The possibility to accelerate a Polarized Beam of p, d, t, ^{3}He at JINR nuclotron

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PLAN

- the dynamic of the spin vector in JINR nuclotron
- strength of spin resonances
- spin resonance crossing methods for JINR nuclotron

Basic Nuclotron Parameters

Charge to mass ratio ions		0.33 – 0.5, 1
Injection energy	nuclei protons	5 MeV/u 20 Mev
Maximum energy	for nuclei with q/A=0.5 for protons	6 GeV/u 12.8 GeV
Transition energy		7.6 GeV
Circumference		251.52 m
Duration of acceleration		1 sec
Magnetic rigidity	at injection maximum	0.647 T·m 45.83 T·m
Betatron tunes	ν_x ν_z	6.8 6.85
Nuber of superperiods	P	8
Normalized emittance		4.5π·mm·mrad

Spin Motion at Circular Accelerator

$$\frac{d\vec{S}}{d\theta} = [\vec{W} \times \vec{S}] \, ,$$

Tomas-BMT equation

$\theta -$ **particle azimuth**

[Ya.S. Derbenev, A.M. Kondratenko, 1970-1973] :

Equilibrium closed orbit

$\vec{n}(\theta + 2\pi) = \vec{n}(\theta) -$ **periodical axis of precession**

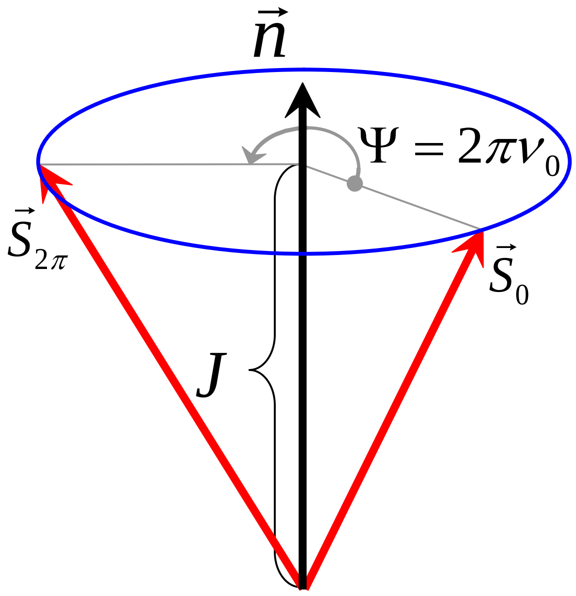
$$\vec{S} = J \cdot \vec{n} + \vec{S}_\perp \, , \qquad J = \vec{S} \cdot \vec{n}, \qquad \vec{S}_\perp \perp \vec{n}$$

Spin vector rotate around n-axis:

If $\vec{S}_0 \parallel \vec{n} \;\Rightarrow\; \vec{S}_{2\pi} = \vec{S}_0$

If $\vec{S}_0 \perp \vec{n} \;\Rightarrow\; \vec{S}_{2\pi} \perp \vec{n}, \;\; \angle\left(\vec{S}_0, \vec{S}_{2\pi}\right) = \Psi = 2\pi\nu_0$

$\nu_0 -$ **spin precession tune**



Not equilibrium orbit

$\vec{n}(\theta + 2\pi, I_i, \Psi_i + 2\pi) = \vec{n}(\theta, I_i, \Psi_i)$

$I_i, \Psi_i -$ **act-phase variables of betatron motion**

$J = \vec{S} \cdot \vec{n} -$ **Spin Adiabatic Invariant**

$\vec{w} = \Delta \vec{W} -$ **spin deviation**

$$\Rightarrow \begin{cases} \Delta \vec{n} - \text{spread of n-axises} \\ \Delta \nu - \text{spread of spin tune} \end{cases}$$

At Nuclotron $\quad \vec{n} = \vec{e}_z \, , \qquad \nu_0 = \gamma G$

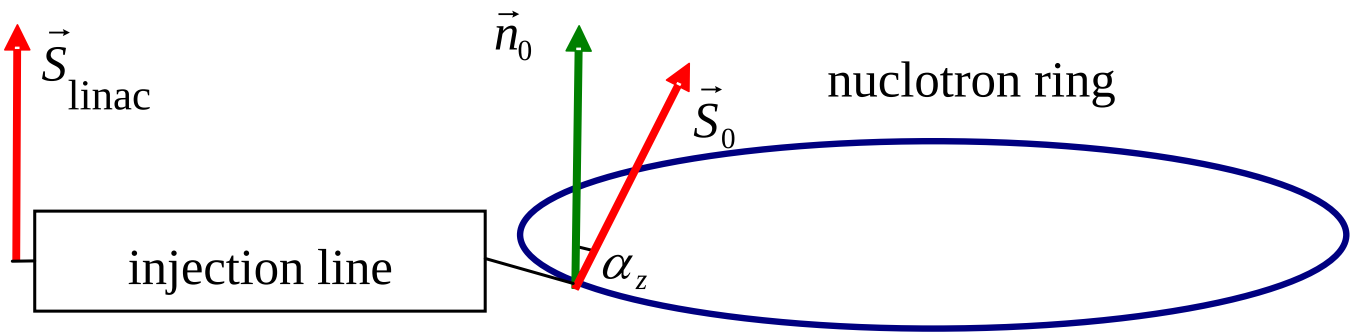
$G = (g-2)/2 -$ **gyromagnetic anomaly**

