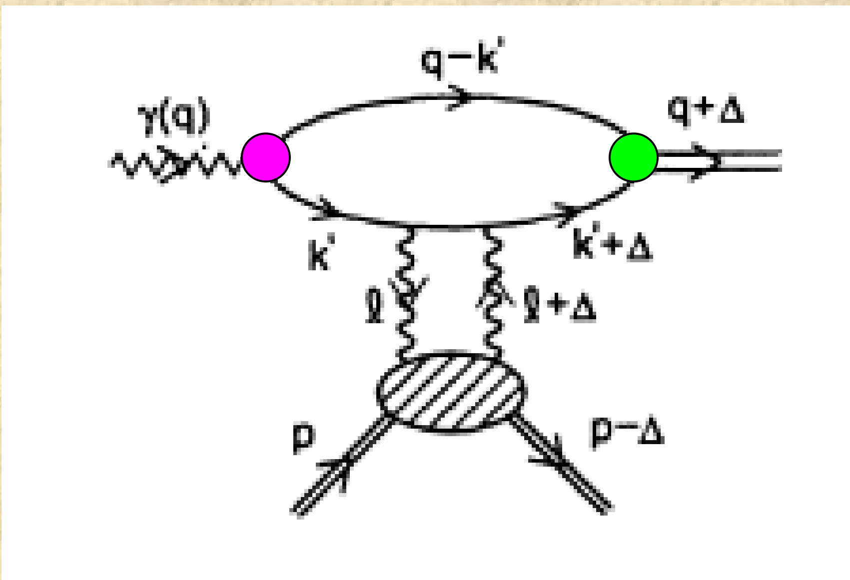
Vector Meson Distribution Amplitudes and their Production

A.E. Dorokhov *(JINR, Dubna)*

- *Introduction*
- *Instanton Model of QCD vacuum*
- ***Definitions of Rho and Photon DAs***
- *Production of Rho Mesons*
- *Conclusions*

Diffractive leptoproduction of Vecor Mesons in QCD



$$\mathcal{M}_f = \sqrt{N_c} \sum_{\lambda_1,\lambda_2} \int \frac{d^2k_\perp d^2k'_\perp}{(16\pi^3)^2} \int_0^1 dz \int_0^1 dz' \times \psi^{V*}_{\lambda_1\lambda_2}(k'_\perp, z') T_{\lambda_1\lambda_2}(k'_\perp, z'; k_\perp, z) \psi^{\gamma}_{\lambda_1\lambda_2}(k_\perp, z) \,,$$

Where Ψ^V and Ψ^γ are the light cone wave functions and T is the scattering amplitude

Photon DAs

Twist 2

$$\langle 0|\bar{q}(z)\sigma_{\mu\nu}[z,-z]q(-z)|\gamma^{\lambda}(q)\rangle = ie_q\left(q_\mu e_\nu^{(\lambda)} - q_\nu e_\mu^{(\lambda)}\right) f_\gamma^T \varphi_\gamma^T(q\bullet z) + h.t.$$

Twist 3

$$\langle 0|\bar{q}(z)\gamma_\mu[z,-z]q(-z)|\gamma^{\lambda}(q)\rangle = e_q e_{\perp\mu}^{(\lambda)} f_\gamma^V \varphi_\gamma^V(q\bullet z) + h.t.,$$

$$\langle 0|\bar{q}(z)\gamma_\mu\gamma_5[z,-z]q(-z)|\gamma^{\lambda}(q)\rangle = e_q 2i\varepsilon_{\mu\nu\alpha\beta} e_\nu^{(\lambda)} q_\alpha z_\beta f_\gamma^A \varphi_\gamma^A(q\bullet z),$$

where $\varphi_\gamma^T(0) = 1, \quad \varphi_\gamma^V(0) = 0$ and

$$\phi_\gamma^i(x) = \frac{1}{2\pi}\int_{-\infty}^{\infty} d\lambda e^{-i\lambda(2x-1)} \varphi_\gamma^i(\lambda), \qquad i = T, V, A.$$

$$f_\gamma^T = \chi_m \left|\langle \bar{q}q\rangle\right| = 40 \text{ MeV}, \quad f_\gamma^V = f_{3\gamma}$$

Magnetic susceptibility of quark condensate (Ioffe, Smilga)

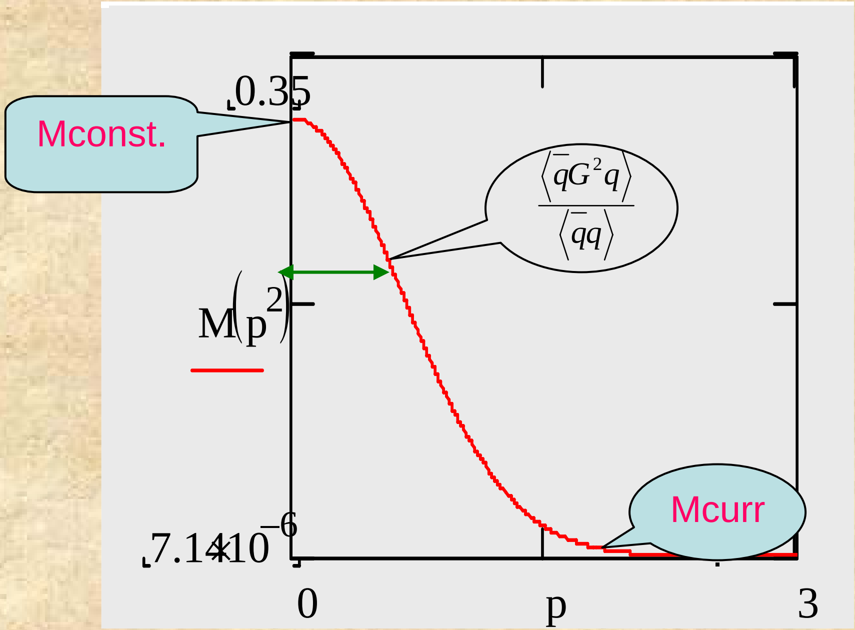
Instanton Liquid Model

The dressed quark propagator is defined as

$$S^{-1}(p) = \hat{p} - M(p), \quad Z(p) = 1$$

$$M(p) = M_q f^2(p)$$

f(k) is quark zero mode in the Instanton field



Conserved Vector current.

The Vector vertex

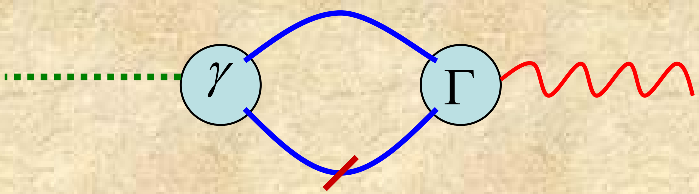
~α_s in pQCD

$q^2 \to \infty$

$$\Gamma^a_\mu(k,q,k'=k+q) = T^a\left[\gamma_\mu - (k+k')_\mu \frac{M(k')-M(k)}{k'^2-k^2} + \rho\text{-pole}\right] \to T^a\gamma_\mu$$

