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About $s - \bar{s}$ and $D_d^{K^+ - K^-}$ in $K^{\pm}$ production in SIDIS

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We know from experiment:

DIS & SIDIS:

$$s+\bar{s}, \quad \Delta s+\Delta\bar{s}, \quad \int dx\,(\Delta s+\Delta\bar{s})<0$$

e^+e^- & SIDIS:

$$D_q^{K^+}, \quad D_q^{K^-}$$

but data is not enough and not precise enough $\Rightarrow$

always assumptions:

$$s-\bar{s}=0, \quad \Delta s-\Delta\bar{s}=0, \quad D_d^{K^+}=D_d^{K^-}\ldots$$

The goal:

How can we justify these assumptions directly?-SIDIS

E. Ch. and E. Leader, EPJ, 2007

In general:

$$s = s_+ + s_-, \quad \bar{s} = \bar{s}_+ + \bar{s}_-, \quad s_\pm \geq 0,\ \bar{s}_\pm \geq 0,$$

$$\Delta s = s_+ - s_-,\ \Delta\bar{s} = \bar{s}_+ - \bar{s}_-, \quad \Delta s \lesseqgtr 0,\ \Delta\bar{s} \lesseqgtr 0$$

The positivity constraints: $|\Delta s| \leq s,\ |\Delta\bar{s}| \leq \bar{s}$

$$s - \bar{s} = (s_+ + s_-) - (\bar{s}_+ + \bar{s}_-) \lesseqgtr 0$$

$$\Delta s + \Delta\bar{s} = (s_+ + \bar{s}_+) - (s_+ + \bar{s}_-) \lesseqgtr 0$$

$$\Delta s - \Delta\bar{s} = (s_+ + \bar{s}_-) - (s_- + \bar{s}_+) \lesseqgtr 0$$

$$\Rightarrow |s - \bar{s}| \leq s + \bar{s}, \quad s - \bar{s} \lesseqgtr 0$$

$$\Rightarrow |\Delta s \pm \Delta\bar{s}| \leq s + \bar{s}, \quad \Delta s \pm \Delta\bar{s} \lesseqgtr 0$$

- exp: $s + \bar{s} \neq 0, \quad x \lesssim 0,5 \quad \Rightarrow$

  1) $x \gtrsim 0,5: \quad s - \bar{s} \simeq 0, \quad \Delta s \pm \Delta\bar{s} \simeq 0$

  2) $\int (s - \bar{s}) = 0 \quad \Rightarrow \quad s - \bar{s}$ changes sign

- no relation between $\Delta s - \Delta\bar{s}$ & $\Delta s + \Delta\bar{s}$

$\Rightarrow$ Our knowledge of $\Delta s + \Delta\bar{s}$ does not help us put limits on $\Delta s - \Delta\bar{s}$!

SIDIS with Kaons sensitive to s-quarks:

$$[K^+ = (\bar{s}u)]$$

SIDIS : $\quad l + N \to l + K^{\pm} + X$

$$\sigma_p^{K^+-K^-} \equiv \sigma_p^{K^+} - \sigma_p^{K^-}$$

$$\sigma_p^{K^+-K^-} = e_u^2\, u_V\, D_u^{K^+-K^-} + e_d^2\, d_V\, D_d^{K^+-K^-} + e_d^2\, (s - \bar{s})\, D_s^{K^+-K^-}$$

$$\Delta\sigma_p^{K^+-K^-} = e_u^2\, \Delta u_V\, D_u^{K^+-K^-} + e_d^2\, \Delta d_V\, D_d^{K^+-K^-} + e_d^2\, (\Delta s - \Delta\bar{s})\, D_s^{K^+-K^-}$$

Why difference cross sections?

