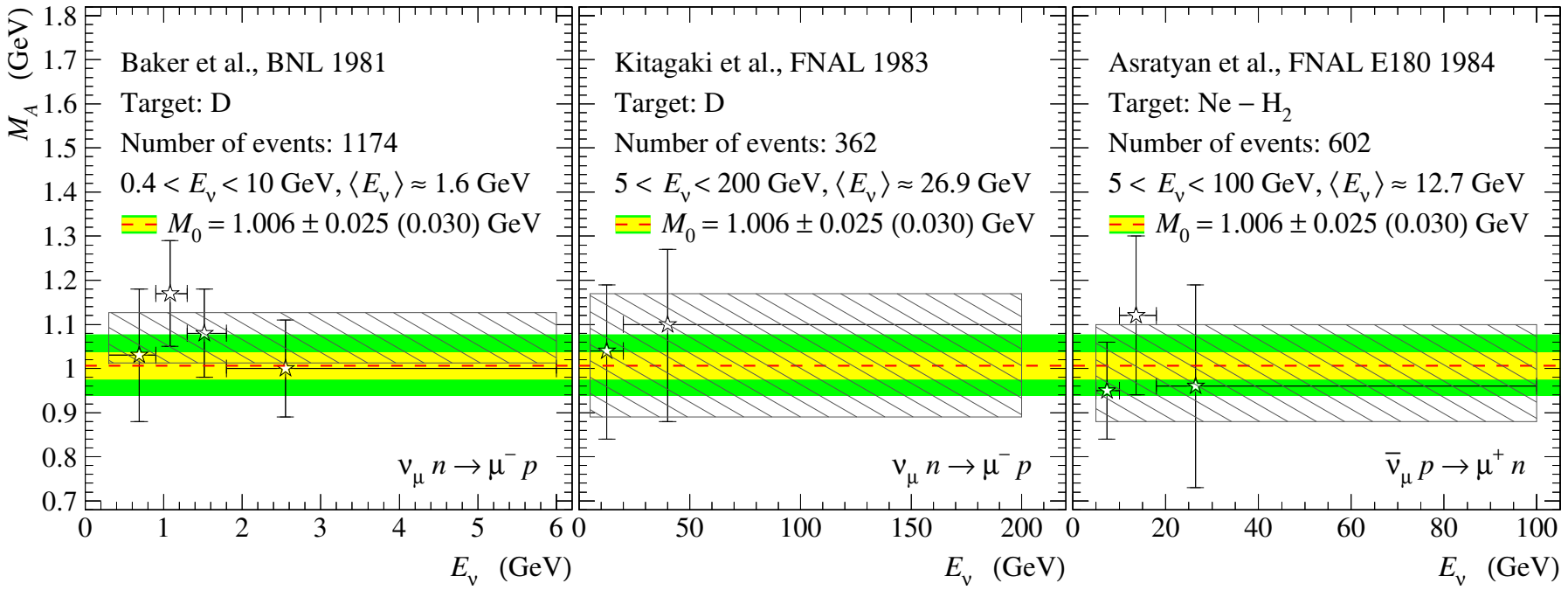



Current axial mass, $M_A$ vs. neutrino energy measured for the $\nu_\mu n \rightarrow \mu^- p$ reaction with the deuterium filled detector at BNL [11]. Rectangle denotes the average value of $M_A$ obtained by the authors of the experiment for the mean neutrino energy $\langle E_\nu \rangle = 1.48$ GeV.

Current axial mass, $M_A$ vs. neutrino energy measured for the $\nu_\mu n \rightarrow \mu^- p$ reaction with the deuterium filled detector at BNL [16]. Rectangle denotes the average value of $M_A$ obtained by the authors of the experiment for the whole neutrino energy range at the mean energy of $\langle E_\nu \rangle = 26.9$ GeV.

Current axial mass, $M_A$ vs. antineutrino energy measured for the $\overline{\nu}_\mu p \rightarrow \mu^+ n$ reaction with the Ne-$H_2$ filled detector in the FNAL experiment E180 [17, 18]. Dashed rectangle denotes the average value of $M_A$ obtained by the authors of the experiment for the whole energy range at the mean energy of $\langle E_\nu \rangle = 15.4$ GeV.

Dashed line and shaded bands in each panel represent the best-fit value of $M_A^D$ and corresponding $1\sigma$ and $2\sigma$ uncertainties, obtained in our global fit to the deuterium and hydrogen CCQE data.

