Forward dispersion relations for Compton scattering and finite energy sum rules for nucleons and light nuclei: New results.

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Content

$\star\bullet$ Prologue: Non-relativistic dipole sum rules for atomic and nuclear photoeffect.

$$\sigma_n(E1) = \int\limits_{thr}^{\infty} d\nu \, \nu^n \, \sigma_{E1}(\nu)$$

Examples: $n = -2 \ \rightarrow$ Kramers-Heisenberg sum rule (SR) for static electric-dipole polarizability of a given quantum system;

$n = -1 \ \rightarrow$ the bremsstrahlung-weighted SR, dependent of charged-"parton" correlation in a given system;

$n = 0 \ \rightarrow$ the famous TRK SR $\rightarrow$ a precusor of Quantum Mechanics.

$$
\begin{aligned}
\sigma_{-2(0)}(E1) &= 4\pi^2\Sigma_n(E_n - E_0)^{\mp 1}\cdot |<n|D_z|0>|^2 \\
&= 2\pi^2\alpha_{E1}, n = -2; \\
&= 2\pi^2 < [D_z[H, D_z]] >, n = 0; \\
\sigma_{-1}(E1) &= \frac{4}{3}\pi^2 < \vec{D}^2 > .
\end{aligned}
$$

Early estimation of the GDR energy in the photonuclear physics (Migdal(1945)):

$$
\bar{E}_{dip} = (\frac{\sigma_0(E1)}{\sigma_{-2}(E1)})^{1/2} \sim A^{-1/3}
$$

another mean value & nucleon corr.problem

$$
\bar{E}^{'}_{dip} = (\frac{\sigma_0(E1)}{\sigma_{-1}(E1)}) \sim \frac{A}{A\cdot A^{2/3} + (corr.terms)}
$$

Disp.Rel.- the beginning of history:

H.A.Kramers (1927); R.Kronig (1946)

Dispersion Relations in QFT(1954-1956):

M.Gell-Mann, M.L.Goldberger, W.Thirring, Phys.Rev.**95**(1954) 1612.

M.L.Goldberger, Phys.Rev.**97**(1955)508. ”Use of causality condition in quantum theory”

R.Karplus, M.A.Ruderman, Phys.Rev. **98**(1955)771. ”Application of causality to scattering”.

N.N. Bogoliubov, Report at Int.Seattle Conference, 1956.

N.N. Bogoliubov, D.V.Shirkov, DAN SSSR,**113** (1957) 527.

Power of DR approach:

- very general underlying assumptions,
- many general properties of the scattering amplitudes may be deduced in a simple way.

In particular:

- GGT: the first example of the ”superconvergent” sum rule(s) for the photon absorption cross sections on nucleons and nuclei.
- Goldberger, Miyazawa,Oehme(1955): dispersion sum rule for pion-nucleon scattering lengths.
- S.G.(1965): dispersion sum rule for spin-dependent photoabsorption cross-section on quantum systems of any spin.

The present report originates from two early papers of author dealing with the total photon-hadron cross-sections:

•S.G., Phys.Lett., **5**(1963) 259. ”Sum rules and high-energy photonuclear absorption”.

• S.G., Phys.Lett. **13**(1964) 240.

”On the Thomas-Reiche-Kuhn sum rule” .

•S.G. in Proc.Int.Conf. on E.m. Interactions at Low and Medium Energies, AN USSR, Moscow, 1972, v.3, p.382.

and aims to present some new experimentally testable and theoretically interesting relations emerging from the dispersion FESR phenomenology.