$\vec{\Pi} = \langle \vec{S} \rangle -$ **vector of polarization,**

$D = 1 - |\vec{\Pi}| -$ **degree of depolarization**

$$\vec{\Pi} = \langle J \, \vec{n} \rangle + \langle \vec{S}_\perp \rangle = \langle J \rangle \langle \vec{n} \rangle$$

Spin matching at nuclotron injection

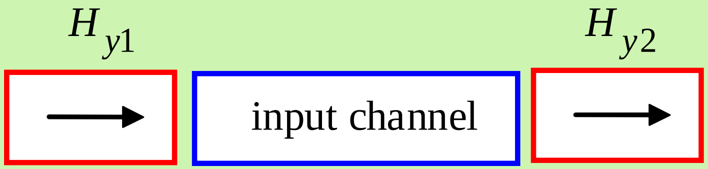


Degree of depolarization due to mismatching

$$D = 2\sin^2\frac{\alpha_z}{2}$$

	^1H	^2H	^3H	^3He
α_z, grad	67	9.8	116	79
D, %	62	1.5	55	81

Scheme of spin matching



	^1H	^2H	^3H	^3He
$(HL)_{y1}$, Tm	0.18	-0.027	-0.12	-0.26
$(HL)_{y2}$, Tm	0.2	0.14	0.04	-0.26

Vector of Polarization and Spin Resonances

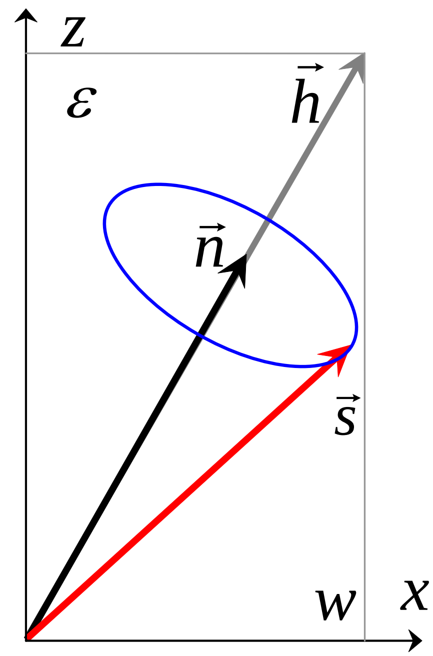
$$\vec{\Pi} = \langle J \rangle \langle \vec{n} \rangle \quad \Rightarrow \quad D = 1 - \left| \langle J \rangle \right| \left| \langle \vec{n} \rangle \right|$$

$\langle J \rangle$ – change of spin integral of motion,

$\langle \vec{n} \rangle$ – spread of n- axises

Vector of polarization is changing in resonance region

Resonance condition: $\nu = \nu_k$, $\quad \nu_k = k + k_x \nu_x + k_z \nu_z + k_\gamma \nu_\gamma$



Resonance coordinate system rotate around n-axis direction with the resonance frequency and spin is moving in the following field:

$$\vec{h} = \varepsilon \vec{e}_z + \vec{w}_k$$

$\varepsilon = \nu_0 - \nu_k$ **is resonance detune**

$$\vec{n} = \frac{\vec{h}}{h} = \frac{\varepsilon}{\sqrt{\varepsilon^2 + w_k^2}} \vec{e}_z + \frac{\vec{w}_k}{\sqrt{\varepsilon^2 + w_k^2}}$$

Far away from the resonance $(|\varepsilon| >> 1)$

$$\left| \langle \vec{n} \rangle \right| = 1$$

At the resonance $(\varepsilon = 0)$

$$\left| \langle \vec{n} \rangle \right| = \left| \langle \vec{w}_k / w_k \rangle \right| \in [0;1]$$

J – **spin adiabatic invariant**

Adiabatic condition:

$\left| \vec{h}' \right| << h^2$ or $\left| \varepsilon' \right| << \varepsilon^2 + w_k^2$

Resonance region: $\left| \varepsilon' \right| \sim h^2 \rightarrow \theta_{\text{res}} \sim 1/\sqrt{\varepsilon'}$

$$\langle J \rangle \neq \text{const}$$

Far away from the resonance region:

