

Teaching neutrino oscillations

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Teaching neutrino oscillations

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Neutrino oscillations are purely quantum mechanical effects that occur over macroscopic time and distance scales. I present the physics of this phenomenon in words, pictures, and analogies rather than mathematics. © 2004 American Association of Physics Teachers.
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I. INTRODUCTION

Neutrino oscillations, first postulated in 1969, have become a reality within the last few years. The consequences for our view of particle physics, astrophysics, and cosmology are profound, and the understanding of the evidence for oscillations and their mechanism can be elusive. This article aims to present the current state of our knowledge of neutrino oscillations in as simple a way as possible. It is a result of preparing for my talk at the American Association of Physics Teachers 2003 winter meeting and teaching fourth-year particle physics course at the University of British Columbia. For brevity and simplicity, I will discuss only the most likely current scenario for neutrino masses and mixings; other possibilities are shamelessly ignored. The hope is that this treatment will be accessible to nonspecialists, and I have tried to minimize the quantity of mathematics. The reader is warned however that this text is dense; most sentences contain some essential ingredient. The bibliography aims at accessibility rather than rigor or completeness.

II. QUARKS AND LEPTONS

The fundamental constituents of matter as we know it come in pairs (doublets) of elementary particles which are very similar to each other except that their charges differ by one fundamental unit. This unit is the magnitude of the charge on the electron, which is the same as that on the proton. The first hint that the universe is structured this way came when it was recognized that atomic nuclei are formed of protons and neutrons, which are more or less identical particles except that one has a charge of +1 and the other a charge of zero. We now understand these particles in terms of their elementary constituents, the quarks. The proton has two up-quarks (u , charge $+2/3$) and one down-quark (d , charge $-1/3$); the neutron has two downs and an up. In addition to these light, stable quarks, there are two more doublets that have larger mass and are unstable, eventually decaying into ups and downs. These are called charm (c , $+2/3$) and strange (s , $-1/3$), top (t , $+2/3$) and bottom (b , $-1/3$). The names have no meaning, except to keep physicists talking the same language. So we have three generations of quark flavors, (u,d), (c,s), (t,b), and this arrangement has deep significance for the structure of the universe. It is too deep to go into here.

The link between the partners in the doublets is the weak interaction, the fundamental force which controls β decay, and, as we shall see, solar fusion reactions. The weak interaction allows a d to turn into a u , etc., when energy consid-

erations permit. Significantly, it allows a free neutron (n) to (β) decay to a proton (p) and an electron (e^- historically known as a β ray),

$$n \rightarrow p + e^- + \bar{\nu}_e. \quad (1)$$

Or, in terms of the quarks (with the spectators in parentheses):

$$u(ud) \rightarrow d(ud) + e^- + \bar{\nu}_e. \quad (2)$$

We'll get to the ν_e and $\bar{\nu}_e$ soon. Suffice it to say that without neutron decay, there would be no free hydrogen in the universe, and we would probably not be around to think about neutrinos.

These flavor states (u,d,c,s,b,t) are called eigenstates and are not rigidly separate. The states in which these quarks propagate are called mass eigenstates, which are each different mixtures of the flavor eigenstates. The mass eigenstates propagate through space at different speeds and so the component waves representing the flavors get out of phase with each other. A quark born as one flavor will soon start looking like another. An s quark traveling through space can turn into a d quark. Likewise all the second members of each doublet can mix between the generations, and any particle made of second- or third-generation quarks can decay into a stable particle made of first-generation quarks. By convention we push all the mixing into the second member of each doublet; we can do this because the weak interaction allows transformations between the two partners within a doublet.

So much for the structure of heavy particles (baryons), those which form much of our mass. What about the light particles (leptons), starting with the electron? There are three charged leptons—the electron, muon, and tau—and three associated very light chargeless neutrinos, separated by one unit of charge, (e^-, ν_e), (μ^-, ν_μ), (τ^-, ν_τ).

Superficially this organization looks very much like the quarks. However, for a long time the neutrinos were thought to be massless (because no mass had been detected). In this case, no mixing is possible, because all neutrinos will propagate at precisely the speed of light, and the mass eigenstates can never become out of phase with each other. An electron neutrino will never change its composition, as all parts will move at the same speed, and so the neutrino can never be detected as any other flavor. In fact, ν_e have at long last been observed to change flavor, and so the simple massless neutrino model cannot be correct.

Mixing of flavors in the charged leptons would be very easy to see. Muons would decay into electrons and gamma rays via a fast electromagnetic process, much faster than the slow weak decay that actually happens,

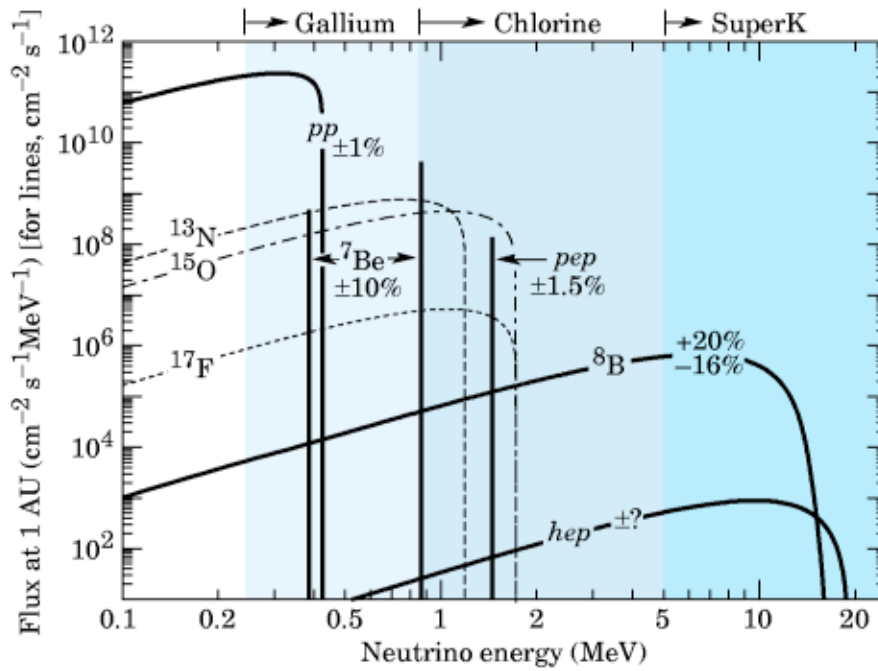


Fig. 1. The solar neutrino spectrum, showing the contribution of the ${}^8\text{B}$ and pp processes, and the thresholds for the three detection mechanisms. The figure is reproduced by courtesy of the Particle Data Group (Ref. 2).

$$\mu^\pm \rightarrow e^\pm + \gamma \quad (\text{electromagnetic decay}), \quad (3)$$

$$\mu^+ \rightarrow e^+ + \bar{\nu}_\mu + \nu_e \quad \text{or} \quad \mu^- \rightarrow e^- + \nu_\mu + \bar{\nu}_e \quad (4)$$

(weak decay).

The first decay in Eq. (4) violates what is called lepton number and has never been observed. The two lepton numbers here are the muon number, $L_\mu = \text{number of } \mu^- \text{ and } \nu_\mu \text{ minus the number of } \mu^+ \text{ and } \bar{\nu}_\mu$, and the electron number, L_e , which is defined similarly. These numbers are defined this way because they appear to be conserved quantities, and the electromagnetic decay would violate both of them. The second decay in Eq. (4) conserves both lepton numbers and is the way all muons seem to decay.

