Breakdown of χPT expansion for Generalize Parton Distributions

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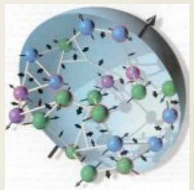

The basis of description of Hard Processes is factorization of hard and soft dynamics:

Amplitude Cross section	= Hard part (ptQCD) $\otimes$ Soft part(npQCD)

Soft part: universal and scale dependent

How to create the most realistic models?

Can we obtain some useful additional information about the soft sector from the low energy effective theory such as χPT?

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Amplitude / Cross section = Hard part (ptQCD) $\otimes$ Soft part(npQCD)

Soft part: universal and scale dependent

How to create the most realistic models?

GPDs:

complicate hybrid of form factors and PDFs

Can we learn smth practical about t-dependence?

How to construct χPT for non-local matrix elements like GPDs?

- χPT is expansion with respect to small momenta and masses: $p/\Lambda_\chi \ll 1$, $\Lambda_\chi \approx 800$MeV
- Main degrees of freedom are Goldstone bosons: π, K and η-mesons
- The baryons can be incorporated in the EFT according to the non-linearly chiral symmetry
- BUT we have to construct the chiral operator which describes in EFT the QCD light-cone operator

This work has been done:

PDF: Ji, Chen, '01
Arndt, Savage '02

GPDs: Kivel, Polyakov '02
Diehl, Manashov, '05,'06
Chen et all, '06

Results for pion PDF with I=1:

$$q(x) \quad = q^{(0)}(x) + a_\chi \, q^{(1)}(x) + a_\chi^2 \, q^{(2)}(x) + \ldots,$$

momentum fraction $\quad x \sim \mathcal{O}(a_\chi^0) \quad\quad a_\chi = (m_\pi / 4\pi F_\pi \;)^2$

?

$$q^{(1)}(x) = \{ \; q^{(0)}(x) - \delta(x) \} \ln\left[1/a_\chi\right] + \mathcal{O}(a_\chi)$$

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PDF: Ji, Chen, '01
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Results for pion PDF, isospin I=1:

$$q(x) \quad = q^{(0)}(x) + a_\chi \, q^{(1)}(x) + a_\chi^2 \, q^{(2)}(x) + \ldots,$$

momentum fraction $x \sim \mathcal{O}(a_\chi^0)$ $\quad a_\chi = (m_\pi / 4\pi F_\pi \,)^2$

$$q^{(1)}(x) = \{\, q^{(0)}(x) - \delta(x)\} \ln[1/a_\chi] + \mathcal{O}(a_\chi)$$

Normalization: $\int_{-1}^{1} dx \; q(x) = 1$

for the case of GPD

$$H^{I=1}(x,\xi)|_{\xi\neq 0,\ t=0} \quad = \quad \mathring{H}^{I=1}(x,\xi)\,(\,1 + a_\chi \ln[1/a_\chi]\,) - a_\chi \ln[1/a_\chi]\,\frac{\theta\,(|x|\leq\xi)}{\xi}\,\mathring{\varphi}_\pi\left(\frac{x}{\xi}\right)$$

Moments:

$$\int dx x^n H^{I=1}(x,\xi) = h_0 + \xi^2 h_2 + \ldots + \xi^n h_n$$

$$h_n = \langle \mathring{H}^{I=1}\rangle_n\,(\,1 + a_\chi \ln[1/a_\chi]) - a_\chi \ln[1/a_\chi]\,\langle\varphi\rangle_n$$

Forward limit

$$\frac{\theta\,(|x|\leq\xi)}{\xi}\,\mathring{\varphi}_\pi\left(\frac{x}{\xi}\right) \stackrel{\xi\to 0}{\to} \delta(x)$$

for the case of GPD

$$H^{I=1}(x,\xi)|_{\xi\neq 0,\ t=0} = \mathring{H}^{I=1}(x,\xi)\,(\,1+a_\chi \ln[1/a_\chi]\,) - a_\chi \ln[1/a_\chi]\,\frac{\theta\,(|x|\le\xi)}{\xi}\,\mathring{\varphi}_\pi\left(\frac{x}{\xi}\right)$$