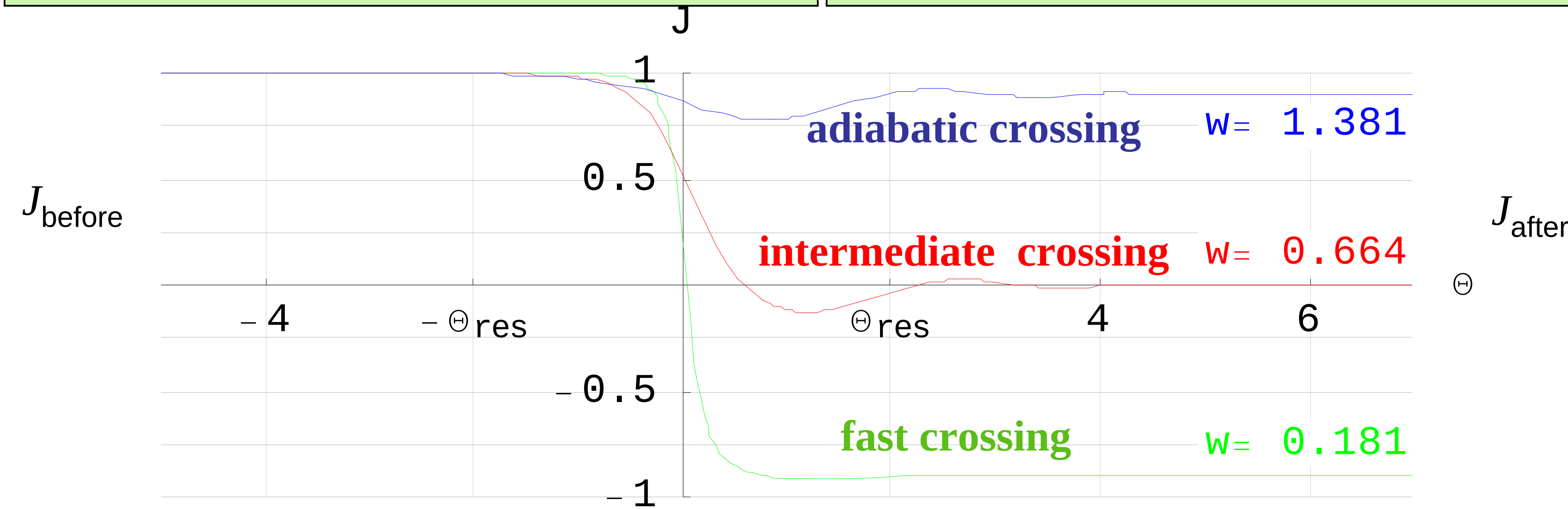
$$\langle J \rangle = \text{const}$$

J change after resonance crossing $\left(\varepsilon' = const\right)$

$\mathrm{w} = \dfrac{w_k}{\sqrt{\varepsilon'}}$ normalized resonance strength

$\Theta = \sqrt{\varepsilon'}\,\theta$ normalized azimuth

$$J_{after} = \left(1 - 2\exp\left(-\frac{\pi \mathrm{w}^2}{2}\right)\right) J_{befor}$$



Far away from the resonance $\varepsilon >> \max\left(w_k, \sqrt{\varepsilon'}\right)$ degree of depolarization is $D = 1 - \left|\left\langle J \right\rangle\right|$

fast crossing

$\left\langle \mathrm{w}^2 \right\rangle << 1$

$D = \pi \left\langle \mathrm{w}^2 \right\rangle << 1$

adiabatic crossing

$\mathrm{w}^2 >> 1$ (coherent), $D \to 0$

$\left\langle \mathrm{w}^2 \right\rangle >> 1$ (not coherent), $D = \dfrac{1}{\pi \left\langle \mathrm{w}^2 \right\rangle} << 1$

intermediate crossing

$\left\langle \mathrm{w}^2 \right\rangle \sim 1$

$D \sim 1$

Linear Spin Resonances at Nuclotron

$|k_x| + |k_z| \le 1$ **linear spin resonance**

Type of spin resonance	Resonance condition	Number of resonance			
		^1H	^2H	^3H	^3He
Intrinsic (P=8)	$\nu = kP \pm \nu_z$	6	−	8	9
Integer	$\nu = k$	25	1	32	37
Non-supereriodical	$\nu = k \pm \nu_z$	44	2	55	64
Coupling resonance	$\nu = k \pm \nu_x$	49	2	63	73

