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High Energy Spin Physics

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# Towards a GPD fitting procedure

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- ***Generalized parton distributions***
- ***How to get a realistic GPD ansatz?***
- ***Ready for a fitting procedure?***
- ***Conclusions***

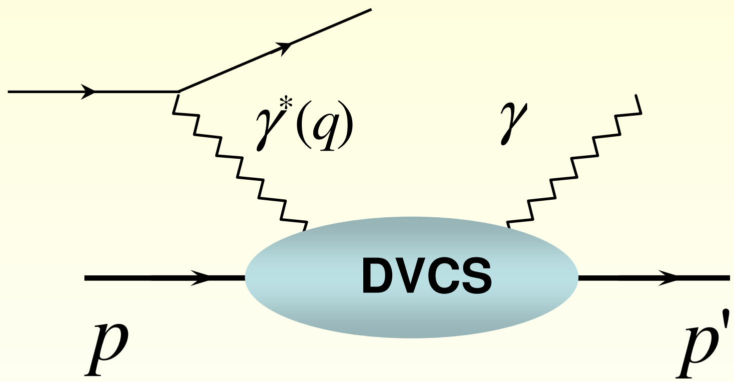
in collaboration with K. Kumerički and K. Passek-Kumerički (Zagreb)

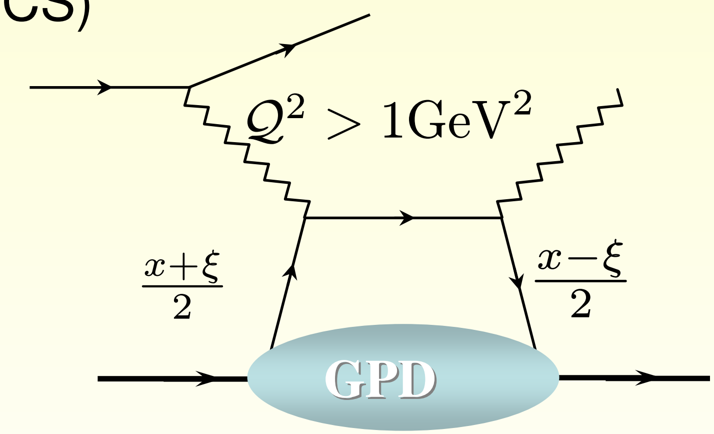
# Generalized parton distributions

DM, Robaschik, Geyer,
Dittes, Hořejši (PhD 92,94)
A. Radyushkin (96)
X. Ji (96)

GPDs appear in various hard exclusive processes,

e.g., hard electroproduction of photons (DVCS)




$t = \Delta^2$ – fix

$$\mathcal{F}(\xi, \mathcal{Q}^2, \Delta^2) = \int_{-1}^{1} dx\, C(x, \xi, \alpha_s(\mu), \mathcal{Q}/\mu) F(x, \xi, \Delta^2, \mu)$$

Compton form factor

observable

hard scattering part

perturbation theory
(our conventions)

GPD

universal
(but conventional)

# Definition of GPDs

Generically, GPDs are defined as matrix elements of light-ray operators

$$F(x,\eta,\Delta^2,\mu^2) = \int_{-\infty}^{\infty} d\kappa\, e^{i\kappa n\cdot P} \langle P_2|\phi(-\kappa n)\phi(\kappa n)_{(\mu^2)}|P_1\rangle\Big|_{\eta=\frac{n\cdot\Delta}{n\cdot P}}\,, \quad n^2 = 0$$

$$P = P_1 + P_2 \qquad \Delta = P_2 - P_1$$

For a nucleon (proton) target (mainly) four different twist-two GPDs appears:

$$\bar\psi_i\gamma_+\psi_i \quad \Rightarrow \quad {}^{i}q^{V} = \bar U(P_2,S_2)\gamma_+ U(P_1,S_1) H_i + \bar U(P_2,S_2)\frac{i\sigma_{+\nu}\Delta^\nu}{2M} U(P_1,S_1) E_i$$

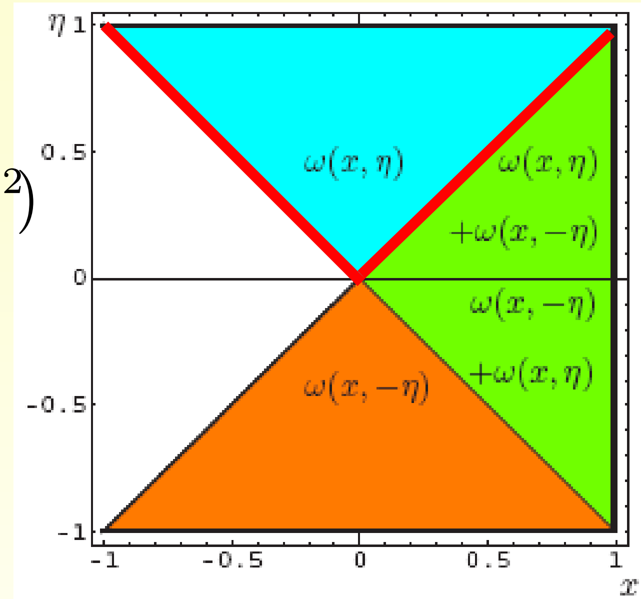
$$\bar\psi_i\gamma_+\gamma_5\psi_i \quad \Rightarrow \quad {}^{i}q^{A} = \bar U(P_2,S_2)\gamma_+\gamma_5 U(P_1,S_1) \widetilde H_i + \bar U(P_2,S_2)\frac{\gamma_5}{2M} U(P_1,S_1) \widetilde E_i$$

shorthand for GPDs: $F = \{H, E, \widetilde H, \widetilde E\}$ & CFFs: $\mathcal{F} = \{\mathcal{H}, \mathcal{E}, \widetilde{\mathcal{H}}, \widetilde{\mathcal{E}}\}$

$$\Delta^2 \equiv t$$

# Support of GPDs – a hint for duality

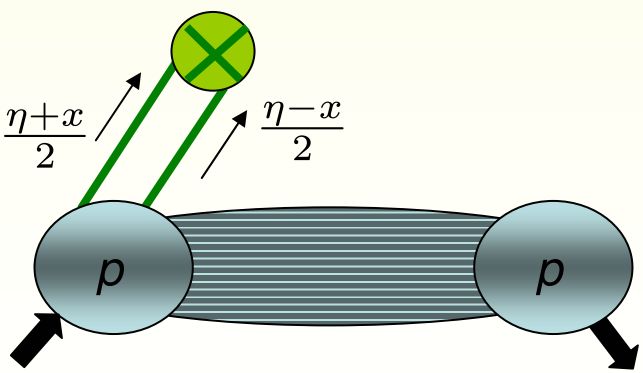
consider a quark GPD (anti-quark $x \to -x$)




$$F = \theta(-\eta \leq x \leq 1)\, \omega\big(x, \eta, \Delta^2\big) + \theta(\eta \leq x \leq 1)\, \omega\big(x, -\eta, \Delta^2\big)$$