Physical amplitudes

$$\Delta_\pi = \int dx\,\frac{\mathring{\varphi}(x)}{1-x}$$

$$\mathcal{F}(\xi) = \int dx\,\frac{H^{I=1}(x,\xi)}{x-\xi+i0} = \mathring{\mathcal{F}}(\xi) - a_\chi \ln[a_\chi]\left(\mathring{\mathcal{F}}(\xi) - \frac{\Delta_\pi}{\xi}\right)$$

?

$$\mathring{\mathcal{F}}(\xi) \sim \xi^{-a},\ \ 0<a<1$$

Can we trust chiral expansion at small values of the momentum fraction x ?

Pion PDF

$$q(x) = \mathring{q}(x) - \{\, q^{(0)}(x) - \delta(x)\} a_\chi \ln [a_\chi] + \mathcal{O}(a_\chi)$$

Assumption: for small $x \sim a_\chi \quad a_\chi \delta(x) = \delta(x/a_\chi) \sim \mathcal{O}(a_\chi^0)$

higher orders $a_\chi^n \delta^{(n)}(x) = \delta^{(n)}(x/a_\chi) \sim \mathcal{O}(a_\chi^0)$

$\Rightarrow$ χPT expansion needs to be reorganized

Result

$$\sum D_n \, \varepsilon^n \delta^{(n)}(x) = f(x\,/\varepsilon)$$

$$\varepsilon = a_\chi \ln [1/a_\chi]$$

Summarize:

$$q(x) = q^{reg}(x) + \delta(x) D_0 + \delta''(x) D_2 + \ldots = q^{reg}(x) + \sum_{i\geq 0,\,\text{even}} D_i\, \delta^{(i)}(x)$$

$$D_i = a_\chi D_i^{(1)} + a_\chi^2 D_i^{(2)} + \ldots$$

In order to check this suggestion one has to compute D_i

Kivel, Polyakov '07

$$D_2 = a_\chi^3 \ln^3 [a_\chi] \langle x^2 \rangle \frac{25}{27}$$

$$\langle x^2 \rangle = \int_{-1}^{1} dx\, x^2 \mathring{q}(x)$$

model independent: does not depend on the subleading chiral constants

Summarize:

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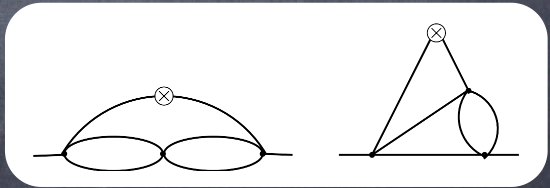
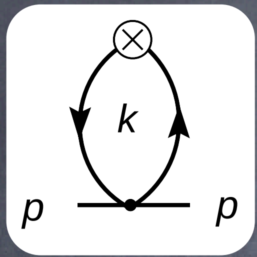
originate from the three loop diagrams:



Contain subdivergencies!

Locality of the UV-countereterms + residue $1/\epsilon^3$

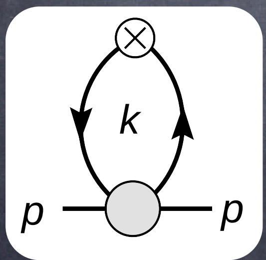
Generation of δ-contributions



$$\sim a_\chi\ \mathring{q}(\beta) * \int dk\ \frac{k_+\delta(xp_+ - \beta k_+)}{m_\pi^2\,[k^2 - m_\pi^2]^2}\,[-4(kp) + \ ...]$$