Nonlocal part

AF



$$\Gamma = \sigma_{\mu\nu}, \gamma_\mu, \gamma_\mu\gamma_5$$

Nonperturbative parameters

$$\langle 0 | \overline{q} \sigma_{\mu\nu} q | 0 \rangle_F = e_q \chi_m \langle 0 | \overline{q} q | 0 \rangle F_{\mu\nu},$$

$$\left\langle 0 \left| \overline{q} g \widetilde{G}_{\mu\nu} \gamma_\alpha \gamma_5 q \right| 0 \right\rangle_F = e_q f_{3\gamma} D_\alpha F_{\mu\nu},$$

$$\langle 0 | \overline{q} q | 0 \rangle = -N_c \int \frac{du}{4\pi^2} \frac{uM(u)}{D(u)}, \quad \chi_m = \frac{N_c}{\langle \bar{q} q \rangle} \int \frac{du}{4\pi^2} \frac{u(M(u) - uM'(u))}{D^2(u)}, \quad f_{3\gamma} = -N_c \int \frac{du}{4\pi^2} \frac{M^2(u)}{D(u)}.$$

$$\langle 0 | \overline{q} q | 0 \rangle |_{\mu_{\rm inst}=1{\rm GeV}} = -(0.24 \text{ GeV})^3, \quad \chi_m |_{\mu_{\rm inst}=1{\rm GeV}} = 2.73 \quad \text{GeV}^{-2},$$
$$f_{3\gamma} |_{\mu_{\rm inst}=1{\rm GeV}} = -0.0035 \quad \text{GeV}^2,$$

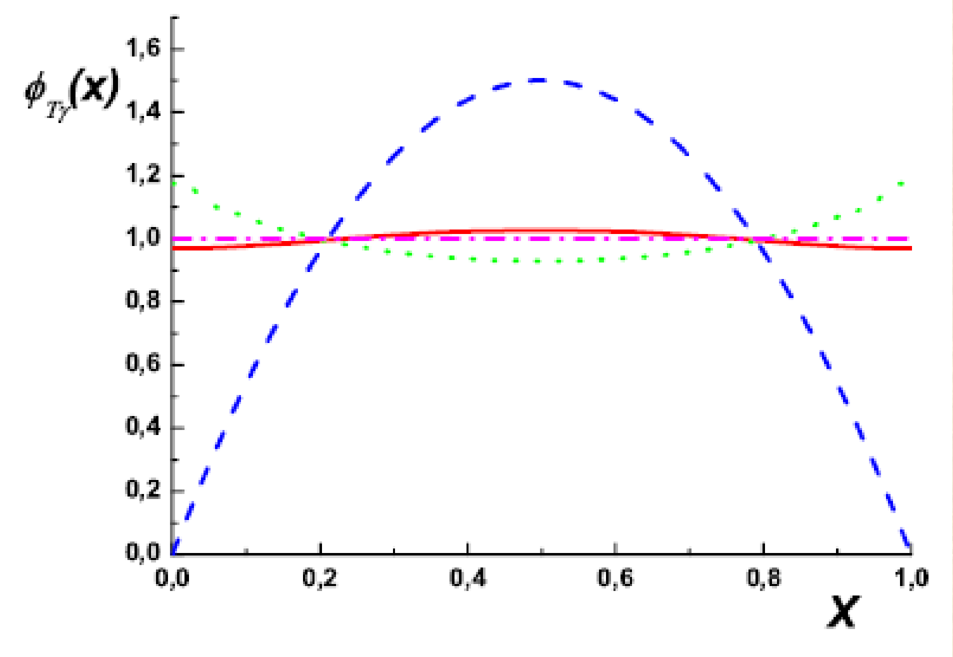
Instanton

$$\langle 0 | \overline{q} q | 0 \rangle |_{\mu=1{\rm GeV}} = -(0.24 \pm 0.02) \text{ GeV}^3, \quad \chi_m |_{\mu=1{\rm GeV}} = (3.15 \pm 0.3) \quad \text{GeV}^{-2},$$
$$f_{3\gamma} |_{\mu=1{\rm GeV}} = -(0.0039 \pm 0.0020) \quad \text{GeV}^2.$$

QCD sr (Braun, Ball, Kivel, 03' Rohrwild 07')

Tensor Photon DA (Twist 2)

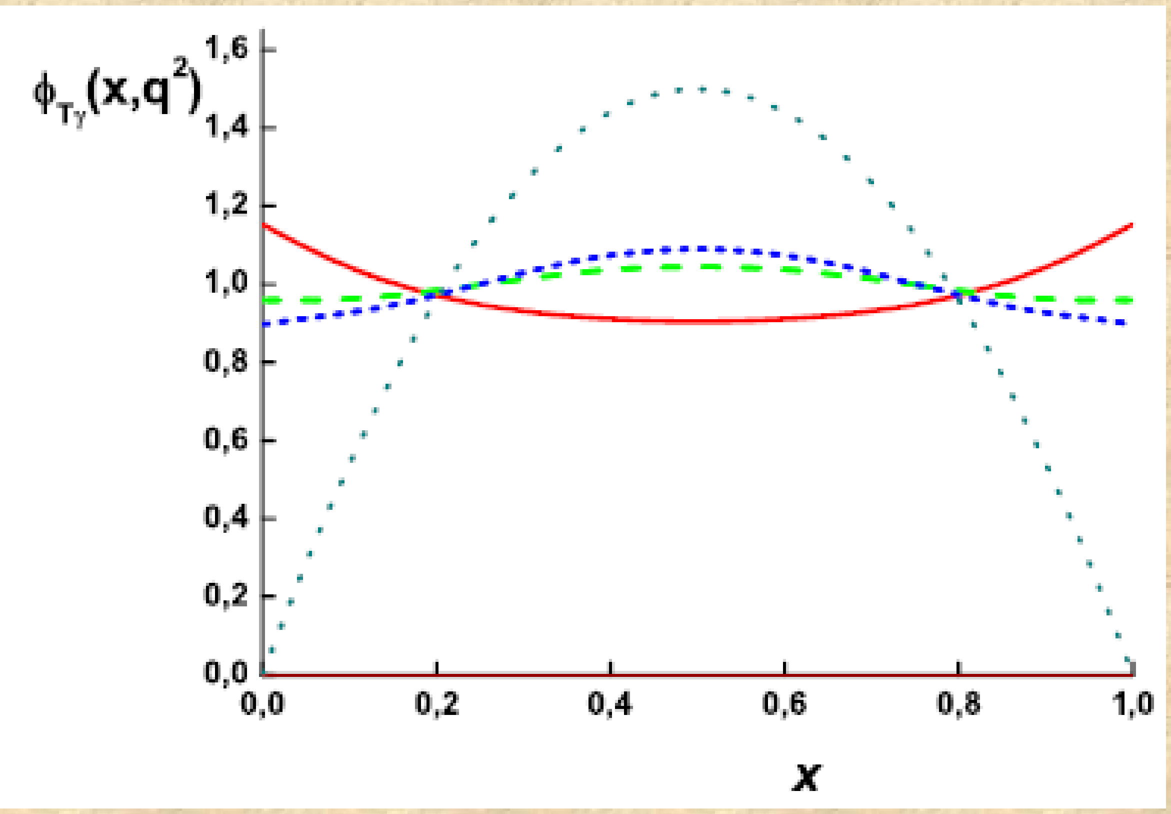
$$\phi_{\perp\gamma}(x,q^2=0)=\frac{1}{\chi_{\rm m}}\frac{N_c}{4\pi^2}\Big[\Theta(\bar{x}x)\int_0^\infty du\frac{M(u)}{D(u)} - \int_0^\infty du\int_{-\infty}^{\infty}\frac{d\lambda}{2\pi}\frac{M_+M_-}{D_+D_-}M^{(1)}(u_+,u_-)\Big].$$



μ_{inst}=0.5 GeV

AD, W. Broniowski, E. Ruiz Arriola PRD 06'