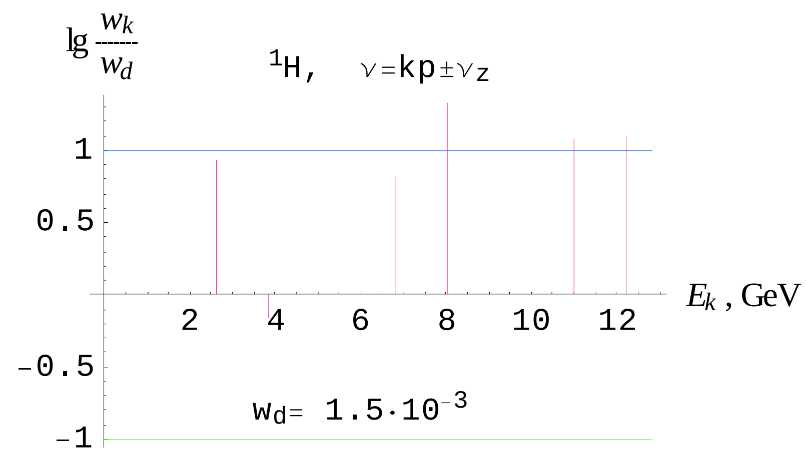
	^1H	^2H	^3H	^3He
G	1.793	-0.143	7.92	-4.184
E_k^{max}, GeV/u	12.84	6.00	3.74	8.28
ε', $(\tau = 0.5\,\text{sec})$	$7.0 \cdot 10^{-6}$	$2.8 \cdot 10^{-7}$	$1.0 \cdot 10^{-5}$	$1.1 \cdot 10^{-5}$
w_d, $(\tau = 0.5\,\text{sec})$	$1.5 \cdot 10^{-3}$	$3.0 \cdot 10^{-4}$	$1.8 \cdot 10^{-3}$	$1.9 \cdot 10^{-3}$

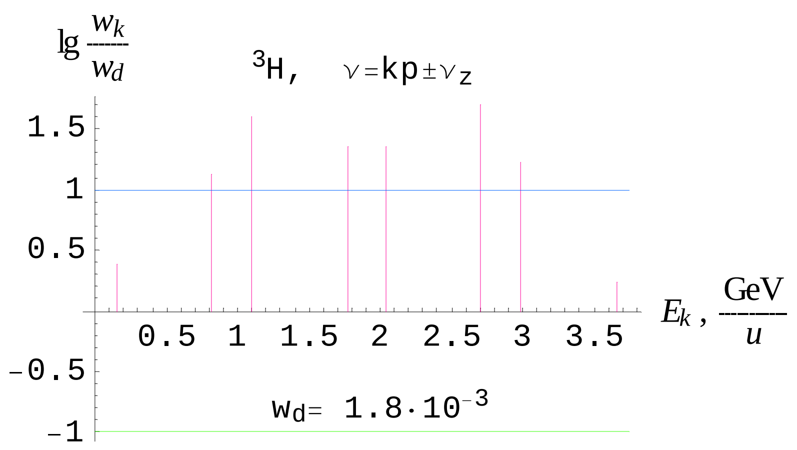
Intrinsic spin resonances at Nuclotron (p, t, ^3He)

