AdS_4 black holes and 3d Gauge Theories

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[work in collaboration with F. Benini, K. Hristov]

F. Benini-AZ; arXiv 1504.03698

F. Benini-K.Hristov-AZ; arXiv 1511.04085

[Thanks to A. Tomasiello for many related discussions]

Introduction

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- I consider supersymmetric black holes in AdS_4
- I relate the counting of their micro-states to a field theory computation

Introduction

One of the success of string theory is the microscopic counting of micro-states for a class of asymptotically flat black holes entropy [Vafa-Strominger'96]

- The black holes are realized by putting together D-branes, extended objects that have gauge theories on the world-volume
- The entropy is obtained by counting states in the corresponding gauge theory

Introduction

No similar result for AdS black holes. But AdS should be simpler and related to holography:

- A gravity theory in AdS_{d+1} is the dual description of a CFT_d

The entropy should be related to the counting of states in the dual CFT. People failed for AdS_5 black holes (states in N=4 SYM).

AdS$_4$ black holes

There are many BPS asymptotically AdS$_4$ static black holes

$$\mathrm{d}s^2 = e^{\mathcal{K}(X)} \left(gr + \frac{c}{2gr} \right)^2 \mathrm{d}t^2 - \frac{e^{-\mathcal{K}(X)} \mathrm{d}r^2}{\left(gr + \frac{c}{2gr} \right)^2} - e^{-\mathcal{K}(X)} r^2 \mathrm{d}s^2_{S^2}$$

- they arise in $\mathcal{N} = 2$ gauged supergravity
- solutions asymptotic to *magnetic* AdS$_4$ and with horizon AdS$_2 \times S^2$
- characterized by a collection of magnetic charges $\int_{S^2} F$

[Cacciatori, Klemm; Gnecchi, Dall'agata; Hristov, Vandoren];

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Some arise in truncation of M theory on AdS$_4 \times S^7$

- four abelian vectors $U(1)^4 \subset SO(8)$ that come from the reduction on S^7.
- vacua of a $N = 2$ gauged supergravity with 3 vector multiplets; one vector is the graviphoton.

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$$F = -2i\sqrt{X^0 X^1 X^2 X^3}$$

$$e^{-\mathcal{K}(X)} = i\left(\bar{X}^\Lambda F_\Lambda - X^\Lambda \bar{F}_\Lambda\right) \;= \sqrt{16 X^0 X^1 X^2 X^3}$$

$$X^i = \frac{1}{4} - \frac{\beta_i}{r}\,, \qquad X^0 = \frac{1}{4} + \frac{\beta_1 + \beta_2 + \beta_3}{r}$$

with arbitrary parameters $\beta_1, \beta_2, \beta_3$.

AdS$_4$ black holes

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The parameters are related to the magnetic charges supporting the black hole

$$\mathfrak{n}_1\,,\ \mathfrak{n}_2\,,\ \mathfrak{n}_3\,,\ \mathfrak{n}_4\,, \qquad \mathfrak{n}_i = \frac{1}{2\pi}\int_{S^2} F^{(i)}\,, \qquad \sum \mathfrak{n}_i = 2$$

by

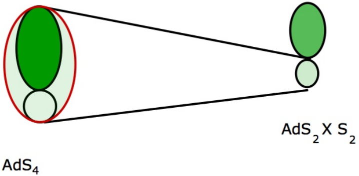
$$\begin{aligned}
\mathfrak{n}_1 &= 8(-\beta_1^2 + \beta_2^2 + \beta_3^2 + \beta_2\beta_3)\ ,\\
\mathfrak{n}_2 &= 8(-\beta_2^2 + \beta_1^2 + \beta_3^2 + \beta_1\beta_3)\ ,\\
\mathfrak{n}_3 &= 8(-\beta_3^2 + \beta_1^2 + \beta_2^2 + \beta_1\beta_2)\ .
\end{aligned}$$

AdS$_4$ black holes

There are many BPS asymptotically AdS$_4$ static black holes

$$\mathrm{d}s^2 = e^{\mathcal{K}(X)}\left(gr + \frac{c}{2gr}\right)^2 \mathrm{d}t^2 - \frac{e^{-\mathcal{K}(X)}\mathrm{d}r^2}{\left(gr + \frac{c}{2gr}\right)^2} - e^{-\mathcal{K}(X)} r^2 \mathrm{d}s^2_{S^2}$$