The relation between neutrinos (ν) and antineutrinos ($\bar{\nu}$) is easy to see in relation to the charged leptons. In reactions and decays neutrinos tend to produce negatively charged leptons (and vice versa), while antineutrinos produce positively charged leptons (see Sec. VII). Although it is easy to tell an electron from a positron, the difference between ν_e and $\bar{\nu}_e$ is much more subtle, especially given the reality of masses and flavor mixing (see Sec. VII). In brief, the Sun produces electron neutrinos, reactors produce electron antineutrinos (from the β decay of fission products), and cosmic rays produce neutrinos and antineutrinos of both electron and muon types. The distinction between neutrino and antineutrino is not crucial in the oscillation debate.

III. EARLY INDICATIONS (1968–1992)

By the 1960s our understanding of the solar interior and of low energy nuclear physics had reached such a stage that the Sun's neutrino output could be predicted with some confidence. In broad terms, this picture remains unchanged to this day. An excellent review of the subject in somewhat more mathematical detail is given in Ref. 1. The Sun produces energy by fusing four hydrogen nuclei into one helium

nucleus. There are many steps and several different possible paths to this process, but ultimately two protons have to become two neutrons. This is a weak interaction process and requires the release of two positrons to conserve charge, and two electron neutrinos to conserve electron number. The positrons annihilate with ambient electrons and contribute to heating the Sun.

For the release of detectable neutrinos, the two most important reactions are as follows. Both are weak interactions and one yields the copious, low energy pp neutrinos, and the other the more scarce high energy ${}^8\text{B}$ neutrinos (see Fig. 1),

$$p + p \rightarrow \text{D} + e^+ + \nu_e \quad (E_\nu = 0 - 0.420 \text{ MeV}), \quad (5)$$

$${}^8\text{B} \rightarrow 2{}^4\text{He} + e^+ + \nu_e \quad (E_\nu = 0 - 14.6 \text{ MeV}). \quad (6)$$

The D is a deuteron, heavy hydrogen ${}^2\text{H}$. The pp neutrinos are very difficult to observe, even by neutrino standards, and can only be detected by radio-chemical means (see below). However their flux (about $10^{15} \text{ m}^{-2} \text{ s}^{-1}$ at the Earth) is 10 000 times that of the ${}^8\text{B}$ neutrinos.

The chance of a neutrino interacting in a detector is proportional to the detector size, the neutrino flux, and rises with increasing neutrino energy. Detection is by means of secondary particles produced as the neutrino is absorbed or scattered. Higher energies make these easier to see; at very low energies only the radiochemical detection of product nuclei works. The first sighting of neutrinos came in 1956 when antineutrinos produced in enormous numbers by a nuclear reactor were detected.³

Neutrino detection: The chargeless neutrino cannot be observed unless it is absorbed by a nucleon and produces a charged lepton or scatters off an electron. In both cases the reaction products can be observed through their electromagnetic properties.

1. *Nucleons*: Electron-type neutrinos can scatter off a neutron to produce a proton and the other half of the neutrino's doublet, the electron. This is a charge-swapping quasi-elastic reaction:

$$\nu_e + n \rightarrow e^- + p + 0.8 \text{ MeV}. \quad (7)$$

This reaction is exothermic; the neutron ($939.6 \text{ MeV}/c^2$) and almost massless neutrino have more mass than the proton ($938.3 \text{ MeV}/c^2$) and the electron ($0.5 \text{ MeV}/c^2$). Hence the reaction is accessible to all solar neutrinos. Unfortunately, free neutrons don't live much longer than a quarter of an hour, and so a macroscopic detector is difficult to build. The reaction doesn't work on protons because the charges cannot be made to match. (Hydrogen would be a convenient detector because it exists in water or hydrocarbons.) It would work for anti-neutrinos, which can make positrons e^+ :

$$\bar{\nu}_e + p \rightarrow e^+ + n. \quad (8)$$

The product positron can be observed at extremely low energies because it captures on ambient electrons, producing γ rays.

The next best way to detect neutrinos is to use neutrons in a nucleus; this is not easy as neutrons are usually bound more tightly in the nucleus than are protons, and so their effective mass decreases and reverses the energy balance of Eq. (7). The tight binding raises the energy threshold for the reaction, in most cases out of reach of solar neutrinos. In addition, the outgoing electron is usually too low in energy to observe, and so one has to rely on a detectable product nucleus. The product nucleus has to be physically removable from the detector and be radioactive with a convenient half-life (preferably days) to accumulate and to observe after removal. In fact, the reaction tends to make a nucleus that is too proton rich, and these tend to decay by the slow process of atomic-electron capture, which produces detectable x rays. The basic decay process is the reverse of neutrino absorption and yields the original nucleus:

$$e^- + p \rightarrow \nu_e + n. \quad (9)$$

In addition, the weakness of the weak interaction means we need hundreds or thousands of tons of target material to get a good signal, so the substance has to be cheap, safe, and pure with very little radioactivity. Three nuclei have been used as detectors to date. The first was chlorine-37, in the form of borrowed cleaning fluid, which has been used by Ray Davis⁴ since the late 1960s:

$$\nu_e + {}^{37}\text{Cl} \rightarrow e^- + {}^{37}\text{Ar} \quad (E_\nu > 0.8 \text{ MeV}). \quad (10)$$

This method of detection is sensitive to most of the ${}^8\text{B}$ spectrum and neutrinos from some intermediate reactions. Radioactive argon, a noble gas, can be bubbled out of the chlorine using helium. The argon atoms decay by the capture of an atomic electron, a process that kicks out further atomic (Auger) electrons which can be detected and counted [Eq. (11)]. However, this detection is clearly not done in real-time, and all direction and most energy information is lost,

$$e^- + {}^{37}\text{Ar} \rightarrow \nu_e + {}^{37}\text{Cl} + \text{Auger electrons}. \quad (11)$$

This was the first successful detection method for solar neutrinos, and as a result, Davis shared the 2002 Nobel Prize in Physics.⁵

The only detector successfully used to date that can observe the pp neutrinos is gallium-71. The first results were recorded by the Soviet-American Gallium Experiment

(SAGE)⁶ (situated in a tunnel underneath Mount Andyrchi in the then Soviet Caucasus, and the Gallex experiment (in the Gran Sasso tunnel, Italy),⁷ both in 1990:

$$\nu_e + {}^{71}\text{Ga} \rightarrow e^- + {}^{71}\text{Ge} \quad (E_\nu > 0.2 \text{ MeV}). \quad (12)$$

Again, timing and directional information is lost, and the reaction in Eq. (12) works like the chlorine reaction except that gallium is converted to radioactive germanium. However, you can't bubble germanium out of liquid gallium without first converting it to a gas. This conversion has been done, but requires much chemistry.

The third useful nucleus is deuterium, but in a different way. The proton and neutron are bound in deuterium so lightly that the electron is energetic enough to be visible. The electron is the only visible particle, because the stable protons move too slowly:

$$\nu_e + \text{D} \rightarrow p + p + e^- \quad (E_\nu > 1.4 \text{ MeV}). \quad (13)$$

The most convenient form of deuterium is heavy water (not inexpensive but at least available⁸). The fast electron emits Cherenkov radiation which can be readily detected. Cherenkov radiation is the result of a charged particle traveling at faster than the speed of light in a medium such as water (in water this speed is $c/1.33$), and a conical pattern of blue and UV photons is emitted which is analogous to a supersonic boom. A little calculation will tell you that the electrons need an energy of more than 0.8 MeV to do this, which is a reasonable threshold in this context (a proton would need 1420 MeV, which is way out of range of solar neutrinos). In practice, the detection threshold in real detectors is higher due to the background caused by natural radioactivity.