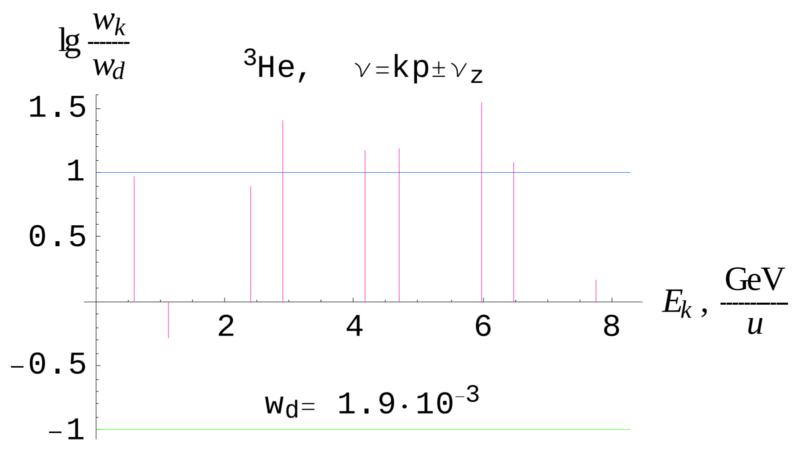
Above blue line – adiabatic crossing

Under green line – fast crossing

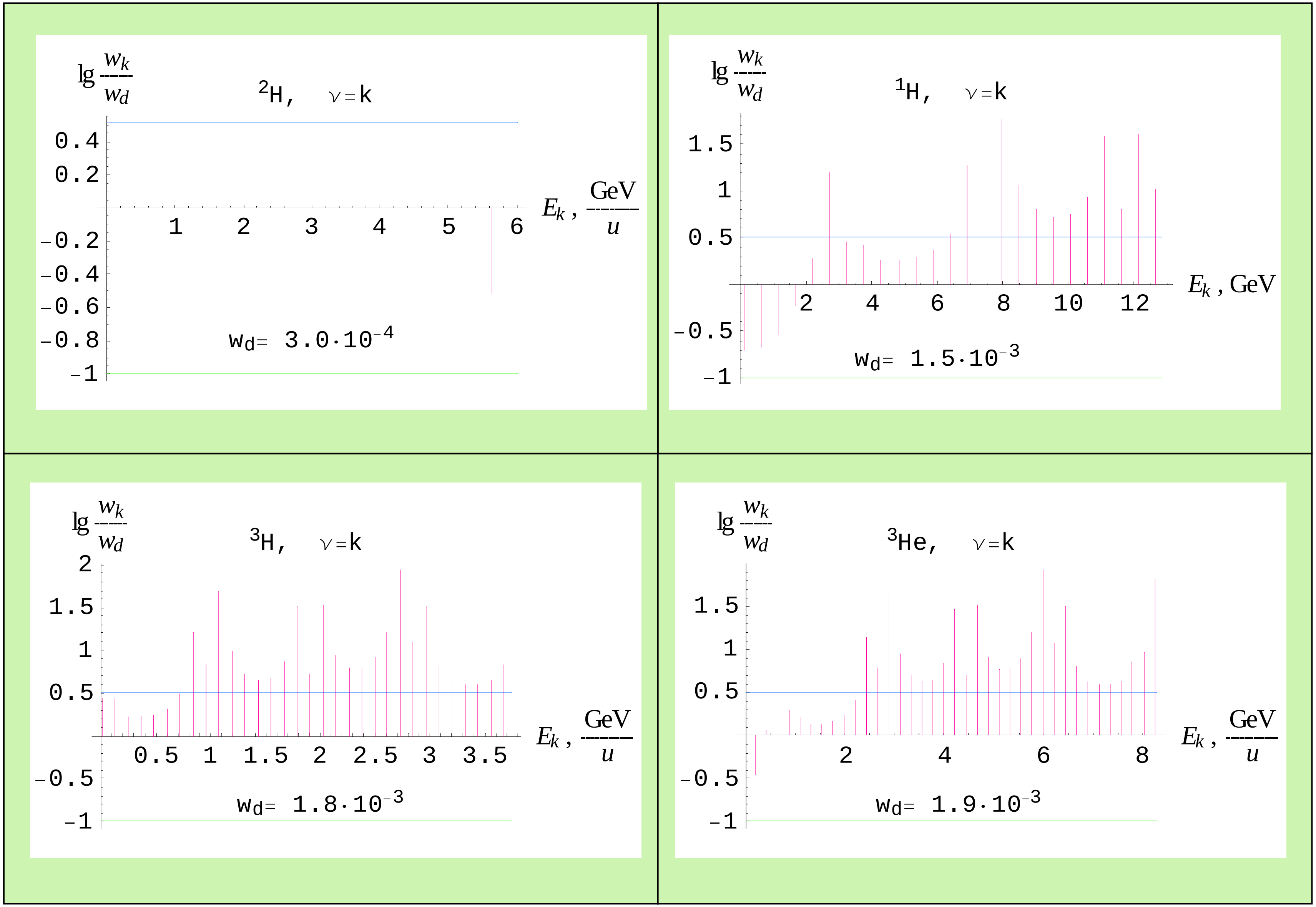
Between lines – intermediate crossing (polarization losing)