Gell-Mann, Goldberger and Thirring (1954) $\rightarrow$ the "superconvergence" sum rule technique:

$$\Delta T = T_{\gamma A}(\nu) - ZT_{\gamma p}(\nu) - NT_{\gamma n}(\nu)$$

$$2\pi^2 \frac{\alpha}{M_n}(\frac{-Z^2}{A} + Z) + \int\limits_{\nu_{\gamma\pi}}^{\infty} d\nu [Z\sigma_{\gamma p}(\nu) + N\sigma_{\gamma n}(\nu)$$
$$-\sigma_{\gamma A}(\nu)] = \int\limits_{\nu_{thr}}^{\nu_{\gamma\pi}} d\nu \sigma_{\gamma A}(\nu)$$

TRK$\rightarrow$ nuclear E1-photoabsorption sum rule:

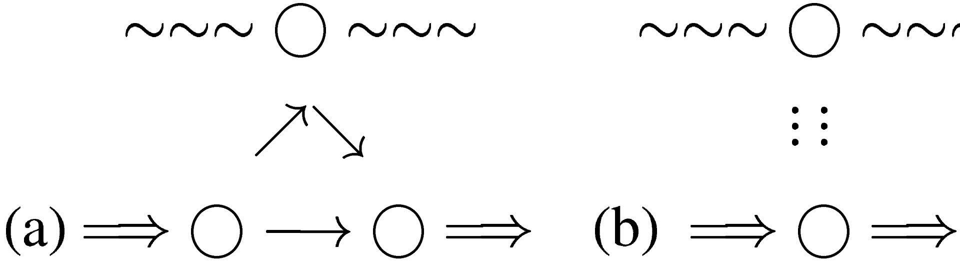
$$\sigma_0(E1) \equiv \int\limits_{\nu_{thr}}^{\infty} \sigma_{E1}(\nu) d\nu = 4\pi^2 \Sigma_n (E_n - E_0) \cdot$$

$$\cdot |\langle n|D_z|A\rangle|^2 = 2\pi^2 < |[D_z[H, D_z]] >$$

$$= (2\pi^2 \alpha NZ)/(AM_n) + 2\pi^2 < [D_z[\hat{V}_{NN}, D_z]] >$$

$$\equiv (2\pi^2 \alpha NZ)/(AM_n)(1 + \kappa(A))$$

2⋆ Role of pion d.o.f. via dispersion approach



Graph (a) represents the impulse approximation (IA), while (b) defines the correction related with the nuclear "collective" pion cloud and that is effective due to short-ranged NN-correlation inside nuclei. Their relative role can qualitatively be characterized by the ratio

$$\frac{t_{cut}(IA)}{t_{cut}(2\pi)} \simeq \frac{8m_n\varepsilon_b}{(A-1)\cdot 4\mu^2}$$

where t_{cut} refers to the beginning of the cut in complex t-plane with the physical region of the momentum trasfers $t \leq 0$. For instance, the ratio is $\sim .22$ (.40 and .66) for the d (^3He and ^4He, respectively). This means that for ^3He and ^4He the "pionic" contributions will be relatively more important compared to deuteron.

$2 \star\star$ The GGT$'\cup$ TRK$'$

It was suggested (S.G.,1964) and perturbatively (to one-loop order) checked later (S.G. and J.Moulin,1974) in scalar, ϕ^3-type "super-renormalized" model, that the generalized Thomas-Reiche-Kuhn is valid for total photoabsorption cross section

$$\sigma_0 = \int d\nu \sigma_{tot}(\nu) = 2\pi^2 \langle \phi_1 | [D[H, D]] | \phi_1 \rangle$$

where the charged scalar field ϕ_1 is locally connected with two scalar fields,the ϕ_2 being charged one and other, ϕ_3,neutral. The double commutator is then interpreted via the known Schwinger-term, *i.e.* the equal-time commutator of the time- and spacial-component of e.m. current operator. Hence, the generalized, "GGT$'$"-sum rule is written (S.G.,1972) in the form

$$\sigma_0^{\gamma A} - Z\sigma_0^{\gamma p} - N\sigma_0^{\gamma n} = 2\pi^2\alpha[\tfrac{NZ}{Am} + \int d\vec{x}(\langle A|\phi^*\phi|A\rangle - \Sigma_i\langle N_i|\phi^*(x)\phi(x)|N_i\rangle)]$$

2• Per aspera ad...$< A|\phi^*\phi|A >$: Why's & Ways.