Off-shell effects on photon DA of leading twist



q^2 =0 GeV^2 dashed line

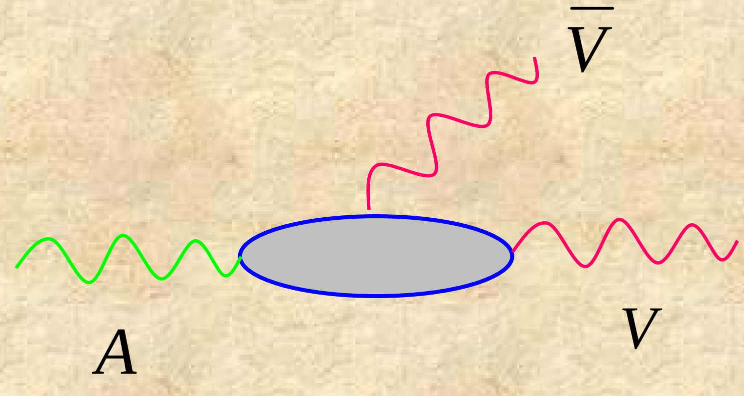
q^2 =0.25 GeV^2 solid line

q^2 =-0.09 GeV^2 short-dashed line

asymptotic DA dotted line

Dependence of the twist-2 tensor component of the photon DA on transverse momentum squared

Magnetic susceptibility in NχQM

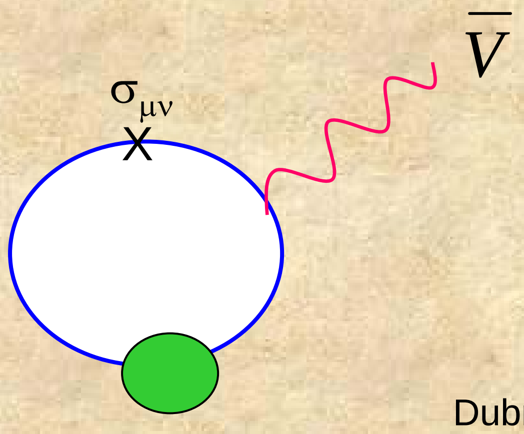


$$\Delta w_L = 2\Delta w_T = \frac{16\pi^2}{3}\frac{m_f\left\langle \bar{q}q_f\right\rangle}{q^4}\chi_f^m$$

In OPE appears as LO twist correction
(Similar to the Second Weinberg SR)

$$e_f\chi_f^m\left\langle \bar{q}_f q_f\right\rangle F_{\mu\nu} = \left\langle \bar{q}_f \sigma_{\mu\nu} q_f\right\rangle,$$

$$\chi_f^m\left\langle \bar{q}_f q_f\right\rangle = 4N_c\int\frac{d^4k}{(2\pi)^4}\frac{M(k)-k^2M'(k)}{\left(k^2+M^2(k)\right)^2}\approx 40\ \mathrm{MeV}$$

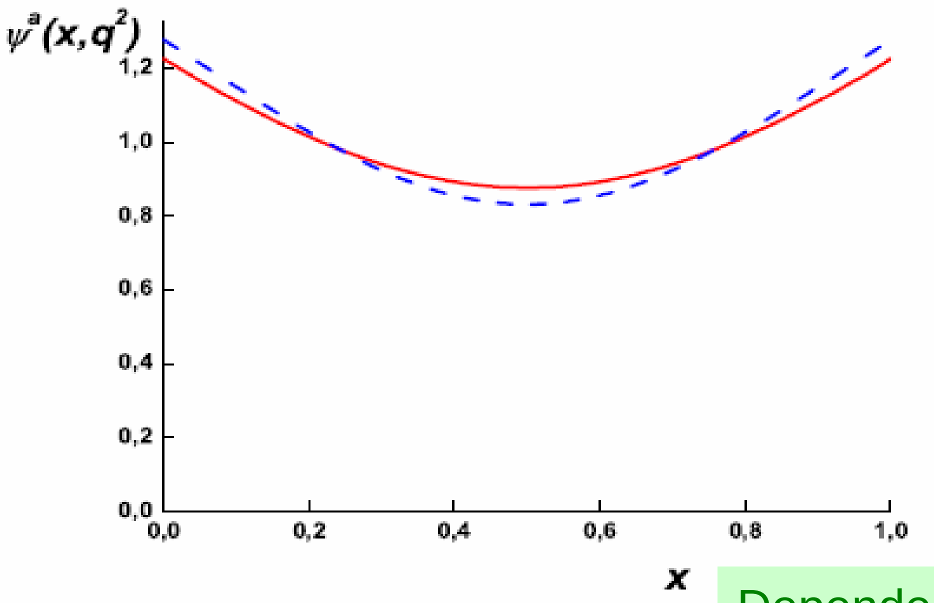


Twist 3 Axial Photon DA

$$\psi_{\gamma}^{(A)}(x) = \frac{1}{f_{3\gamma}} \frac{N_c}{4\pi^2} \int_0^{\infty} du \int_{-\infty}^{\infty} \frac{d\lambda}{2\pi} \frac{x M_-^2 + \overline{x} M_+^2}{D_+ D_-},$$

$$f_{3\gamma} = \frac{N_c}{4\pi^2} \int_0^{\infty} du \frac{M^2(u)}{D(u)}.$$

$$\int_0^1 dx \psi_{\gamma}^{(A)}(x) = 1 + \frac{1}{f_{3\gamma}} \frac{N_c}{4\pi^2} \int du \frac{u^2 M(u) M'(u)}{D^2(u)}.$$



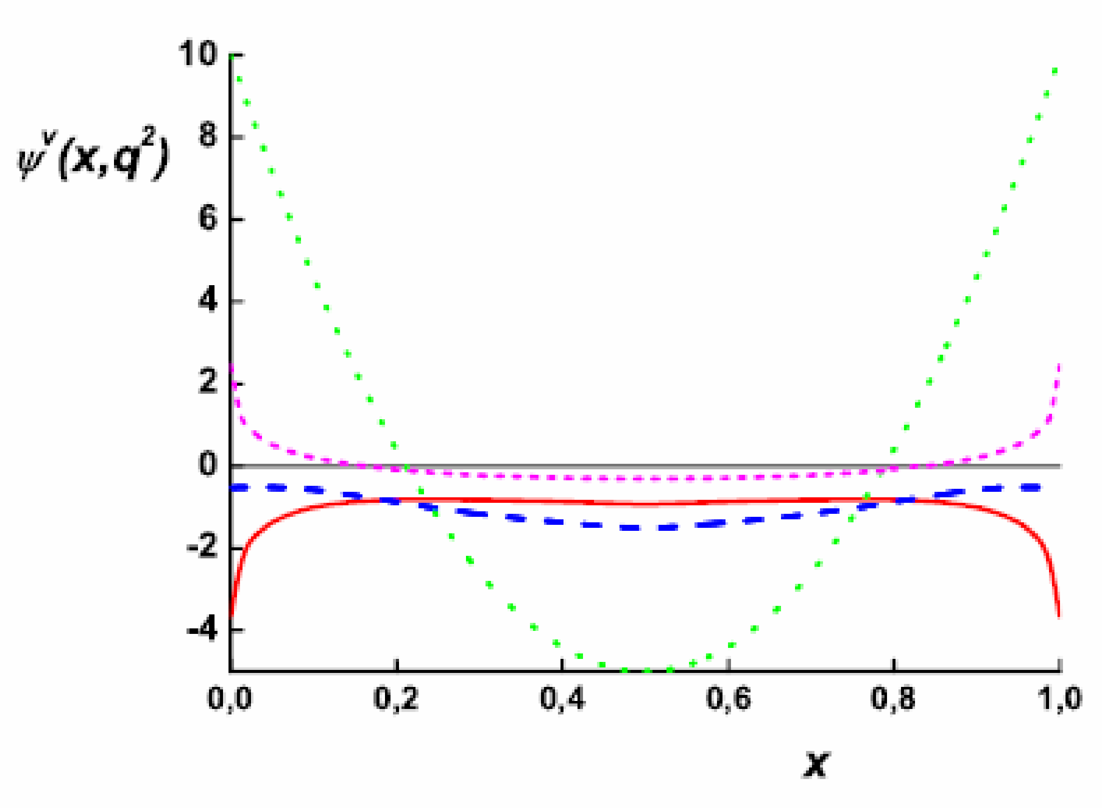
q^2 =0 GeV^2 dashed line

q^2 =0.25 GeV^2 solid line

Dependence of the twist-3 axial-vector component of the photon DA on transverse momentum squared