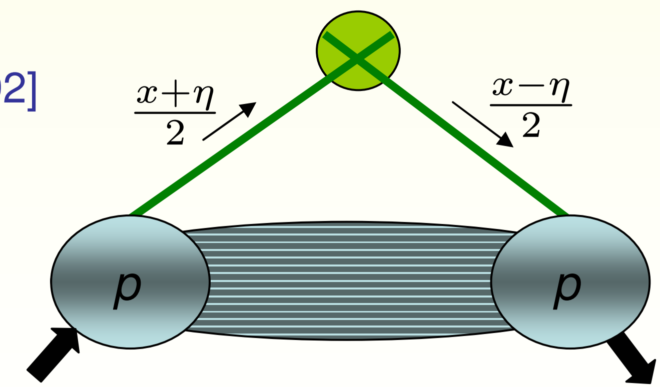
$$\omega\left(x, \eta, \Delta^2\right) = \frac{1}{\eta} \int_0^{\frac{x+\eta}{1+\eta}} dy\, (1-x)^p f(y, (x-y)/\eta, \Delta^2)$$

a naive ***dual*** interpretation on partonic level:




*support extension is unique* [DM et al. 88/92]

*ambiguous* (*D*-term)
[DM, A. Schäfer (05)]




central region $-\eta < x < \eta$

mesonic exchange in *t*-channel

outer region $\eta < x$

partonic exchange in *s*-channel

# Overlap representation of GPDs

QCD bound state problem might be formulated in LC quantization: **Drell, Yan (69) Drell, Brodsky**

$$P^-|P,S\rangle = \frac{M^2}{P^+}|P,S\rangle\,, \quad \text{with} \quad P^- = P^0 - P^3\,,\ P^+ = P^0 + P^3\,,\ \mathbf{P}_\perp = 0$$

***formally***, solution is expanded with respect to *partonic degrees of freedom:*

$$|P,S=\{\uparrow,\downarrow\}\rangle = \sum_{n,\lambda_i}\int [dx\, d^2\mathbf{k}]_n\, \psi^{\uparrow,\downarrow}_{n,\lambda_i}(x_i,\mathbf{k}_\perp,\lambda_i)|n, x_iP^+, x_i\mathbf{P}_\perp + \mathbf{k}_{\perp i},\lambda_i\rangle$$

GPDs defined as overlap of LC-wave functions (outer region): **Diehl, Feldmann, Jakob, Kroll (98)**

$$F(x\geq\eta,\eta,\Delta^2) \propto \sum_{n,\lambda_i}\left(\frac{1-\eta}{1+\eta}\right)^{\frac{2-n}{2}}\int [dx\, d^2\mathbf{k}]_n \delta\!\left(\frac{x+\eta}{1+\eta}-x_1\right)\psi^{\uparrow *}_{n,\lambda_i}(x'_i,\mathbf{k}'_{\perp i})\psi^{\uparrow(\downarrow)}_{n,\lambda_i}(x_i,\mathbf{k}_{\perp i})$$

$$x'_1 = \frac{x-\eta}{1-\eta}\,, \quad \mathbf{k}'_{\perp,1} = \mathbf{k}_{\perp,1} - \frac{1-x}{1-\eta}\mathbf{\Delta}_\perp$$

***Note:*** $x'_1 \to 0 \text{ for } x \to \eta$

***positivity constraints*** **[Pobylitsa (02)]** ***are satisfied***

***if Lorentz symmetry is correctly implemented, central region follows from duality***

**D.S. Hwang D.M., to appear**

# Constraints on GPDs

! polynomiality conditions arise from *hidden* Lorentz covariance

$$\int_{-\eta}^{1} dx\, x^n F(x,\eta,t) = \text{polynom of order } n \text{ or } n+1 \text{ in } \eta$$

satisfied within spectral representation (*D*-term is misleading)

$$F(x,\eta,t) = (1-x)^p \int_0^1 dy \int_{-1+y}^{1-y} dz\; \delta(x-y-z\eta) f(y,z,t),\; p=\{0,1\}$$

! lowest moment reduction to partonic form factor – related to observables

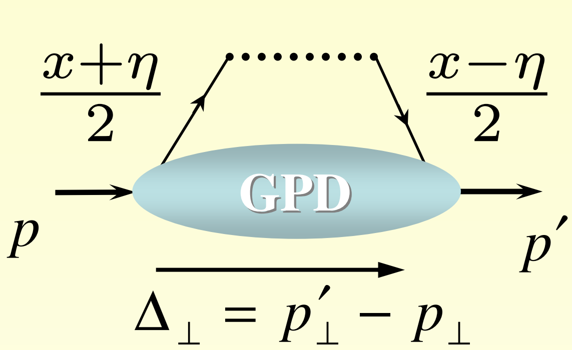
! first moment is given by the expectation value of the energy-momentum tensor

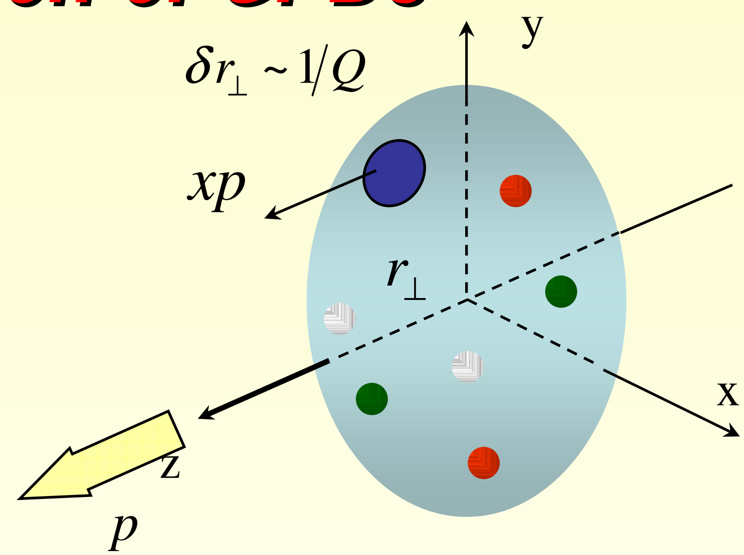
! reduction to parton densities (PDs)

$$q(x) = \lim_{\Delta\to 0} H(x,\eta,t), \quad \Delta q(x) = \lim_{\Delta\to 0} \widetilde{H}(x,\eta,t)$$

