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Feynman diagrams in nuclear physics at low and intermediate energies

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Several aspects of using Feynman diagram formalism in nuclear theory at low and intermediate energies are considered, including extrapolation of differential cross sections to the pole position, derivation of the generalized folding potential and investigation of the asymptotic behavior of nuclear wave functions. Recent results on using the deuteron exchange diagram to resolve the problem of the quadrupole moment of ^{6}Li are briefly outlined.

INTRODUCTION

Nowadays diagram methods are extensively used not only in elementary particle physics, but in nuclear physics as well. The subject matter of this paper is related to bound nuclear systems and nuclear reactions at low and intermediate energies when one can use non-relativistic kinematics. In that case a distinction should be made between four-dimensional and three-dimensional diagrams. Four-dimensional non-relativistic diagrams are usual Feynman diagrams, in which an individual non-relativistic propagator $(E_i - \vec{k}_i^2/2m_i + i0)^{-1}$ corresponds to each virtual particle i and integration over 4- momentum corresponds to each closed contour. On the other hand, in three-dimensional diagrams, a propagator $(E - H_0 + i0)^{-1}$ and integration over 3-momentum correspond to each intermediate state, E and H_0 being the total energy and the free Hamiltonian of the system, respectively. Note that if one considers processes in a three-body system, then four-dimensional diagrams can be reduced to three-dimensional ones by performing trivial integrations over energy variables. For four- (or more) body systems the one-to-one correspondence between four- and three-dimensional diagrams does not exist.

In what follows several aspects of using Feynman diagram formalism in nuclear theory at low and intermediate energies are considered.

I. EXTRAPOLATION OF DIFFERENTIAL CROSS SECTIONS TO THE POLE POSITION

Consider the $A + x \to B + y$ reaction where composite bound systems A and y can be represented as $A = B + c, y = x + c$, c being a nucleon or a cluster. The amplitude of this reaction has a pole in $z = \cos\theta$ at $z = z_0$, z_0 lying in the unphysical region $|z_0| > 1$ (θ is the c.m. scattering angle). This pole corresponds to the diagram of Fig. 1.

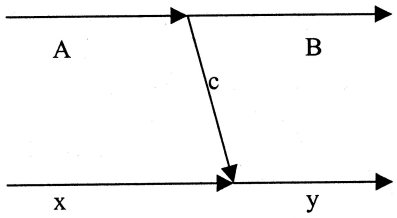

Fig. 1

The differential cross section of the reaction has a pole of the second order and can be written as

$$\sigma(z) = \frac{a_0}{(z-z_0)^2} + \frac{a_1}{z-z_0} + a_2(z) \tag{1}$$

Consider a quantity $\varphi(z) = (z-z_0)^2\sigma(z)$. At $z = z_0$ one has $\varphi(z_0) = a_0$. a_0 can be expressed in terms of the product squared $G^2_{ABc}G^2_{yxc}$ of the vertex constants corresponding to the vertices of the diagram in Fig. 1. The vertex constant G_{ABc} is the on-shell matrix element of the virtual process $A \leftrightarrow B + c$. It is an important nuclear characteristic related directly to the asymptotic normalization coefficient of the wave function of the A nucleus in the $B + c$ channel. To find a_0, $\varphi(z)$ is written approximately as $\varphi(z) = \sum_{i=1}^{N} c_i R_i(z)$ where $R_i(z)$ are some known polynomials, for example, the Hermite polynomials. Coefficients c_i are fitted by χ^2 procedure to experimental values of $\varphi(z)$ in the physical region $-1 \le z \le 1$. Then $\varphi(z)$ is extrapolated to the pole position by putting $z = z_0$. The method under discussion allows one to extract the valuable information on asymptotic normalization coefficients of bound-state nuclear wave functions. It is of interest to extend that method to the case of singularities other than a pole one, in particular, to logarithmic singularities due to triangle Feynman diagrams.

II. DERIVATION OF THE GENERALIZED FOLDING POTENTIAL

The four-dimensional covariant formalism of non-relativistic Feynman diagrams was used for the derivation of the general expression for the generalized folding potential (GFP), describing the interaction of a particle with a composite system (say, a nucleus) [1]. The input, which is required to construct the GFP, is the same as that for the standard folding potential (SFP). This input includes the potentials of the pair interaction of the incoming particle with the constituents (nucleons or clusters) of the composite system as well as the inner wave function of this system. The GFP corresponds to the amplitude of the triangle Feynman diagram containing three vertices: i) the vertex of the virtual decay of the target nucleus into constituents; ii) the vertex of the reverse process of the virtual synthesis of the nucleus and iii) the vertex describing the above-mentioned pair interaction. In the simple case that target A consists of two constituents b and c , the GFP is the sum of two terms, one of which is represented in Fig. 2. The second term is obtained from Fig. 2 by the substitution $b \leftrightarrow c$.