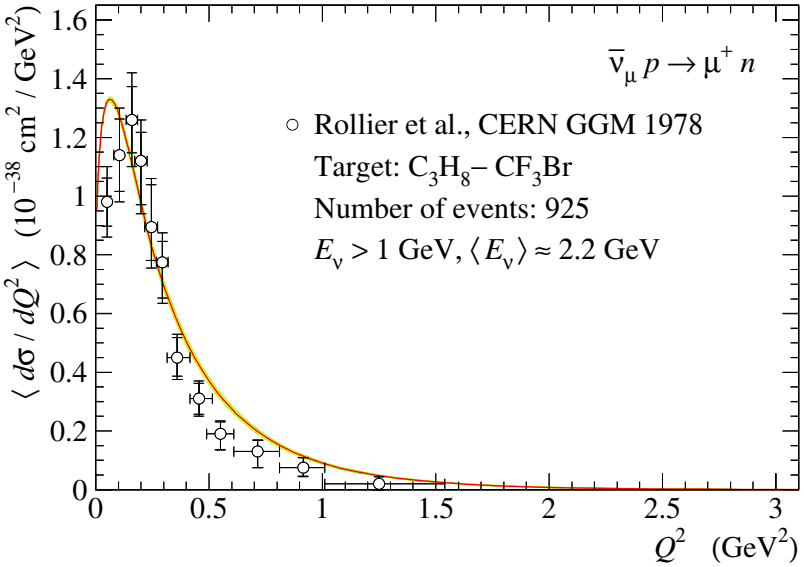


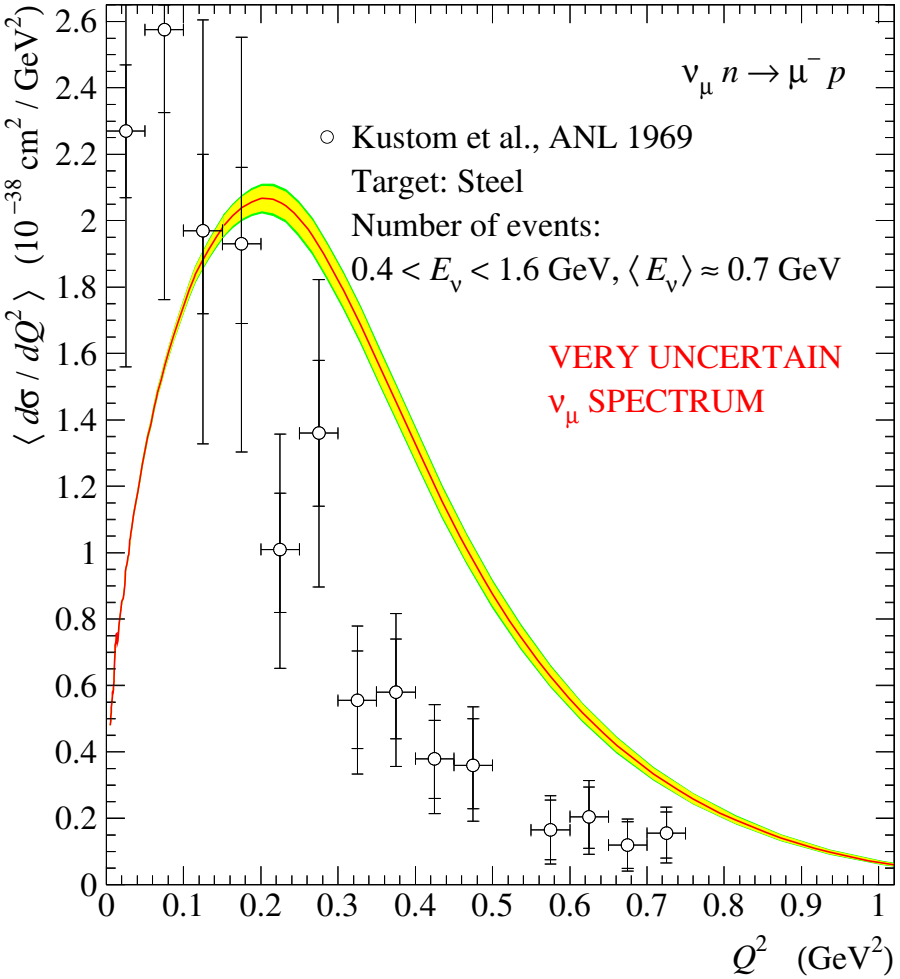

The same as in the previous slide except the lines and bands which now represent the best-fit value of the parameter $M_0$ (which can be identified with the current axial mass of the nucleon, $M_A$) and corresponding $1\sigma$ and $2\sigma$ uncertainties, obtained in our global fit to all CCQE data described in slide 69.

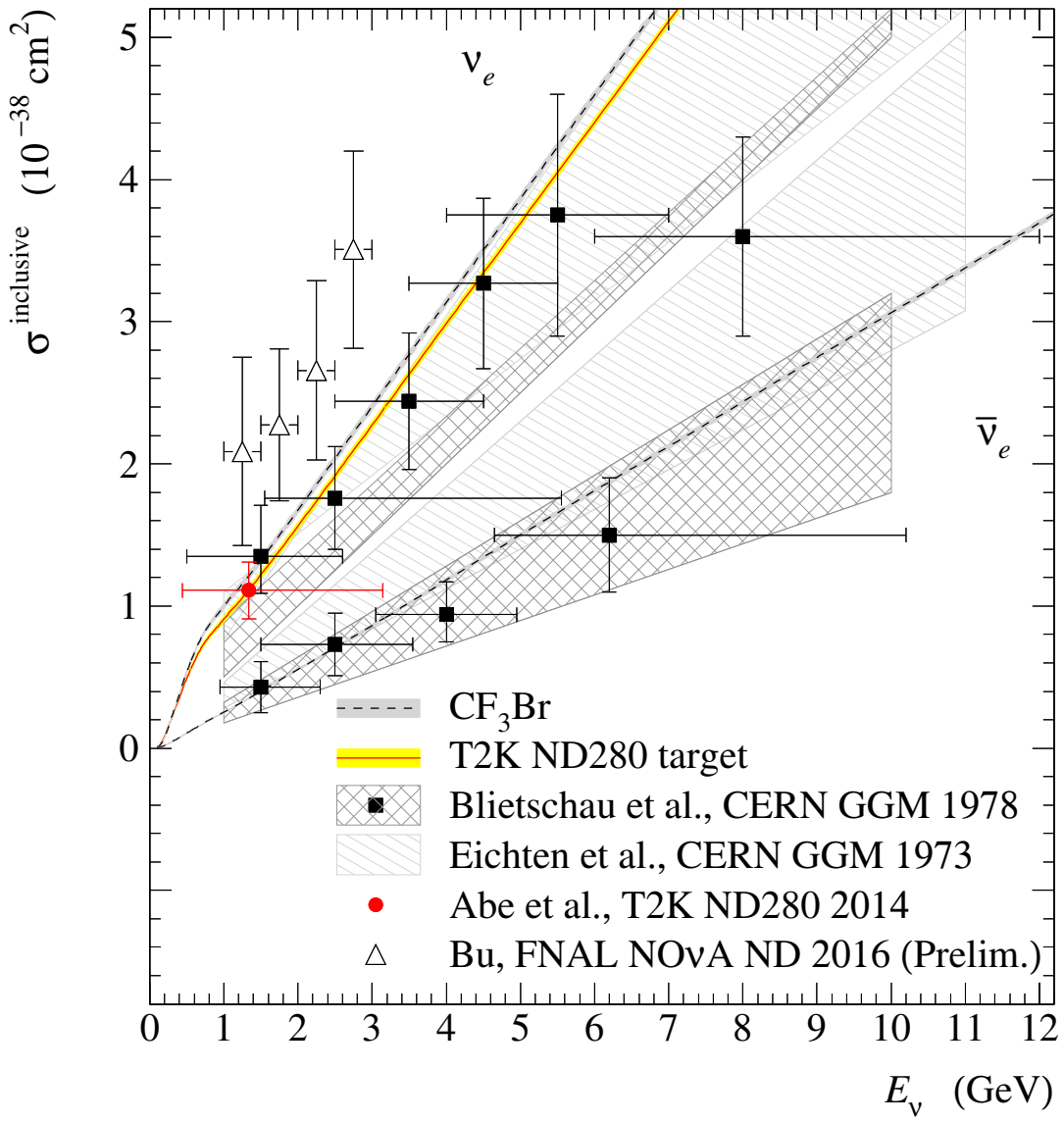
In all our fits, the nuclear corrections for the $\nu - d$ CCQE scattering were accounted for within the impulse approximation and by using the closure over the final dinucleon states and meson exchange current correction at low $Q^2$; the Reid hard core potential and Hulthen wave function for the deuteron were applied.[a]

[a] See S.K. Singh and H. Arenhovel, *"Pion exchange current effects in* $\nu_\mu + d \to \mu^- + p + p$*,"* Z. Phys. A **324**, 347 (1986) and references therein.