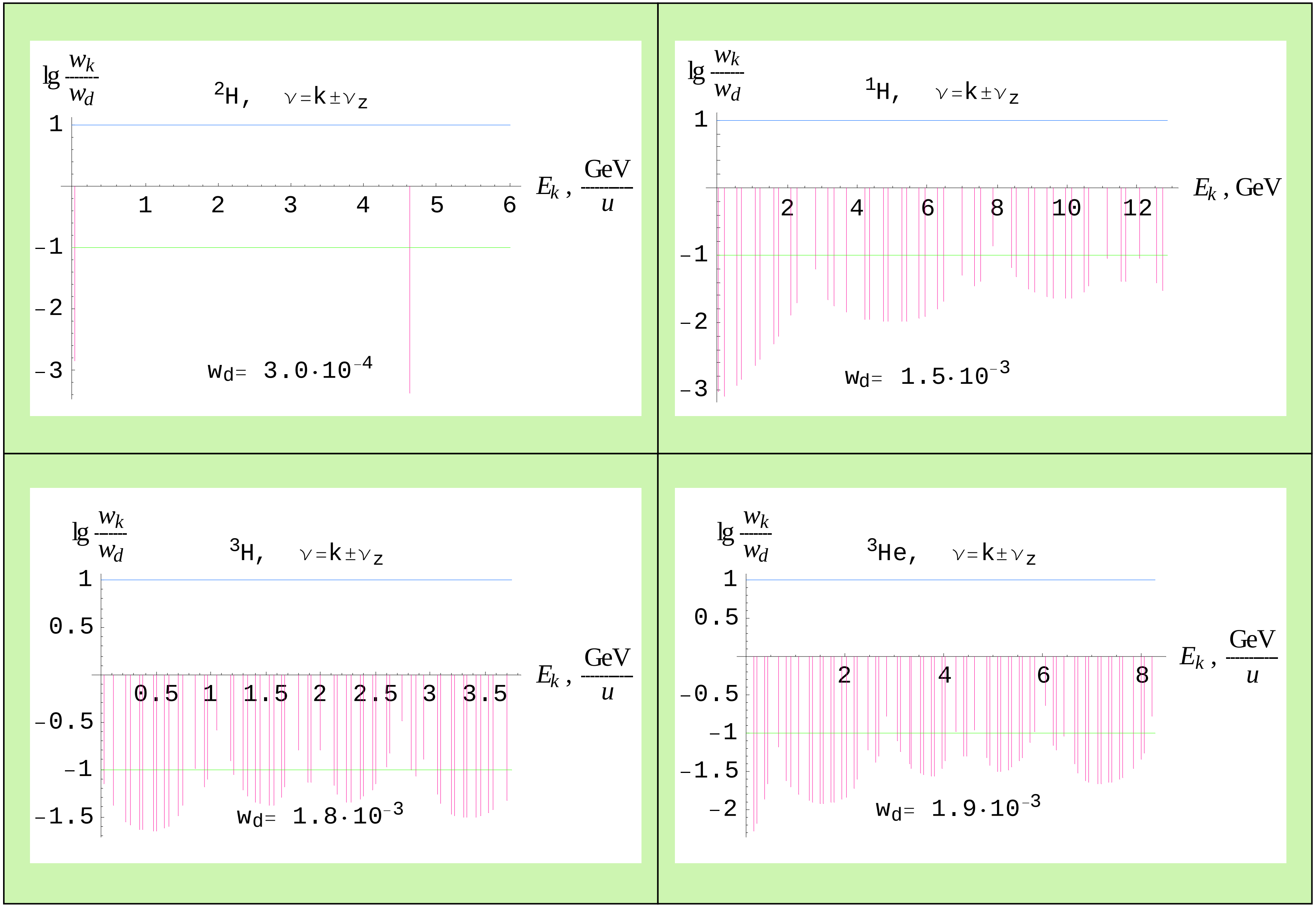




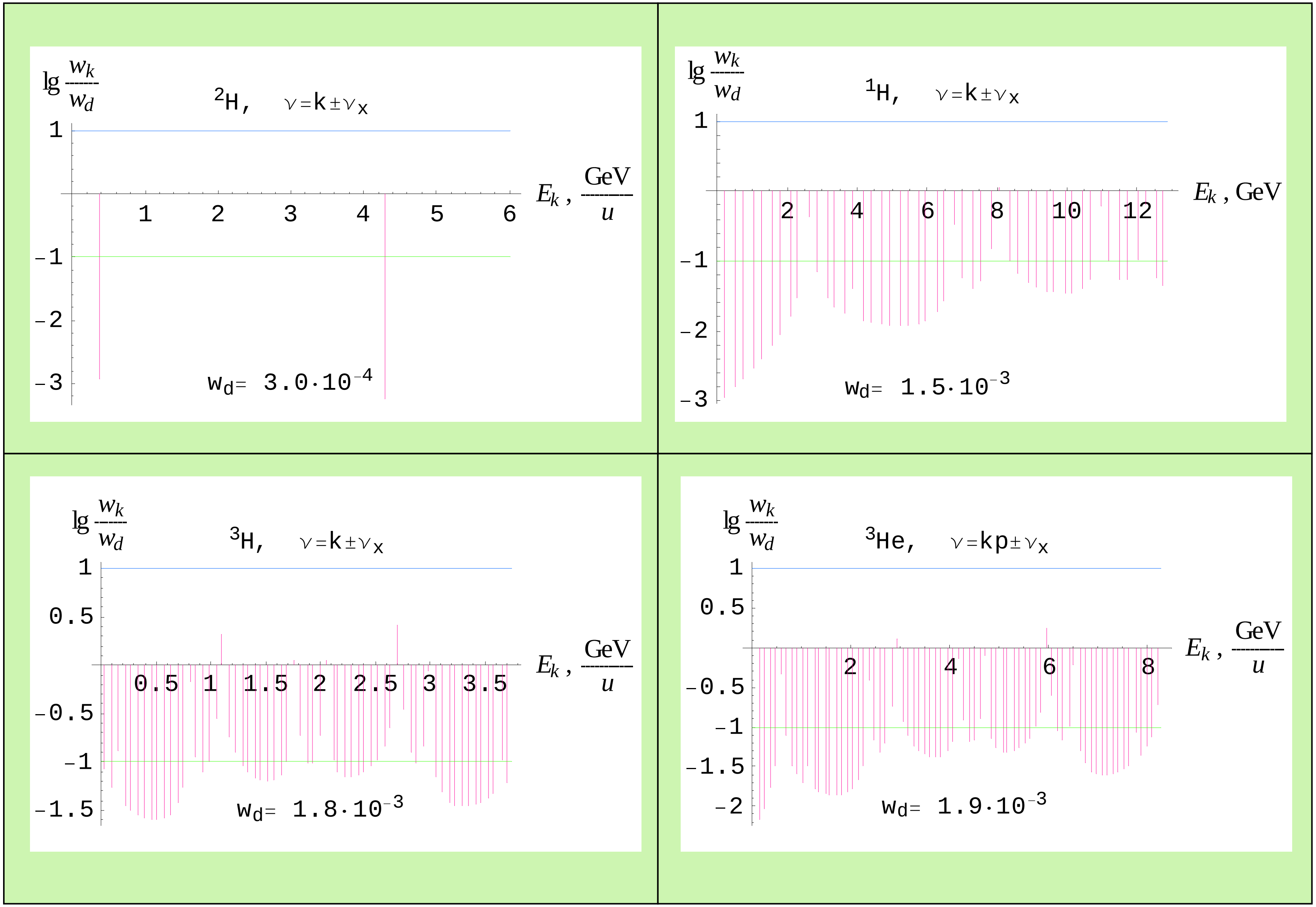
Integer spin resonances at Nuclotron (D,p, t, ^{3}He)