The photonuclear sum rule including the terms $< A|\phi^*\phi|A >$ and $< N|\phi^*\phi|N >$, represented by the Feynman diagram (a) was rediscovered and widely discussed by M.Ericson et al.,Nucl.Phys. **A663 & 664**(2000) 369c. The matter is that the matrix element corresponding to graph (b) is directly connected with the Σ_π-term,i.e. the chiral symmetry breaking and its restoration.

$$
\begin{array}{ccc}
\sim\sim\sim \bullet \sim\sim\sim & \qquad & \otimes \\
\vdots\,\vdots & & \vdots\,\vdots \\
(a)\Longrightarrow \bigcirc \Longrightarrow & & (b)\Longrightarrow \bigcirc \Longrightarrow
\end{array}
$$

•• FESR and problem of ”Big Circle”

The standard FESR techniques enable us to confine ourselves with amplitudes in the finite region of the complex energy plane

$$f(\nu) = \frac{1}{2\pi\imath}\oint dz \frac{f(z)}{z-\nu}$$

where $f(\nu)$ is the spin-averaged, forward Compton scattering amplitude and the integration contour along the real axis and the large circle in the complex energy-plane. Our choice of the ”superconvergent” combination of Compton amplitudes $f_{\gamma A(p,n)}$ is different from GGT.It includes amplitudes of two nuclei with $A_1 = Z_1 + N_1$, $A_2 = Z_2 + N_2$:

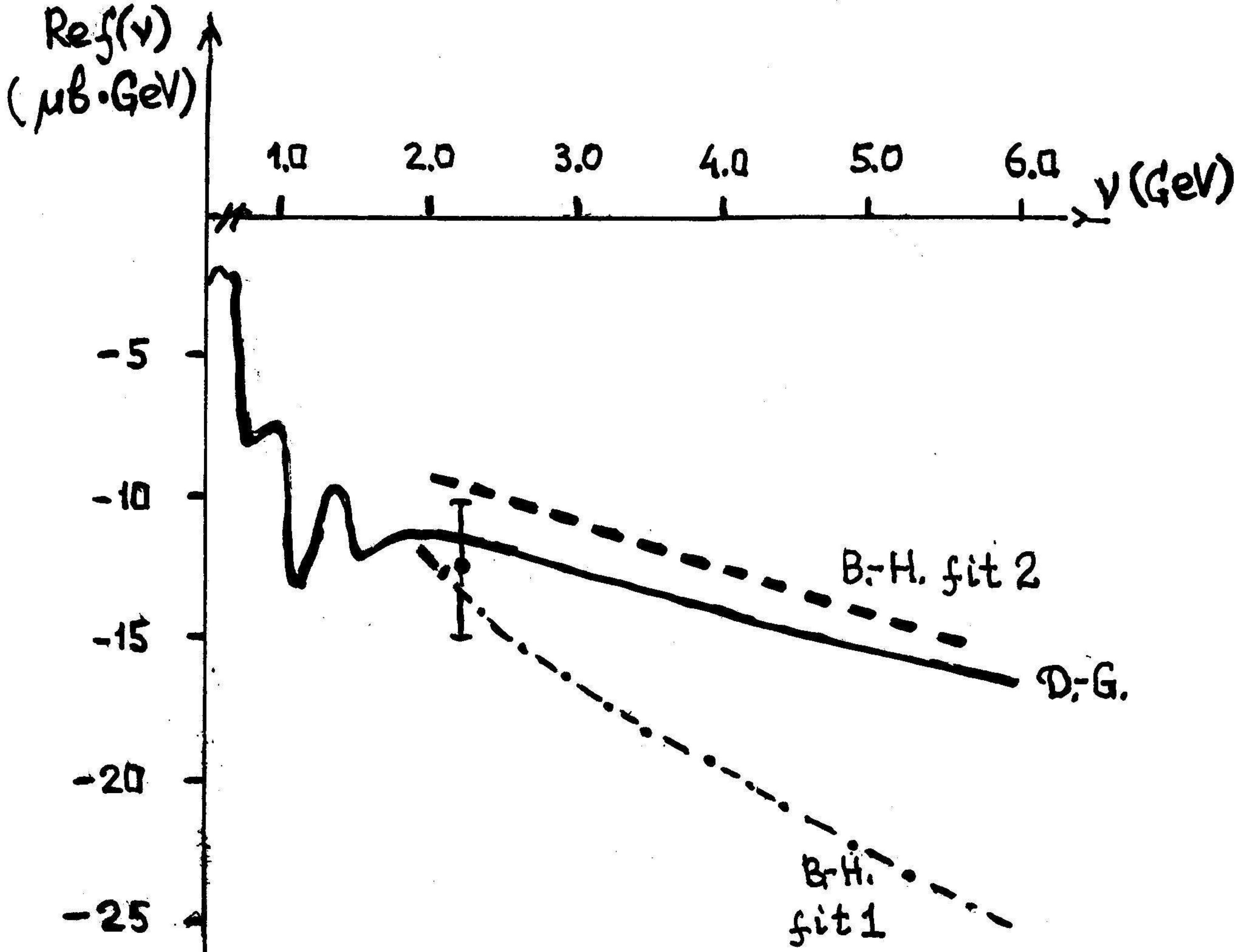
$$\frac{1}{A_1} f_{A_1} - \frac{1}{A_2} f_{A_2} = \frac{Z_1 N_2 - N_1 Z_2}{A_1 A_2}(f_p - f_n)$$