In Fig. 2 particles x and x' are on energy shell and A and A' are off shell. The potential thus obtained is non-local and energy-dependent. It becomes complex above the inelastic threshold. As distinct from the SFP, the GFP, being inserted into the Schrödinger or Lippmann-Schwinger equations, leads to the solutions accounting for the contribution not only from the elastic channel but from inelastic channels as well. The difference between the SFP and the GFP is due to the different way of going off energy shell. On shell, the expression for the GFP

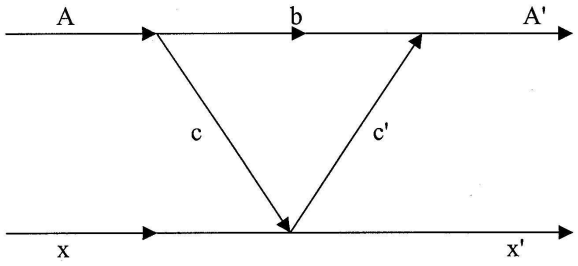

Fig. 2

goes over into that for the SFP. This property implies, in particular, that, within the first Born approximation, the scattering amplitudes corresponding to the GFP and the SFP coincide.

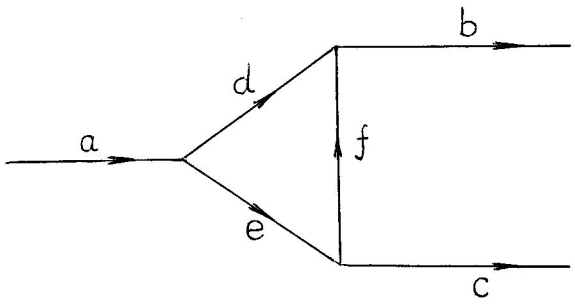

Fig. 3

The results described above can easily be extended to the case of the double-folding potential. In that case the expression for the GFP is written as a product of the amplitudes of two diagrams of the type of Fig. 2.

III. ASYMPTOTIC BEHAVIOR OF BOUND STATE WAVE FUNCTIONS

Consider a non-relativistic composite bound system a , say, a nucleus, which can be divided into two subsystems (fragments) b and c. Define the overlap integral (OI) $I(\vec{r})$ for the virtual process $a \to b + c$

$$I(\vec{r}) = \int \Psi_b^+(\tau_b)\,\Psi_c^+(\tau_c)\,\Psi_a(\tau_a, \tau_b, \tau_c) d\tau_b d\tau_c, \tag{2}$$

where $\Psi_i(\tau_i)$ is the internal wave function of the system i. For brevity we neglect the long-range Coulomb interaction and consider spinless systems a, b and c, hence $I(\vec{r}) = I(r)$, r being the distance between b and c. In the general case the results given below are valid for radial OI's $I_{ls}(r)$ corresponding to specific values of a channel orbital momentum l and channel spin s [2]. For a wide class of processes, main contributions to their amplitudes come from the values of OITs at large r. In particular, the asymptotics of OITs determines the cross sections for certain reactions, which are of interest for nuclear astrophysics [3]. Relating the OI by

a Fourier transform to the form factor $G(q)$ for the $a \to b+c$ vertex (q being the relative momentum of b and c fragments) and making use of the Cauchy theorem, one obtains the asymptotic form of $I(r)$ as [4]

$$I(r) = c_0 e^{-\kappa r}/r + c_1 e^{-\kappa_1 r}/r^p, \quad r \to \infty, \tag{3}$$

where c_0 and c_1 are some constants, κ is related to the binding energy in the $a \to b+c$ vertex, $\kappa = \kappa_{abc}$, $\kappa_{ijk} = \sqrt{2\mu_{jk}(m_j + m_k - m_i)}$, $\mu_{jk} = m_j m_k/(m_j + m_k)$, m_i is a mass of a particle i, $i\kappa_1$ is the position of the singularity of $G(q)$ in the complex q plane, which is the nearest to the physical region, and $p > 1$ is determined by the type of this singularity. The first term in Eq.(2) corresponds to the usually adopted УnormalФ asymptotics of the OI. If the system a consists of three or more constituents, then, as a rule, $i\kappa_1$ is a proper singularity of one of triangle Feynman diagrams of the type of Fig.3 where b,c,d,e and f are certain fragments into which the composite system a can virtually be divided.

For such a diagram $\kappa_1 = (m_b/m_d)(\kappa_{ade} + \kappa_{bdf})$ [4,5] and $p = 2$. Apart from that diagram, one may consider generalized triangle diagrams, which are obtained from Fig. 3 by substituting two or more inner lines for lines d, e or f. According to the general theory of the singularities of Feynman diagrams [6], the position $i\kappa_1$ of the singularity of such a diagram is obtained from that of an ordinary triangle diagram by the substitution of the corresponding sums of masses for the masses of d, e and f. If the total number of inner lines in a generalized triangle diagram is n, then in Eq.(3) the exponent $p = (3n-5)/2$. The examples of generalized triangle diagrams are shown in Fig. 4 ($n = 4$, $p = 7/2$) and Fig. 5 ($n = 5$, $p = 5$).