The first such heavy water Cherenkov detector is the Sudbury Neutrino Observatory (SNO).⁹ It started operating in a northern Ontario nickel mine in 1999, a decade after the first light water Cherenkov detector, Kamiokande (see the following section).

2. *Electrons*: Conceptually, the simplest way to detect neutrinos is via elastic scattering off electrons, which are plentiful in any material. One has to choose a cheap, safe, purifiable, transparent medium so the Cherenkov light from neutrino-scattered electrons can be distinguished from the inevitable trace radioactivity. Water works well, although using it in practice is far from simple. Alternatively, liquid scintillator rather than pure water can be used (see Sec. V C). The scintillation light produced by charged particles is more intense than Cherenkov light, so it is sensitive to lower energies. However, it is more expensive than even ultra-pure water, and the trace radioactivity levels have to be much lower to allow use of this increased sensitivity.

The masters of the water-Cherenkov technique are the Japanese Kamiokande collaboration and its successor, Super-Kamiokande. Kamiokande announced the first real-time, directional detection of solar neutrinos in 1989.¹⁰ Masatoshi Koshiba shared the 2002 Nobel Prize in Physics for initiating this series of experiments.⁵ The reaction is

$$\nu_e + e^- \rightarrow \nu_e + e^-. \quad (14)$$

There is zero energy threshold for the reaction, but the recoil electron requires some energy to be detected, as before. Undergraduates have complained to me that Eq. (14) is not a real reaction because the left-hand side is the same as the right. They have obviously been made to sit through too many chemistry lectures. The left-hand electron is stationary, while the right-hand one is recoiling from the neutrino and is

Table I. Summary of solar neutrino data circa 1990. The detection medium is given, as is the threshold for neutrino detection, the predominant solar reaction observed, and the fraction of the expected standard solar model flux seen.

Detection medium	Threshold	Predominant neutrino source	Fraction observed
Gallium	0.2 MeV	$pp + {}^8\text{B}$	55%
Chlorine	0.8 MeV	${}^8\text{B}$	34%
Light water	9 MeV, later 5 MeV	${}^8\text{B}$	48%

moving close to the speed of light, which makes it visible. The directionality comes from the fact that the recoil electron tends to travel in line with the original neutrino. This method of detection is sensitive to the upper end of the ${}^8\text{B}$ spectrum.

There is a slight but significant complication here. Electron scattering also works for muon and tau neutrinos, for example,

$$\nu_\mu + e^- \rightarrow \nu_\mu + e^- \quad (15)$$

The cross section (that is, sensitivity) of this reaction is only 15% of that for electron neutrinos.

3. *The Data:* By 1990, the chlorine, light water, and gallium experiments (in that order) had all reported seeing only a small fraction of the expected signal. The numbers are listed in Table I. Note that all these experiments have to be done deep underground to get away from cosmic rays which would easily swamp the tiny neutrino signal on the surface.

There are 10%–20% errors in these fractions due to experimental and theoretical uncertainties, but it is clear that none of these numbers is consistent with 100%, nor with each other. By this time, the solar astrophysicists were becoming very confident of their flux predictions, and so this energy-dependent deficit became known as the solar neutrino problem. When physicists use the word “problem,” they mean “research opportunity.”

IV. PONTECORVO’S IDEA

All of the above detection reactions are sensitive only to ν_e (with the one exception noted). In 1969 Bruno Pontecorvo reasoned that if neutrinos had small and different masses, and the flavors mixed, then ν_e ’s born in the Sun might reach the Earth as, say ν_μ (ν_τ ’s were not known then) and be undetectable.¹¹ The resulting ν_e deficit might be the answer to the solar neutrino problem.

For simplicity, consider two neutrino species. Neutrinos are born and detected via the weak interaction as flavor eigenstates, for example, e and μ . However they propagate as mass eigenstates which have a distinct velocity, labeled, for example, 1 and 2. If flavor is not rigorously conserved (there is no particular reason why it should be, except that is the way the charged leptons seem to behave), and if the masses m_1 and m_2 are slightly different, these two pairs of states may not be one and the same, but may be mixed:

$$\nu_1 = \nu_e \cos \theta + \nu_\mu \sin \theta, \quad (16a)$$

$$\nu_2 = -\nu_e \sin \theta + \nu_\mu \cos \theta. \quad (16b)$$

We will define the phrase “slightly different” later.

The use of sines and cosines in Eq. (16) ensures that the mixing produces neither more nor less neutrinos than we started with. Its called a unitary transformation because the

particle number depends on the square of the amplitude terms ($\cos^2 + \sin^2$ is always 1). Consider a neutrino that was created as an electron neutrino. The probability that it will be detected as an electron neutrino a distance L away is (see for example, Refs. 12 and 13):

$$P_{ee} = P_{\mu\mu} = 1 - \sin^2 2\theta \sin^2 k\Delta m^2 L/E. \quad (17)$$

The constant $k = 1.27 \text{ MeV}/(\text{m}\cdot\text{eV}^2)$: $\Delta m^2 = m_2^2 - m_1^2$ is measured in $(\text{eV}/c^2)^2$, the energy E in MeV, and the distance from the source L in meters. Equation (17) describes classical “vacuum oscillations,” and also works with GeV and km units. The detection probability for parameters we now associate with solar and reactor neutrinos are shown in Fig. 2.

With three species, the mass and flavor basis states are linked via a 3×3 matrix with two mass splittings, three angles like θ and an extra one that makes life different for neutrinos and antineutrinos (a CP-violating phase).¹⁴

V. THE HARD EVIDENCE, 1992–2002

There are now two widely recognized pieces of evidence for neutrino oscillations: the atmospheric muon-neutrino deficit, and the solar electron-neutrino deficit. The first was formally announced by the Kamiokande collaboration in 1992.¹⁵ It was declared to be evidence of nonzero neutrino mass (consistent with oscillations) in 1998 by that collaboration’s successor, Super-Kamiokande.¹⁶ Evidence for a non-zero neutrino mass had been building steadily over 30 years, but compelling evidence came in 2001/2 from the Sudbury Neutrino Observatory collaboration,¹⁷ who showed that solar astrophysics could not be to blame for the deficit, and that the “missing” neutrinos were arriving at the Earth as other flavor states. The particular oscillation mechanism suggested by the solar experiments was confirmed in December 2002 by the KamLAND reactor-neutrino detector (Sec. VC), a fact that removed any lingering worries about uncertainties due to solar astrophysics.

We will call the neutrino parameters revealed by atmospheric neutrinos Δm_A^2 and θ_A , and those revealed by solar (and reactor) neutrinos Δm_S^2 and θ_S . The remaining unknown angle is now very much sought. The CP-violating phase is even hotter property, but it will be very difficult and expensive to find.¹⁸

A. Atmospheric neutrinos

Atmospheric neutrinos are made by pion and kaon decays resulting from cosmic-ray interactions in the upper atmosphere. The numerology of these decays leads us to expect two muon neutrinos and antineutrinos for each electron neutrino and antineutrino. It is very difficult experimentally to distinguish between atmospheric neutrinos and antineutrinos, and we generally count them together. The following reaction sequence is typical:

$$p + {}^{14}\text{N} \rightarrow \pi^+ + X, \quad (18)$$

$$\pi^+ \rightarrow \mu^+ + \nu_\mu, \quad (19)$$

$$\mu^+ \rightarrow e^+ + \nu_e + \bar{\nu}_\mu, \quad (20)$$

where X represents nuclear fragments.