! positivity constraints (requirement on GPDs and scheme) **[Pobylitsa (02)]**
are automatically satisfied in the overlap representation

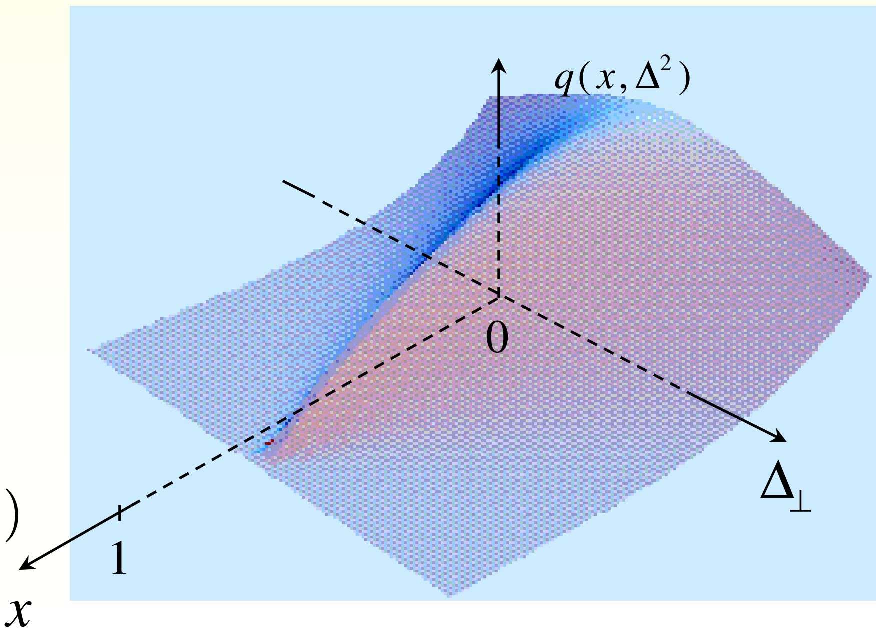
# Partonic interpretation of GPDs




GPDs simultaneously carry information on both *longitudinal* and *transverse* distribution of partons in a proton

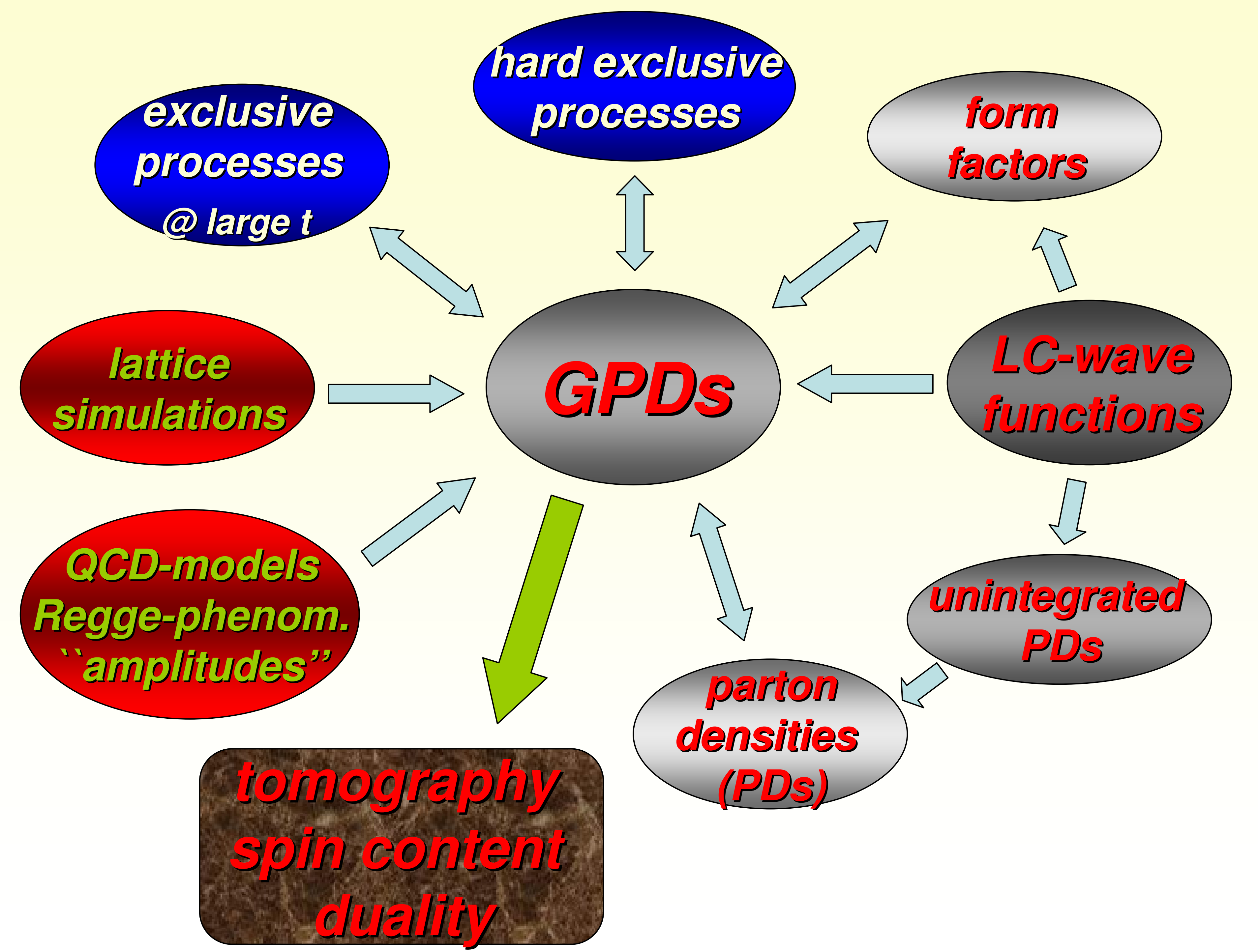
for $\eta$=0 they have a probabilistic interpretation (infinite momentum frame) **[Burkhardt (00)]**




GPDs contain also information on partonic angular momentum **[X. Ji (96)]**

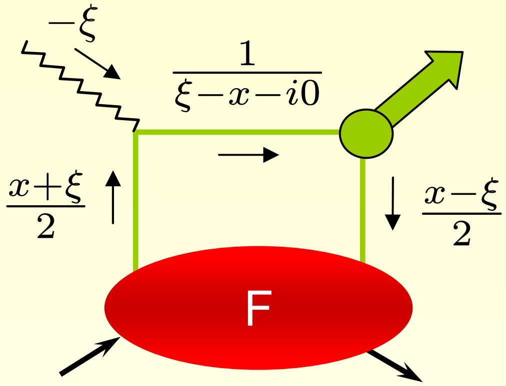
$$\frac{1}{2} = \sum_{a=q,G} J_a^z$$

$$J_a^z = \lim_{\Delta\to 0} \frac{1}{2} \int_{-1}^{1} dx\, x\, (H_a + E_a)\,(x, \eta, \Delta^2)$$


hard exclusive processes
exclusive processes @ large t
form factors
lattice simulations
GPDs
LC-wave functions
QCD-models
Regge-phenom.
``amplitudes''
unintegrated PDs
parton densities (PDs)
tomography
spin content
duality

