The Null Energy Condition and its violation

V.A. Rubakov

Institute for Nuclear Research
of the Russian Academy of Sciences

Department of Particle Physics and Cosmology
Physics Faculty, Moscow State University

The Null Energy Condition, NEC

$$T_{\mu\nu}n^{\mu}n^{\nu} > 0$$

for any null vector n^{μ}, such that $n_{\mu}n^{\mu} = 0$.

- Quite robust
- In the framework of classical General Relativity implies a number of properties:
  - Penrose theorem:

Penrose' 1965

Once there is trapped surface, there is singularity in future.

Assumptions:

(i) The NEC holds

(ii) Cauchi hypersurface non-compact

Trapped surface:

a closed surface on which outward-pointing light rays actually converge (move inwards)

Spherically symmetric examples:

$$ds^2 = g_{00}dt^2 + 2g_{0R}dtdR + g_{RR}dR^2 - R^2 d\Omega^2$$

$4\pi R^2$: area of a sphere of constant t, R.

Trapped surface: R decreases along all light rays.

- Sphere inside horizon of Schwarzschild black hole
- Hubble sphere in contracting Universe $\Longrightarrow$

  Hubble sphere in expanding Universe = anti-trapped surface $\Longrightarrow$ singularity in the past.

  - No-go for bouncing Universe scenario and Genesis

Related issue: Can one in principle create a universe in the laboratory?

- Question raised in mid-80’s, right after invention of inflationary theory

  Berezin, Kuzmin, Tkachev’ 1984; Guth, Farhi’ 1986

  Idea: create, in a finite region of space, inflationary initial conditions $\Longrightarrow$ this region will inflate to enormous size and in the end will look like our Universe.

- Do not need much energy: pour little more than Planckian energy into little more than Planckian volume.
  At that time: negaive answer [In the framework of classical General Relativity]:

  Guth, Farhi’ 1986; Berezin, Kuzmin, Tkachev’ 1987

  Inflation in a region larger than Hubble volume, $R > H^{-1} \Longrightarrow$ Singularity in the past guaranteed by Penrose theorem

Meaning:

- Homogeneous and isotropic region of space: metric

$$ds^2 = dt^2 - a^2(t)d\mathbf{x}^2 \ .$$

Local Hubble parameter $H = \dot{a}/a$.

Wish to create region whose size is larger than H^{-1}.

This is the definition of a universe.

Hubble size regions evolve independently of each other

$\Longrightarrow$ legitimate to use eqs. for FLRW universe

- A combination of Einstein equations:

$$\frac{dH}{dt} = -4\pi G(\rho + p)$$

$\rho = T_{00}$ = energy density; $T_{ij} = \delta_{ij} p$ = effective pressure.

- The Null Energy Condition:

$T_{\mu\nu} n^\mu n^\nu \geq 0$, $n^\mu = (1, 1, 0, 0) \Longrightarrow \rho + p > 0 \Longrightarrow dH/dt < 0$,

Hubble parameter was greater early on.

At some moment in the past, there was a singularity, $H = \infty$.

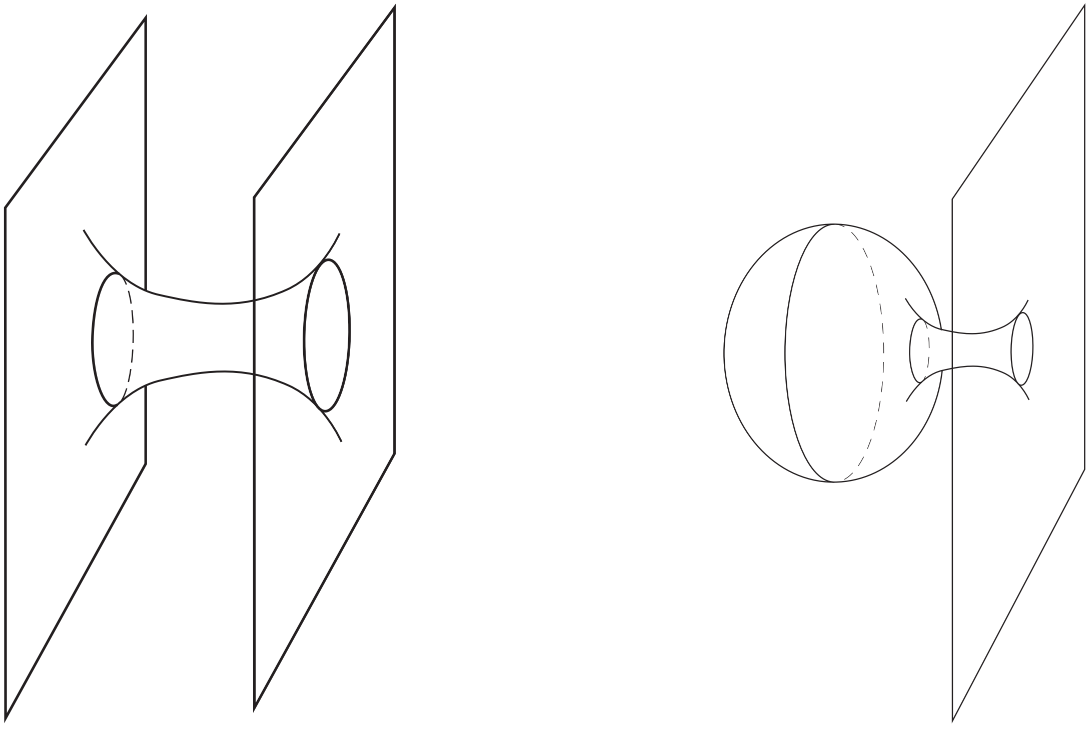
- Another side of the NEC: Covariant energy-momentum conservation:

$$\frac{d\rho}{dt} = -3H(\rho + p)$$

NEC: energy density decreases during expansion, except for $p = -\rho$, cosmological constant.

Many other facets of the NEC

No-go for Lorentzian wormholes



Can the Null Energy Condition be violated in classical field theory?

- Folklore until recently: NO!

  Pathologies:

  - Ghosts:

$$E = -\sqrt{p^2 + m^2}$$

  Example: theory with wrong sign of kinetic term,

$$\mathscr{L} = -(\partial\phi)^2 \quad \Longrightarrow \quad \rho = -\dot{\phi}^2 - (\nabla\phi)^2 \,, \qquad p = -\dot{\phi}^2 + (\nabla\phi)^2$$

$$\rho + p = -2\dot{\phi}^2 < 0$$

  Catastrophic vacuum instability

  NB: Can be cured by Lorentz-violation

  (but hard! – even though Lorentz-violation is inherent in cosmology)

Other pathologies

- Gradient instabilities:

$$E^2 = -(p^2 + m^2) \implies \varphi \propto \mathrm{e}^{|E|t}$$

- Superluminal propagation of excitations

Theory cannot descend from healthy Lorentz-invariant UV-complete theory

Adams et. al.' 2006

No-go theorem for theories with Lagrangians involving first derivatives of fields only (and minimal coupling to gravity)

Dubovsky, Gregoire, Nicolis, Rattazzi' 2006

$$L = F(X^{IJ}, \pi^I)$$

with $X^{IJ} = \partial_\mu \pi^I \partial^\mu \pi^J \Longrightarrow$

$$T_{\mu\nu} = 2\frac{\partial F}{\partial X^{IJ}} \partial_\mu \pi^I \partial_\nu \pi^J - g_{\mu\nu} F$$

In homogeneous background

$$T_{00} \equiv \rho = 2\frac{\partial F}{\partial X^{IJ}} X^{IJ} - F$$

$$T_{11} = T_{22} = T_{33} \equiv p = F$$

and

$$\rho + p = 2\frac{\partial F}{\partial X^{IJ}} X^{IJ} = 2\frac{\partial F}{\partial X^{IJ}} \dot{\pi}^I \dot{\pi}^J$$

NEC-violation: matrix $\partial F / \partial X_c^{IJ}$ non-positive definite.

But

Lagrangian for perturbations $\pi^I = \pi_c^I + \delta\pi^I$

$$L_{\delta\pi} = U_{IJ}\, \partial_t \delta\pi^I \cdot \partial_t \delta\pi^J - \frac{\partial F}{\partial X_c^{IJ}}\, \partial_i \delta\pi^I \cdot \partial_i \delta\pi^J + \dots$$

Gradient instabilities and/or ghosts

NB. Loophole: $\partial F / \partial X_c^{IJ}$ degenerate.

Higher derivative terms (understood in effective field theory sense) become important and help.

Ghost condensate

Arkani-Hamed et. al.' 2003

Can the Null Energy Condition be violated in a simple and healthy way?

- Folklore until recently: NO!