Twist 3 Vector Photon DA

$$\psi^{(V)}_{\gamma\parallel}(x) = \frac{\delta(x)+\delta(\overline{x})}{2} - (\Theta(x) - \Theta(\overline{x})) + \frac{1}{f_{3\gamma}}\frac{N_c}{8\pi^2}\int_0^\infty du \int_{-\infty}^{\infty}\frac{d\lambda}{2\pi}\frac{(M_+ - M_-)^2}{D_+ D_-},$$



$$\int_0^1 dx \psi^{(V)}_{\gamma\parallel}(x) = 0,$$

q^2 =0 GeV^2 dashed line

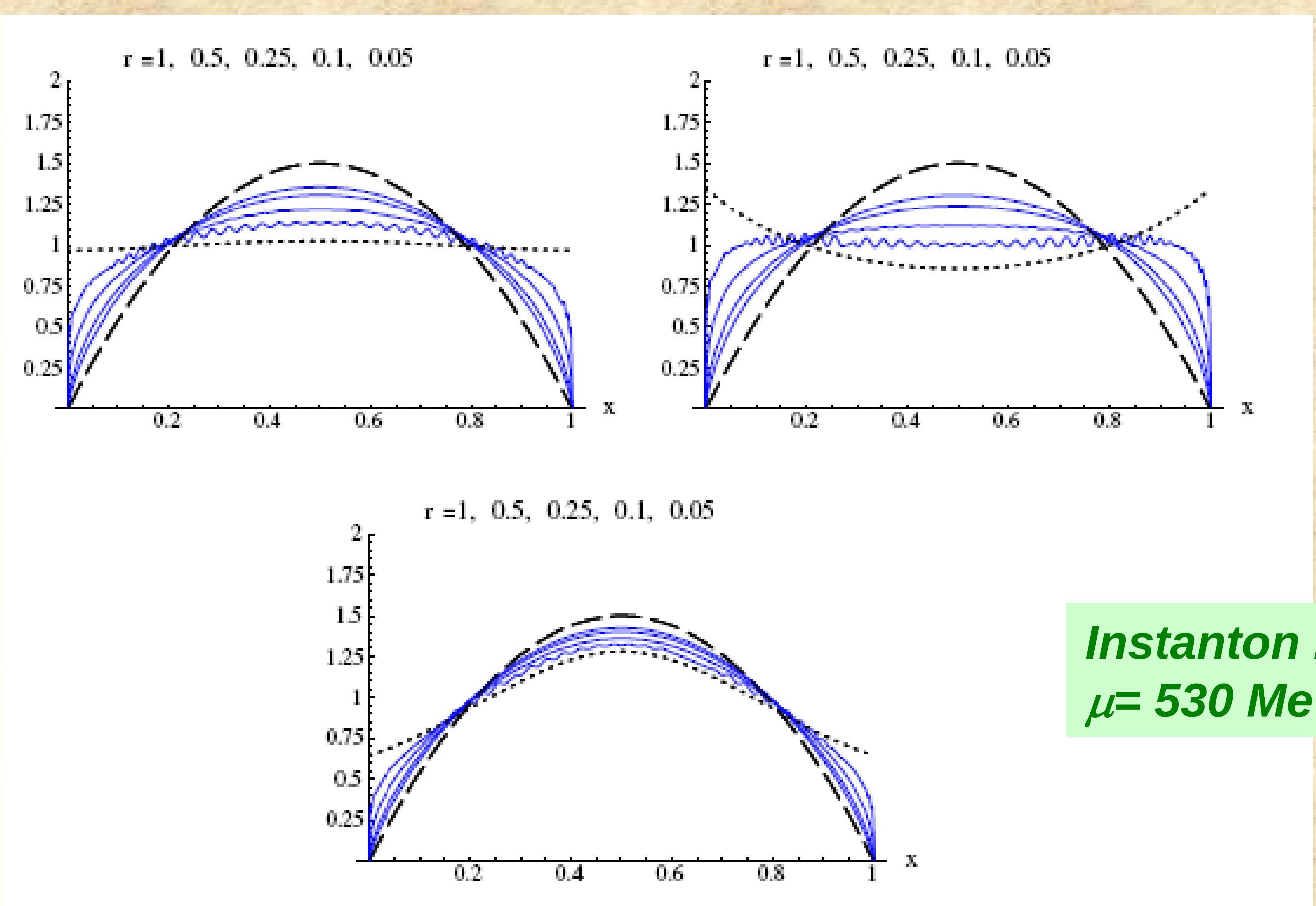
q^2 =0.25 GeV^2 solid line

Wandzura-Wilczek approximation

Conformal Expansion model

Dependence of the twist-3 vector component of the photon DA on transverse momentum squared

Effects of QCD evolution



Instanton model scale
μ= 530 MeV

The LO ER-BL evolution of the nonlocal model predictions for the leading-twist tensor photon DA. Top left: the real photon DA, top right: the virtual photon at q2 =0.25 GeV2; bottom: the virtual photon at q2=-0.09 GeV2. The dashed lines show the asymptotic DA. Initial conditions are indicated by dotted lines. The solid lines correspond to evolved DAs at scales Q= 1, 2.4, 10, and 1000 GeV.

r_T dependence of Photon DA

$$\varphi_{\gamma^*}^{T,L}\left(x,\vec{r}_\perp\right)\sim K_0\left(\varepsilon r_\perp\right),\qquad \varepsilon^2=m_q^2+x\left(1-x\right)Q^2$$

Perturbative approach
(N.Nikolaev,B.Zakharov)