# *Which partonic information can be accessed?*

Real and imaginary part of CFFs have to LO the following partonic interpretation:




$$\Im\mathrm{m}\mathcal{F}(\xi,\mathcal{Q}^2,\Delta^2) = \pi F(x=\xi,\xi,\Delta^2,\mathcal{Q}^2)$$

$$\Re\mathrm{e}\mathcal{F}(\xi,\Delta^2,\mathcal{Q}^2) = \mathrm{PV}\int_{-1}^{1} dx \frac{1}{\xi - x} F(x,\xi,\Delta^2,\mathcal{Q}^2)$$

Real part is given by a dispersion relation:

$$\Re\mathrm{e}\mathcal{F}(\xi,\Delta^2,\mathcal{Q}^2) = \mathrm{PV}\int_{-1}^{1} d\xi' \frac{1}{\xi - \xi'} F(x=\xi',\xi',\Delta^2,\mathcal{Q}^2) + \mathcal{C}(\Delta^2,Q^2)$$

CFFs to LO are entirely determined:

$$\mathcal{F}(\xi,\mathcal{Q}^2,\Delta^2) \Leftrightarrow F(x=\xi,\eta=\xi,\Delta^2,\mathcal{Q}^2),\ \theta(|x|\leq\xi)D(x/\xi,\Delta^2,\mathcal{Q}^2)$$

***How strong is the skewness dependence?***

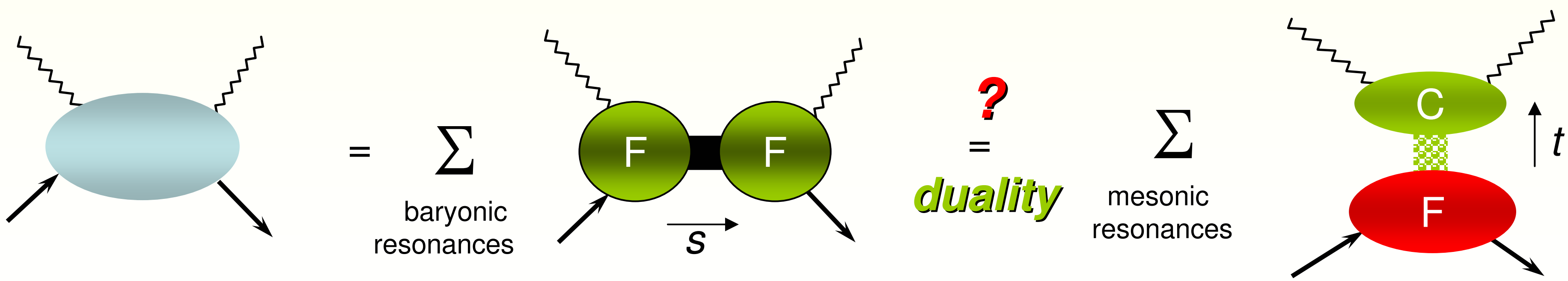
in DIS: $q(x,\mathcal{Q}^2) = F(x,\eta=0,\Delta^2=0,\mathcal{Q}^2)$

# How to get a realistic GPD model?

- lattice simulations of GPD moments (first few, heavy pion world) [QCDSF,LHPC,…]
- bag model [Ji et al.], quark soliton model [Göke et al,…], BS-equation [Miller,…], ….
- overlap of LC wave functions [Brodsky, Feldmann, Diehl, Hwang, Jakob, Kroll]
- models for amplitudes (perhaps better understanding as for GPDs)
  - resumming *s*-channel resonances [Close, Zhao]
  - vector dominance & Regge inspired description [Guidal et al., M. Capua et al., …]

*s*-channel contributions (resonance region, large *x*)

*t*-channel contributions (Regge phenomenology, small *x*)




⇨ take models (`knowledge') for the amplitude and extract GPDs

# Ansatz for partonic partial wave amplitudes

We work in **conformal Mellin-space** and use **SO(3) *t*-channel partial waves**

$$F_j(\eta,\Delta^2,\mu^2) = \int_{-1}^{1} dx\, \eta^j C_j^{3/2}(x/\eta) F(x,\eta,\Delta^2,\mu^2) = \sum_J F_j^J(\Delta^2,\mu^2)\hat{d}_J(\eta)$$

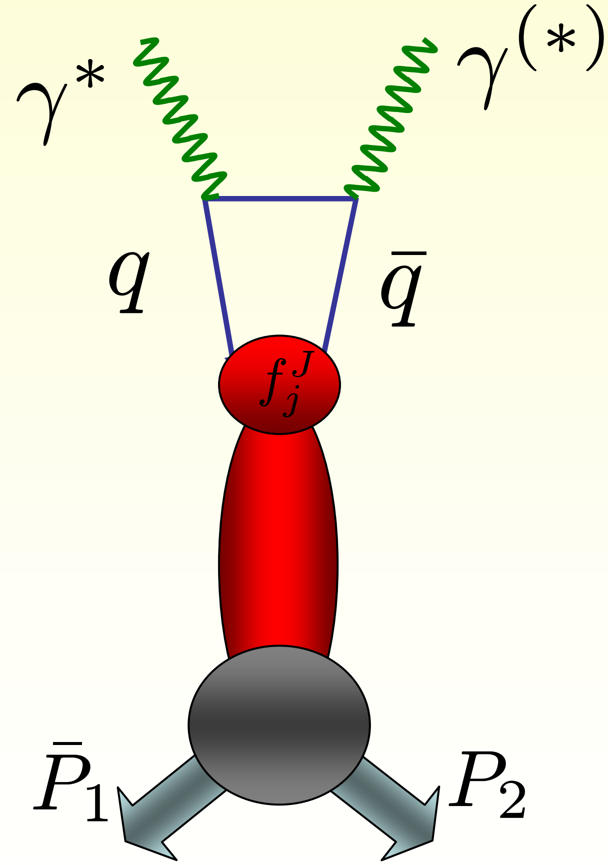
- at short distance a quark/anti-quark state is produced, labeled by ***conformal spin*** *j+2*

- they form an intermediate mesonic state with total angular momentum ***J*** strength of ***coupling*** is $f_j^J,\ J \leq j+1$