  Today: YES,

Senatore' 2004;
V.R.' 2006;
Creminelli, Luty, Nicolis, Senatore' 2006

General property of non-pathological NEC-violating field theories:

Non-standard kinetic terms

Example: scalar field $\pi(x^\mu)$,

$$L = K_0(X,\pi) + K_1(X,\pi)\cdot\Box\pi$$

$$\Box\pi \equiv \partial_\mu\partial^\mu\pi\,, \qquad X = (\partial_\mu\pi)^2$$

- Second order equations of motion (but L cannot be made first order by integration by parts)
- Generalization: Horndeski theory (1974) rediscovered many times

Fairlie, Govaerts, Morozov' 91;

Nicolis, Rattazzi, Trincherini' 09, ...

$$L_n = K_n(X,\pi)\partial^{\mu_1}\partial_{[\mu_1}\pi\cdot\dots\cdot\partial^{\mu_n}\partial_{\mu_n]}\pi$$

Five Lagrangians in 4D, including K_0

Generalization to GR: L_0, L_1 trivial, $L_{n>1}$ non-trivial

Deffayet, Esposito-Farese, Vikman' 09

Simple playground

$$L = F(Y) \cdot \mathrm{e}^{4\pi} + K(Y) \cdot \Box\pi \cdot \mathrm{e}^{2\pi}$$

$$\Box\pi \equiv \partial_\mu \partial^\mu \pi \, , \qquad Y = \mathrm{e}^{-2\pi} \cdot (\partial_\mu \pi)^2$$

Deffayet, Pujolas, Sawicki, Vikman' 2010
Kobayashi, Yamaguchi, Yokoyama' 2010

- Second order equations of motion
- Scale invariance: $\pi(x) \to \pi'(x) = \pi(\lambda x) + \ln\lambda$.

(technically convenient)

Example: homogeneous solution in Minkowski space (attractor)

$$e^{\pi_c} = \frac{1}{\sqrt{Y_*}(t_* - t)}$$

- $Y \equiv e^{-2\pi_c} \cdot (\partial_\mu \pi_c)^2 = Y_* =$ const, a solution to

$$Z(Y_*) \equiv -F + 2Y_*F' - 2Y_*K + 2Y_*^2K' = 0$$

$$' = d/dY \, .$$

Energy density

$$\rho = e^{4\pi_c} Z = 0$$

Effective pressure T_{11}:

$$p = e^{4\pi_c}(F - 2Y_*K)$$

Can be made negative by suitable choice of $F(Y)$ and $K(Y)$
$\Longrightarrow \rho + p < 0$, violation of the Null Energy Condition.

Turning on gravity

$$p = \mathrm{e}^{4\pi_c}\,(F - 2Y_* K) = -\frac{M^4}{Y_*^2 (t_* - t)^4}\,, \qquad \rho = 0$$

M: mass scale characteristic of π

- Use $\dot{H} = -4\pi G(p+\rho) \Longrightarrow$

$$H = \frac{4\pi}{3}\frac{M^4}{M_{Pl}^2 Y_*^2 (t_* - t)^3}$$

NB:

$$\rho \sim M_{Pl}^2 H^2 \sim \frac{1}{M_{Pl}^2 (t_* - t)^6}$$

Early times $\Longrightarrow$ weak gravity, $\rho \ll p$.

Expansion, $H \neq 0$, is negligible for dynamics of π.

Perturbations about homogeneous Minkowski solution

$$\pi(x^\mu) = \pi_c(t) + \delta\pi(x^\mu)$$

- Quadratic Lagrangian for perturbations:

$$L^{(2)} = \mathrm{e}^{2\pi_c} Z' (\partial_t \delta\pi)^2 - V(\vec{\nabla}\delta\pi)^2 + W(\delta\pi)^2$$

$V = V[Y; F, K, F', K', K'']$. Absence of ghosts:

$$Z' \equiv dZ/dY > 0$$

Absence of gradient instabilities and of superluminal propagation

$$V > 0\,; \qquad V < \mathrm{e}^{2\pi_c} Z'$$

Can be arranged.

What is this good for?

- Application # 1: cosmology

Non-standard scenario of the start of cosmological expansion: Genesis, alternative to inflation

Creminelli, Nicolis, Trincherini' 2010

Have $\rho + p < 0$ and GR $\Longrightarrow dH/dt > 0, \quad d\rho/dt > 0$.