This is due to the need to include in the parameterization of the Compton amplitudes the constant, energy-independent terms reflecting the possible presence of the non-Regge, fixed $j = 0$-pole contributions to the real parts of γN-amplitudes at high energies. This problem has a long and not finished story yet. For the γp-amplitude a rather solid evidence for the presence of $j = 0$-pole residue of the order $C_p \simeq -3\mu b \cdot GeV$ giving contribution to $Ref_p(\nu)$ has been claimed by Damashek and Gilman ('70).However their high-energy form of the γp-amplitude seems to be at variance with modern ones, including the log-rising terms. In the work by M.M.Block and F.Halzen(Phys.Rev.**D 70**(2004) 091901(R); "Evidence for the saturation of the Froissart bound") a provocative idea was advanced about the smooth join of the Compton scattering amplitude including the log-rising terms to experimetally measurable amplitude at rather low energy of photons of order $\sim$ 2 GeV. It would be very interesting to confront this hypothesis with the new and improved measurements of the real part of Compton scattering. Two proposed fits:

$\sigma_{\gamma p} = c_0 + c_1 log(\nu/m) + c_2 log^2(\nu/m) + b_R(\nu/m)^{-.5}$

$c_0 = 105.6(92.5); c_1 = -4.74 \pm 1.17(-.46 \pm 2.88); c_2 = 1.17 \pm .16(.8 \pm .3); b_R = 64(78)$

give different energy dependence of the real part $Ref_p(\nu)$ at the photon energy of 2-6 GeV.



Re f(ν)
(μb·GeV)
1.0
2.0
3.0
4.0
5.0
6.0
ν (GeV)
-5
-10
-15
-20
-25
B.-H. fit 2
D.-G.
B.-H.
fit 1

•• FESR and problem of the "Big Circle" modelling (cont'd)

- M.Damashek and F.J.Gilman, Phys.Rev.**D1** (1970)1319. "Forward Compton Scattering"

Assumed parameterization of $f_p(\nu)$:

$$Imf(\nu) = (\nu/4\pi)\sigma_{tot}(\nu) = \Sigma_{i,\alpha_i(0)} c_i \nu^{\alpha_i(0)}$$

$$Ref(\nu) = (1/4\pi)\Sigma_i c_i(-cot\frac{\pi\alpha_i}{2})\nu^{\alpha_i(0)} + C_p$$

If $\sigma_(tot) = 96.6 + 70.2/\nu^{1/2} : C_p = -2.5\mu b \cdot GeV$

- H.Alvensleben, et al. Phys.Rev.Lett. **30**('73)328; "Experimental verification of the Kramers-Kronig Relation at High Energy".