- mesons propagate with $\frac{1}{m^2(J)-t} \propto \frac{1}{J-\alpha(t)}$

- decaying into a nucleon anti-nucleon pair with given spin *S* and angular momentum *L*, described by an ***impact form factor***

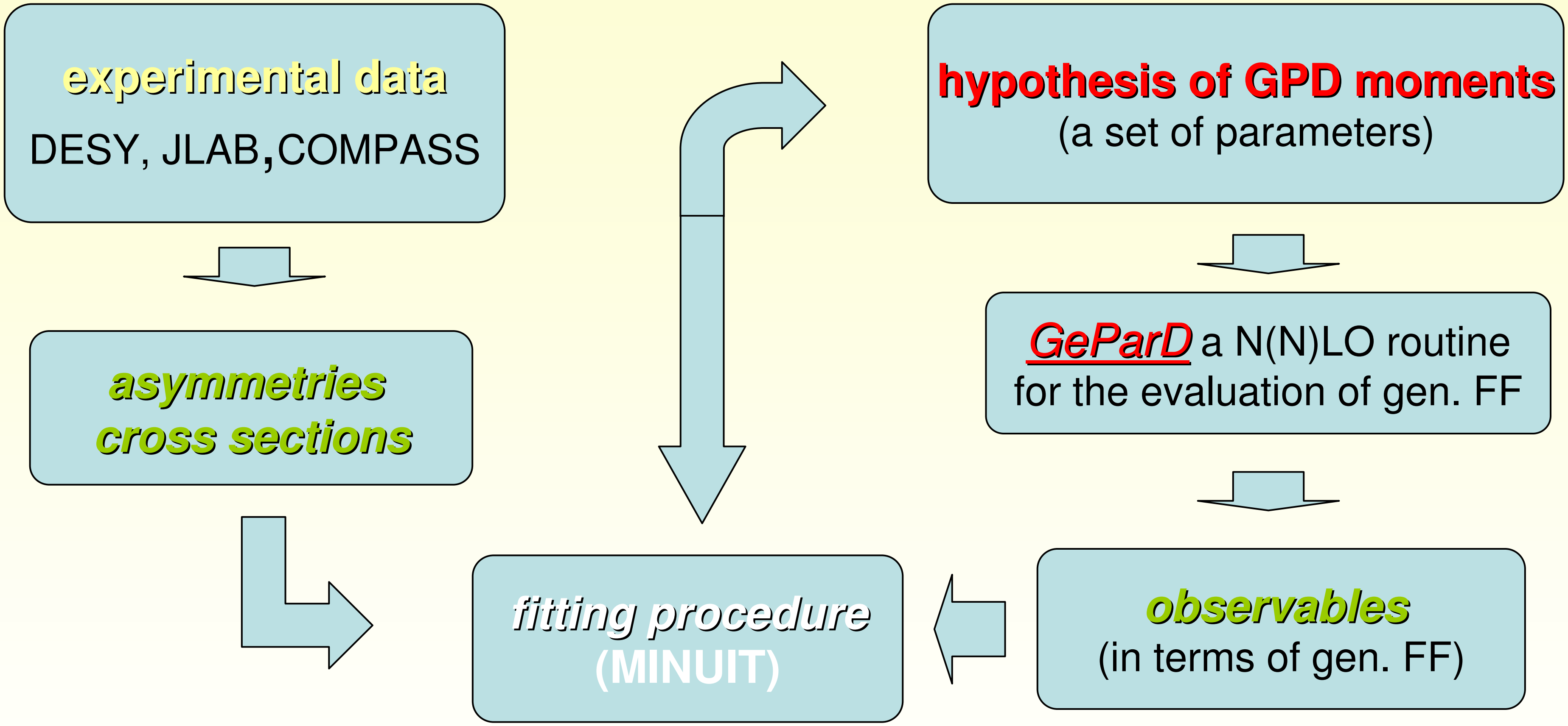
$$F_j^J(\Delta^2) = \frac{f_j^J}{J-\alpha(\Delta^2)} \frac{1}{(1-\frac{\Delta^2}{M^2(J)})^p}$$




**!** *D*-term arises from the SO(3) partial wave *J=j+1* *(j → -1)*

# Ready for a GPD fitting procedure?

[K. Kumerički, D.M., K. Passek-Kumerički, hep-ph/0703179]




partially *YES* but it is *NOT* completed yet:

- reasonable well motivated hypotheses of GPD moments must be implemented
- some technical, however, straightforward work is left
  (like a reevaluation of observables)

# Lessons from DVCS fits for H1 and ZEUS data

DVCS cross section has been measured in the small $\xi = \mathcal{Q}^2/(2W^2+\mathcal{Q}^2)$ region

$$40\mathrm{GeV} \lesssim W \lesssim 150\mathrm{GeV}, \quad 2\mathrm{GeV}^2 \lesssim \mathcal{Q}^2 \lesssim 80\mathrm{GeV}^2, \quad |t| \lesssim 0.8\mathrm{GeV}^2$$

and it is predicted by

$$\frac{d\sigma}{d\Delta^2}(W,\Delta^2,\mathcal{Q}^2) \approx \frac{4\pi\alpha^2}{\mathcal{Q}^4}\frac{W^2\xi^2}{W^2+\mathcal{Q}^2}\left[|\mathcal{H}|^2 - \frac{\Delta^2}{4M_{\mathrm{p}}^2}|\mathcal{E}|^2 + \left|\widetilde{\mathcal{H}}\right|^2\right](\xi,\Delta^2,\mathcal{Q}^2)\Big|_{\xi=\frac{\mathcal{Q}^2}{2W^2+\mathcal{Q}^2}}$$

suppressed contributions *<<0.05>>* relative *O(ξ)*

LO [Belitsky, DM, Kirchner (01), Guzey, Teckentrup (06)]

data are described within ***questionable*** *t*-slope parameters

NLO [Freund, M. McDermott (02)]

results strongly depend on used parton density parameterization

⇨ ***do a simultaneous fit to DIS and DVCS***

# Ansatz for conformal GPD moments

$$H_j^{\Sigma}(\eta, \Delta^2, \mu_0^2) = N_{\Sigma} \frac{B(1-\alpha_{\Sigma}(0)+j, 8)}{B(2-\alpha_{\Sigma}(0), 8)} \frac{1}{1-\frac{\Delta^2}{(m_j^{\Sigma})^2}} \frac{1}{\left(1-\frac{\Delta^2}{(M_j^{\Sigma})^2}\right)^3} + \mathcal{O}(\eta^2)$$

PD momentum fraction

PD Mellin moments

Regge inspired
*t*-dependence

impact form factor
(counting rules, lattice)

some simplifications in the ansatz:

- neglecting $\eta$ dependence
- only designed for small *x* (no momentum sum rule, $N_{\Sigma}, N_G$ free parameters)
- flavor non-singlet contribution is neglected (< 5% effect)
- fixed numbers of quarks *($n_f$=4)*

parameters @ fixed input scale $Q^2 = 4\ GeV^2$

- 2x normalization *N*, 2x intercept *α*, 2x cut-off mass $M_0$
- little sensitivity of slope *α'* (=0.15/GeV$^2$ )
- little sensitivity on *j*-dependence in $M_j$

| order (scheme) | $\alpha_s(M_Z)$ | $N_\Sigma$ | $\alpha_\Sigma(0)$ | $M^2_\Sigma$ | $N_G$ | $\alpha_G(0)$ | $M^2_G$ | $\chi^2$ | $\chi^2/\text{d.o.f.}$ | $\chi^2_{\Delta^2}$ |
|---|---|---|---|---|---|---|---|---|---|---|
| LO | 0.130 | 0.157 | 1.17 | 0.228 | 0.527 | 1.25 | 0.263 | 100 | 0.85 | 38.5 |
| NLO ($\overline{\text{MS}}$) | 0.116 | 0.172 | 1.14 | 1.93 | 0.472 | 1.08 | 4.45 | 109 | 0.92 | 4.2 |
| NLO ($\overline{\text{CS}}$) | 0.116 | 0.167 | 1.14 | 1.34 | 0.535 | 1.09 | 1.59 | 95 | 0.80 | 2.2 |
| NNLO ($\overline{\text{CS}}$) | 0.114 | 0.167 | 1.14 | 1.17 | 0.571 | 1.07 | 1.39 | 91 | 0.77 | 2.2 |

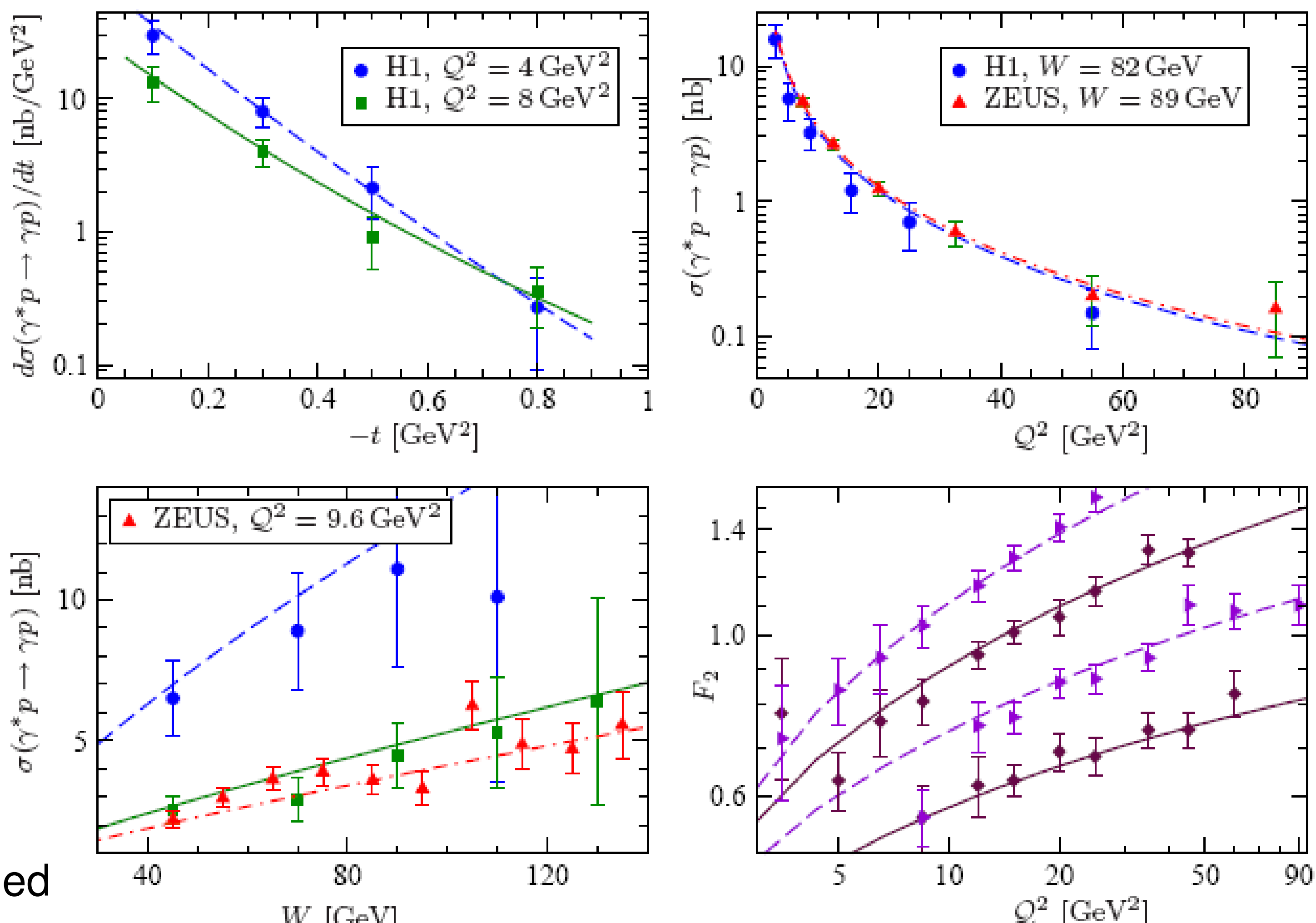
***simultaneous NNLO fit to DVCS and DIS***

LO & MS NLO fits are not optimal

missing parameter

CS beyond LO yields good fits

neglecting $\eta$ is justified ? just luck



# Can one do better?

***Yes***, introduce a distribution of SO(3) partial waves in conformal GPD moments

toy example: take two partial waves

$\eta$ dependence can be safely neglected

$$F_j(\eta,\Delta^2) = \frac{f_j^{j+1}}{(1-\frac{\Delta^2}{M^2(j+1)})^p}\left(\frac{1}{j+1-\alpha(\Delta^2)}\hat{d}_{j+1}(\eta) + \frac{s\,\eta^2}{j-1-\alpha(\Delta^2)}\hat{d}_{j-1}(\eta)\right)$$

*effective relative strength of remaining partial waves*

now we get a very good LO fit:

- fixed $s_G = 0$, $M = M_G = M_\Sigma$
- $X^2$/d.o.f. = 0.52, $s_\Sigma = -0.75$,
- *other parameters are consistent with previous fits*