Not superperiodical spin resonances at Nuclotron (D,p, t, ^3He)



Coupling spin resonances at Nuclotron (D,p, t, ^{3}He)



Methods of spin resonance crossing

There are various methods for spin resonances crossing. These methods are based on:

- increasing the velocity of the spin-resonance crossing due to "jump" of the betatron tune;
- increasing the velocity of the spin-resonance crossing due to "jump" of the spin tune;
- resonance-strength compensation;
- an increase of the spin-resonance strength by means of specially introduced magnetic fields for adiabatic crossing of spin resonance (can be used for integer resonances crossing at nuclotron) ;
- decrease of the spin-resonance crossing velocity.

Methods of spin resonance crossing
1.Increasing of the spin-resonance strength for adiabatic crossing

Synchrotron modulation of energy

$$\nu = \gamma G = \nu_0 + \sigma_\gamma \cos \Psi_\gamma$$

σ_γ – energy spread srm of spin tune

$\nu_\gamma = \Psi'_\gamma$ – synchrotron tune

$$\varepsilon = \varepsilon_0 + \varepsilon_{\text{synch}}$$

$\varepsilon'_{\text{synch}} = \sigma_\gamma \nu_\gamma$ – synchrotron tuning velocity

The adiabatic condition

$$|\varepsilon'| << w_k^2$$

The adiabatic condition with synchrotron modulation

$$w_k^2 >> \max(\varepsilon'_0, \sigma_\gamma \nu_\gamma)$$

At nuclotron

$$w_{\text{adiab}} \sim 10^{-2}$$

The following field integrals are necessary

	^1H	^2H	^3H	^3He
$(HL)_{y1}$, Tm	1.0	3.4	0.3	0.9

$$B\rho = 45\ T \cdot m$$

Integer resonances

$$H_y, \quad L_y$$

The resonance strength due to solenoid

$$w_k = \frac{\varphi_y}{2\pi} = \frac{1+G}{2\pi} \frac{H_y L_y}{B\rho}$$

$B\rho$ – magnetic rigidity

Methods of spin resonance crossing
2.The method of compensation degree of depolarization