$$\sim\ a_\chi\ \delta(x) \int dk\ \frac{k^2}{m_\pi^2\,[k^2 - m_\pi^2]^2},$$

higher orders: $\sim\ a_\chi^{i+1}\,\ln^{i+1}[a_\chi]\ \delta^{(i)}(x)$



$$\frac{1}{\varepsilon^i} a_\chi^{i+1}\ \mathring{q}(\beta) * \int dk\ \frac{k_+\delta(xp_+ - \beta k_+)}{m_\pi^{2(i+1)}\,[k^2 - m_\pi^2]^2}[(k\cdot p)^{(i+1)} + ...]$$

$$\sim\ \frac{1}{\varepsilon^i} a_\chi^{i+1}\ \langle x^i\rangle \delta^{(i)}(x) \int dk\ \frac{k^{2(1+i)}}{m_\pi^{2(i+1)}\,[k^2 - m_\pi^2]^2},$$

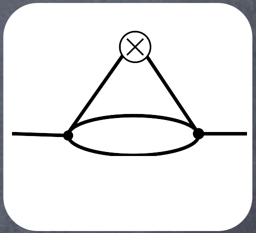
pion PDFs **Kivel, Polyakov '07**

isospin I=1 (non-singlet operator)

$$q(x) = q^{reg}(x) + a_\chi \ln[a_\chi] \; \underline{\delta(x)} + \frac{25}{27}\langle x^2\rangle \; a_\chi^3 \ln^3[a_\chi] \; \underline{\delta''(x)} + \mathcal{O}(a_\chi^4)$$

isospin I=0 (singlet operator)

$$Q(x) = Q^{reg}(x) + \frac{5}{6}\langle x\rangle \; a_\chi^2 \ln^2[a_\chi] \; \underline{\delta'(x)} + \mathcal{O}(a_\chi^3)$$



The mechanism is quite general and will work at arbitrary order of χPT

GPDs, t=0 for simplicity

$$H^{I=0}(x,\xi) = H^{I=0}_{reg}(x,\xi) - \frac{5}{6}\ a_\chi^2 \ln^2\left[a_\chi\right]\ \frac{\theta\left(|x|\le\xi\right)}{\xi^2}\ D\left(\frac{x}{\xi}\right)$$

D-term

$$H^{I=1}(x,\xi) = H^{I=1}_{reg}(x,\xi)$$
$$+\,\theta\left(|x|\le\xi\right)\left\{a_\chi \ln\left[a_\chi\right]\frac{1}{\xi}\mathring{\varphi}_\pi\left(\frac{x}{\xi}\right) - \frac{25}{27}a_\chi^3\ln^3\left[a_\chi\right]\frac{1}{\xi^3}\ \psi'\left(\frac{x}{\xi}\right)\right\},$$

$$D(z) = 2F^{I=0}(\beta,\alpha)\ *\delta(\alpha+\beta-z)$$

$$\psi'\left(z\right) = F^{I=1}(\beta,\alpha)\ *\beta\ \delta'\left(\alpha+\beta-z\right)$$

$$|\alpha|+|\beta|<1$$

The inverse moments $\mathcal{F}^I(\xi) = \int_{-1}^{1} dx \frac{H^I(x,\xi)}{x-\xi}$

$$\mathcal{F}^{I=0}(\xi) = \mathcal{F}^{I=0}_{reg}(\xi) + a_\chi^2 \ln^2 [a_\chi] \; \frac{5}{6} \; \frac{\Delta_D}{\xi^2} + ...$$

$$\mathcal{F}^{I=1}(\xi) = \mathcal{F}^{I=1}_{reg}(\xi) + a_\chi \ln[a_\chi] \frac{\Delta_\pi}{\xi} + a_\chi^3 \ln^3[a_\chi] \frac{25}{27} \frac{\Delta_{\psi'}}{\xi^3} + ...$$

where $\Delta_{\pi,D,\psi'} = \int_{-1}^{1} dx \frac{\{\varphi_\pi, D, \psi'\}(x)}{1-x}$