# A fly in the ointment (examples of doubtful agreement)

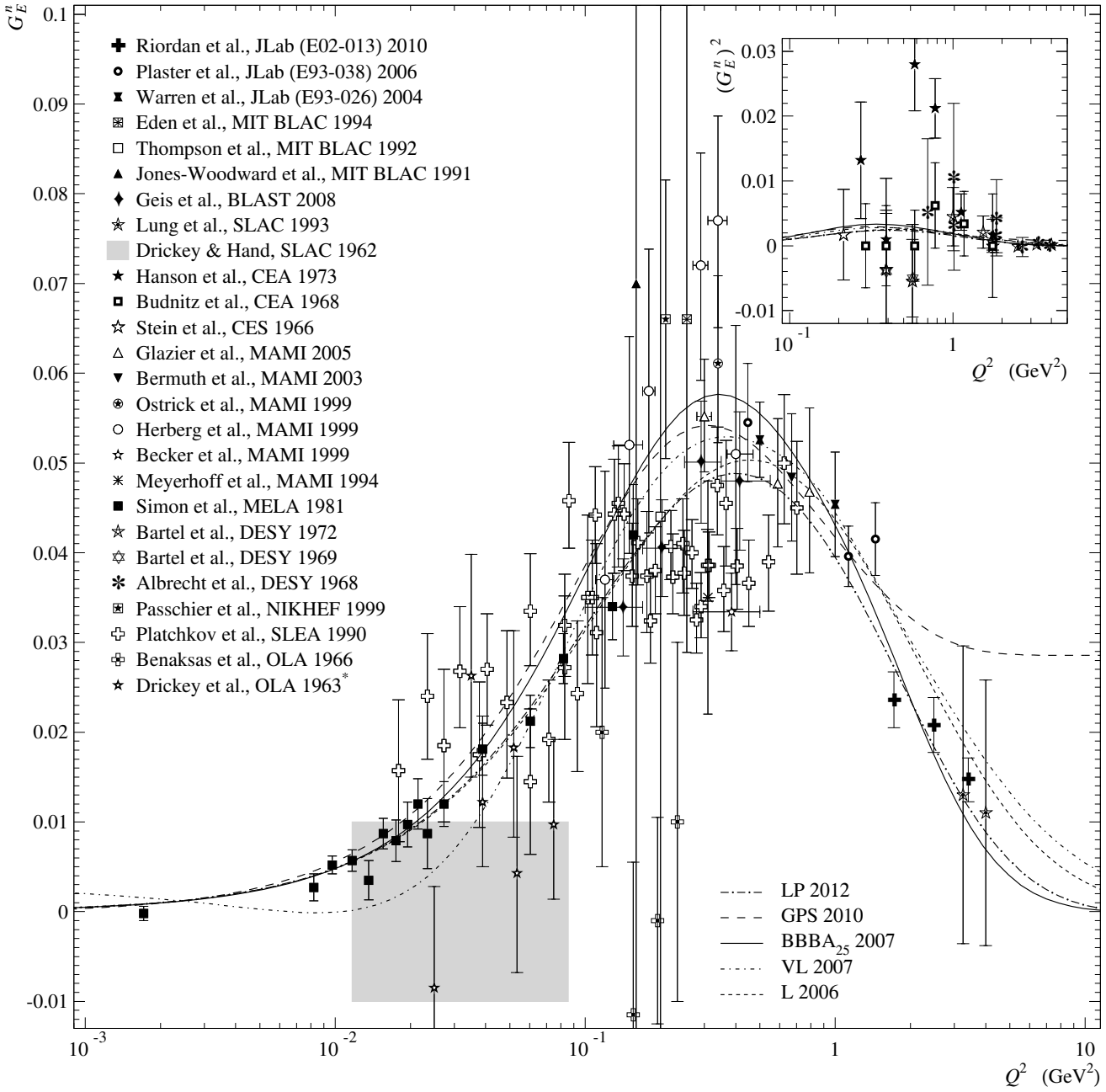


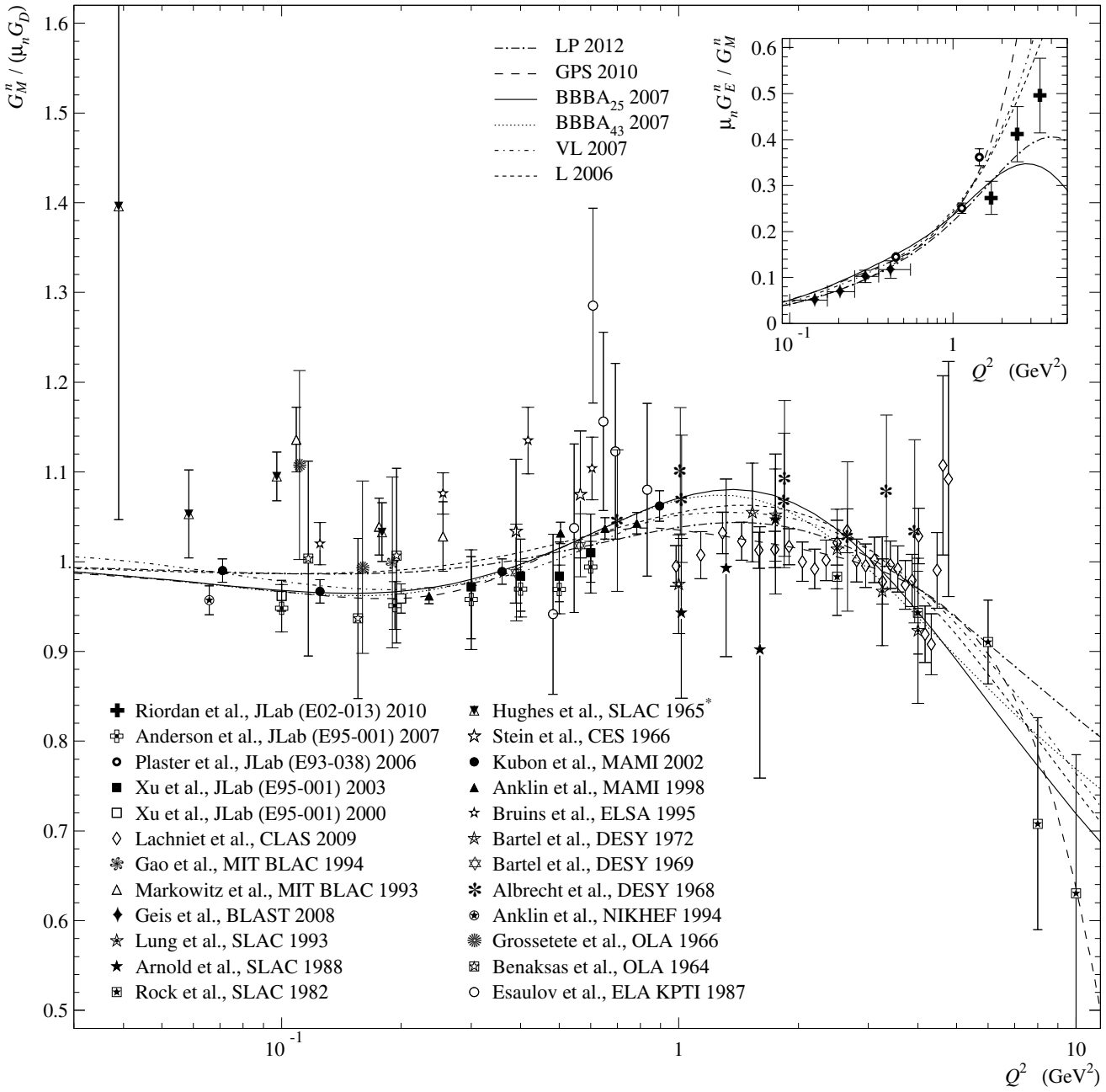



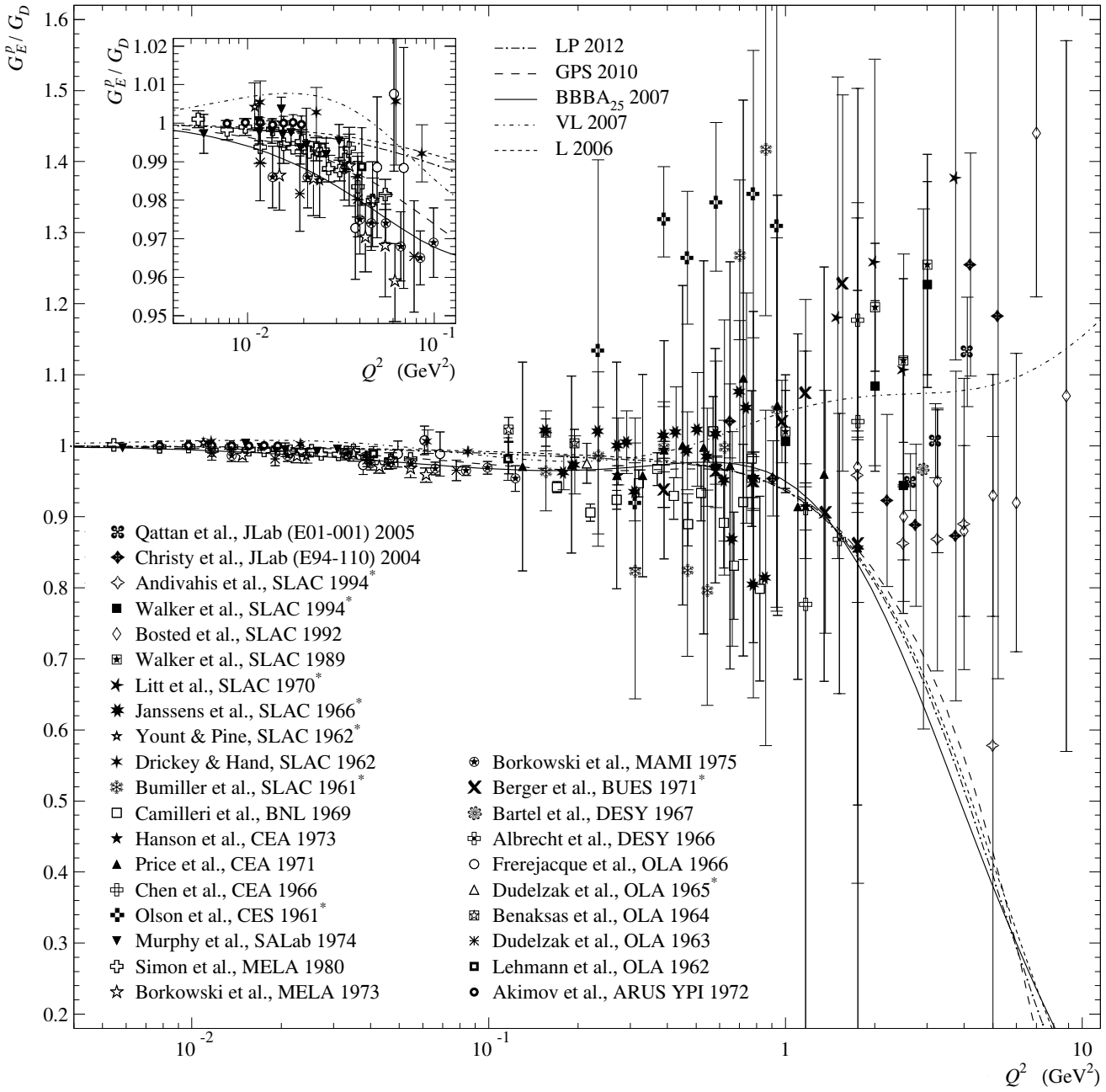


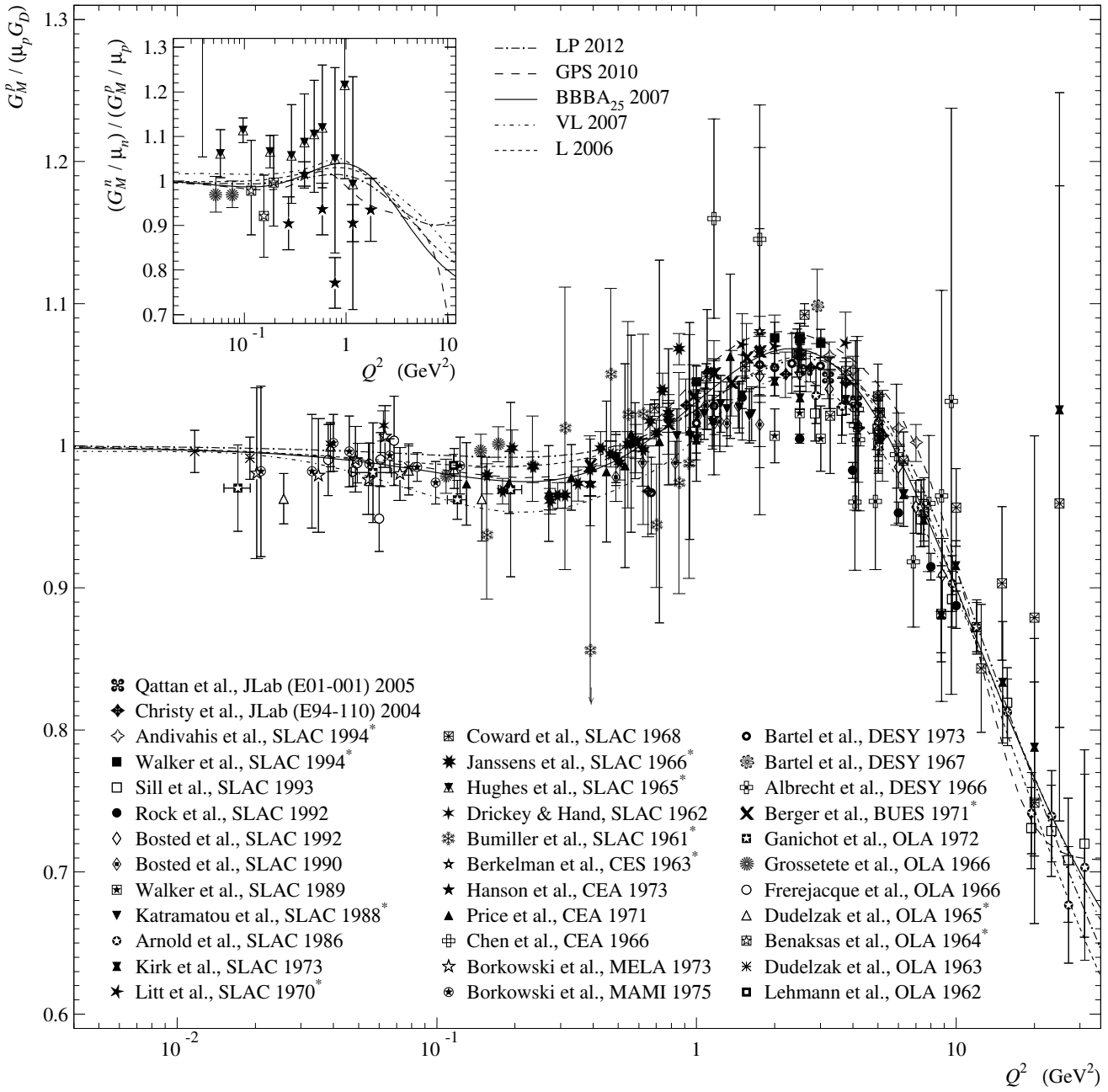

Total inclusive CC cross sections for $\nu_e$ and $\overline{\nu}_e$ at low energies with a bit surprising (*preliminary*) data from NO$\nu$A Near Detector (X. Bu, arXiv:1601.01213 [hep-ex]).