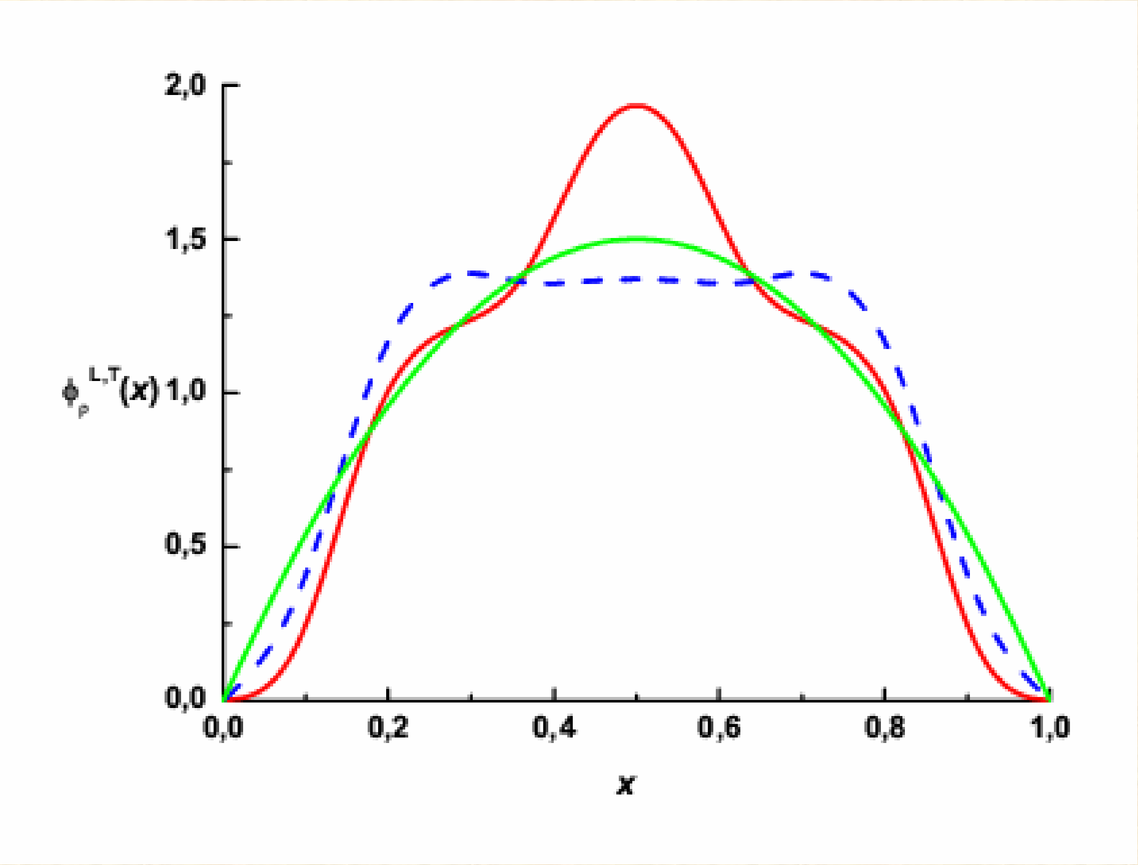
$$r_{q\bar{q}}\sim\frac{1}{\varepsilon}=\frac{1}{\sqrt{m_q^2+Q^2x\left(1-x\right)}}$$

For asymmetric qq configurations the transverse size is huge

$$\varphi_{\gamma^*}^{T,L}\left(x,\vec{r}_\perp\right)\sim\frac{m_\rho\left(Q^2+m_\rho^2\right)\left(1+\varepsilon r_\perp\right)}{\varepsilon^3}e^{-\varepsilon r_\perp},\qquad \varepsilon^2=\frac{1}{4}m_\rho^2+x\left(1-x\right)Q^2$$

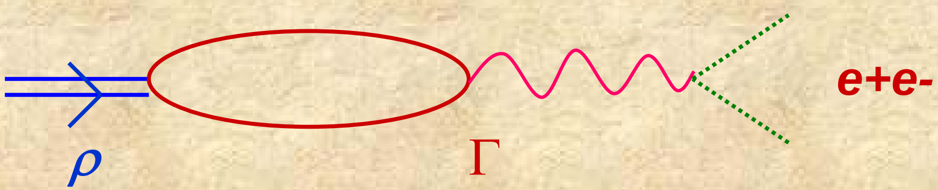
Nonperturbative approach
(AD, Broniowski, Ruiz Arriola)

Leading twist Transverse and Longitudinal ρ–meson DAs



Strong suppression of hadronic (rho) DAs at endpoint region
Photon DA is always wider than Asymptotic DA!

Photon-Rho Meson Coupling, $\rho\rightarrow$e+e-



$$\langle 0 | J^{\mu a} | \rho_s^b \rangle = -f_\rho \delta^{ab} \varepsilon_s^\mu, \qquad \Gamma_\rho = \frac{8\pi}{3}\alpha^2 m_\rho f_\rho^2$$

$$f_\rho = \frac{N_c g_{\rho qq}}{4\pi^2}\int \frac{d^4k}{\pi^2}\frac{\sqrt{m_+ m_-}}{D_+ D_-}\left[m_+ m_- + k_+ k_- - \frac{2}{3}k_\perp^2\left(1+m^{2(1)}\right)\right],$$

where $k_\pm = k \pm p/2, m_\pm = m\left(k_\pm\right), D_\pm = k_\pm^2 + m_\pm^2$

$$f_\rho = 0.114 \text{ GeV}^2$$

$$f_\rho^{\exp} = 0.1177 \text{ GeV}^2$$

Differential cross section for ρ0L production

$$\left.\frac{d\sigma}{dt}\right|_{t=0}\left(\gamma^{*}N\to\rho^{0}N\right)=\frac{1}{Q^{6}}8\pi^{4}\alpha_{em}\alpha_{s}^{2}\left(Q\right)f_{\rho}^{2}\eta_{\rho}^{2}\left[xG\left(x,Q\right)\right]^{2}$$

Brodsky,Frankfurt,Gunion, Mueller, Strikman

$$\eta_{\rho}=\frac{1}{2}\frac{\int\left[dx/x\left(1-x\right)\right]\varphi_{\rho}\left(x\right)}{\int dx\varphi_{\rho}\left(x\right)}$$

Inverse moment of ρ Distribution Amplitude controling the Leading Twist contribution to the Leptoproduction Amplitude

$$\eta_{\rho}^{As}=3\ \text{ for }\ \varphi_{\rho}^{As}\sim x\left(1-x\right),$$

$$\eta_{\rho}^{NP}=3.3-3.5\ \text{for }\ \varphi_{\rho}^{NP}\ \text{ which is wider than }\varphi_{\rho}^{As}$$

$$\eta_{\rho}^{Exp}=3.5-4$$

Pion DAs in QCD

Twist 2

$$\left\langle 0\left|\bar{d}(z)\gamma_\mu\gamma_5[z,-z]u(-z)\right|\pi^+(p)\right\rangle = -ip_\mu f_\pi^A \varphi_\pi^A(p\bullet z) + O\left(m_\pi^2\right),$$

Twist 3

$$\left\langle 0\left|\bar{d}(z)\gamma_5[z,-z]u(-z)\right|\pi^+(p)\right\rangle = f_\pi^P \varphi_\pi^P(p\bullet z),$$

$$\left\langle 0\left|\bar{d}(z)\sigma_{\mu\nu}[z,-z]u(-z)\right|\pi^+(p)\right\rangle = -2i\varepsilon_{\mu\nu\alpha\beta} p_\alpha z_\beta f_\pi^T \varphi_\pi^T(p\bullet z),$$

where $z^2=0,\ \varphi_\pi^i(0)=1,$ and

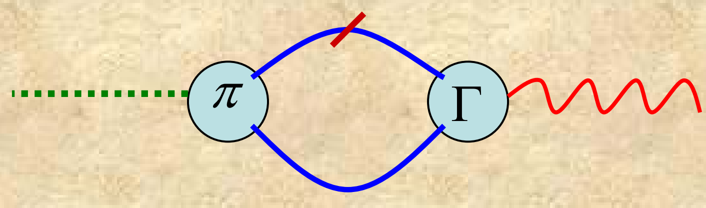
$$\phi_\pi^i(x) = \frac{1}{2\pi}\int_{-\infty}^{\infty} d\lambda e^{-i\lambda(2x-1)}\varphi_\pi^i(\lambda), \qquad i = A, P, T.$$

$$[z,-z] = P\exp\left(i\int_{-z}^{z} dx_\mu A^\mu(x)\right)$$

$$f_\pi^A = f_\pi, \quad f_\pi^P = \left|\left\langle \bar{q}q\right\rangle\right|/f_\pi$$