Amplitude $\mathcal{F}^I(\xi) \sim \left(\frac{a_\chi \ln(1/a_\chi)}{\xi} \right)^n, \quad n = 1, 2, 3 \ldots$

$\Rightarrow$ at small one has to perform resummation of such contributions

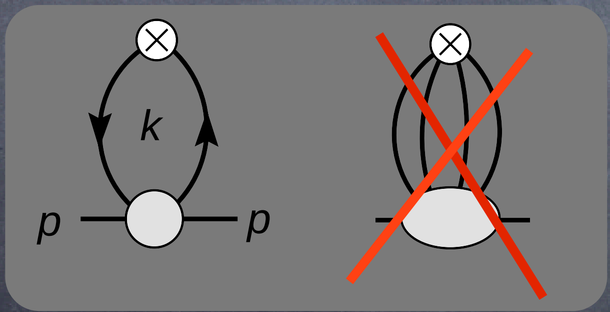
resummation of δ-contributions

consider only the leading Log's terms $\sim a_\chi^n \ln^n[a_\chi]$

☞ $$\mathcal{L}_2 = \frac{1}{4} F_\pi^2 \,\mathrm{tr}\left(\partial^\mu \mathrm{U} \partial_\mu \mathrm{U}^\dagger + \chi^\dagger \mathrm{U} + \chi \mathrm{U}^\dagger \right)$$

assumption: only the two particle operator vertexes contribute

PDFs



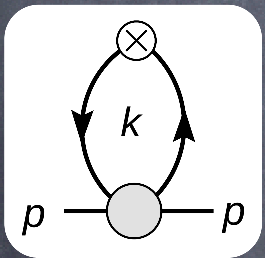
☞ restricts the chiral representation of the operator

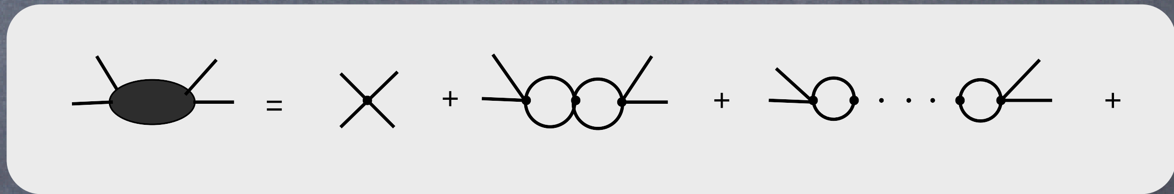
Introduce the auxiliary parameter : large-N

$$\mathcal{L}_2 = \frac{1}{4} F_\pi^2 \, \mathrm{tr}\left(\partial^\mu \mathrm{U} \partial_\mu \mathrm{U}^\dagger + \chi^\dagger \mathrm{U} + \chi \mathrm{U}^\dagger \right)$$

$$SU(2) \times SU(2) = O(4) \rightarrow O(N+1)$$

Isospin I=1 (non-singlet case)





$$q(x) = q_{reg}(x) - \frac{1}{N}\, \theta(|x| < N\varepsilon) \int_{|x|/(N\varepsilon)}^{1} \frac{dz}{z}\; \mathring{q}(z) + \mathcal{O}(1/N^2)$$

$$\varepsilon = a_\chi \ln\left[1/a_\chi\right]$$

$$q(x) = q_{reg}(x) - \frac{1}{N}\,\theta(|x| < N\varepsilon)\int_{|x|/(N\varepsilon)}^{1}\frac{dz}{z}\;\mathring{q}(z) + \mathcal{O}(1/N^2)$$