$A = A_c(\gamma p \to \gamma^* p \to l^+ l^- p) + A_{BG} : Ref_p(2.2GeV) = -12.3 \pm 2.4\ \mu bGeV$

- M.M.Block and F.Halzen, Phys.Rev.**D 70**(2004) 091901(R); "Evidence for the saturation of the Froissart bound". Smooth join to DG, two fits, but additional constant in $Ref(\nu)$ taken, however, *ad hoc*.

• Our resulting sum rule for nucleons

Parameters:

$$Imf_p(\nu) - Imf_n(\nu) = (\nu/4\pi)(\sigma_p^{tot} - \sigma_n^{tot}) = b_{a_2}\nu^{1/2}$$

$$Re(f_p(\nu) - f_n(\nu)) = (1/4\pi)b_{a_2}(-\nu^{1/2}) + C_p - C_n$$

$\sigma_p^{tot}(\nu) - \sigma_n^{tot}(\nu) = 24.6/\nu^{1/2}$

$C_p = -3.0\mu b \cdot GeV; C_n$-is a free parameter.

The energy interval considered: $\nu_{min} = \nu_{thr}(\pi)$

$\nu_{max} = 1.64\ GeV$, corresponding to $s(\gamma N) \simeq 2\ GeV^2$ The meson photoproduction cross-sections on the neutron are largely unknown and should be extracted *e.g.* from the deuteron data. Of all possible photo-meson reactions, the best known is the single pion photoproduction. Therefore we treat the neutron cross-sections entering our sum rules as follows. The $\sigma_{\gamma n}^{tot}(\nu)$ is split into two parts: $\sigma_{\gamma n}^{tot} = \sigma(\gamma n \to \pi N) + \sigma(\gamma n \to 2\pi N + \dots)$

The single pion production cross-section is taken according to theoretical calculation with fairly good multipole amplitudes of the MAID Collaboration. The detailed experimental study of the meson photoproduction on the deuteron target is planned at the MAMI electron accelerator (Mainz, Germany) up to photon energies $\sim 1.5\ GeV$. So, anticipating the appearance of the γn-data, needed for the checking of FESR sum rule for the difference of the γp- and γn- Compton amplitudes and extracting the value $C_p - C_n$ required further for definition of the nuclear sum rules, we present first the dependence of the experimentally measurable ratios $R^{tot}_{n/p}(R^{non-res}_{n/p})$, defined as:

$$\frac{\sigma_0^{tot}(\gamma n \to 2\pi + X)}{\sigma_0^{tot}(\gamma p \to 2\pi + X)}\left(\frac{\sigma_0^{non-res}(\gamma n \to 2\pi + X)}{\sigma_0^{non-res}(\gamma p \to 2\pi + X)}\right)$$

as the function of several plausible values of C_n, taking $C_p = -3.0\ \ \mu b GeV$ for granted. The results are presented in Table.