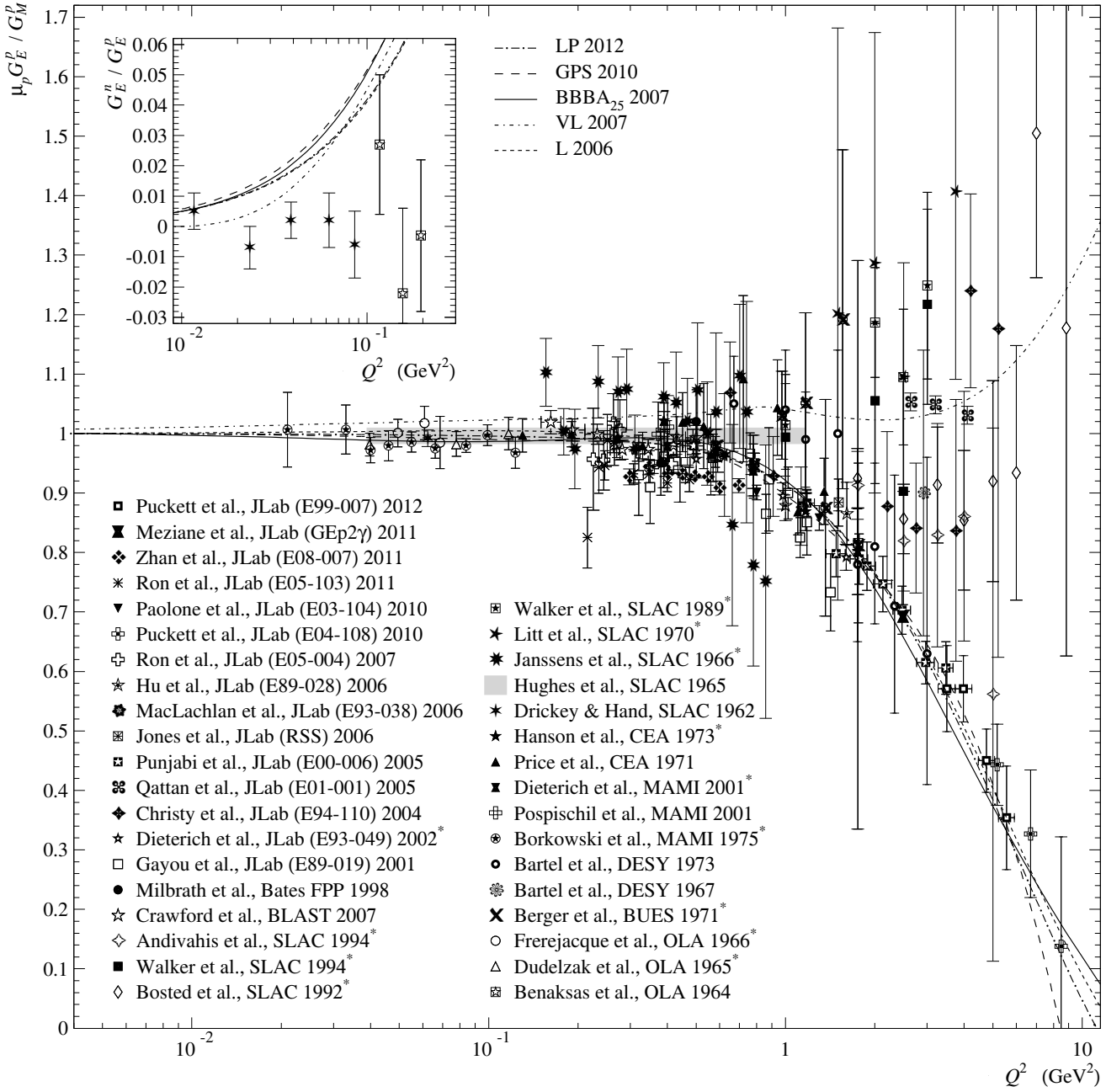
# Comparison of several models for the nucleon electromagnetic form factors with data


$G_E^n$
Riordan et al., JLab (E02-013) 2010
Plaster et al., JLab (E93-038) 2006
Warren et al., JLab (E93-026) 2004
Eden et al., MIT BLAC 1994
Thompson et al., MIT BLAC 1992
Jones-Woodward et al., MIT BLAC 1991
Geis et al., BLAST 2008
Lung et al., SLAC 1993
Drickey & Hand, SLAC 1962
Hanson et al., CEA 1973
Budnitz et al., CEA 1968
Stein et al., CES 1966
Glazier et al., MAMI 2005
Bermuth et al., MAMI 2003
Ostrick et al., MAMI 1999
Herberg et al., MAMI 1999
Becker et al., MAMI 1999
Meyerhoff et al., MAMI 1994
Simon et al., MELA 1981
Bartel et al., DESY 1972
Bartel et al., DESY 1969
Albrecht et al., DESY 1968
Passchier et al., NIKHEF 1999
Platchkov et al., SLEA 1990
Benaksas et al., OLA 1966
Drickey et al., OLA 1963*
LP 2012
GPS 2010
BBBA$_{25}$ 2007
VL 2007
L 2006
$Q^2$ (GeV$^2$)
$(G_E^n)^2$
$Q^2$ (GeV$^2$)

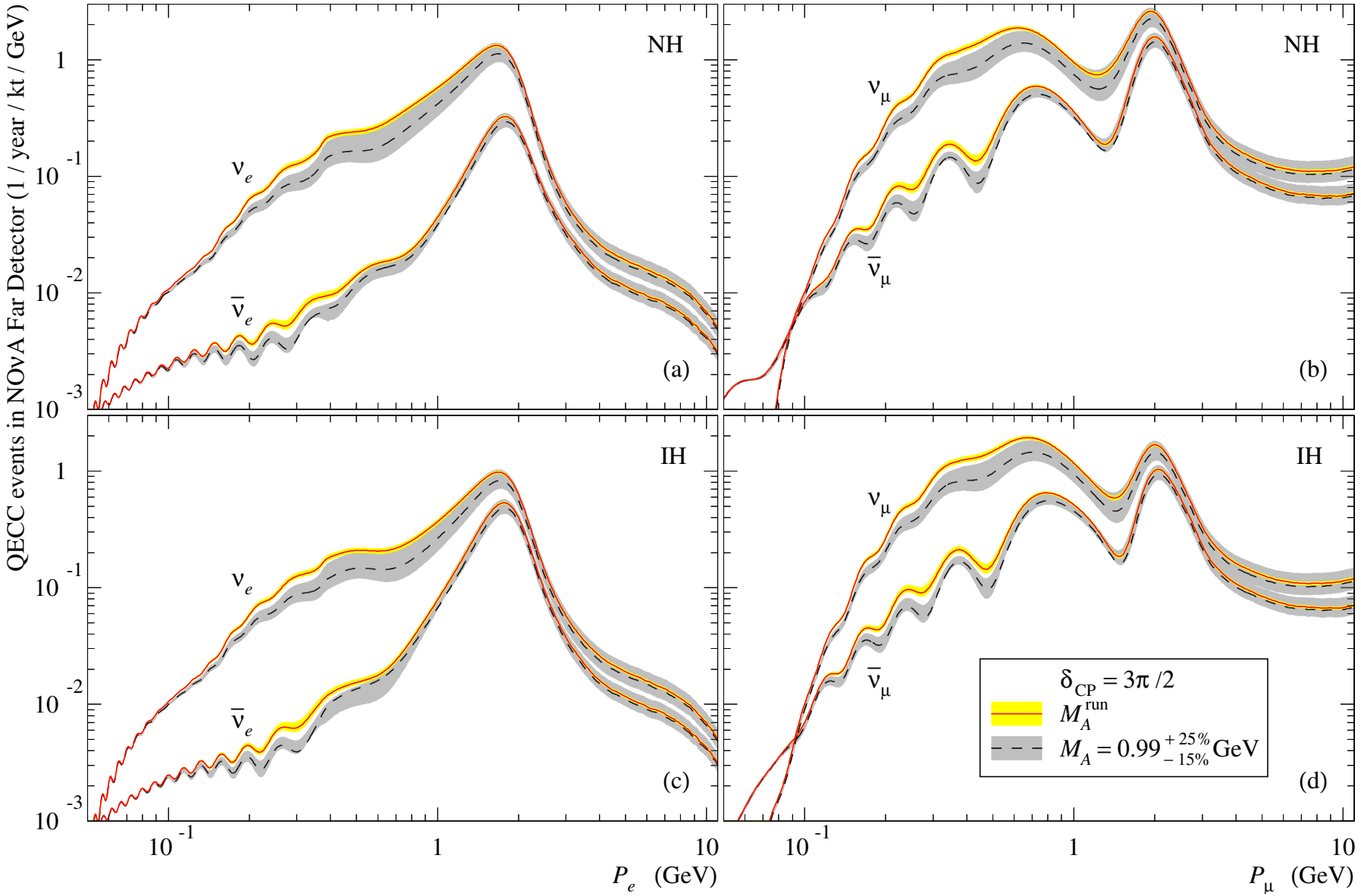

$G_M^n / (\mu_n G_D)$
LP 2012
GPS 2010
BBBA$_{25}$ 2007
BBBA$_{43}$ 2007
VL 2007
L 2006
$\mu_n G_E^n / G_M^n$
$Q^2$ (GeV$^2$)
Riordan et al., JLab (E02-013) 2010
Anderson et al., JLab (E95-001) 2007
Plaster et al., JLab (E93-038) 2006
Xu et al., JLab (E95-001) 2003
Xu et al., JLab (E95-001) 2000
Lachniet et al., CLAS 2009
Gao et al., MIT BLAC 1994
Markowitz et al., MIT BLAC 1993
Geis et al., BLAST 2008
Lung et al., SLAC 1993
Arnold et al., SLAC 1988
Rock et al., SLAC 1982
Hughes et al., SLAC 1965*
Stein et al., CES 1966
Kubon et al., MAMI 2002
Anklin et al., MAMI 1998
Bruins et al., ELSA 1995
Bartel et al., DESY 1972
Bartel et al., DESY 1969
Albrecht et al., DESY 1968
Anklin et al., NIKHEF 1994
Grossetete et al., OLA 1966
Benaksas et al., OLA 1964
Esaulov et al., ELA KPTI 1987