The observed μ/e ratio is more like unity on the average and is strongly dependent on the zenith angle, that is, the distance the neutrino has traveled since birth (see Fig. 3). It

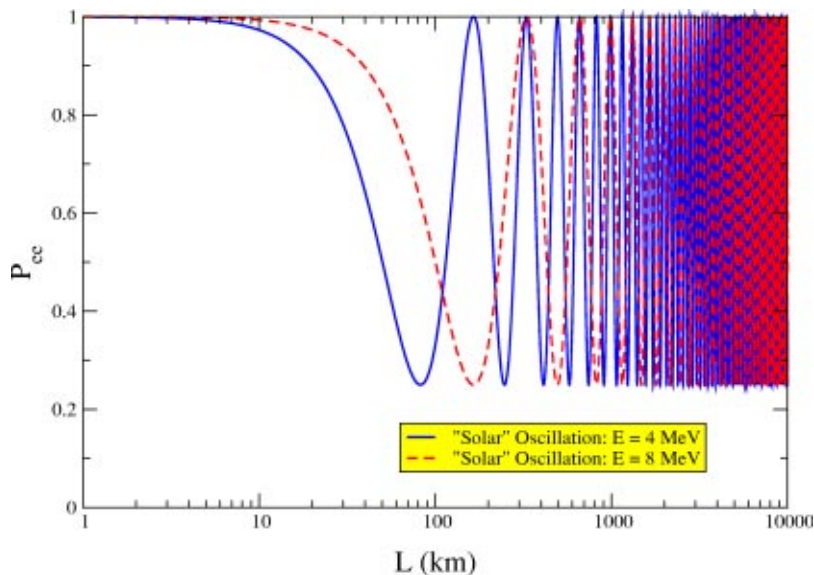


Fig. 2. The survival probability for ν_e (P_{ee}) is plotted against the distance between the source and detection for two monoenergetic electron neutrinos, at 4 MeV (typical detected reactor neutrino energy) and at 8 MeV (typical detected solar neutrino energy). Both have the currently accepted solar oscillation parameters of $\Delta m_{21}^2 = 6 \times 10^{-5}$ and $\theta = \pi/6$. It is clear that a reactor experiment, with a neutrino spectrum several MeV wide, will see no effect below 10 km, but at 100 km or greater will see about 60% of the expected $\bar{\nu}_e$ flux.

seems that the further a ν_μ travels in the Earth, the less chance it has of being detected. One might wonder if the deficit is due to neutrino absorption in the dense interior of the Earth. However we understand neutrino absorption very well, and the mean free path of an atmospheric neutrino in rock must be about 10^{12} m. Thus only a tiny fraction interact in the Earth, which has a diameter of only 10^7 m. Therefore they must be disappearing by some other means.

The measured flux of ν_μ 's is about half the expected value, while that for the ν_e 's is about right. There are strong indications from Super-Kamiokande that the missing ν_μ 's are showing up as ν_τ 's (which are very difficult to observe). We will assume that this explanation is the case.

A detailed analysis in terms of the path length through the Earth yields an L/E dependence as expected from the Eq. (17) with the parameters:¹⁹

$$\Delta m_A^2 \approx 3 \times 10^{-3} \text{ eV}^2, \quad \theta_A \approx \pi/4. \quad (21)$$

Atmospheric neutrinos are typically a few GeV in energy, so Δm_A^2 is more or less fixed by the geometry of the earth; otherwise, the effect would be unobservable. The data sample is divided into two distinct angular regimes: the neutrinos come either from above, with path lengths of tens of kilometers, or from below with path lengths of thousands of kilometers (see Fig. 3). Simple solid-angle considerations tell us that not many neutrinos come from around the horizontal, with path lengths of hundred of kilometers. One reason why this evidence is so strong, despite the complexity of the particle interactions in the atmosphere, is that we observe no oscillation effects above the horizontal, and plenty below it. The only possible complications are those of the Earth's geometry and magnetic field (which distorts the isotropy of the cosmic rays), but these are now well understood. Hence, oscillations will be visible for 1000 km path lengths and 1 GeV neutrinos if Δm_A^2 is $\approx 10^{-3} \text{ eV}^2$ and the mixing angle is big, as can readily be seen from Eq. (17) and Fig. 4.

The Super-Kamiokande collaboration announced this result as the discovery of neutrino oscillations in 1998. The big surprise was the largeness of the mixing angle, which may be maximal (45°). The mixings previously observed in the

quark sector are very small (the biggest is 13°). So, two of the mass eigenstates (1 and 3) are likely an equal mix of ν_μ and ν_τ , with very little ν_e in ν_3 .

The size of the neutrino source (a few kilometer layer in the upper atmosphere) is small compared to flight distances (20–13 000 km), and ν_μ and ν_τ interact identically in the earth (that is, no matter effects), so it is useful to think in terms of vacuum oscillations. Things are rather different in the solar case.

B. Solar neutrinos

The last piece of evidence for solar neutrino oscillations came in 2002 when the Sudbury Neutrino Observatory col-

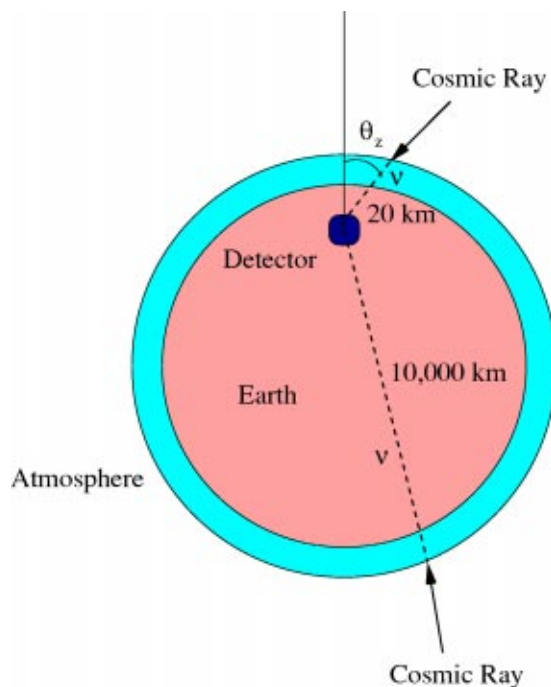


Fig. 3. The production mechanism and geometry for the detection of atmospheric neutrinos.