Table

C_p	C_n	$R^{tot}_{n/p}$	$R^{non-res}_{n/p}$
0	0	.95	.98
-3	-2	.72	.62
-3	0	.60	.39

For illustrative reasons, we indicate the results of the modelling the neutron-to-proton ratios as following from the ratios of the electric dipole moment fluctuation in the lowest hadronic Fock-components of the nucleon with at least one charged pion: $N \leftrightarrow \pi + N, 2\pi + N, \pi + \Delta(1231)$,$\varepsilon = (m_\pi / m_N$:

$$\frac{\langle \vec{D}^2(n \leftrightarrow p\pi^-)\rangle}{\langle \vec{D}^2(p \leftrightarrow n\pi^+)\rangle} \simeq 1 + 2\varepsilon$$

$$\frac{\langle \vec{D}^2(n \leftrightarrow n\pi^+\pi^-)\rangle}{\langle \vec{D}^2(p \leftrightarrow p\pi^+\pi^-)\rangle} \simeq r_{uncorr}$$

$$r_{uncorr} \simeq \frac{(1+4\varepsilon)\langle \vec{r}^{\,2}_{\pi^+\pi^-}\rangle}{(1+2\varepsilon)\langle \vec{r}^{\,2}_{\pi^+\pi^-}\rangle + 2\varepsilon\langle(\vec{r}_{\pi^+\pi^-}\cdot\vec{r}_{p\pi^-})\rangle} \geq 1$$

$$r_{corr} \simeq \frac{\langle \vec{D}^2(n \leftrightarrow \pi\Delta)\rangle}{\langle \vec{D}^2(p \leftrightarrow \pi\Delta)\rangle} \simeq \ .66\ (.41)$$

The first value in r_{corr} refers to the sum over all possible charge $\pi\pi$-states produced in the final decay stage $\pi\Delta \to \pi\pi N$, while the second ratio corresponds to the selection of the $\pi^+\pi^-$ final states. The numerical relevance of r_{corr} to the last two rows in the Table testifies on the crucial importance of the correlation of the valence and nonvalence partonic composites of the nucleon in producing of the ultimate characteristics of the Compton scattering amplitude.

$\star\star$ Photoabsorption sum rule on lightest nuclei

As an example of the generalized nuclear sum rule we choose a pair of lightest nuclei - the deuteron and ^3He. While in the deuteron case the total photoabsorption cross section is known well above our taken $\nu_{max} \simeq 1.6\ GeV$, the $\sigma_{tot}(\gamma^3He)$ is known to $.8\ GeV$, hence, in this case we should use $\nu_{max} = .8\ GeV$. For arbitrary $A_1 = Z_1 + N_1$ and $A_2 = Z_2 + N_2$ our general sum rule reads:

$$2\pi^2[\frac{f_{A_1}(\nu{=}0)+S_\pi(A_1)}{A_1}-\frac{f_{A_2}(\nu{=}0)+S_\pi(A_2)}{A_2}+\frac{Z_1N_2-Z_2N_1}{A_1A_2}\cdot(\frac{2b_{a_2}\nu_{max}^{1/2}}{2\pi^2}-C_p+C_n)]$$
$$=\frac{\sigma_0^{\nu max}(\gamma A_1)}{A_1}-\frac{\sigma_0^{\nu max}(\gamma A_2)}{A_2}$$

where $f_{A_i}(\nu=0)\simeq-(\alpha Z_i^2)/(A_i m_n)$ is the Thompson zero-energy amplitude, $S_\pi(A_i)$ is defined below

$$S_\pi(A_i)\simeq\frac{\alpha}{3}\int d^3x\langle A_i|\vec{\phi}(x)\vec{\phi}(x)|A_i\rangle$$

and the integration in $\sigma_0^{\nu_{max}}$ extends from the photodisintegration threshold to the upper bound ν_{max}.

The main aim of the using the nuclear sum rules appears to be the extraction of information about the value of difference of the nuclear matrix elements:

$\Delta\Sigma_\pi=\int d\vec{x}(\mu^2/2)[\frac{1}{A_1}<A_1|\vec{\phi}(x)\cdot\vec{\phi}(x)|A_1>--\frac{1}{A_2}<A_2|\vec{\phi}(x)\cdot\vec{\phi}(x)|A_2>].$

The term $\Delta\Sigma_\pi$ can thus be measured via experiments to give useful information on the chiral symmetry characteristics in real nuclei.