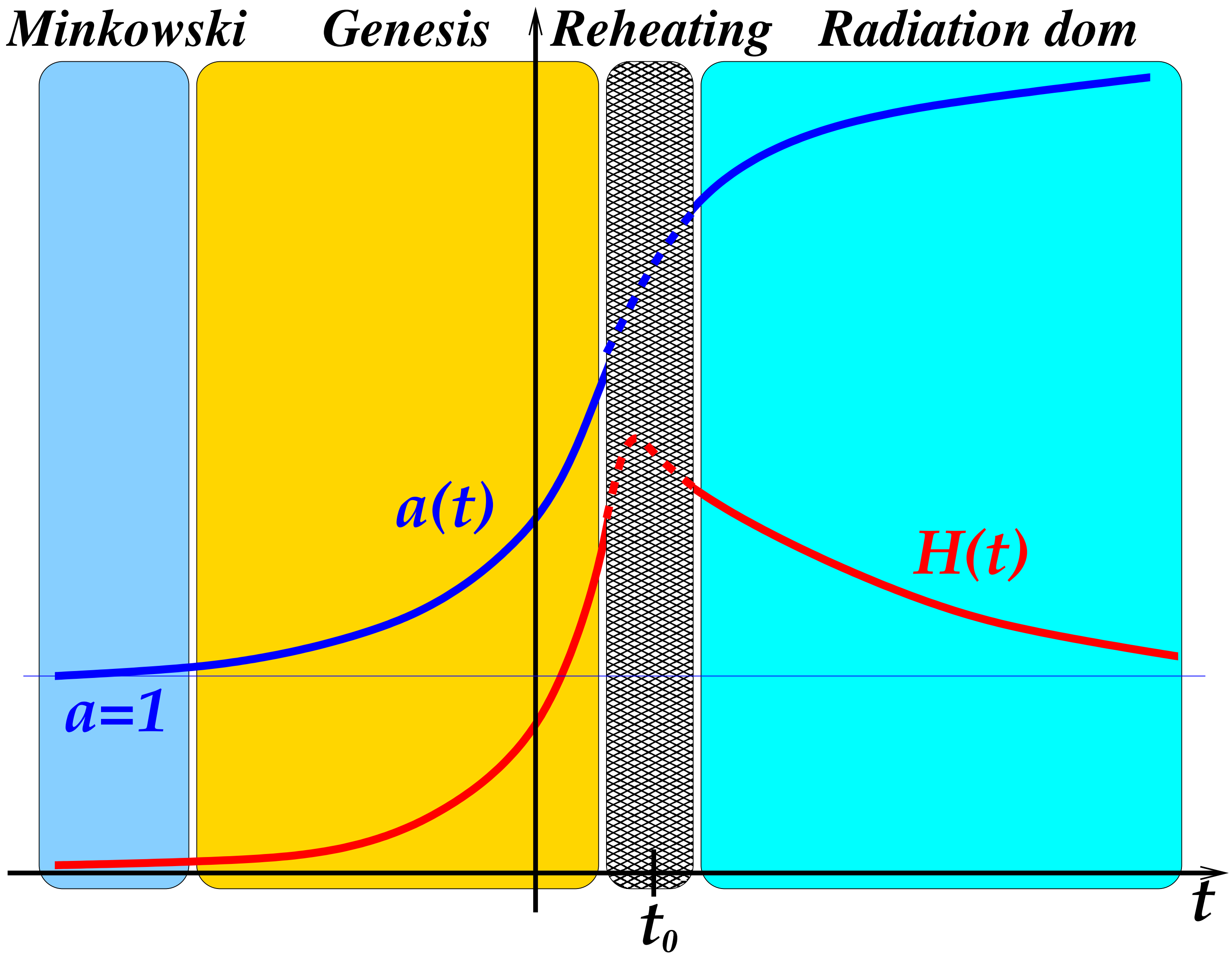
The Universe starts from Minkowski,

expansion slowly accelerates,

energy density builds up.

Expansion speeds up and at some point energy density of the field π is converted into heat (defrosting), hot epoch begins.

Genesis



- Another cosmological scenario: bounce
  Collapse $\longrightarrow$ expansion, also alternative to inflation

Qui et. al.' 2011;
Easson, Sawicki, Vikman' 2011;
Osipov, V.R.' 2013

- In either case: there may be enough symmetry to arrange for nearly flat power spectrum of density perturbations.