- all terms are NS's $\Rightarrow$ in PD's & FF's
  - no $g(x)$ and $D_g^h(z)$
  - in NLO, NNLO ... no new PD's & FF's
  - in Q^2 evolution – no new PD's & FF's

E. Ch. and E. Leader,

Nucl. Phys. B607 (2001); SPIN2004, Trieste

$s - \bar{s} = 0$? LO:

$$\sigma_{p+n}^{K^+-K^-} = (u_V + d_V)(4D_u + D_d)^{K^+-K^-} + 2(s-\bar{s})D_s^{K^+-K^-}$$

$$\sigma_{p-n}^{K^+-K^-} = (u_V - d_V)(4D_u - D_d)^{K^+-K^-}$$

Consider the measurable quantity:

$$R_+(x,z) = \frac{\sigma_d^{K^+} - \sigma_d^{K^-}}{u_V + d_V} = (4D_u + D_d)\left[1 + \frac{s-\bar{s}}{2(u_V+d_V)}\left(\frac{D_s}{D_u}\right)^{K^+-K^-}\right]$$

$$\text{if} \quad s - \bar{s} \simeq 0 \quad \Leftrightarrow \quad R_+(x,z) = (4D_u + D_d)(z)$$

$$\text{Recall}: \quad \left(\frac{D_s}{D_u}\right)^{K^+-K^-} (z) > 1$$

We examine the x-dependence $\longrightarrow$ it is present only if $s - \bar{s} \neq 0$:

1. If $\quad R_+(x, z_0) \neq R_+(z_0) \longrightarrow s - \bar{s} \neq 0$

2. If $\quad R_+(x, z_0) = R_+(z_0) \longrightarrow s - \bar{s} \simeq 0$

- Independently of our knowledge of the FF's we can test if $s - \bar{s} \simeq 0$ is a good approximation!
- If $D_q^{K\pm}$ known at some z_0 we can put limits on $s - \bar{s}$ - the FFs: *EMC 1989; de Florian, Sassot & Stratmann, 2007*

Limits on $s - \bar{s}$, LO

If there is no dependence on x, it is always within a certain error of the measurements:

$$\begin{aligned} \underline{\underline{\text{th :}}} \quad R_+(x, z) &= (4D_u + D_d) + \frac{s - \bar{s}}{u_V + d_V} D_s^{K^+ - K^-} \\ \underline{\underline{\text{exp :}}} \quad &= r_+(z_0) \pm \delta r_+(z_0), \\ & \quad r_+ >> \delta r_+ \end{aligned}$$

The limits depend on $\delta r_+ / r_+$ and on our knowledge of the FFs at $z = z_0$:

$$\left| \frac{(s - \bar{s})}{2(u_V + d_V)} \left(\frac{D_s}{D_u} \right)^{K^+ - K^-} \right| \leq \frac{\delta r_+}{|r_+|}$$

LO?

$$(\sigma_p - \sigma_n)^{K^+ - K^-}(x, z) = (u_V - d_V)\,(4\,D_u - D_d)^{K^+ - K^-}$$

Consider the measurable quantity:

$$R_-(x, z) = \frac{(\sigma_p - \sigma_n)^{K^+ - K^-}}{u_V - d_V} = (4\,D_u - D_d)^{K^+ - K^-}(z)$$

We examine the x-dependence $\longrightarrow$ present only if NLO:

1. If $R_-(x, z_0) = R_-(z_0) \Rightarrow$ LO!

2. If $R_-(x, z_0) \neq R_-(z_0) \Rightarrow$ NLO!

- Independent on our knowledge of the FFs

$D_d^{K^+-K^-} = 0?$

LO:

$$\begin{cases} R_+(x,z) = (4\,D_u + D_d)^{K^+-K^-} + \frac{2(s-\bar{s})}{(u_V+d_V)}\,D_s^{K^+-K^-} \\ R_-(x,z) = (4\,D_u - D_d)^{K^+-K^-} \end{cases}$$

Then:

$$R_+ - R_- = 2D_d^{K^+-K^-} + \frac{2(s-\bar{s})}{(u_V+d_V)}D_s^{K^+-K^-}$$

Suppose there is no x-dependence in both $R_\pm(x,z_0)$:

$$\begin{cases} R_+(x,z_0) = R_+(z_0) & \Rightarrow \quad (s-\bar{s}) \simeq 0 \\ R_-(x,z_0) = R_-(z_0) & \Rightarrow \quad \text{LO!} \end{cases}$$