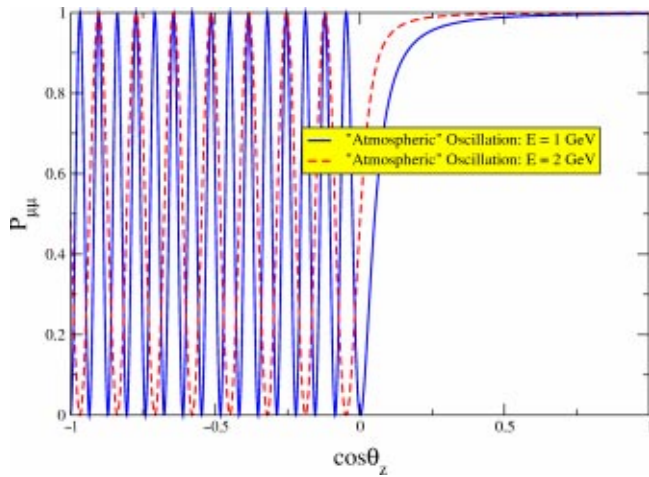


Fig. 4. The survival probability for ν_μ ($P_{\mu\mu}$) is plotted against the cosine of the zenith angle θ for an atmospheric neutrino experiment. Neutrinos coming from directly above have $\cos\theta = +1$, those from the nadir have $\cos\theta = -1$. The energies plotted are 1 and 2 GeV (typical values in this case) with the accepted atmospheric oscillation parameters of $\Delta m_{21}^2 = 3 \times 10^{-3}$ and $\theta_A = \pi/4$. It is easy to see that with an atmospheric neutrino spectrum a few GeV wide, there will be no effect on ν_μ flux above the horizontal, but only 1/2 of the expected ν_μ flux will be seen below the horizontal.

laboration reported the flux of electron neutrinos measured by reaction 13 and the total flux of all neutrinos flavors measured by a reaction unique to deuterium, which is totally blind to neutrino flavor ($x = e, \mu, \tau$):

$$\nu_x + D \rightarrow p + n + \nu_x. \quad (22)$$

Here the neutron is captured by another deuterium nucleus, which produces a detectable gamma ray. The analysis of the data for this reaction yields precisely the flux of neutrinos expected from standard solar astrophysics. However, comparison with events of the type shown in Eq. (13) show that only 34% of these neutrinos still have electron flavor by the time they reach us. The remaining 66% are in all likelihood an equal mix of ν_μ and ν_τ if the atmospheric interpretation is correct. It should also be noted that there is some evidence that the electron fraction increases at night, when the neutrinos have passed through thousands of kilometers of the Earth to reach the detector.

In retrospect, the relatively large signal in the light water detectors was a good clue [by this time the Super-Kamiokande measured fraction had hardened to $(47 \pm 2)\%$]; the difference between this result and SNO's is due to ν_μ and ν_τ . Once the ν_μ and ν_τ contributions are taken into account, the measured ν_e fraction varies only slightly with energy. The Sudbury Neutrino and Super-Kamiokande observatories see no distortion; only the ultra-low threshold gallium experiments see a slightly larger fraction. This weak energy dependence is not what you would expect from vacuum oscillations, which have a strong energy dependence. However, another process is at play, known as the large mixing angle (LMA) solution,²⁰ which relies on the behavior of neutrinos in the dense interior of the Sun. For a more mathematical approach than what follows here, see Ref. 21 (although this article was written before SNO's results were released).

Neutrinos in dense matter: Mikheyev, Smirnov, and Wolfenstein^{20,13} first explained how neutrino mixing would be affected by the presence of dense matter, the solar core for example, or perhaps the interior of the Earth. Because ordi-

nary matter contains electrons and not μ or τ , electron neutrinos behave in a subtly different manner in matter than does the ν_μ or ν_τ . This difference distorts the masses and flavor components of the mass eigenstates when neutrinos move between a vacuum and dense matter. The region of parameter space in which these effects are important is shown in Fig. 5. It is fairly narrowly defined in terms of Δm^2 . If Δm^2 is too big, then matter effects become negligibly small; the critical value is fixed by fundamental physics and the density of the solar core to be around 10^{-4} eV^2 . If Δm^2 is too small, then the vacuum oscillation length becomes much bigger than the solar core, and so this region of dense matter starts to look too small, from the neutrino's perspective, to have an effect.

In Fig. 5 plausible solutions to the solar neutrino problem are shown as they appeared in 1987, when the idea of matter effects arose. The small mixing angle solution was the theoretical favorite, as small mixing angles were familiar from the behavior of quarks. However, the small mixing angle and the vacuum solution produced strong spectral distortions that we just don't observe. The LMA solution reduces the ν_e flux evenly across the spectrum, and by 2002 it had emerged as the likely explanation.

A detailed analysis yields the following oscillation result:¹⁷

$$\Delta m_{21}^2 \approx 6 \times 10^{-5} \text{ eV}^2, \quad \theta_{12} \approx \pi/6. \quad (23)$$

We can now see from Eq. (17) that the vacuum oscillation length is a few 100 km. This length is small compared to the 35 000 km radius of the solar core where these neutrinos are born. Thus vacuum oscillation effects are totally washed out. So, for understanding solar neutrinos: think mass eigenstates!

The flavor composition of mass eigenstates in the LMA scenario is shown in Fig. 6. The simplest expression that is consistent with the data for the composition of neutrino mass eigenstates in a vacuum is²²

$$\nu_1 \approx \sqrt{\frac{3}{4}} \nu_e - \sqrt{\frac{1}{8}} (\nu_\mu - \nu_\tau), \quad (24a)$$

$$\nu_2 \approx \sqrt{\frac{1}{4}} \nu_e + \sqrt{\frac{3}{8}} (\nu_\mu - \nu_\tau), \quad (24b)$$

$$\nu_3 \approx \sqrt{\frac{1}{2}} (\nu_\mu + \nu_\tau). \quad (24c)$$

Whether this scenario is approximately or exactly correct is one of the biggest questions in neutrino physics at present. Evidence, or lack thereof, for a ν_e component in ν_3 is crucial, as noted above. To understand how we come to this result, we have to back track the neutrinos from a vacuum to the heart of the Sun. As the rising density starts to single out the ν_e as special, nearly all the electron flavor is piled into ν_2 , whose effective mass rises because of the preferential interaction between ν_e and electrons. This effect is shown in Fig. 7; it is somewhat energy dependent, but above a neutrino energy of 5 MeV, the ν_e component of ν_2 has risen from a vacuum value of 25% to $>99\%$. (For a more mathematical account, see Ref. 23). Hence when a high energy ν_e is born in a fusion or decay reaction, it becomes mostly ν_2 , with a bit of ν_1 , and possibly a tiny bit of ν_3 . These mass states then proceed out of the Sun and on to the Earth. In doing so, ν_2 becomes a fairly equal mix of all flavors, and

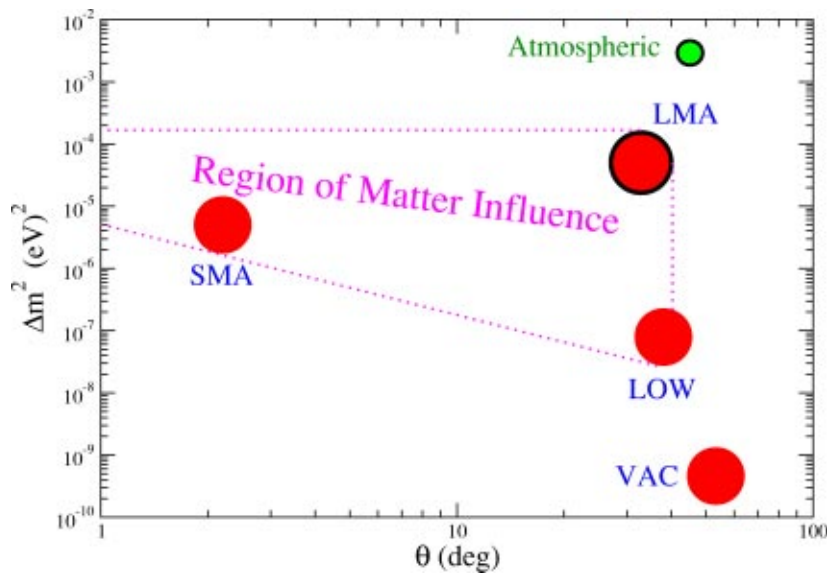


Fig. 5. Various solutions to the solar neutrino which were popular prior to SNO's 2002 papers, plotted in $\Delta m^2 - \theta$ space. The vacuum, low (LOW) and small mixing angle solutions are now thought to be much less likely than the large mixing angle (LMA) solution. The region where matter effects come into play is shown, as is the solution to the atmospheric neutrino problem, for comparison.

that is basically what we measure (along with possibly tiny contributions from ν_1 and ν_3).