  Particularly powerful: conformal symmetry

First mentioned by Antoniadis, Mazur, Mottola' 97
Concrete models: V.R.' 09;
Creminelli, Nicolis, Trincherini' 10

What if our Universe started off from or passed through
an unstable conformal state
and then evolved to much less symmetric state we see today?

Specific shapes of non-Gaussianity, statistical anisotropy.

No gravity waves

Example #2: Creating a universe in the laboratory

VR' 13

Idea

- Prepare quasi-homogeneous initial configuration.

  Large sphere, $Y = Y_*$ inside, $\pi =$ const (Minkowski) outside,

  smooth interpolation in between.

  Spatial derivatives small compared with time derivatives.

- Initial state: energy density and pressure small everywhere, geometry nearly Minkowskian. No antitrapped surface. Possible to create.

- Evolution: Genesis inside the sphere, Minkowski outside

Done?

Not quite!

Obstruction

to both Genesis/bouncing cosmology and "creation of a universe"

- Energy density:

$$\rho = \mathrm{e}^{4\pi_c} Z$$

  $Z = 0$ in both Genesis regime $\mathrm{e}^{\pi} = -1/t$
  and Minkowski $\pi = \text{const} \Longrightarrow$
  dZ/dY negative somewhere in between.

- On the other hand: absence of ghosts requires

$$dZ/dY > 0$$

  Hence, there are ghosts somewhere in space $\equiv$ instability

- This is a general property of theories of one scalar field with
  - Second order field equations
  - Scale invariance: $\pi(x) \to \pi'(x) = \pi(\lambda x) + \ln\lambda$.

Proof

- Equation for homogeneous field always coincides with energy conservation (Noether theorem)

$$\frac{\delta S}{\delta \pi} \propto -\dot{\rho} = 0$$

This is second order equation, hence ρ contains first derivatives only, hence by scale invariance

$$\rho = \mathrm{e}^{4\pi} \cdot Z[\mathrm{e}^{-2\pi} (\partial \pi)^2]$$

- Write $\pi = \pi_c + \delta\pi$, then eqn. for $\delta\pi$ is

$$-Z' \ \partial_t^2 \delta\pi + \text{lower time derivatives} = 0$$

Hence

$$\mathscr{L}(\delta\pi) \propto Z' (\partial_t \delta\pi)^2 + \dots$$

$\rho \propto Z = 0$ both at Genesis and Minkowski $\Longrightarrow Z' < 0$ somewhere in between. QED

Ways out

- Give up scale invariance.

Elder, Joyce, Khoury' 13

A lot more technically demanding.

$$L = K_0(X,\pi) + K_1(X,\pi) \cdot \Box\pi$$

$$\Box\pi \equiv \partial_\mu \partial^\mu \pi \,, \qquad X = (\partial_\mu \pi)^2$$

Cook up K_0 and K_1.

NEC-satisfying $\Longrightarrow$ NEC-violating cosmology

Not Genesis yet: need

NEC-violating $\Longrightarrow$ NEC-satisfying

Work in progress

- Give up single field, make model more complicated.

  But keep dynamics simple.

In the context of creation of a universe in the lab:

- Make the Lagrangian for π explicitly dependent on radial coordinate r.

  To this end, introduce a new field whose background configuration is $\varphi(r)$