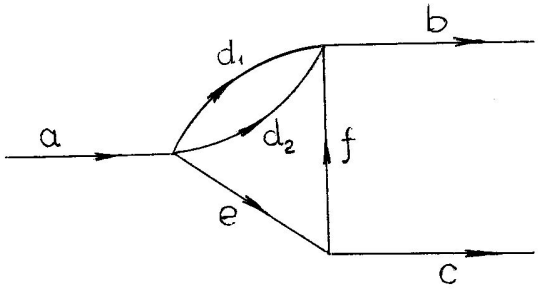

Fig. 4

It is possible that $\kappa_1 < \kappa$; in that case the asymptotics of $I(r)$ is determined by the second ("anomalous") term in Eq.(2).

The typical examples of the anomalous asymptotics of the OI ($\kappa_1 < \kappa$) are as follows: $^{16}\mathrm{O} \to ^{13}\mathrm{N} + ^{3}\mathrm{H}$ and $^{20}\mathrm{Ne} \to ^{17}\mathrm{O} + ^{3}\mathrm{He}$ (Fig. 3), $^{9}\mathrm{Be} \to ^{6}\mathrm{Li} + ^{3}\mathrm{H}$ and $^{12}\mathrm{C} \to ^{9}\mathrm{Be} + ^{3}\mathrm{He}$ (Fig. 4), and $^{16}\mathrm{O} \to ^{9}\mathrm{Be} + ^{7}\mathrm{Be}$ (Fig. 5).

IV. TENSOR INTERACTION IN THE $d\alpha$ SYSTEM

There exists a long-standing problem of describing tensor interaction in the $d\alpha$ system. In particular, calculations of the ^{6}Li quadrupole moment Q within various versions of a three-body model (n,p,α) result in large positive values ($Q_{\mathrm{theor}} \approx 4 \div 6$ mb), whereas the experimental value is small negative ($Q_{\mathrm{exp}} = 0.82$ mb) [7]. Note that these models predict excellently all

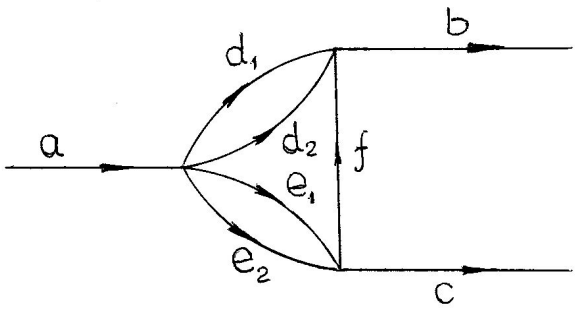

Fig. 5

other properties of $A = 6$ nuclei for ground and low-lying levels. The recent results [8] show that the problem under discussion could be solved by allowing for the exchange mechanism, which is described by the diagram of deuteron transfer (see Fig. 6).

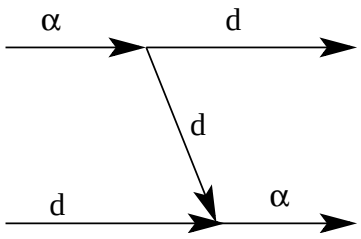

Fig. 6

The amplitude V of this diagram could be considered as an exchange potential of $d\alpha$ interaction, which is non-local and energy-dependent. This potential has a tensor component due to the D state admixture in the $d + d$ channel of the α particle wave function. In the momentum space potential V assumes the form

$$V(\vec{k},\vec{k}') = \Gamma_1 \Gamma_2 G_d, \tag{4}$$

where Γ_1 and Γ_2 are the vertex functions corresponding to the virtual $\alpha \leftrightarrow d + d$ processes and G_d is the non-relativistic deuteron propagator. The vertex functions Γ_i depend on the relative momenta of two deuterons and are expressed by well known way [9] through the Fourier component of the overlap integral $< dd|\alpha >$, for which the authors [8] used the results of realistic four-nucleon calculations as well as some phenomenological results derived from the analyses of (d,α) and (α,d) reactions on heavy targets.

The exchange potential thus constructed leads to coupled-channel integral equations for the ^{4}He-d system. Numerical solution of these equations gives both the S-wave and D-wave functions for the ^{4}He-d relative motion, which, being substituted into the matrix element for the quadrupole operator, result in *negative* contribution to the Q value. The calculations, which used several types of the ^{4}He modern four-body wave function, have demonstrated for the first time the almost complete cancellation between the *negative* contribution of exchange origin and the *positive* contribution (due to the direct ^{4}He-d tensor interaction of folding type) to the value of Q. So that, in agreement with the experiment, the resulting value of Q is quite small

in magnitude and should be very sensitive to fine details of the ^{4}He wave function and of the deuteron distortion in ^{6}Li.

It is worth noting that, though the exchange potential under discussion is not taken into account in the existing three-body $\alpha - n - p$ models of ^{6}Li, it can be included in these models by adding the corresponding projector onto the deuteron state. The potential thus obtained represents the three-body $\alpha - n - p$ force.

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