Asymptotic to AdS$_4$ for $r \gg 1$ and with horizon AdS$_2 \times S^2$ at some $r = r_h$



$$ds^2 \sim \frac{dr^2}{r^2} + r^2(-dt^2 + d\theta^2 + \sin\theta^2 d\phi^2)) \qquad ds^2 \sim -(r - r_h)^2 dt^2 + \frac{dr^2}{(r - r_h)^2} + (d\theta^2 + \sin\theta^2 d\phi^2)$$

AdS$_4$ black holes

There are many BPS asymptotically AdS$_4$ static black holes

$$\mathrm{d}s^2 = e^{\mathcal{K}(X)}\left(gr + \frac{c}{2gr}\right)^2 \mathrm{d}t^2 - \frac{e^{-\mathcal{K}(X)}\mathrm{d}r^2}{\left(gr + \frac{c}{2gr}\right)^2} - e^{-\mathcal{K}(X)} r^2 \mathrm{d}s^2_{S^2}$$

The entropy is the area of S^2

$$S = -\frac{\pi}{G_4}\sqrt{X_1(r_h)X_2(r_h)X_3(r_h)X_4(r_h)}\sum_a \frac{\mathfrak{n}_a}{X_a(r_h)}$$

for example, for $\mathfrak{n}_1 = \mathfrak{n}_2 = \mathfrak{n}_3$

$$\sqrt{-1 + 6\mathfrak{n}_1 - 6\mathfrak{n}_1^2 + (-1 + 2\mathfrak{n}_1)^{3/2}\sqrt{-1 + 6\mathfrak{n}_1}}$$

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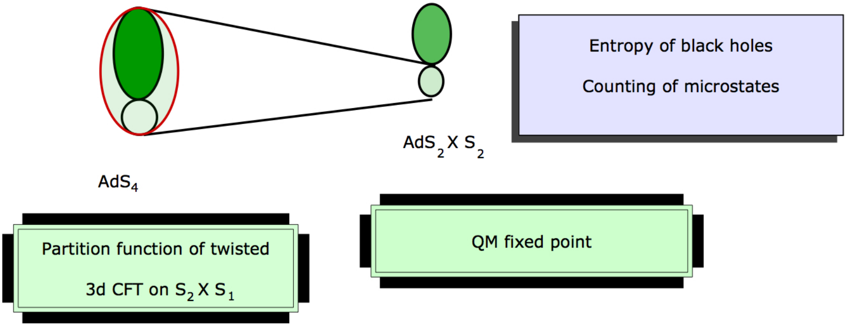
Supersymmetry is preserved by a twist

$$(\nabla_\mu - iA_\mu)\epsilon = \partial_\mu \epsilon \qquad \Longrightarrow \qquad \epsilon = \text{cost}$$

AdS$_4$ black holes

AdS black holes are dual to a topologically twisted CFT on $S^2 \times S^1$ with background magnetic fluxes for the global symmetries



In one dimension more: [Benini-Bobev]

The background

Consider an $\mathcal{N} = 2$ gauge theory on $S^2 \times S^1$

$$ds^2 = R^2\big(d\theta^2 + \sin^2\theta\, d\varphi^2\big) + \beta^2 dt^2$$

with a background for the R-symmetry proportional to the spin connection:

$$A^R = -\frac{1}{2}\cos\theta\, d\varphi = -\frac{1}{2}\omega^{12}$$

so that the Killing spinor equation

$$D_\mu\epsilon = \partial_\mu\epsilon + \frac{1}{4}\omega_\mu^{ab}\gamma_{ab}\epsilon - iA_\mu^R\epsilon = 0 \qquad \Longrightarrow \qquad \epsilon = \mathrm{const}$$

The background

This is just a topological twist. [Witten '88]

The result becomes interesting when supersymmetric backgrounds for the flavor symmetry multiplets (A^F_μ, σ^F, D^F) are turned on:

$$u^F = A^F_t + i\sigma^F \,, \qquad\qquad q^F = \int_{S^2} F^F = iD^F$$

and the path integral, which can be exactly computed by localization, becomes a function of a set of magnetic charges q^F and chemical potentials u^F.

[Benini-AZ; arXiv 1504.03698]

A topologically twisted index

The path integral can be re-interpreted as a twisted index: a trace over the Hilbert space $\mathcal{H}$ of states on a sphere in the presence of a magnetic background for the R and the global symmetries,

$$\mathrm{Tr}_{\mathcal{H}} \left((-1)^F e^{iJ_F A^F} e^{-\beta H} \right)$$

$Q^2 = H - \sigma^F J_F$

holomorphic in u^F

where J_F is the generator of the global symmetry.