$G_E^p / G_D$
$Q^2$ (GeV$^2$)
LP 2012
GPS 2010
BBBA$_{25}$ 2007
VL 2007
L 2006
Qattan et al., JLab (E01-001) 2005
Christy et al., JLab (E94-110) 2004
Andivahis et al., SLAC 1994*
Walker et al., SLAC 1994*
Bosted et al., SLAC 1992
Walker et al., SLAC 1989
Litt et al., SLAC 1970*
Janssens et al., SLAC 1966*
Yount & Pine, SLAC 1962*
Drickey & Hand, SLAC 1962
Bumiller et al., SLAC 1961*
Camilleri et al., BNL 1969
Hanson et al., CEA 1973
Price et al., CEA 1971
Chen et al., CEA 1966
Olson et al., CES 1961*
Murphy et al., SALab 1974
Simon et al., MELA 1980
Borkowski et al., MELA 1973
Borkowski et al., MAMI 1975
Berger et al., BUES 1971*
Bartel et al., DESY 1967
Albrecht et al., DESY 1966
Frerejacque et al., OLA 1966
Dudelzak et al., OLA 1965*
Benaksas et al., OLA 1964
Dudelzak et al., OLA 1963
Lehmann et al., OLA 1962
Akimov et al., ARUS YPI 1972

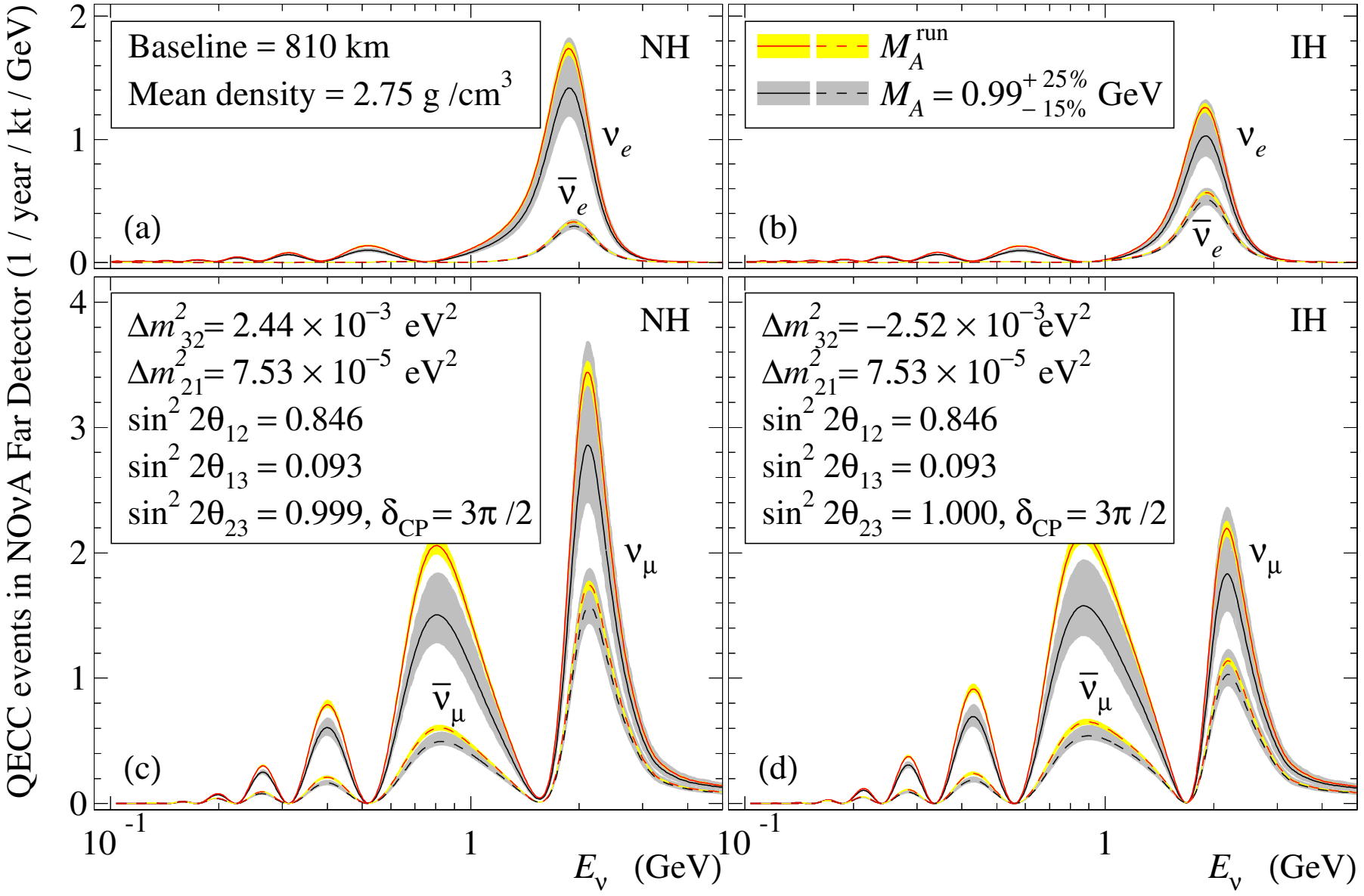

$G_M^p / (\mu_p G_D)$
$Q^2$ (GeV$^2$)
$(G_M^n / \mu_n) / (G_M^p / \mu_p)$
LP 2012
GPS 2010
BBBA$_{25}$ 2007
VL 2007
L 2006
Qattan et al., JLab (E01-001) 2005
Christy et al., JLab (E94-110) 2004
Andivahis et al., SLAC 1994*
Walker et al., SLAC 1994*
Sill et al., SLAC 1993
Rock et al., SLAC 1992
Bosted et al., SLAC 1992
Bosted et al., SLAC 1990
Walker et al., SLAC 1989
Katramatou et al., SLAC 1988*
Arnold et al., SLAC 1986
Kirk et al., SLAC 1973
Litt et al., SLAC 1970*
Coward et al., SLAC 1968
Janssens et al., SLAC 1966*
Hughes et al., SLAC 1965*
Drickey & Hand, SLAC 1962
Bumiller et al., SLAC 1961*
Berkelman et al., CES 1963*
Hanson et al., CEA 1973
Price et al., CEA 1971
Chen et al., CEA 1966
Borkowski et al., MELA 1973
Borkowski et al., MAMI 1975
Bartel et al., DESY 1973
Bartel et al., DESY 1967
Albrecht et al., DESY 1966
Berger et al., BUES 1971*
Ganichot et al., OLA 1972
Grossetete et al., OLA 1966
Frerejacque et al., OLA 1966
Dudelzak et al., OLA 1965*
Benaksas et al., OLA 1964*
Dudelzak et al., OLA 1963
Lehmann et al., OLA 1962