Then:

1. If $(R_+ - R_-)(z_0) \neq 0 \quad \longrightarrow \quad D_d^{K^+-K^-}(z_0) \neq 0$

2. If $(R_+ - R_-)(z_0) = 0 \quad \longrightarrow \quad D_d^{K^+-K^-}(z_0) \simeq 0$

• Independently of our knowledge of the FF's we can say if $D_d^{K^+-K^-}(z_0) \simeq 0$ is a reasonable approximation!

Limits on $D_d^{K^+-K^-}$

$$1)\ \underline{\underline{th:}} \begin{cases} R_+ = (4D_u + D_d)\left\{1 + \frac{2\,(s-\bar{s})}{u_V+d_V}\,D_s\right\} \\ R_+ - R_- = 2D_d + \frac{2\,(s-\bar{s})}{u_V+d_V}\,D_s \end{cases}$$

$$\Rightarrow R_+(z) - R_-(z) \neq 0 \quad \longrightarrow D_d^{K^+-K^-} \neq 0$$

$$\Rightarrow R_+(z) - R_-(z) \simeq 0 \quad \longrightarrow \quad \text{limits} \quad \text{on} \quad D_d^{K^+-K^-}$$

$$2)\ \underline{\underline{exp:}} \begin{cases} R_+(x, z_0) = r_+(z_0) \pm \delta r_+(z_0), \quad r_+ >> \delta r_+ \\ (R_+ - R_-)(x, z_0) = 0 \pm \delta r(z_0) \end{cases}$$

Then:

$$|\frac{(s-\bar{s})}{(u_V + d_V)}\left(\frac{D_s}{2D_u}\right)^{K^+-K^-}| \leq \frac{\delta r_+}{|r_+|}$$

$$|\left(\frac{D_d}{2D_u}\right)^{K^+-K^-}| \leq \frac{\delta r_+}{|r_+|} + \frac{\delta r}{2|D_u^{K^+-K^-}|}$$

We solve the two inequalities:

$$|a| \leq \delta_1$$

$$|a + b| \leq \delta_2$$

$$\longrightarrow \quad \delta_1 - \delta_2 \leq |b| \leq \delta_1 + \delta_2$$

NLO:

The analytic express. contain the same PD's and FF's as in LO, only in NLO they enter in convolutions:

$$(\sigma_p + \sigma_n)^{K^+-K^-} =$$

$$= [(u_V + d_V) \otimes (4D_u + D_d) + 2(s - \bar{s}) \otimes D_s] \otimes (1 + \alpha_s C_{qq})$$

$$(\sigma_p - \sigma_n)^{K^+-K^-} = (u_V - d_V) \otimes (1 + \alpha_s C_{qq}) \otimes (4D_u - D_d)$$

If both: $s - \bar{s} \simeq 0$ & $D_d^{K^+-K^-}(z_0) \simeq 0$ then we must succeed to fit both $(\sigma_p \pm \sigma_n)^{K^+-K^-}$ with the same $D(z)$:

$$\begin{cases} (\sigma_p + \sigma_n)^{K^+-K^-}(x,z) = (u_V + d_V) \otimes (1 + \alpha_s C_{qq}) \otimes D(z) \\ (\sigma_p - \sigma_n)^{K^+-K^-}(x,z) = (u_V - d_V) \otimes (1 + \alpha_s C_{qq}) \otimes D(z) \end{cases}$$

$$\Downarrow$$

$$D(z) = D_u^{K^+-K^-}(z)$$

If no such fit can be obtained $\Rightarrow$ either

1) $s - \bar{s} \neq 0$, or

2) $D_d^{K^+-K^-}(z_0) \neq 0$ or

3) both $s - \bar{s} \neq 0$ & $D_d^{K^+-K^-}(z_0) \neq 0$

- No knowledge of $D_{u,s}^{K^+-K^-}$ required!