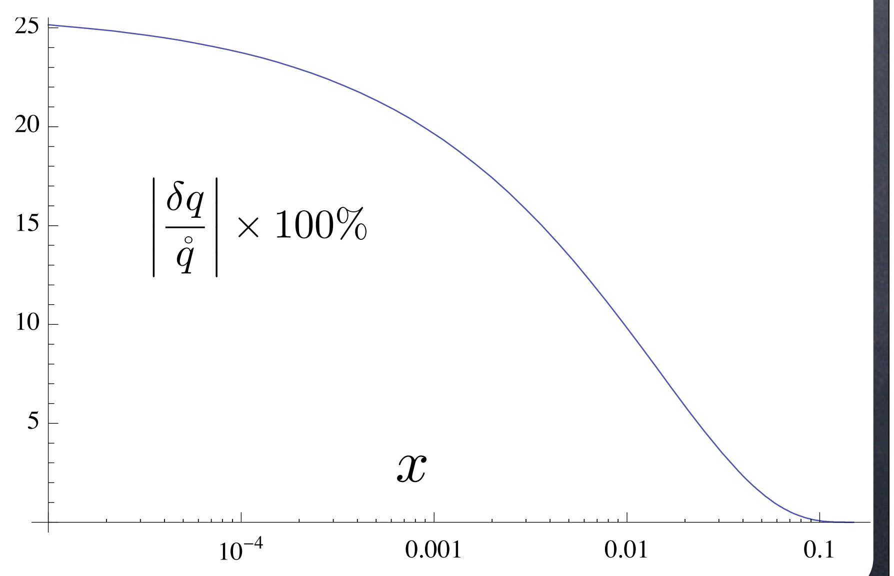
$$\varepsilon = a_\chi \ln\left[1/a_\chi\right]$$

$$\frac{1}{N}\,\theta(|x| < N\varepsilon)\int_{|x|/(N\varepsilon)}^{1}\frac{dz}{z}\;\mathring{q}(z) = \frac{1}{N}\sum_{n\geq 1}(N\varepsilon)^n\langle x^{n-1}\rangle\frac{\delta^{(n-1)}(x)}{n!}$$

$$|x| < 3a_\chi \ln[\Lambda_\chi^2/m_\pi^2] \simeq 0.15$$

$$\Lambda_\chi \simeq 800\text{Mev}$$

$$\mathring{q}(x) \simeq N|x|^{-0.5}(1-|x|)^2$$



Conclusions

Chiral expansions of the PDFs and GPDs is broken for the small values of the momentum fraction $x \sim (m_\pi/F_\pi)^2$

$$q(x) = q^{reg}(x) + \delta(x) D_0 + \delta''(x) D_2 + \ldots = q^{reg}(x) + \sum_{i \geq 0,\,\mathrm{even}} D_i\, \delta^{(i)}(x)$$

$$\mathcal{F}^I(\xi) = \int_{-1}^{1} dx \frac{H^I(x,\xi)}{x-\xi}$$

$$\mathcal{F}^{I=0}(\xi) = \mathcal{F}^{I=0}_{reg}(\xi) + a_\chi^2 \ln^2[a_\chi] \; \frac{5}{6} \; \frac{\Delta_D}{\xi^2}$$

$$\mathcal{F}^{I=1}(\xi) = \mathcal{F}^{I=1}_{reg}(\xi) + a_\chi \ln[a_\chi] \frac{\Delta_\pi}{\xi} + a_\chi^3 \ln^3[a_\chi] \frac{25}{27} \frac{\Delta_{\psi'}}{\xi^3} + \ldots$$

Possible solution is resummation of the singular terms:

$$q(x) = q^{reg}(x) + \delta(x) D_0 + \delta''(x) D_2 + \ldots = q^{reg}(x) + \sum_{i \geq 0,\, \text{even}} D_i\, \delta^{(i)}(x)$$

↓ "Large-N"

$$q(x) = q_{reg}(x) - \frac{1}{N}\, \theta(|x| < N\varepsilon) \int_{|x|/(N\varepsilon)}^{1} \frac{dz}{z}\, \mathring{q}(z) + \mathcal{O}(1/N^2)$$

$$\varepsilon = a_\chi \ln\left[1/a_\chi\right]$$

GPDs : the work in progress ...