$\mu_p G_E^p / G_M^p$
$G_E^n / G_E^p$
$Q^2$ (GeV$^2$)
LP 2012
GPS 2010
BBBA$_{25}$ 2007
VL 2007
L 2006
Puckett et al., JLab (E99-007) 2012
Meziane et al., JLab (GEp2γ) 2011
Zhan et al., JLab (E08-007) 2011
Ron et al., JLab (E05-103) 2011
Paolone et al., JLab (E03-104) 2010
Puckett et al., JLab (E04-108) 2010
Ron et al., JLab (E05-004) 2007
Hu et al., JLab (E89-028) 2006
MacLachlan et al., JLab (E93-038) 2006
Jones et al., JLab (RSS) 2006
Punjabi et al., JLab (E00-006) 2005
Qattan et al., JLab (E01-001) 2005
Christy et al., JLab (E94-110) 2004
Dieterich et al., JLab (E93-049) 2002*
Gayou et al., JLab (E89-019) 2001
Milbrath et al., Bates FPP 1998
Crawford et al., BLAST 2007
Andivahis et al., SLAC 1994*
Walker et al., SLAC 1994*
Bosted et al., SLAC 1992*
Walker et al., SLAC 1989*
Litt et al., SLAC 1970*
Janssens et al., SLAC 1966*
Hughes et al., SLAC 1965
Drickey & Hand, SLAC 1962
Hanson et al., CEA 1973*
Price et al., CEA 1971
Dieterich et al., MAMI 2001*
Pospischil et al., MAMI 2001
Borkowski et al., MAMI 1975*
Bartel et al., DESY 1973
Bartel et al., DESY 1967
Berger et al., BUES 1971*
Frerejacque et al., OLA 1966*
Dudelzak et al., OLA 1965*
Benaksas et al., OLA 1964

# Some applications to NO$\nu$A experiment




An application to NO$\nu$A experiment (for detail, see Ref. [72]).




An application to NO$\nu$A experiment (for detail, see Ref. [71]).

# References


### ANL EXPERIMENTS

[1] W. A. Mann *et al.*, in: Proceedings of the 16th International Conference on High Energy Physics, National Accelerator Laboratory, Chicago-Batavia, Illinois, September 6–13, 1972, edited by J. D. Jackson and A. Roberts (National Accelerator Laboratory, Batavia, Illinois, 1973), paper #784.
[2] W. A. Mann *et al.*, Phys. Rev. Lett. **31**, 844 (1973).
[3] D. H. Perkins, in: Proceedings of the 1975 International Symposium on Lepton and Photon Interactions at High Energies, Stanford University, August 21–27, 1975, edited by T. W. Kirk (Stanford Linear Accelerator Center, Stanford, 1975), p. 571.
[4] S. J. Barish *et al.* (ANL–Purdue Collaboration), preprints COO-1428-428, ANL-HEP-CP-75-38 (unpublished).
[5] S. J. Barish *et al.*, Phys. Rev. D **16**, 3103 (1977).
[6] R. A. Singer, in: Proceedings of the International Conference on Neutrino Physics and Astrophysics, "Neutrino'77," Baksan Valley, USSR, June 18–24, 1977, edited by M. A. Markov, G. V. Domogatsky, A. A. Komar, and A. N. Tavkhelidze (Publishing office "Nauka", Moscow, USSR, 1978), Vol. **2**, p. 95.
[7] K. L. Miller *et al.*, Phys. Rev. D **26**, 537 (1982).
[8] R. L. Kustom *et al.*, Phys. Rev. Lett. **22**, 1014 (1969).

### BNL EXPERIMENTS

[9] A. M. Cnops *et al.*, in: Proceedings of the Topical Conference on Neutrino Physics at Accelerators, Oxford, UK, July 4–7, 1978, edited by A. G. Michette and P. B. Renton (Rutherford Lab, Chilton, 1978), p. 62. see also preprint BNL-24848, OG431, Brookhaven National Laboratory, 1978 (unpublished).
[10] G. Fanourakis *et al.*, Phys. Rev. D **21**, 562 (1980).
[11] N. J. Baker *et al.*, Phys. Rev. D **23**, 2499 (1981).
[12] L. A. Ahrens *et al.*, Phys. Rev. D **31**, 2732 (1985); L. A. Ahrens *et al.* (Japan–USA Neutrino Collaboration), BNL-36726, CONF-850375-15, 1985.
[13] L. A. Ahrens *et al.*, Phys. Lett. B **202**, 284 (1988).
[14] T. Kitagaki *et al.*, Phys. Rev. D **42**, 1331 (1990)
[15] K. Furuno *et al.*, A talk at the 2nd International Workshop on Neutrino-Nucleus Interactions in the few-GeV Region, (NuInt 2002), University of California, Irvine, December 12–15, 2002; KEK Preprint 2003-48, September, 2003 (unpublished). With reference to H. Sagawa, Ph. D. Thesis, Tohoku University, 1985 (unpublished); T. Kitagaki *et al.*, Phys. Rev. D **34**, 2554 (1986).

### FNAL EXPERIMENTS

[16] T. Kitagaki *et al.*, Phys. Rev. D **28**, 436 (1983).
[17] A. E. Asratyan *et al.*, Yad. Fiz. **39**, 619 (1984) [Sov. J. Nucl. Phys. **39**, 392 (1984)].
[18] A. E. Asratyan *et al.*, Phys. Lett. **137** B, 122 (1984).
[19] V. V. Ammosov *et al.* (IHEP–ITEP–MPEI Collaboration), Z. Phys. C **36**, 377 (1987).
[20] N. Suwonjandee, Ph. D. Thesis, University of Cincinnati, Cincinnati, 2004, FERMILAB-THESIS-2004-67, Fermi National Accelerator Laboratory, Illinois, 2004; UMI 31-20857 (unpublished).
[21] A. A. Aguilar-Arevalo *et al.* (MiniBooNE Collaboration), Phys. Rev. Lett. **100**, 032301 (2008) [arXiv:0706.0926 [hep-ex]].
[22] T. Katori, AIP Conf. Proc. **1189**, 139 (2009) [arXiv:0909.1996 [hep-ex]].
[23] J. L. Alcaraz-Aunion and J. Walding, AIP Conf. Proc. **1189**, 145 (2009) [arXiv:0909.5647 [hep-ex]]; see also J. L. Alcaraz-Aunion, Ph. D. Thesis, University of Barcelona, Barcelona, 2010, FERMILAB-THESIS-2010-45, Fermi National Accelerator Laboratory, Illinois, 2010 (unpublished).
[24] A. A. Aguilar-Arevalo *et al.* (MiniBooNE Collaboration), Phys. Rev. D **81**, 092005 (2010) [arXiv:1002.2680 [hep-ex]].
[25] A. A. Aguilar-Arevalo *et al.* (MiniBooNE Collaboration), Phys. Rev. **D** 88, 032001 (2013) [arXiv:1301.7067 [hep-ex]].
[26] J. Chvojka, Ph. D. Thesis., University of Rochester, 2012, FERMILAB-THESIS-2012-22, Fermi National Accelerator Laboratory, Illinois, 2012 (unpublished).
[27] G. A. Fiorentini *et al.*, Phys. Rev. Lett. **111**, 022502 (2013) [arXiv:1305.2243 [hep-ex]].
[28] L. Fields *et al.*, Phys. Rev. Lett. **111**, 022501 (2013) arXiv:1305.2234 [hep-ex].

