Mixed symmetry multiplets & higher-spin curvatures

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Scuola Normale Superiore & INFN

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24 November 2015

SCUOLA NORMALE SUPERIORE

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➡ *free local Lagrangians, however, are usually required to be generated by 2nd order kinetic tensors*

➡ *still, free equations naturally appear in higher-derivative form, once they are formulated à la Bargmann-Wigner*

we investigated further the Bargmann-Wigner program
extending it to the case of *multi-particle representations*

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extending it to the case of *multi-particle representations*

→ *alternative to more conventional single-particle equations*

→ *akin to massless hsp as emerging from tensionless strings*

Back to basics:
wave equations for particles with zero mass

~

wave equations for particles with zero mass

two options:

gauge dependent

gauge independent

Wave equations for m=0, s=2

$\sim$

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s.t.

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gauge equivalence: finite spin

same tensor as
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Fierz 1939

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Connecting the two descriptions:

$$\partial_{[\lambda}\mathcal{R}_{\mu\nu],\,\rho\sigma} = 0 \quad \longrightarrow \quad \mathcal{R}_{\mu\nu,\,\rho\sigma}(h) = \partial_\mu\partial_\rho h_{\nu\sigma} + \ldots$$

Poincaré Lemma

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➡

- $\partial_{[\lambda} \mathcal{R}_{\mu\nu], \rho\sigma}(h) \equiv 0$

- $\eta^{\mu\rho} \mathcal{R}_{\mu\nu,\rho\sigma}(h) = 0$ corresponds to the vanishing of the linearised Ricci tensor, that can be written

$$\Box h_{\mu\nu} = \partial_{(\mu} \Lambda_{\nu)}(h)$$

so as to stress that it reduces to $P^2 = 0$ upon partial gauge fixing

Wave equations for $m = 0$, spin s

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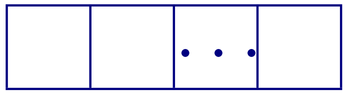
$$\varphi \equiv \varphi_{\mu_1 \ldots \mu_s} \sim$$ 

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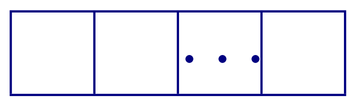
$\varphi \equiv \varphi_{\mu_1 \ldots \mu_s} \sim$ 

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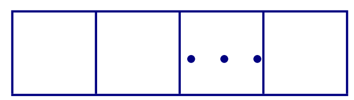
$$\varphi \equiv \varphi_{\mu_1 \ldots \mu_s} \sim$$ 

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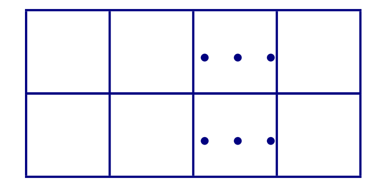
$$\mathcal{R} \equiv \mathcal{R}_{\mu_1 \nu_1, \ldots, \mu_s \nu_s} \sim$$ 

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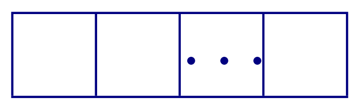
$\varphi \equiv \varphi_{\mu_1 \ldots \mu_s} \sim$ 

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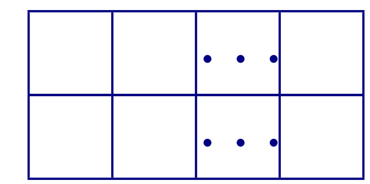
$\mathcal{R} \equiv \mathcal{R}_{\mu_1 \nu_1, \ldots, \mu_s \nu_s} \sim$ 

s.t.

$$d\,\mathcal{R} = 0$$

$$\mathcal{R}' = 0$$

Wave equations for spin s

Connecting the two descriptions:

$$d\mathcal{R} = 0 \longrightarrow \mathcal{R}_{\mu_1\nu_1,\ldots,\mu_s\nu_s} = \partial_{\mu_1}\ldots\partial_{\mu_s}\varphi_{\nu_1\ldots\nu_s} + \ldots$$