Fixed by chiral symmetry

Symmetries do not define f^T_π

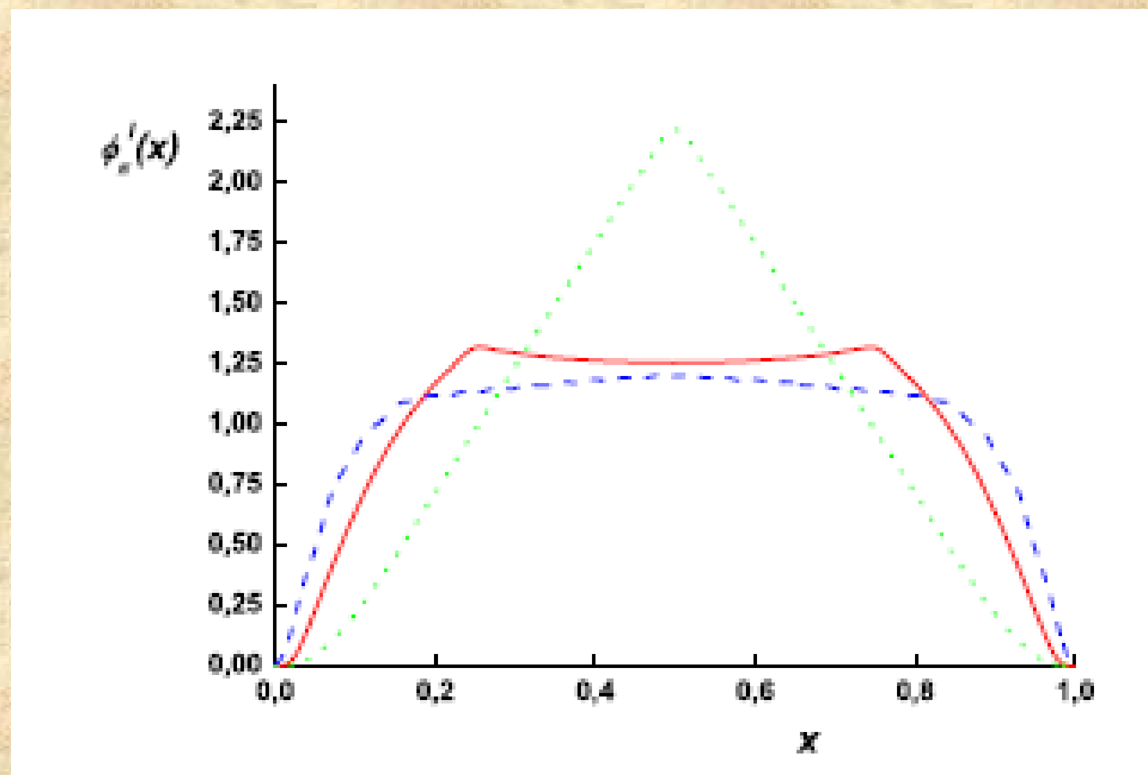


The pion vertex

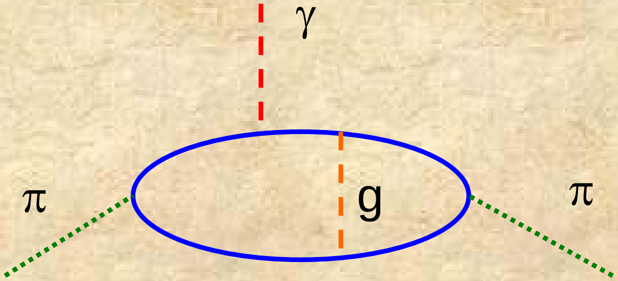
$$\Gamma_{\pi}^{a}\left(k,k'\right)=ig_{\pi qq}\gamma_5\tau^a f\left(k\right)f\left(k'\right)$$

$$\Gamma=\gamma_\mu\gamma_5,\gamma_5,\sigma_{\mu\nu}$$

Pion DAs of twists 2 and 3



Axial-Vector (leading twist 2)
Pseudo-Scalar (twist 3)
Tensor (twist 3)



Important that DAs are suppressed at edge points x=0,1,
since $\alpha(x(1-x)q^2)$ is not small there

The result for couplings is

$$f_\pi^A = f_\pi, \quad f_\pi^P = \left|\langle \bar{q}q \rangle\right| / f_\pi$$

$$f_\pi^T = \frac{1}{3 f_\pi} \frac{N_C}{4\pi^2} \left\{ -\frac{1}{2} \int \frac{u^2 M'}{D^2(u)} du - \int 2M \frac{uM\left(M - uM'/2\right)}{D(u)} du + \int \frac{uM}{D(u)} du, \right\}$$

$$\left|\langle \bar{q}q \rangle\right| = \frac{N_C}{4\pi^2} \int \frac{uM(u)}{D(u)} du, \qquad f_\pi^2 = \frac{N_C}{4\pi^2} \int \frac{uM(u)\left(M - uM'/2\right)}{D(u)} du,$$

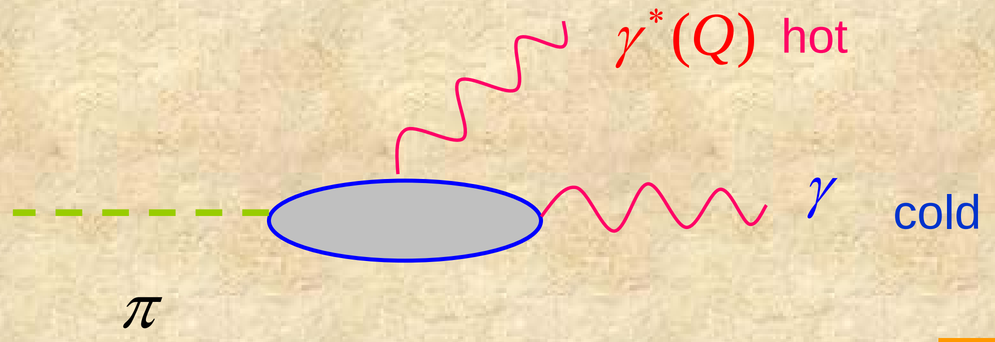
$$D(u) = u + M^2(u)$$

$$f_\pi^A = 92 \text{ MeV}, \quad f_\pi^P = (390 \text{ MeV})^2, \; f_\pi^T = (550 \text{ MeV})^2$$

Conclusions:

1) Twist 2,3 and 4 pion, photon and rho DAs are found within the instanton model at low momenta scale.
2) At this scale they differ essentially from asymptotic shape and thus QCD evolution is important for phenomenology.
3) Hadronic DAs (π,ρ) are suppressed at edge points x=0,1 reflecting nonlocal structure of QCD vacuum. It make pQCD to be applicable to some hard processes $\alpha_s(x(1-x)Q^2)$
4) At the same time hadronic component of photon DAs is almost constant because it couples to currents locally. Photon DAs are calculated for real and virtual photons with transverse and longitudinal polarizations.
5) Vector Meson Production is a way to investigate nonperturbative regime of QCD

Hadronic Distribution Amplitudes (DA)



Twist expansion

$$F_{\pi\gamma}\left(Q^2\right) = \frac{J_2}{Q^2} + \frac{J_4}{Q^4} + \dots$$

$$J_2 = \int_0^1 dx \frac{\varphi_\pi^{A(2)}(x)}{x(1-x)}, \qquad J_4 = \int_0^1 dx K_4(x) \varphi_\pi^{A(4)}(x) \dots$$