There are hints in the Sudbury Neutrino Observatory data that the ν_e fraction in observed solar neutrinos is slightly higher at night. The LMA scenario can easily explain this increase by allowing the ν_2 to become a little more electron-like after taking a long run through the dense Earth at night.

A word of caution regarding the interpretation of Fig. 7. The electron component of ν_3 is not confirmed and might be very small. The mass scale assumes that m_1 is zero, which might not be the case. In addition, the smaller mass difference may be between eigenstates two and three rather than one and two; we cannot tell as yet. In any case, these mixings are fundamental and the next challenge is to figure out why they are this way.

C. Confirmation from reactor neutrinos

Because of the potential uncertainties surrounding the generation of solar neutrinos and the importance of the neutrino

oscillations, it was vital to confirm the solar results with a controllable, terrestrial source of low energy neutrinos. The only source intense enough for neutrino detection at distances large enough to see oscillations (see Fig. 2) is a commercial nuclear reactor, or, better still, a group of them. This idea was the origin of the Japanese ultralong-baseline reactor experiment, KamLAND.²⁴ The KamLAND detector sits in the old Kamiokande cavity and can “see” several nuclear reactors, which sit an average of 180 km away. It consists of 3000 tons of liquid scintillator, which allows the reactor antineutrinos to be absorbed by hydrogen nuclei [the reaction in Eq. (8)] and produce positrons. The liquid scintillator produces much more light per unit energy than does the Cherenkov process, enabling KamLAND to see much lower energy reactions than the Super-Kamiokande or Sudbury Neutrino Observatory. This feature is very important given the low energy of many reactor neutrinos and the fact that the oscillation effects increase with decreasing energy [Eq. (17)]. The solar results were confirmed and refined by Kam-

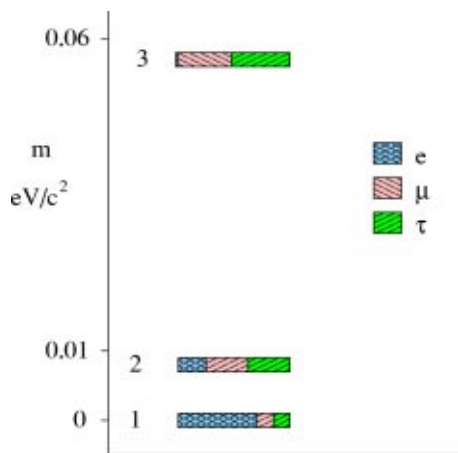


Fig. 6. The flavor composition of mass eigenstates assuming the LMA solution for solar neutrino oscillation. The electron component of ν_3 is not confirmed and may be very small. The mass scale assumes that m_1 is zero, which may not be the case. The smaller mass difference may be 2–3 rather than 1–2.

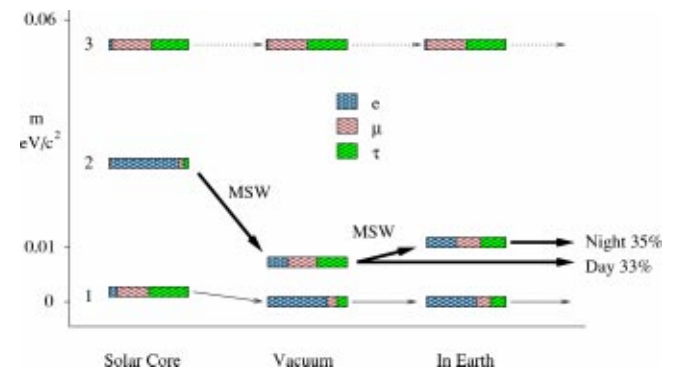


Fig. 7. A cartoon of the LMA solution for solar neutrino oscillation. We show here the big picture of solar neutrino physics—including what we know about atmospheric neutrinos—in terms of mass eigenstates and their flavor components. Representing the flavor components as fixed fractions is only really useful here because the oscillations are washed out due to the source size. I assume a noninverted hierarchy and $m_1 = 0$. At the moment there is no evidence for the electron component in ν_3 ; shown here is roughly the maximum possible value. The electron component of ν_1 and the nonelectron component of ν_2 are likely very small. I have also exaggerated the regeneration of ν_e in the earth; the effect may be very tiny.

LAND in December 2002 when the collaboration reported seeing only 60% of the expected no-oscillation antineutrino signal.

VI. BIG QUESTIONS

A. Why don't charged electrons and muons oscillate?

Electrons, muons (and taus) are charged. The things that we know oscillate (neutrinos and K^0 's) are neutrals. We know two things about neutrals: we can deduce that they were made (by observing other reaction products), and we can deduce that they died (by decay or absorption products). What goes on in between is anyone's guess. You can figure out that you just made an electron neutrino, and somewhere down the beam pipe figure out that a muon neutrino just interacted and died. The electron and muon neutrino do not have well-defined masses (they are not mass eigenstates), but the mass splittings are tiny, so whatever oscillation occurs, we do not have to worry about mismatches in measured energies or momenta.²⁵ In between life and death, however, they travel as mass eigenstates ($\nu_{1,2,3}$).

Charged particles are different, we can track them through a drift chamber. Like Schrödinger's cat, they interact too much with the environment to be in an ill-defined state. In addition the mass differences are huge (at least 100 MeV, a billion times the biggest neutrino mass difference), so if you watch a muon suddenly become an electron for a few meters and then revert to a muon, there would be major accounting difficulties on the energy/momentum front. For muons and electrons, the mass eigenstates and flavor eigenstates are one and the same thing.

B. Why are the mixings so large?

We don't know. A clue will come when we measure the electron component in ν_3 at the Japanese Hadron Facility (now known as JPARC) in the next decade.¹⁸ This angle may be merely small or tiny enough to be a clue to some new physics. The value of this angle will also tell us whether the $\nu_\mu - \nu_\tau$ mixing is actually or merely approximately maximal, and thus whether the angles are somehow randomly chosen or in some way special.

C. If neutrinos exit the Sun without interaction, how does the density of the solar core affect them?

This question is the most difficult to explain to the person at the bus stop. It is about amplitudes and probabilities. Probabilities (of interaction, say) can be expressed as the squares of quantum mechanical amplitudes. A solar neutrino can pass through a light year of lead with only a small chance of interaction, so the probability here is tiny. But the amplitude (its square root) is not that small. The way the electron density in the solar core affects neutrinos depends, however, on amplitudes. Hence, it is possible to alter the neutrino states while the probability of interaction is negligibly small. The characteristic distance required for this skewing of neutrino states is only about 100 km in the core of the Sun, and a few thousands of kilometers in the Earth.¹³ This large distance is one reason why the day-night asymmetry, if it can be measured at all, is very small. The Sun never dips too far below the horizon even at the most southerly solar neutrino detector (Super-Kamiokande, 36°N), and therefore the path length of the neutrinos in the Earth is seldom large enough to alter the