  $N_\Sigma = 0.14$, $\alpha_\Sigma = 1.20$, $N_G = 0.8$, $\alpha_G = 1.16$
- $X^2_t = 2.61$, $M^2 = 0.86$

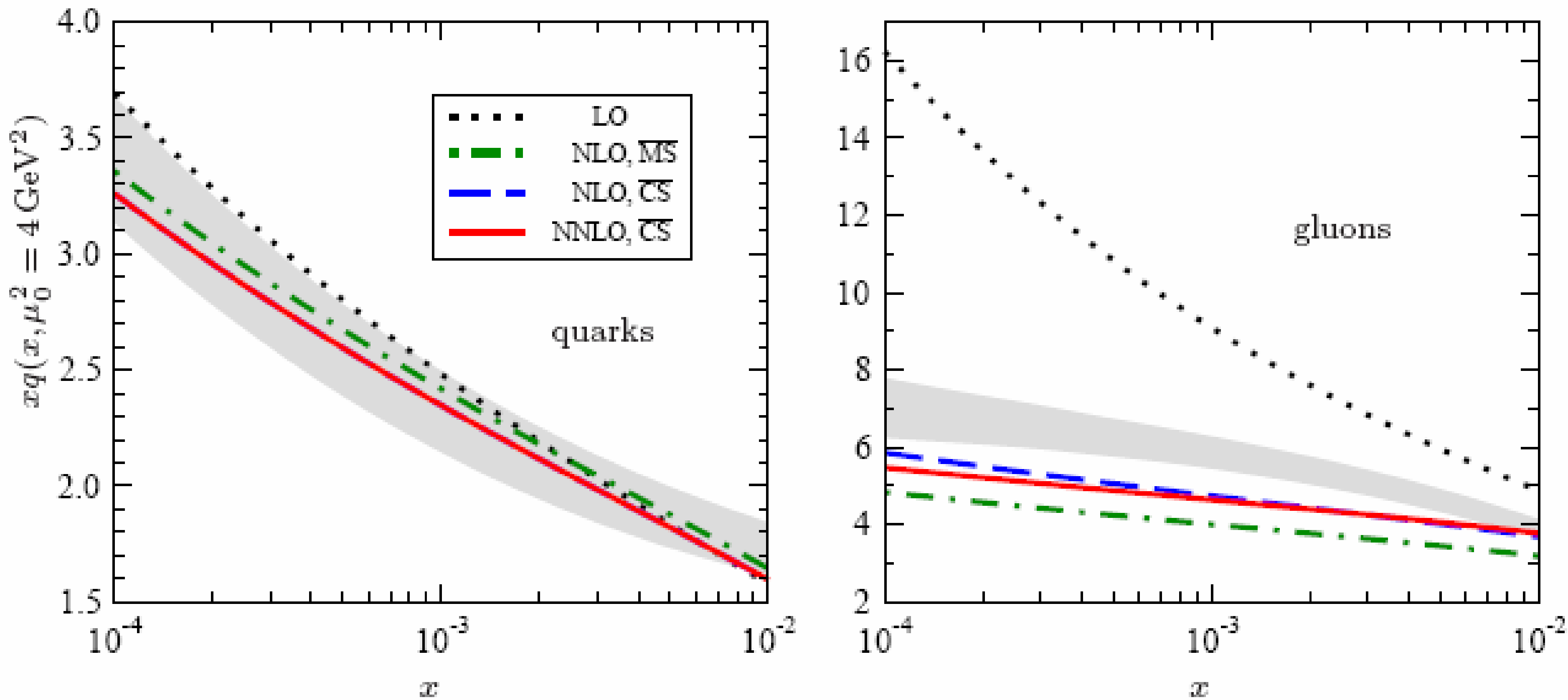
⇒ ***'negative'*** skewness dependence is ***required*** at LO

# Partonic picture: longitudinal degrees

our fits are ***compatible*** with Alekhin's NLO PDF parameterization:

✓ central value of our quark densities lies in Alekhin's error band

✓ gluons are less constrained by DIS fit
(error bands would overlap)

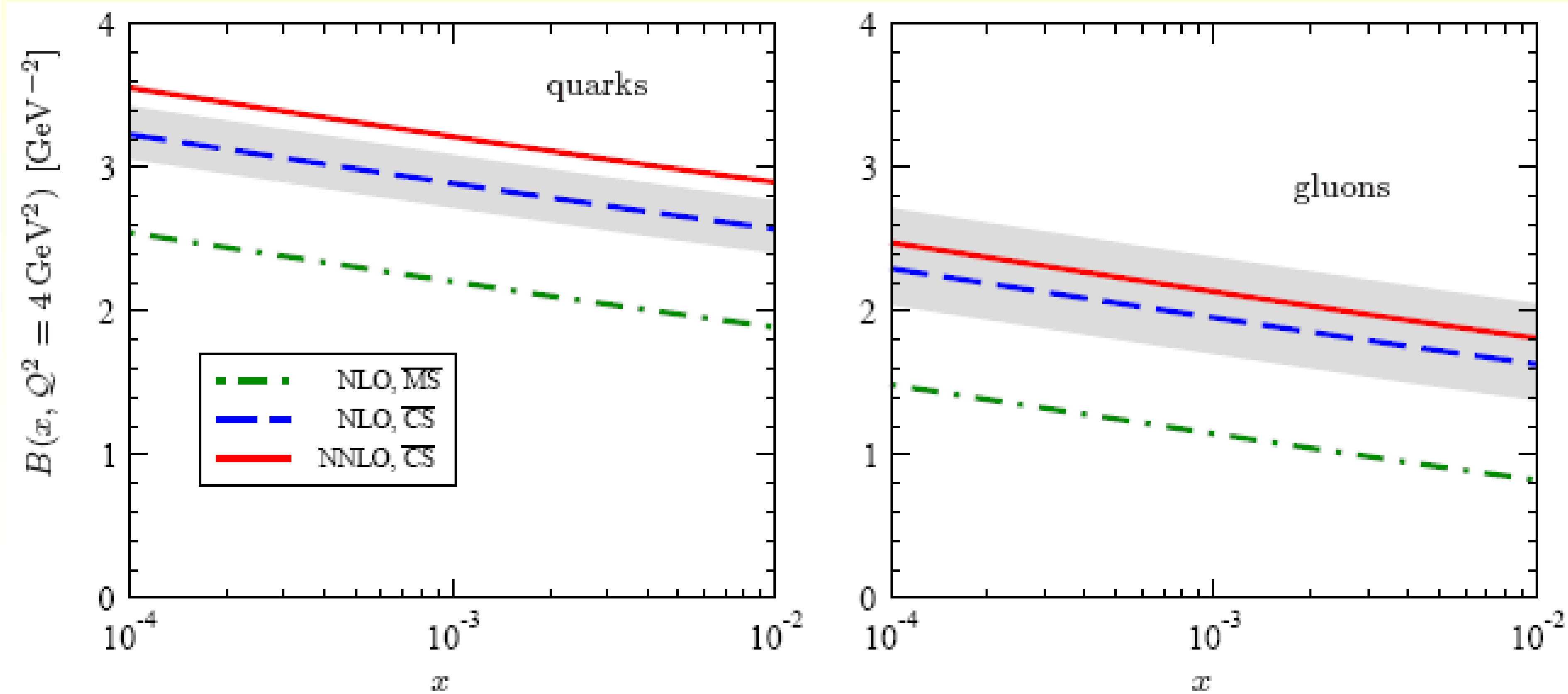


# Partonic picture: transversal degrees

transversal distribution of partons in the infinite momentum frame:

$$H(x,\vec{b}) = \int \frac{d^2\vec{\Delta}}{(2\pi)^2}\, e^{-i\vec{b}\cdot\vec{\Delta}} H(x,\eta=0,\Delta^2=-\vec{\Delta}^2)$$

the average distance of partons is: $\langle \vec{b}^2\rangle(x,\mathcal{Q}^2) = \frac{\int d\vec{b}\,\vec{b}^2 H(x,\vec{b},\mathcal{Q}^2)}{\int d\vec{b}\, H(x,\vec{b},\mathcal{Q}^2)} = 4B(x,\mathcal{Q}^2)$



# Conclusions

❖ useful to consider GPDs as overlap of wave functions

➔ a tool to probe the wave functions of nucleon, hadrons, and nuclei

➔ this point of view allows

i. to connect uninegrated parton densities and GPDs
ii. yields the question for the appropriate parameterization of wave functions

❖ only “realistic” GPD parameterizations provide insight into the proton

- tomography -- 3D picture (realistic to do at present/future)
- angular momentum of partons (a very long way)

❖ dual parameterization of GPDs based on *t*-channel exchanges formulated in conformal Mellin space

✓ parameterization of all degrees of freedoms of GPDs

✓ numeric is ***fast*** and ***reliable*** [even at NLO for MS scheme]

✓ perturbative expansion in DVCS ***works*** – except for ***evolution*** at ***small x***

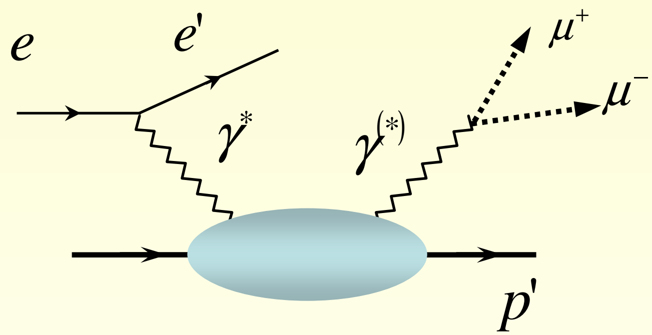
✓ fitting procedure (better than comparing model A, B, ..., with data) ***can be set up***

✓ a \`global’ analysis of GPD related data ***requires NLO***

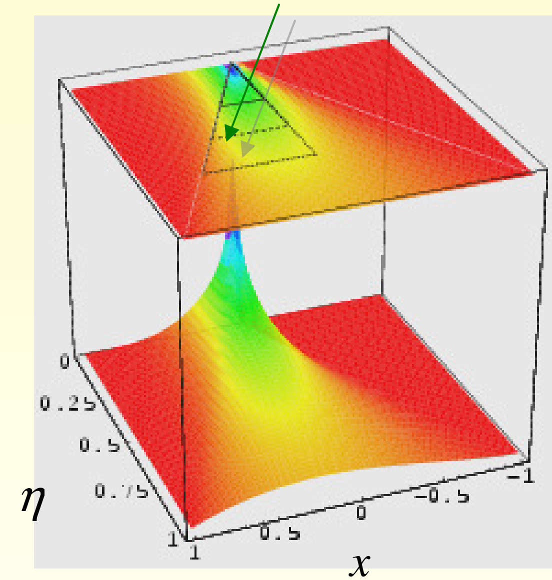
# GPD related hard exclusive processes

- Deeply virtual Compton scattering
  (clean probe)

$$ep \to e'p'\gamma$$
$$ep \to e'p'\mu^+\mu^-$$
$$\gamma p \to p'e^+e^-$$



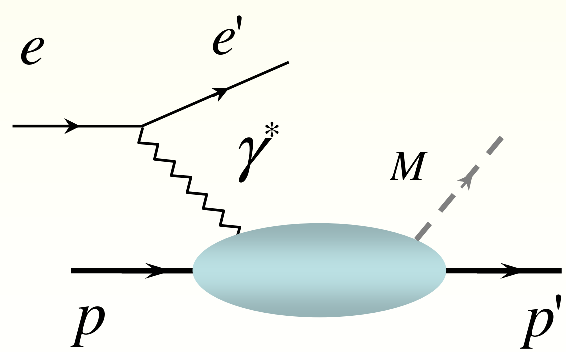

scanned area of the surface as a functions of lepton energy




$ep \to e'\,p'\,\mu^+\mu^-$

- Hard exclusive meson production
  (flavor filter)

$$ep \to e'p'\pi$$
$$ep \to e'p'\rho$$
$$ep \to e'n\pi^+$$
$$ep \to e'n\rho^+$$




<u>twist-two observables:</u>

cross sections

transverse target spin asymmetries

- etc.

measured from H1, ZEUS, HERMES; Hall A & B (CLAS) @ JLAB

# Perturbative and higher twist corrections

- ***perturbative next--to--leading order corrections*** [conformal approach D.M. (94)]
  - ✓ hard scattering part for photon/meson electroproduction [A. Belitsky, D.M. (00,01)
  - ✓ flavor singlet part for meson electroproduction [D. Ivanov, L. Szymanowski (04) ]
  - ✓for all then flavor singlet twist--two anomalous dimensions [A. Belitsky, D.M. (98)]
  - ✓and flavor singlet twist--two evolution kernels [A. Belitsky, D.M., A. Freund (99,00)]
- ***evaluation of higher twist contributions***
  - ✓completing the twist-three sector [A. Belitsky, D.M. (00)]
  - ❖ target mass corrections (twist-4) to photon electroproduction [A.Belitsky,D.M.(01)
  - ❖ WW-approximation to helicity flip DVCS contribution [N. Kivel, L. Mankiewicz (01)]

  o power suppressed corrections are not well understood
- ***perturbative next--to--next--to--leading order corrections to DVCS***
  [D.M. (05); K.Kumerićki, K.Passek-Kumerićki, D.M., A. Schäfer (06/07)]