Coefficient function

Distribution Amplitude

DA (source) is the probability amplitude to find qq pair in the Pion with x_q=x

QCD is the interaction theory of hadrons in terms of fundamental degrees of freedom – massless gluons and massive quarks

$$L = -\frac{1}{4}\int d^4x F^a_{\mu\nu}(x) F^a_{\mu\nu}(x) + \int d^4x \bar{q}(x)\left[\gamma_\mu\left(i\partial_\mu + gA^a_\mu(x)\frac{\lambda^a}{2}\right) - m_q\right]q(x)$$

$$F^a_{\mu\nu}(x) = \partial_\mu A^a_\nu(x) - \partial_\nu A^a_\mu(x) + gf^{abc}A^b_\mu(x)A^c_\nu(x)$$

Perturbative series are asymptotic one

$$A = \sum_{n=0}^{\infty} a_n g^n$$

There are nonperturbative phenomena, non analytical in g

$$M(g) = M_0 \exp(c/g)$$

Instanton is nonperturbative solution of classical Yang-Mills equations.

$$A^a_\mu = 2\eta^a_{\mu\nu}\frac{x_\nu}{x^2+\rho^2},$$

$$\left(i\partial_\mu - A^a_\mu(x)\frac{\lambda^a}{2}\right)\gamma^\mu\psi(x) = 0,$$

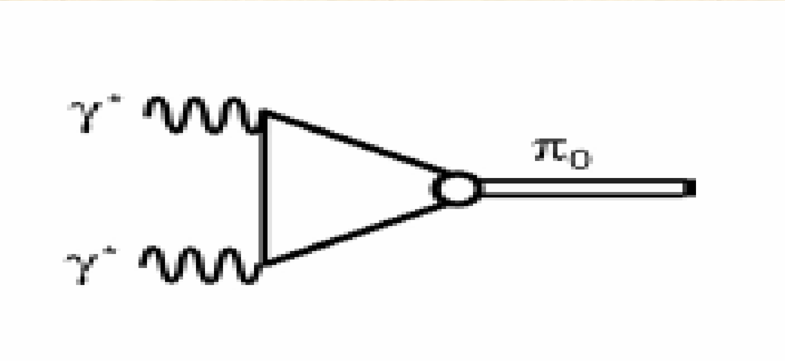
$$\psi_0(x) = \chi\frac{\rho}{\pi|x|\left(x^2+\rho^2\right)^{3/2}}$$

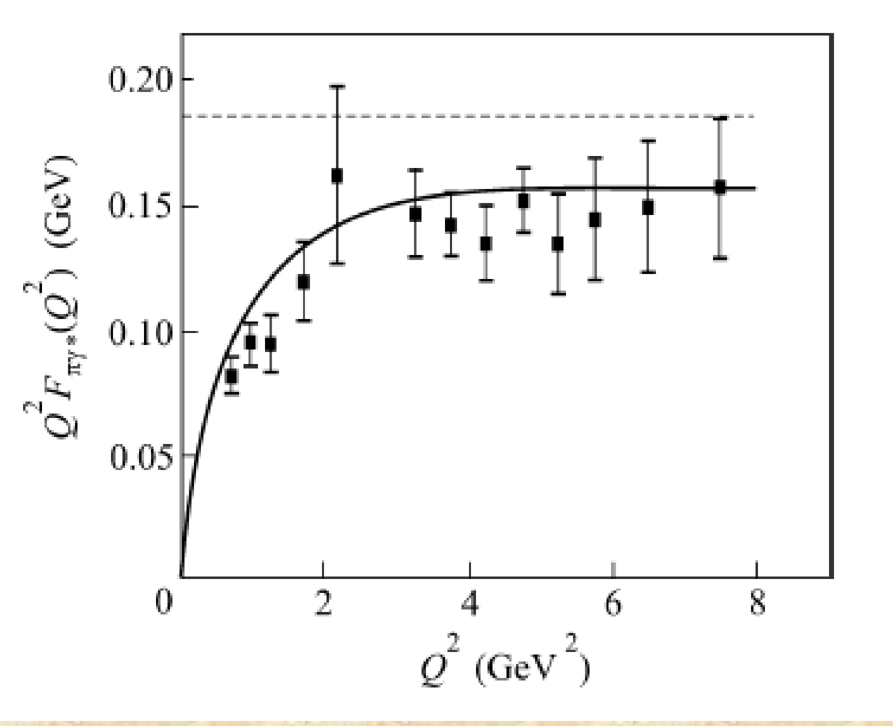
Instanton

$\rho=0.3$ fm is the size of instanton

Quark Zero Mode Solution

Pion Transition Form Factor (CLEO data vs Instanton Model)





The points are CLEO data

The curve is Instanton Model prediction

$$Q^2 F_{\pi\gamma}\left(Q^2\right) = J_2 + \ldots$$

$$J_2 = \frac{f_\pi}{3}\int_0^1 dx \frac{\varphi_\pi^{A(2)}(x)}{x(1-x)}$$

A number Applications of DAs

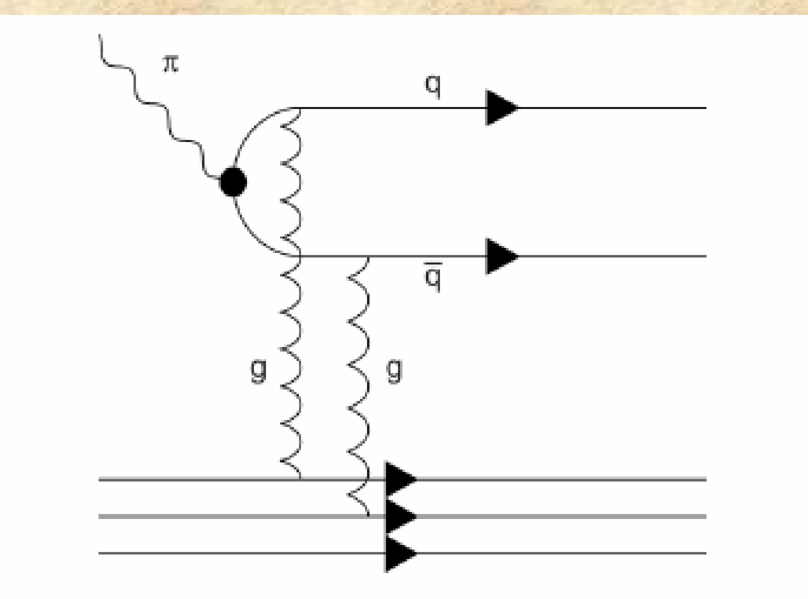
Pion and γ may be measured in the as a transition form factor like $\gamma^* \to \pi\gamma$ (JLAB, USA)