Measurabilty of $s - \bar{s}$ and $D_d^{K^+ - K^-}$

1) difference cross sections: $\Rightarrow$ high precisions needed

2) data in bins in both x and z required

HERMES 2005: SIDIS $\Rightarrow \sigma_d^{K^\pm}$, $\sigma_p^{K^\pm}$, $[x_i, z_j]$
presented in 4 z-bins as functions of x:

$$0,023 \leq x \leq 0,300 \quad \rightarrow \quad in \quad 7 \quad x - bins$$

For 2 of the z-bins:

$$0,350 \leq z \leq 0,450$$
$$0,450 \leq z \leq 0,600$$

the precision is

$$\sigma_d^{K^+ - K^-} \simeq (7 - 13)\%$$
$$\sigma_{p-n}^{K^+ - K^-} \simeq (10 - 15)\%$$

- We can form R_+ and R_-
(u_V and d_V are known well enough) and test
$s - \bar{s} \simeq 0$ for $0,023 \leq x \leq 0,300$ with this precision!

$\underline{\underline{\Delta s - \Delta\bar{s} = 0?}}$

COMPASS: $\overrightarrow{l} + \overrightarrow{N} \to l + h^{\pm} + X, \quad A_d^{h-\bar{h}} = \frac{\Delta\sigma_d^{h-\bar{h}}}{\sigma_d^{h-\bar{h}}}$

If: $\overrightarrow{l} + \overrightarrow{N} \to l + K^{\pm} + X, \quad A_d^{K^+-K^-} = \frac{\Delta\sigma_d^{K^+-K^-}}{\sigma_d^{K^+-K^-}} =$

$$= \frac{(\Delta u_V + \Delta d_V)(4D_u + D_d) + 2(\Delta s - \Delta\bar{s})D_s}{(u_V + d_V)(4D_u + D_d) + 2(s - \bar{s})D_s}$$

$$\simeq \frac{\Delta u_V + \Delta d_V}{u_V + d_V}\left\{1 + \left(\frac{\Delta s - \Delta\bar{s}}{\Delta u_V + \Delta d_V} - \frac{s - \bar{s}}{u_V + d_V}\right)\left(\frac{D_s}{2D_u}\right)^{K^+-K^-}\right\}$$

Study the z-dependence of $A_d^{K^+-K^-}(x_0, z)$ at $x = x_0$:

- If $\Delta s - \Delta\bar{s} \neq 0$ or/& $s - \bar{s} \neq 0 \Leftrightarrow z$-dependence
- If $s - \bar{s} \simeq 0$ we obtain info. about $\Delta s - \Delta\bar{s} \simeq 0$:

1) If there is z-dep. $\Rightarrow \Delta s - \Delta\bar{s} \neq 0$

2) If there is no z-dep. $\Rightarrow \Delta s - \Delta\bar{s} \simeq 0$ is a good approxim.

- No knowledge of the FFs required!

$\underline{\underline{A_d^{h-\bar{h}}}}$

$$A_d^{h-\bar{h}} = \frac{\Delta\sigma_d^{h-\bar{h}}}{\sigma_d^{h-\bar{h}}} \quad \Leftarrow \quad COMPASS$$

Note: $A_d^{h-\bar{h}}$ cannot give info. about s-quarks $\rightarrow A_d^{h-\bar{h}}$ has low sensitivity to s-quarks:

$$\frac{\sum_h D_s^{h-\bar{h}}}{\sum_h (4D_u + D_d)^{h-\bar{h}}} \simeq \frac{D_s^{K^+-K^-}}{3D_u^{\pi^+-\pi^-} + 4D_u^{K^+-K^-}} \ll 1$$

Recall:

$$D_q^{h+} = D_q^{\pi^+} + D_q^{K+} + D_q^{p} + D_q^{res} \simeq D_q^{\pi^+} + D_q^{K+}$$

$$SU(2): \qquad D_s^{\pi^+-\pi^-} = 0$$

$\underline{\underline{\text{But:}}}$

- $$A_d^{h-\bar{h}} = A_d^{K^+-K^-} \quad \Rightarrow \quad \left(\frac{\Delta s - \Delta\bar{s}}{\Delta u_V + \Delta d_V} - \frac{s - \bar{s}}{u_V + d_V}\right) \simeq 0$$