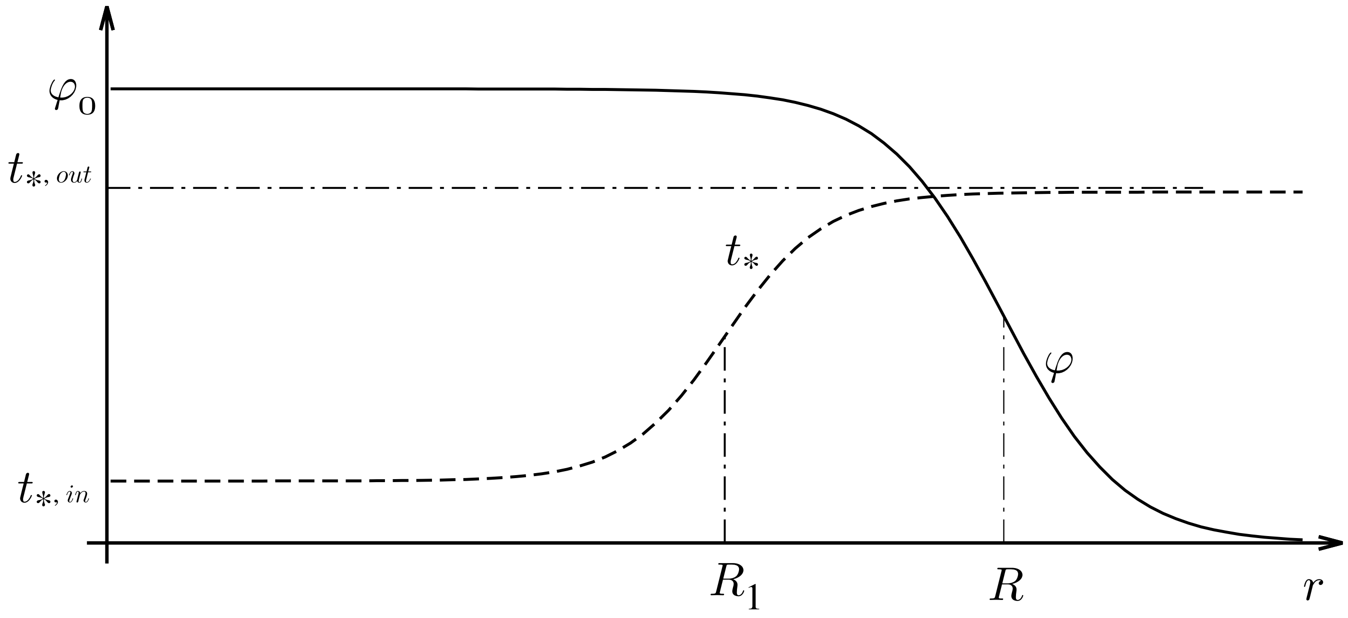
- Example:
$$F = a(\varphi) + b(\varphi)(Y - \varphi) + \frac{c(\varphi)}{2}(Y - \varphi)^2$$
$$K = \kappa(\varphi) + \beta(\varphi)(Y - \varphi) + \frac{\gamma(\varphi)}{2}(Y - \varphi)^2$$

  Choose functions $a(\varphi)$, ..., and initial condition for π in such a way that quasi-homogeneous solution is

$$e^{\pi} = \frac{1}{\sqrt{\varphi_0}t_*(r) - \sqrt{\varphi(r)}t}$$

$$e^{\pi} = \frac{1}{\sqrt{\varphi_0}\, t_*(r) - \sqrt{\varphi(r)}\, t}$$

- Interior: $Y = \varphi_0 \Longrightarrow$ Genesis $t_{*,in}$ small $\Longrightarrow$ quick start
  Exterior $\dot{\pi} = 0 \Longrightarrow Y = 0 \Longrightarrow$ Minkowski



Initial conditions, $t = 0$: at $r < R$ pressure

$$p_{in} = \frac{M^4}{Y_0^2 t_{*,in}^4}$$

Require $p_{in} R^3 / M_{Pl}^2 \ll R \Longrightarrow$ weak gravity, gravitational potentals small everywhere.
Together with $t_{*,in} \ll R$ this guarantees

$$H_{in} = \frac{4\pi M^4}{3 M_{Pl}^2 Y_0^2 t_{*,in}^3} \ll R^{-1}$$

No antitrapped surfaces initially. Anti-trapped surface (Hubble size) gets formed when

$$(t_{*,in} - t_1) \sim \left(\frac{M^4 R}{M_{Pl}^2 Y_0^2} \right)^{1/3}$$

Gravity is still weak at that time. No black hole (yet?).

Creation of a universe in controlled, weak gravity regime

Why question mark?

- What do spatial gradients do?
- Where does the system evolve once gravity is turned on?
  What is the global geometry?
  Does a black hole get formed?
- Explicit (numerical) solution needed

How about Lorentzian wormholes?