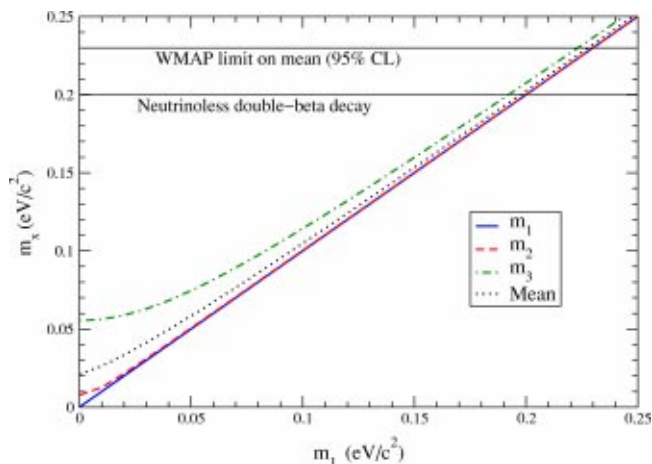


Fig. 8. A combination of the neutrino oscillation evidence with absolute mass limits from measurements of the cosmic microwave background (WMAP) and double-beta decay. The masses $m_{1,2,3}$ are plotted for varying values of m_1 , from zero up to the absolute mass limit.

flavor content. The current best measurement indicates that the nocturnal ν_e flux in Sudbury, Ontario (46°N) is $(7 \pm 5)\%$ greater than in the day.¹⁷

D. What are the absolute masses of the neutrinos?

We're closing in on this question; the masses are plotted as a function of m_1 in Fig. 8.²⁶ The minimum possible value of m_3 is about $0.06 \text{ eV}/c^2$, but oscillation measurements cannot tell us what the absolute masses are.

Physicists have been trying to measure neutrino masses directly by looking at β -decay spectra ever since neutrinos were first conceived. A nonzero mass would reduce the high energy end-point of the spectrum. Although direct, this approach is limited by apparently unavoidable statistical and systematic errors and is not sensitive to masses less than a few eV/c^2 .

Enlightenment may well come from less direct measurements. Observation of ripples in the cosmic microwave background²⁷ have sensitivity to the masses of the primordial (the Big Bang) neutrinos which abound everywhere in the universe (reckoned to be about a billion per m^3). The current best limit from this source is $0.23 \text{ eV}/c^2$. In addition, a variation on the β -decay theme known as neutrinoless double-beta decay²⁸ can potentially go below the crucial $0.06 \text{ eV}/c^2$ and also say something about the fundamental nature of neutrinos (see Sec. VII). The current limit from this source is about $0.2 \text{ eV}/c^2$. Another decade of hard work should close this gap.

E. What is the mass of an electron (or mu or tau) neutrino?

The ν_e doesn't have a well-defined mass; it is a thorough mix of two (or three) neutrinos states which do have well-defined, but different masses. If evidence of a neutrino mass is seen in β -decay spectra, it will be an averaged value (depending on the mixing angles) of these mass eigenstates.

F. Will we be able to see the decay $\mu \rightarrow e \gamma$?

No. By the tenets of the uncertainty principle a muon can turn into a heavy W (the mediator of the weak interaction) and a ν_μ for a very short period of time (see Fig. 9). This ν_μ can in principle oscillate into a ν_e , which can coalesce with the W into an electron. The γ ray carries away the excess energy and momentum and everyone is happy. However, Heisenberg tells us we can borrow 80 GeV for a W for only 10^{-26} s or so. But we already know that it takes thousands of kilometers for a 100 MeV (the muon's mass) neutrino to oscillate. That is many milliseconds (an eternity) at the speed of light, so it is never going to happen.

VII. A BRIEF NOTE ON NEUTRINOS AND ANTINEUTRINOS

While neutrinos and antineutrinos were thought to be massless, the distinction between the two was fairly clear. Massless particles are constrained to move at the speed of light and have definite helicity. Helicity is the orientation of the particle's intrinsic angular momentum (spin) with respect to its velocity. If these two vectors are aligned, the particle is said to be right-handed; if anti-aligned, left-handed. There are no other possibilities. A dazzlingly clever experiment by Maurice Goldhaber and collaborators in 1958 (the last great desktop experiment in particle physics)²⁹ showed us that neutrinos are left-handed, and antineutrinos right-handed. This simple spin assignment seemed adequate at the time, but now we know that neutrinos have tiny masses, and so cannot travel at precisely the speed of light. Hence, one can in principle boost into a reference frame moving faster than the neutrino. In this case the neutrino will reverse its momentum direction but its spin will remain unchanged, and so its helicity will reverse itself. Therefore a left-handed neutrino in one frame will look like a right-handed (anti-?)neutrino in

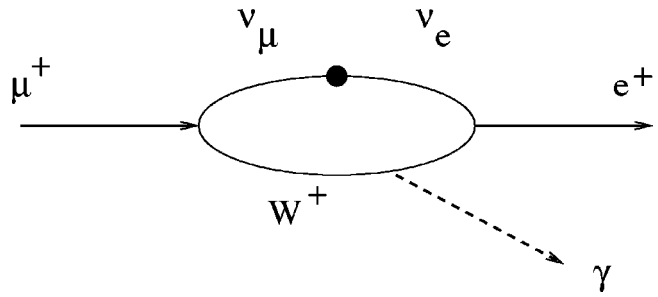


Fig. 9. Feynman diagram for $\mu \rightarrow e + \gamma$. Imagine time running left to right. The heavy W^+ can link a muon with a muon neutrino, or an electron with an electron neutrino, and also—because it is charged—shake off a gamma ray. Hence if the neutrinos mix, so must the muon and electron. However, the big mass-energy of the W^+ can only be borrowed for a brief 10^{-26} s, too short for the leisurely neutrino oscillation to take place.

another, although a simple change of frame should not change its intrinsic nature and it should still be a neutrino.

There are two possibilities. Either right-handed neutrinos exist (and left-handed antineutrinos), or somehow neutrinos and antineutrinos are the same thing, and we only distinguish them by the helicity we measure. Here the measuring tool is the weak interaction itself, which only allows for different reactions depending on the helicity of the particle (which we may have mistakenly identified as “neutrinos” and “antineutrinos”). In the first case we refer to “Dirac neutrinos,” ($\nu \neq \bar{\nu}$) and in the second, “Majorana neutrinos” ($\nu = \bar{\nu}$), named for Paul Dirac and Ettore Majorana, respectively, two prominent early theorists in the field.

Neutrinoless double beta decay²⁸ is a two-step β -decay process in which two neutrinos annihilate each other before emerging. An observation of this process would not only give us a good idea of the neutrino mass scale, it would also tell us that neutrinos are Majorana particles and capable of