### CERN EXPERIMENTS

[29] M. M. Block *et al.*, Phys. Lett. **12**, 281 (1964).
[30] H. Burmeister *et al.*, in: Proceedings of the Informal Conference on Experimental Neutrino Physics, CERN, Geneva, January 20–22, 1965, edited by C. Franzinetti, CERN Yellow Report No. 65-32, European Organization for Nuclear Research, Geneva, 1965, p. 25.
[31] C. Franzinetti, Lecture given at the Chicago Meeting of the American Physical Society, Chicago, October 28, 1965, CERN Yellow Report No. 66-13, European Organization for Nuclear Research, Geneva, March 1966 (unpublished).
[32] E. C. M. Young, Ph. D. Thesis, CERN Yellow Report No. 67-12, European Organization for Nuclear Research, Geneva, 1967 (unpublished).
[33] A. Orkin-Lecourtois, C. A. Piketty, Nuovo Cim. **50** A, 927 (1967).
[34] M. Holder *et al.*, Nuovo Cim. A **57**, 338 (1968).
[35] I. Budagov *et al.*, Lett. Nuovo Cim. **2**, 689 (1969).
[36] T. Eichten *et al.*, Phys. Lett. **46** B, 274 (1973).
[37] M. Haguenauer (for the Aachen–Brussels–CERN–Paris–Milano–Orsay–London Collaboration), in: Proceedings of the 17th International Conference on High Energy Physics, London, July 1–10, 1974, edited by J. R. Smith (Rutherford High Energy Laboratory, Didcot, Berkshire, UK, 1975), p. IV-95.
[38] M. Rollier (for the Aachen–Bruxelles–CERN–Ecole Polytechnique–Orsay–London Collaboration), in: Proceedings of the International Colloquium on High Energy Neutrino Physics, Paris, France, March 18–20, 1975, (Editions du CNRS, École Polytechnique, 1975), p. 349.
[39] S. Bonetti *et al.*, Nuovo Cim. **38** A, 260 (1977).
[40] P. Musset and J.-P. Vialle, Phys. Rept. **39**, 1 (1978).
[41] M. Rollier (for the Gargamelle Antineutrino Collaboration, Bari–Milano–Strasbourg–Torino–University College London), in: Proceedings of the Topical Conference on Neutrino Physics at Accelerators, Oxford, UK, July 4–7, 1978, edited by A. G. Michette and P. B. Renton (Rutherford Lab, Chilton, 1978), p. 68.
[42] M. Pohl *et al.* (Gargamelle Neutrino Propane Collaboration), Lett. Nuovo Cim. **26**, 332 (1979).
[43] N. Armenise *et al.*, Nucl. Phys. B **152**, 365 (1979).
[44] D. Allasia *et al.* (Amsterdam–Bergen–Bologna–Padova–Pisa–Saclay–Torino Collaboration), Nucl. Phys. B **343**, 285 (1990).
[45] S. K. Singh and E. Oset, Nucl. Phys. A **542**, 587 (1992).
[46] A. Martinez de la Ossa Romero, arXiv:hep-ex/0703026.
[47] V. V. Lyubushkin *et al.*, Eur. Phys. J. C **63**, 355 (2009) [arXiv:0812.4543[hep-ex]].
[48] V. V. Lyubushkin, Ph. D. Thesis, JINR, Dubna, 2009 (unpublished).

### IHEP EXPERIMENTS

[49] S. V. Belikov *et al.* (IHEP–ITEP Collaboration), preprint IFVE 81-146 ONF SERP-E-45, Serpukhov, 1981 (unpublished).
[50] S. V. Belikov *et al.*, Yad. Fiz. **35**, 59 (1982) [Sov. J. Nucl. Phys. **35**, 35 (1982)].
[51] S. V. Belikov *et al.*, Z. Phys. A **320**, 625 (1985); S. V. Belikov *et al.*, Yad. Fiz. **41**, 919 (1985) [Sov. J. Nucl. Phys. **41**, 589 (1985)].
[52] V. V. Makeev *et al.*, Pisma v Zh. Eksp. Teor. Fiz. **34**, 418 (1981) [JETP Lett. **34**, 397 (1981)].
[53] H. J. Grabosch *et al.* (SKAT Collaboration), preprints PHE 86-11, Berlin-Zeuthen, 1986 and IFVE 86-221 ONF SERP-E-107, Serpukhov, 1986 (unpublished).
[54] H. J. Grabosch *et al.*, Yad. Fiz. **47**, 1630 (1988) [Sov. J. Nucl. Phys. **47**, 1032 (1988)].
[55] V. V. Ammosov *et al.*, Fiz. Elem. Chast. Atom. Yadra **23**, 648 (1992) [Sov. J. Part. Nucl. **23**, 283 (1992)].
[56] J. Brunner *et al.* (SKAT Collaboration), Z. Phys. C **45**, 551 (1990).

### K2K & T2K EXPERIMENTS

[57] T. Kikawa (on behalf of the T2K collaboration), XXIV Workshop on Weak Interactions and Neutrinos, September 16 – 21, 2013, Natal, Brazil.
[58] D. Hadley (for the T2K Collaboration), PoS(EPS-HEP 2013)008.
[59] K. Abe *et al.*, Phys. Rev. D **87**, 092003 (2013) [arXiv:1302.4908 [hep-ex]].
[60] R. Gran *et al.* (K2K Collaboration), Phys. Rev. D **74**, 052002 (2006) [arXiv:hep-ex/0603034].

[61] K.Abe *et al.* (T2K Collaboration), Phys. Rev. D **92**, 112003 (2015) [arXiv:1411.6264 [hep-ex]].
[62] K.Abe *et al.* (T2K Collaboration), Phys. Rev. Lett. **113**, 241803 (2014) [arXiv:1407.7389 [hep-ex]].
[63] K. Abe *et al.* (T2K Collaboration), Phys. Rev. D. 91, 112002 (2015) [arXiv:1503.07452 [hep-ex]].
[64] X. Espinal and R. Sánchez, AIP Conf. Proc. **967**, 117 (2007).
[65] K. Nitta *et al.*, Nucl. Instrum. Meth. A **535**, 147 (2004) [arXiv:hep-ex/0406023].
[66] M. H. Ahn *et al.* (K2K Collaboration), Phys. Rev. D **74**, 072003 (2006) [arXiv:hep-ex/0606032].
[67] Y. Ashie *et al.* (Super-Kamiokande Collaboration), Phys. Rev. D **71**, 112005 (2005) [arXiv:hep-ex/0501064].
[68] K. Abe *et al.* (T2K Collaboration), Phys. Rev. Lett. **56**, 1107 (1986); erratum – *ibid.* **56**, 1883 (1986).
[69] K. Abe *et al.* (T2K Collaboration), Phys. Rev. D **93**, 112012 (2016) [arXiv:1602.03652 [hep-ex]].

**Papers on $M_A^{\mathbf{run}}$ and its applications**

[70] L. D. Kolupaeva, K. S. Kuzmin, O. N. Petrova, and I. M. Shandrov, *"Some uncertainties of neutrino oscillation effect in the NO$\nu$A experiment,"* Mod. Phys. Lett. A **31**, 1650077 (2016) [arXiv:1603.07451 [hep-ph]].
[71] K. S. Kuzmin, V. A. Naumov, and O. N. Petrova, *"Running axial mass of the nucleon for NO$\nu$A experiment,"* presented by O. N. Petrova at the 52'nd Winter School of Theoretical Physics "Theoretical aspects of Neutrino Physics", 14–21 Febraury 2016, Lądek Zdrój, Poland. Accepted by Acta Phys. Polon. B (in press).
[72] K. S. Kuzmin, V. A. Naumov, and O. N. Petrova, *"Quasielastic interactions of neutrinos with nuclei in the empirical model of running nucleon axial mass,"* presented by O. N. Petrova at the International Session-Conference of the Section of Nuclear Physics of PSD RAS, April 12–15, 2016, JINR Dubna. Accepted by Phys. Part. Nucl. (in press).
[73] K. S. Kuzmin and V. A. Naumov, *"Running axial-vector mass of the nucleon for a precise evaluation of the quasielastic (anti)neutrino–nucleus cross sections,"* in preparation.