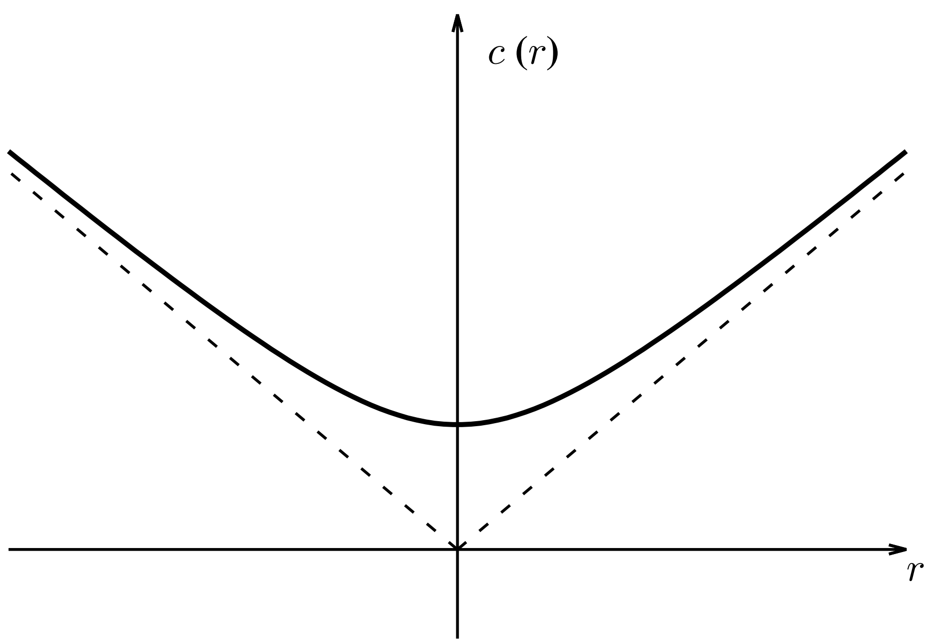
V.R.' 2015

Static, sphericaly symmetric wormhole in $(d+2)$-dimensional space-time:

$$ds^2 = a^2(r)dt^2 - dr^2 - c^2(r)d\Omega_2^2$$

Asymptotics

$$a \to a_{\pm}\ , \quad c(r) \to \pm r\ , \quad \text{as}\ r \to \pm\infty$$



Einstein equiations $\Longrightarrow$ averaged NEC violation, ANECV

$$\int_{-\infty}^{+\infty} dr\, \frac{c^{\alpha}}{a} \left(T_0^0 - T_r^r\right) < 0 \quad \text{for all } \alpha \leq 1$$

Also, for <u>monotonous</u> $c'(r)$

$$\int_{-\infty}^{+\infty} dr\, a c^{d-2} \left(T_0^0 - T_r^r\right) < 0$$

Try the Lagrangian $L = K_0(X, \pi) + K_1(X, \pi) \cdot \Box \pi$,
search for solution $\pi = \pi(r)$. Necessary cond. for stability

$$\int_{-\infty}^{+\infty} dr\, \frac{c^{d}}{a} \left(T_0^0 - T_r^r\right) > 0$$

$$\int_{-\infty}^{+\infty} dr\, a^{2\beta-1} c^{d-2\beta} \left(T_0^0 - T_r^r\right) > 0 \quad \text{for all } 0 \leq \beta \leq 1$$

3-dim. space-time, $d = 1$

No go: ANECV with $\alpha = 1$

$$\int_{-\infty}^{+\infty} dr \, \frac{c}{a} \left(T_0^0 - T_r^r \right) < 0$$

Stability

$$\int_{-\infty}^{+\infty} dr \, \frac{c}{a} \left(T_0^0 - T_r^r \right) > 0$$

$D > 3$-dim. space-time

- Tension between ANECV and stability in general, e.g. in 4-dim space-time

$$\int_{-\infty}^{+\infty} dr\, \frac{c}{a} \left(T_0^0 - T_r^r\right) < 0 \,,$$

$$\int_{-\infty}^{+\infty} dr\, c \left(T_0^0 - T_r^r\right) > 0$$

- No wormholes with monotonous c': ANECV

$$\int_{-\infty}^{+\infty} dr\, a c^{d-2} \left(T_0^0 - T_r^r\right) < 0$$

Stability with $\beta = 1$

$$\int_{-\infty}^{+\infty} dr\, a c^{d-2} \left(T_0^0 - T_r^r\right) > 0$$

Wormholes of simple shapes ruled out.

To conclude

- There exist field theory models with healthy violation of the Null Energy Condition
- This opens up new opportunities for cosmology: Genesis, bouncing Universe.
- Removes obstruction for creating a universe in the laboratory.

  A concrete scenario is fairly straightforward to design.
- Obtaining stable Loretzian wormholes is not so simple, if at all possible
- Are there appropriate fields in Nature?

  Hardly. Still, we may learn at some point that our Universe went through Genesis or bounce phase. This will mean that the Null Energy Condition was violated in the past by some exotic fields.