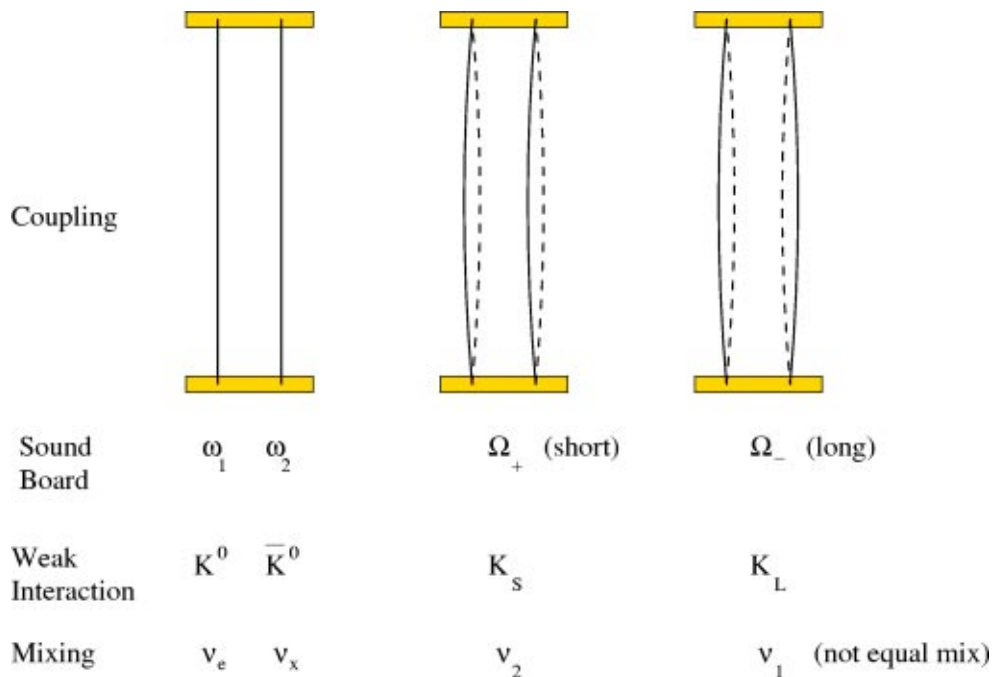


Fig. 10. The analogy between piano strings, neutral kaons, and vacuum oscillations. That for neutral kaons is rather exquisite, but for neutrinos we have to remember that the mixture of e and $x = \mu - \tau$ in $\nu_{1,2}$ is not equal but in the ratio $\frac{1}{4} :: \frac{3}{4}$.

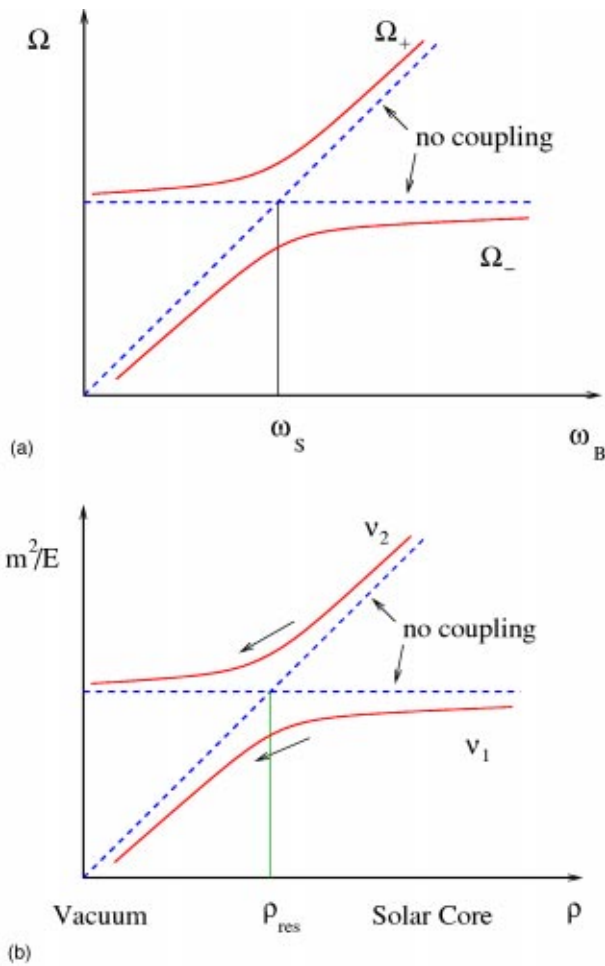


Fig. 11. (a) Resonant frequencies Ω_{\pm} of a system consisting of a sound board with a characteristic frequency ω_B and two identical strings with frequency ω_S . The dashed line shows the case where there is no coupling between string and sound board. (b) The effective mass-squared of two neutrino species as a function of the electron density of the medium. The ν_2 eigenstate is largely ν_e inside the Sun; the ν_1 eigenstate is largely a non-electron flavor.

self-annihilating. If not observed even with a mass-sensitivity below the minimum limits set by oscillations, then neutrinos are probably Dirac particles. There have been discussions of this issue in Ref. 30.

VIII. A MUSICAL ANALOGY

Consider two adjacent piano strings that are tuned to the frequencies, $\omega_1 \approx \omega_2$, and attached to a sound board. If we pluck or strike one string, soon both strings will be oscillating with a mixture of the two modes, one with the strings in phase, one with them in antiphase. The frequencies of these modes are Ω_+ and Ω_- as shown in Fig. 10. The secret of the piano sound is that the Ω_+ mode couples very strongly to the sound board and makes a loud but rapidly decaying sound. The Ω_- mode couples only weakly to the sound board (the motion of one string negates the other) and produces a soft but sustained sound.³¹

The two strings are a useful approximate analogy for neutrino vacuum oscillations. Each string can be considered a flavor eigenstate, but the time development is in terms of the Ω_{\pm} modes. Averaged over time, half the energy appears in

each string, regardless of which string is struck. However, the neutrino mixing is not 50–50 (at least not for solar neutrinos), and neutrinos do not decay, as does the sound of a string.

As an aside, this analogy is much more elegant when used to explain the other significant oscillation phenomenon in particle physics, that of neutral kaons. Neutral kaons are made as flavor eigenstates K^0 and \bar{K}^0 , but these are not mass eigenstates (just like neutrinos). Whichever kaon we make, it propagates as a mix ($K^0 + \bar{K}^0$ or $K^0 - \bar{K}^0$) of two states which have radically different decay times (called K-long and K-short). In analogy with the strings, we can regenerate the rapidly decaying component by damping one string in the same manner as one regenerates K-short particles by placing a thin piece of material in the beam, thus preferentially “damping out” the K^0 's.

If we consider a sound board to have a characteristic frequency ω_B and two identical strings with frequency ω_S , then we find that the resonant frequencies of the whole system Ω_{\pm} behave as shown in the upper part of Fig. 11.³² Now look at the way neutrino masses are usually plotted, as m^2/E against the density of the medium in the lower part of Fig. 11,^{21,14} and the similarity is clear.

With no coupling we have two frequencies, ω_B and ω_S . With coupling the same is true so long as ω_B is either much higher or much lower than ω_S . The interesting part occurs when the two frequencies are similar, and their behavior is very similar to that of ν_2 as they travel out from the solar core. At birth in the dense core, ν_2 is dictated by fundamental physics and the electron density (that is, the sound board frequency ω_B). As it leaves the Sun, this state passes through the resonance region and emerges as the higher of the two mass states (Ω_+), the one with 25% electron content.

Incidentally, the small mixing angle solution involves a rapid passage through the resonance region which allows a transition from the Ω_+ to Ω_- states, even with a tiny coupling. As we have seen, however, nature did not go this route.

IX. SUGGESTED PROBLEM

For simplicity, consider all atmospheric muon neutrinos to have an energy of 1 GeV. In an underground detector you observe these neutrinos coming from above and below the horizontal, and compare the numbers with what you expect from a computer model of their production. Assume that they all are produced at 10 km altitude in the atmosphere. If you see $(95 \pm 5)\%$ of the expected number above the horizontal and $(50 \pm 5)\%$ coming from below, you conclude that oscillations are at work. What possible values can the oscillation parameters Δm_A and θ_A have? Hint: Convert the length in Eq. (17) to an observed zenith angle and replot Fig. 3 for different oscillation parameters. A spreadsheet calculation is adequate. Given that equal intervals in $\cos \theta_z$ are equivalent to equal solid-angle areas of the sky, it should be easy to estimate the observed fractions above and below the horizontal.

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