Localization

Exact quantities in supersymmetric theories with a charge $Q^2 = 0$ can be obtained by a saddle point approximation

$$Z = \int e^{-S} = \int e^{-S+t\{Q,V\}} \underset{t\gg 1}{=} e^{-\bar{S}|_{class}} \times \frac{\det_{fermions}}{\det_{bosons}}$$

Very old idea that has become very concrete recently, with the computation of partition functions on spheres and other manifolds supporting supersymmetry.

The partition function

The path integral for an $\mathcal{N}=2$ gauge theory on $S^2 \times S^1$ with gauge group G localizes on a set of BPS configurations specified by data in the vector multiplets $V = (A_\mu, \sigma, \lambda, \lambda^\dagger, D)$

- A magnetic flux on S^2, $\mathfrak{m} = \frac{1}{2\pi}\int_{S^2} F$ in the co-root lattice
- A Wilson line A_t along S^1
- The vacuum expectation value σ of the real scalar

Up to gauge transformations, the BPS manifold is

$$(u = A_t + i\sigma, \mathfrak{m}) \in \mathcal{M}_{\text{BPS}} = \big(H \times \mathfrak{h} \times \Gamma_{\mathfrak{h}}\big)/W$$

The partition function

The path integral reduces to a the saddle point around the BPS configurations

$$\sum_{\mathfrak{m}\in\Gamma_{\mathfrak{h}}} \int du d\bar{u}\, \mathcal{Z}^{\text{cl }+\text{1-loop}}(u,\bar{u},\mathfrak{m})$$

- The integrand has various singularities where chiral fields become massless
- There are fermionic zero modes

The two things nicely combine and the path integral reduces to an r-dimensional contour integral of a meromorphic form

$$\boxed{\frac{1}{|W|}\sum_{\mathfrak{m}\in\Gamma_{\mathfrak{h}}} \oint_C Z_{\text{int}}(u,\mathfrak{m})}$$

The partition function

- In each sector with gauge flux $\mathfrak{m}$ we have a a meromorphic form

$$Z_{\text{int}}(u,\mathfrak{m}) = Z_{\text{class}} Z_{\text{1-loop}}$$

$$\boxed{Z^{\text{CS}}_{\text{class}} = x^{k\mathfrak{m}}} \qquad x = e^{iu}$$

$$\boxed{Z^{\text{chiral}}_{\text{1-loop}} = \prod_{\rho\in\mathfrak{R}} \left[\frac{x^{\rho/2}}{1-x^{\rho}}\right]^{\rho(\mathfrak{m})-q+1}} \qquad q = \text{R charge}$$

$$\boxed{Z^{\text{gauge}}_{\text{1-loop}} = \prod_{\alpha\in G} (1-x^{\alpha})\,(i\,du)^{r}}$$

- Supersymmetric localization selects a particular contour of integration C and picks some of the residues of the form $Z_{\text{int}}(u,\mathfrak{m})$.

[Jeffrey-Kirwan residue - similar to Benini,Eager,Hori,Tachikawa '13; Hori,Kim,Yi '14]

A Simple Example: SQED

The theory has gauge group $U(1)$ and two chiral Q and $\tilde{Q}$

$$Z = \sum_{\mathfrak{m}\in\mathbb{Z}} \int \frac{dx}{2\pi i\, x} \left(\frac{x^{\frac{1}{2}} y^{\frac{1}{2}}}{1-xy}\right)^{\mathfrak{m}+\mathfrak{n}} \left(\frac{x^{-\frac{1}{2}} y^{\frac{1}{2}}}{1-x^{-1}y}\right)^{-\mathfrak{m}+\mathfrak{n}}$$

	$U(1)_g$	$U(1)_A$	$U(1)_R$
Q	1	1	1
$\tilde{Q}$	-1	1	1

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	$U(1)_g$	$U(1)_A$	$U(1)_R$
Q	1	1	1
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Consistent with duality with three chirals with superpotential XYZ

$$Z = \left(\frac{y}{1-y^2}\right)^{2\mathfrak{n}-1} \left(\frac{y^{-\frac{1}{2}}}{1-y^{-1}}\right)^{-\mathfrak{n}+1} \left(\frac{y^{-\frac{1}{2}}}{1-y^{-1}}\right)^{-\mathfrak{n}+1}$$

A Simple Example: SQED

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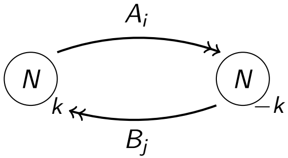
	$U(1)_g$	$U(1)_A$	$U(1)_R$
Q	1	1	1
$\tilde{Q}$	-1	1	1

The index adds to and complete the list of existing tools (superconformal indices, sphere partition functions) for testing dualities.