- $$A_p^{K^+-K^-}(x,z) = \frac{\Delta u_V}{u_V}(x) \quad \Rightarrow$$
  $$D_d^{K^+-K^-} \simeq 0\,, \qquad (s-\bar{s}) \simeq (\Delta s - \Delta\bar{s}) \simeq 0$$

$l + N \rightarrow l + K^{\pm}, K_s^0 + X$

LO: $K^{\pm}$ and K_s^0 allows to determine $D_q^{K^+ + K^-}$ solely from SIDIS

important difference:

e^+e^- : $s \simeq m_Z^2 \quad \Rightarrow \quad Z$-exchange

SIDIS: $q^2 \ll m_Z^2 \quad \Rightarrow \quad \gamma$-exchange

Q^2-evolution brings uncertainties: $D_g^{K^+ + K^-} = ?$

$\Rightarrow K_s^0$ allows to avoid them:

$$SU(2): \quad D_q^{K^+ + K^-} \leftrightarrow D_d^{K^0 + \bar{K}^0}$$

$$\begin{aligned} \sigma_{p-n}^{K^+ + K^- + 2K_s^0} &= ((u + \bar{u}) - (d + \bar{d}))D_{u+d}^{K^+ + K^-} \\ \sigma_p^{K^+ + K^- + 2K_s^0} &\simeq [e_u^2(u + \bar{u}) + e_d^2(d + \bar{d})]D_{u+d}^{K^+ + K^-} \\ &\quad + 2e_d^2(s + \bar{s})D_s^{K^+ + K^-} \end{aligned}$$

in all QCD orders only $D_{u+d}^{K^+ + K^-}$ and $D_s^{K^+ + K^-} \Leftarrow$ SU(2)

$$\sigma_{p,n}^{K^+ + K^- - 2K_s^0} \simeq [e_u^2(u + \bar{u}) - e_d^2(d + \bar{d})]D_{u-d}^{K^+ + K^-}$$

NLO: $D_g^{K^+ + K^-}$ enters $\Rightarrow \quad e^+e^- \rightarrow K^{\pm} + X$ needed

Conclusions

We suggest some tests for $s - \bar{s} = 0$ and $D_d^{K^+ - K^-} = 0$

- $$R_+(x, z_0) = \frac{\sigma_d^{K^+ - K^-}}{u_V + d_V} \quad \Rightarrow \quad s - \bar{s} \neq 0?$$

no knowledge of the FFs required!

if $(D_s/D_u)^{K^+ - K^-}(z_0)$ known $\Rightarrow$ limits on $s - \bar{s}$

- $$R_-(x, z_0) = \frac{(\sigma_p - \sigma_n)^{K^+ - K^-}}{u_V - d_V} \quad \Rightarrow \quad D_d^{K^+ - K^-} \neq 0?$$

no knowledge of the FFs required!

- $R_-(x, z_0) \quad \Rightarrow \quad LO?$

- NLO: fitting both

$$\sigma_d^{K^+ - K^-}(x, z) \quad \& \quad (\sigma_p - \sigma_n)^{K^+ - K^-}(x, z)$$

with the same $D(z)$ $\Rightarrow$ $s - \bar{s} \simeq 0$ & $D_d^{K^+ - K^-}(z_0) \simeq 0$

$$\Rightarrow \quad D(z) = D_u^{K^+ - K^-}$$

no knowledge of the FFs required!

- HERMES 2005 has presented very precise data on SIDIS $\sigma_d^{K^\pm}$ and $\sigma_p^{K^\pm}$ which allows to form the above ratios and test $s - \bar{s} \simeq 0$ with $\lesssim 10\%$ accuracy in the x-range: $0.023 \leq x \leq 0.300$