Stochastic geometric model for a system of n fermions

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We consider a "finite dimensional system of Fermions without spin" represented by an element of the exterior algebra of the n-dimensional complex space. We associate invariant vector fields on $SO(2n + 1)$ to the Fermionic creation-annihilation operators. These vector fields implement the regular representation of the Lie algebra $\mathfrak{so}(2n + 1)$. As such, they do not satisfy the canonical anti-commutation relations in general, however once they have been projected onto an appropriate subspace of $L^2(SO(2n + 1))$ these relations are satisfied. We choose a symmetric positive-definite quadratic form in the creation-annihilation operators as Hamiltonian for our system of Fermions. The realization of Fermionic creation-annihilation operators in terms of (invariant) vector fields allows us to interpret the (Wick rotated) time evolution of the system of Fermions as a stochastic process generated by a hypoelliptic operator.