Going back to the black hole

The dual field theory to $\mathrm{AdS}_4 \times S^7$ is known: is the ABJM theory with gauge group $U(N) \times U(N)$



with quartic superpotential

$$W = A_1 B_1 A_2 B_2 - A_1 B_2 A_2 B_1$$

defined on twisted $S^2 \times \mathbb{R}$ with magnetic fluxes $\mathfrak{n}_i$ for the R/global symmetries

$$SU(2)_A \times SU(2)_B \times U(1)_B \times U(1)_R \subset SO(8)$$

.

The dual field theory

The ABJM twisted index is

$$Z = \frac{1}{(N!)^2} \sum_{\mathfrak{m},\widetilde{\mathfrak{m}} \in \mathbb{Z}^N} \int \prod_{i=1}^{N} \frac{dx_i}{2\pi i x_i} \frac{d\tilde{x}_i}{2\pi i \tilde{x}_i} x_i^{k\mathfrak{m}_i} \tilde{x}_i^{-k\widetilde{\mathfrak{m}}_i} \times \prod_{i\neq j}^{N} \left(1 - \frac{x_i}{x_j}\right)\left(1 - \frac{\tilde{x}_i}{\tilde{x}_j}\right) \times$$

$$\times \prod_{i,j=1}^{N} \left(\frac{\sqrt{\frac{x_i}{\tilde{x}_j} y_1}}{1 - \frac{x_i}{\tilde{x}_j} y_1}\right)^{\mathfrak{m}_i - \widetilde{\mathfrak{m}}_j - \mathfrak{n}_1 + 1} \left(\frac{\sqrt{\frac{x_i}{\tilde{x}_j} y_2}}{1 - \frac{x_i}{\tilde{x}_j} y_2}\right)^{\mathfrak{m}_i - \widetilde{\mathfrak{m}}_j - \mathfrak{n}_2 + 1}$$

$$\left(\frac{\sqrt{\frac{\tilde{x}_j}{x_i} y_3}}{1 - \frac{\tilde{x}_j}{x_i} y_3}\right)^{\widetilde{\mathfrak{m}}_j - \mathfrak{m}_i - \mathfrak{n}_3 + 1} \left(\frac{\sqrt{\frac{\tilde{x}_j}{x_i} y_4}}{1 - \frac{\tilde{x}_j}{x_i} y_4}\right)^{\widetilde{\mathfrak{m}}_j - \mathfrak{m}_i - \mathfrak{n}_4 + 1}$$

where $\mathfrak{m}, \widetilde{\mathfrak{m}}$ are the gauge magnetic fluxes and y_i are fugacities for the three independent $U(1)$ global symmetries ($\prod_i y_i = 1$)

The dual field theory

Strategy:

- Re-sum geometric series in $\mathfrak{m}, \widetilde{\mathfrak{m}}$.

$$Z = \int \frac{dx_i}{2\pi i x_i} \frac{d\tilde{x}_i}{2\pi i \tilde{x}_i} \frac{f(x_i, \tilde{x}_i)}{\prod_{j=1}^N (e^{iB_i} - 1) \prod_{j=1}^N (e^{i\widetilde{B}_j} - 1)}$$

- Find the zeros of denominator $e^{iB_i} = e^{i\tilde{B}_j} = 1$ at large N
- Evaluate the residues at large N

$$Z \sim \sum_I \frac{f(x_i^{(0)}, \tilde{x}_i^{(0)})}{\det \mathbb{B}}$$

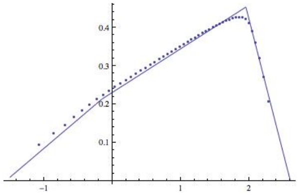
The large N limit

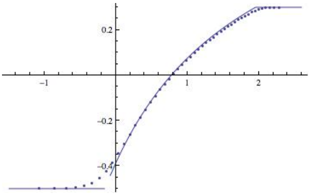
Step 2: solve the large N Limit of algebraic equations giving the positions of poles