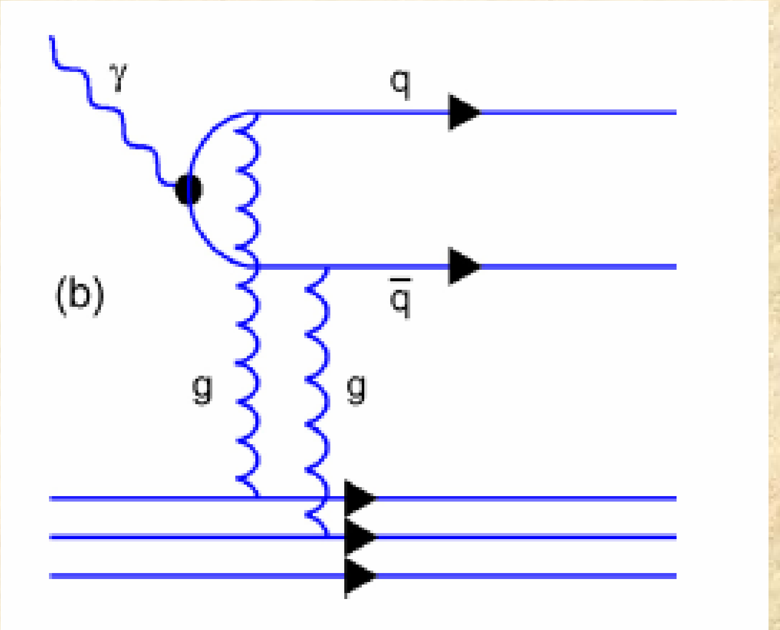
In diffractive double-jet production (HERA; FNAL)

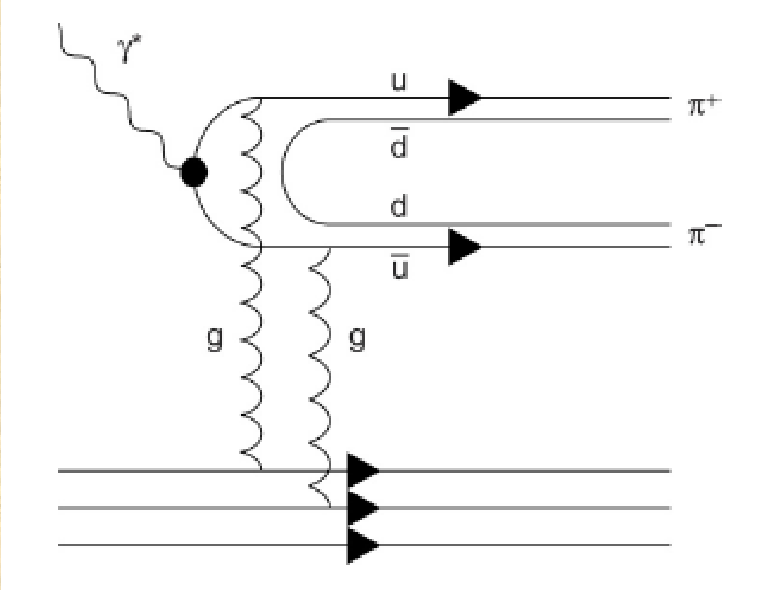
γ and ρ are important for diffractive ρ meson production (HERA)

B-meson exclusive decays $B \to \pi\pi$, $\rho\gamma$ (KEK)

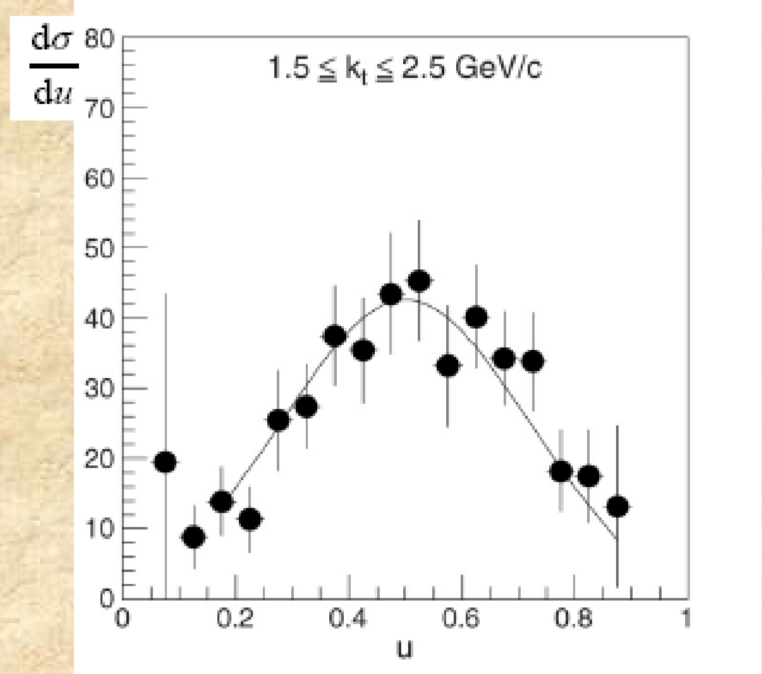
Pion and Photon dissociation to *q q* via two-gluon (Pomeron) exchange





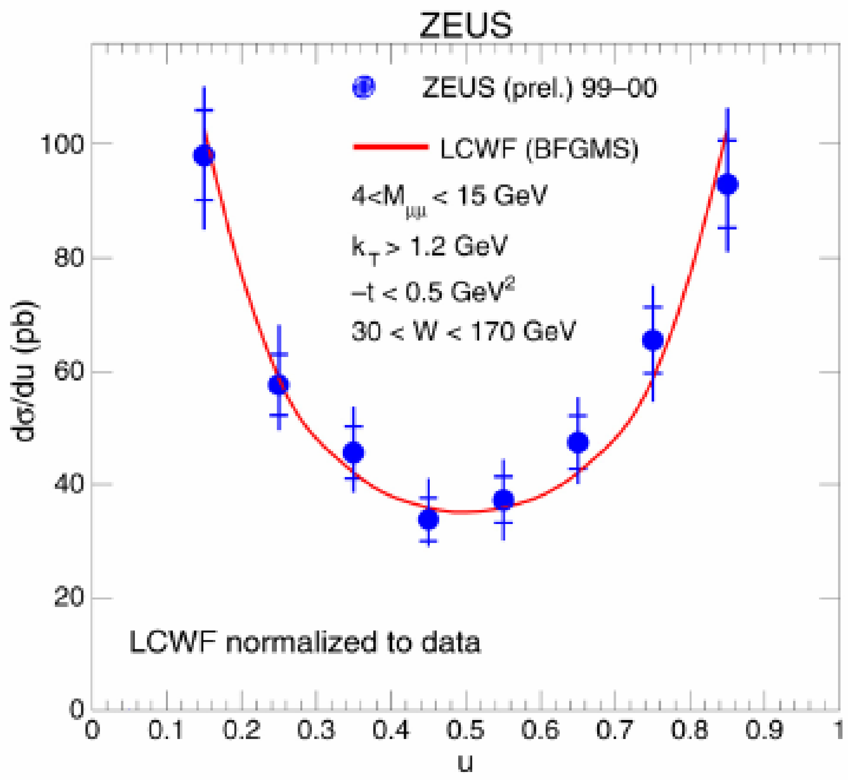


Fermilab E791 collaboration



$$\frac{d\sigma}{du} \propto \phi_\pi^2(u, Q^2) = N \cdot u^2(1-u)^2 \left(1.0 + a_2 C_2^{3/2}(2u-1) + a_4 C_4^{3/2}(2u-1)\right)^2$$

$$a_2 = 0.30 \pm 0.05,\ a_4 = (0.5 \pm 0.1) \times 10^{-2}.$$



Differential cross section $d\sigma/du$ for the $\gamma \to \mu^+\mu^-$ process measured for $30 < W < 170$ GeV, $4 < M_{\mu\mu} < 15$ GeV, $k_T > 1.2$ GeV$/c$ and $-t < 0.5$ (GeV$/c)^2$. The inner error bars show the statistical uncertainty; the outer error bars show the statistical and systematics added in quadrature. The data points are compared to the prediction of LCWF theory [16]. The theory is normalized to data.