• Discussion

• To have a qualitative idea about the scale of the $\Delta\Sigma_\pi$ it is reasonable to compare the value of $(4\pi^2\alpha)/(3\mu^2)[(1/3)\Sigma_\pi(^3He) - (1/2)\Sigma_\pi(d)] \simeq 7.75\ (1.17)\ \mu b \cdot GeV$ for $C_p = -3, C_n = -2(0)$ with the value of $60 \cdot [(1/3)(2/3)\kappa_{^3He} - (1/2) \cdot (1/2)\kappa_d] = (1/3) \cdot 40 \cdot (.75 \pm .10) - (1/2) \cdot 30 \cdot (.37 \pm .11) \simeq 4.4 \pm 2.1\ \mu b \cdot GeV$, representing the "potential parts" in the difference of non-relativistic TRK sum rules

$$2\pi^2\alpha[(1/3) <^3 He|[D,[V_{NN}]]|^3He> -$$

$$-(1/2) < d|[D,[V_{NN}]]|d >]$$

. Qualitatively, this correspondence looks rather satisfactory because the non-relativistic value lyes in between two following from more general sum rule with the differing values of C_n. And one cannot but draw attention on the strong dependence of mentioned estimations on two chosen numerical values of C_n.

$\bullet\star$ Considering the difference of "weighted" Compton amplitudes on $He-4$ and d we left with the difference of only $\bar{S}_\pi(He-4)-\bar{S}_\pi(d)$ because the isovector Regge-exchanges are absent for isoscalar nuclei with $Z=N$. Therefore

$$
\begin{aligned}
2\pi^2\cdot(\bar{S}_\pi{}^{bound}(N)-\bar{S}_\pi{}^{free}(N)) &= \bar{S}_\pi(He-4)-\bar{S}_\pi(d)-15\cdot(\kappa(He-4)-\kappa(d)) \\
&= 7.9-15\cdot(1-.4)=-1.1\ \ mb\cdot MeV
\end{aligned}
$$

The resulting sign gives some (weak) evidence for in-medium effect in the $He-4$ in favour of so-called partial restoration of the chiral symmetry in nuclear matter.

$\bullet\star\star$ A rough estimation gives approximately equal mean nuclear density and mean internucleon distances in nuclei $Li-7$ and $He-3$.This, in turn, invites to neglect the difference $\bar{S}_\pi(Li-7)-\bar{S}_\pi(He-3)$ and to try to estimate C_p-C_n from our sum rule and photonuclear data available for these nuclei. With assumed mean accuracy of measurements $\pm1(\pm5)\%$ one can obtain $C_p-C_n=-1.7\pm.5(\pm2.9)\ \ mb\cdot MeV$ from evaluated $\frac{1}{2\pi^2}[\frac{1}{7}\sigma_0(Li-7)-\frac{1}{3}\sigma_0(He-3)]=.31\pm.11(\pm.56)mb\cdot MeV$

- Conclusion to the σ_0-type sum rules

In view of the carried out discussion, the following is assumed to be important:

• To compare the two-pion (and more, if possible!) relative yields in γp- and γn - reactions somehow singled out from γd -collisions.

• To extend measurements of the total photoabsorption on ^3He - and ^4He -nuclei at least up to energy of photons $1.5 \div 2.0 \ \ GeV$.

• New direct measurements of the $Ref_p(\nu)$ in the region $4 \leq \nu \leq 6$ with the method used earlier by group of S.Ting (PRL,1978) would be very helpful and interesting.

• From the nuclear NRQM side: $< A|[D[T+V,D]]|A >$, $A =^{3(4)} He$, $V = AV_{18}$ + rel.corr's+ isobar config's+...

- The accumulation of more detailed and accurate neutron data is highly desirable to diminish the uncertainties of the presented results.

- After all, maybe, the direct study of $l(\gamma)n$ - interactions with the lepton- <u>and</u> neutron-storage rings (?!)..