$$1 = x_i^k \prod_{j=1}^{N} \frac{\left(1 - y_3 \frac{\tilde{x}_j}{x_i}\right)\left(1 - y_4 \frac{\tilde{x}_j}{x_i}\right)}{\left(1 - y_1^{-1} \frac{\tilde{x}_j}{x_i}\right)\left(1 - y_2^{-1} \frac{\tilde{x}_j}{x_i}\right)} = \tilde{x}_j^k \prod_{i=1}^{N} \frac{\left(1 - y_3 \frac{\tilde{x}_j}{x_i}\right)\left(1 - y_4 \frac{\tilde{x}_j}{x_i}\right)}{\left(1 - y_1^{-1} \frac{\tilde{x}_j}{x_i}\right)\left(1 - y_2^{-1} \frac{\tilde{x}_j}{x_i}\right)}$$

with an ansatz

$$\log x_i = i\sqrt{N} t_i + v_i \,, \qquad \log \tilde{x}_i = i\sqrt{N} t_i + \tilde{v}_i$$





The large N limit

Step 3: plug into the partition function. The final result is surprisingly simple

$$\mathbb{R}\mathrm{e} \log Z = -\frac{1}{3} N^{3/2} \sqrt{2k\Delta_1\Delta_2\Delta_3\Delta_4} \sum_a \frac{\mathfrak{n}_a}{\Delta_a} \qquad\qquad y_i = e^{i\Delta_i}$$

The main result

The index is:

$$\mathbb{R}\mathrm{e}\log Z = -\frac{1}{3} N^{3/2} \sqrt{2k\Delta_1\Delta_2\Delta_3\Delta_4} \sum_a \frac{\mathfrak{n}_a}{\Delta_a} \qquad\qquad y_i = e^{i\Delta_i}$$

This function can be extremized with respect to the Δ_i and

$$\boxed{\mathbb{R}\mathrm{e}\log Z|_{crit}(\mathfrak{n}_i) = \mathrm{BH\,Entropy}(\mathfrak{n}_i)}$$

$$\boxed{\Delta_i|_{crit} \sim X^i(r_h)}$$

The main result

Compare the field theory formula

$$\mathbb{R}\mathrm{e} \log Z = -\frac{1}{3} N^{3/2} \sqrt{2k\Delta_1\Delta_2\Delta_3\Delta_4} \sum\nolimits_a \frac{\mathfrak{n}_a}{\Delta_a}$$

with the gravity one

$$S = -\frac{\pi}{G_4} \sqrt{X_1(r_h) X_2(r_h) X_3(r_h) X_4(r_h)} \sum_a \frac{\mathfrak{n}_a}{X_a(r_h)}$$

Attractor mechanism

The BPS equations at the horizon imply that, in our gauge, the quantity

$$\mathcal{R} = \left(F_\Lambda p^\Lambda - X^\Lambda q_\Lambda\right)$$

with (q, p) electric and magnetic charges, is extremized with respect to the scalar fields at the horizon and

$$S = |\mathcal{R}|$$

For us $p^\Lambda = \mathfrak{n}^\Lambda$ and $q_\Lambda = 0$ and

$$F_\Lambda = -\frac{i}{X^\Lambda}\sqrt{X^0 X^1 X^2 X^3}$$

R-symmetry mixing

The extremization reflects exactly what's going on in the bulk. The graviphoton field strength depends on r

$$T_{\mu\nu} = e^{K/2} X^{\Lambda} F_{\Lambda,\mu\nu}$$

suggesting that the R-symmetry is different in the IR and indeed

$$\boxed{\Delta_i|_{crit} \sim X^i(r_h)}$$

R-symmetry extremization

Some QFT extremization is at work? symmetry enhancement at the horizon AdS_2

$$\mathrm{QM}_1 \to \mathrm{CFT}_1$$

The twisted index depends on Δ_i because we are computing the trace

$$\mathrm{Tr}_{\mathcal{H}}(-1)^F e^{i\Delta_i J_i} \equiv \mathrm{Tr}_{\mathcal{H}}(-1)^R$$

where $R = F + \Delta_i J_i$ is a possible R-symmetry of the system.

- R is the exact R-symmetry at the superconformal point
- Natural thing to extremize: in even dimensions central charges are extremized, in odd partition functions...

Conclusions

The main message of this talk is that you can related the entropy of a class of AdS_4 black holes to a microscopic counting of states.

- first time for AdS black holes

Conclusions

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- first time for AdS black holes

But don't forget that we also gave a general formula for the topologically twisted path integral of 2d (2,2), 3d $\mathcal{N} = 2$ and 4d $\mathcal{N} = 1$ theories.

- Efficient quantum field theory tools for testing dualities.
- With many field theory questions/generalizations