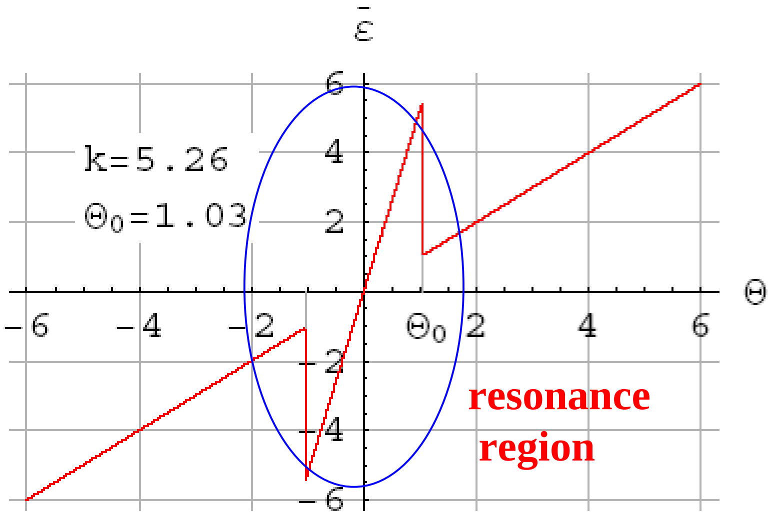
The compensation condition

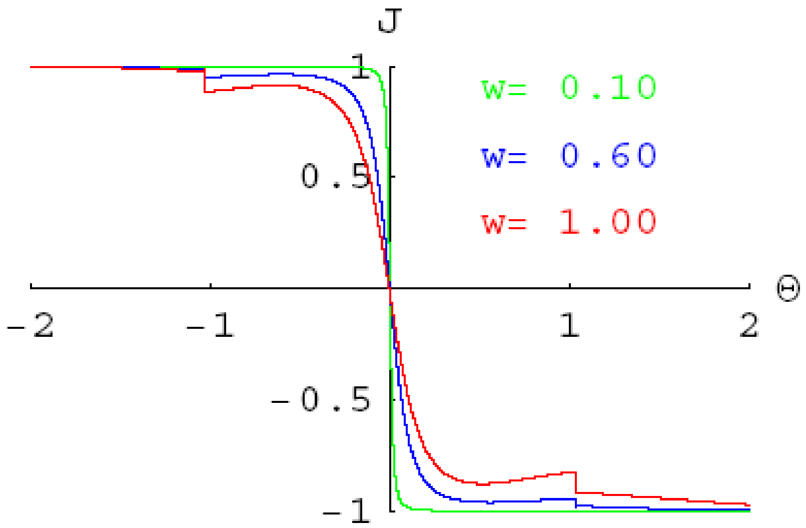
$$J_{after} = \mp J_{befor}$$

The example of compensation degree of depolarization

$$J_{after} = -J_{befor}$$

$$k = \frac{\varepsilon'_{in}}{\varepsilon'_0}, \quad \Theta = \theta\sqrt{\varepsilon'_0}, \quad \bar{\varepsilon} = \frac{\varepsilon}{\sqrt{\varepsilon'_0}}$$





Compensation condition for the fast crossing:

$$\int_{-\infty}^{\infty} \exp\left(\int_0^{\theta} \varepsilon(\theta)\, d\theta\right) d\theta = 0$$

Conservation of the polarization will occur with high accuracy as

w^6

Methods of spin resonance crossing
2.The method of compensation degree of depolarization (cont.)

The structure to manipulate of spin detune ε during resonance crossing

$\frac{\varphi_x}{2}$	φ_y	$-\varphi_x$	$-\varphi_y$	$\frac{\varphi_x}{2}$
$H_x, \frac{L_x}{2}$	H_y, L_y	$-H_x, L_x$	$-H_y, L_y$	$H_x, \frac{L_x}{2}$

Changing of the spin tune is $\Delta\nu = \frac{\varphi_x \varphi_y}{2\pi}$ **Vertical deviation of orbit is** $\Delta z_{\max} = \frac{H_x L_x}{8 B\rho}\left(4L_x + 5L_y\right)$

For implementation this method following parameters are necessary

$\Delta\nu \sim 4\sqrt{\varepsilon'}, \quad \Delta t \sim 1/\left(\sqrt{\varepsilon'}\omega_0\right)$ $\omega_0 -$ **revolution frequency**

Magnet length

$L_x = L_y = 40\,\mathrm{cm}$

One structure length

$L_{\mathrm{str}} = 160\,\mathrm{cm}$

Total structures length

$L_{\mathrm{tot}} = 4 \times 160\,\mathrm{cm}$

Parameters	^1H	^2H	^3H	^3He
$\Delta\nu$	$1.1\cdot 10^{-2}$	$2.1\cdot 10^{-3}$	$1.2\cdot 10^{-2}$	$1.3\cdot 10^{-2}$
$\Delta t,\ \mu s\ \ (\beta = 1)$	50	250	40	40
$H_x,\ T$	0.7	3.5	0.55	0.55
$H_y,\ T$	4.3	14	1.35	3.8
$\Delta z_{\max}\cdot\gamma,\ \mathrm{cm}$	4	10	1.1	2.1

Conclusions

- Tasks to accelerate polarized beams of p, t, ^{3}He from the technical point of view are equivalent and differ for the beam of D .
- During injection of the polarized beams the vector of polarization must be directed along n-axis.
- Additional magnetic insertions must be included into the nuclotron lattice. Additional elements do not influence to the beam dynamic at nuclotron.
- For calculation of spin resonance strengths the software is developed.
- Developed methods can be applied for other accelerators, as example for NICA project, COSY, AGS and s.o.