Generalised Poincaré Lemma

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Wave equations for spin s

Bekaert Boulanger
2002, 2003

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Generalised Poincaré Lemma

- $$d\mathcal{R}(\varphi) \equiv 0$$

- The higher-derivative equation $\mathcal{R}' = 0$ can be proven to be equivalent to the wave equation

$$\Box\varphi = \partial\Lambda(\varphi)$$

where the r.h.s. can be gauge fixed to zero. (! *Note*: this is not the Fronsdal equation)

Goal of this talk

~

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- *For spin 2: Ricci = 0*
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standard hsp theories are ``Ricci-like"

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Spin one (and p-forms)

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Our goal:

we wish to extend the Bargmann-Wigner program

to encompass the Maxwell-like equations

$$\partial \cdot \mathcal{R}\,(\varphi) \;=\; 0$$

for all spins, in any D, i.e. including tensors with mixed symmetry

Plan

§ Maxwell-like equations à la Bargmann-Wigner

§ Curvatures & wave operators for gauge potentials

§ Reducible multiplets and tensionless strings

Based on

- ***J.Phys.A: Math.Theor. 48 (2015)*** ***(with X. Bekaert and N. Boulanger)***

- ***Class.Quant.Grav. 29 (2012)***

see also

- ***Nucl.Phys. B881 (2014) 248-268*** ***(with S. Lyakhovic and A. Sharapov)***
- ***JHEP 1303 (2013) 168*** ***(with A. Campoleoni)***
- ***Prog.Theor.Phys.Suppl. 188 (2011)***
- ***Phys.Lett. B690 (2010)***
- ***J.Phys.Conf. Ser. 222 (2010)***

Maxwell-like equations à la Bargmann-Wigner

spin 2

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$$P^2 = 0 \longrightarrow p_\mu = (p_+, 0, \ldots, 0)$$

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$$P^{2} = 0 \longrightarrow p_{\mu} = (p_{+},\, 0,\, \ldots,\, 0)$$

$$\partial^{\mu}\mathcal{R}_{\mu\nu,\rho\sigma} = 0 \longrightarrow \mathcal{R}_{-\nu,\rho\sigma} = 0$$

$$\partial_{[\lambda}\mathcal{R}_{\mu\nu],\,\rho\sigma} = 0 \longrightarrow \mathcal{R}_{ij,\,kl} = 0$$

spin 2

→ *The only non-vanishing components of* $\mathcal{R}_{\mu\nu,\,\rho\sigma}$ *are*

$$\mathcal{R}_{+i,\,+j} \sim h_{ij}$$

i.e. they define a symmetric tensor of GL(D-2)

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- In terms of particles (irreps of O(D-2)) this means

$$\partial_{[\lambda}\mathcal{R}_{\mu\nu],\,\rho\sigma} = 0 \quad \longrightarrow \quad \text{one particle with } m = 0,\ s=2$$

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Maxwell-like eqs propagate reducible multiplets

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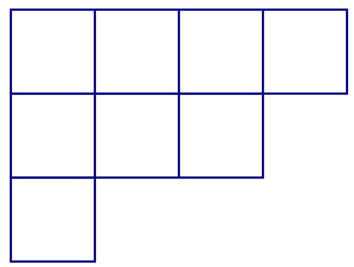
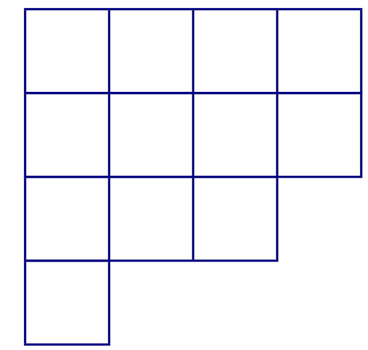
General case: consider an arbitrary tableau in GL(D-2) and build its Bargmann-Wigner counterpart, by adding a row on its top

$$Y_{GL(D-2)} =$$



$$\longrightarrow \qquad \mathcal{R}_{GL(D)} =$$



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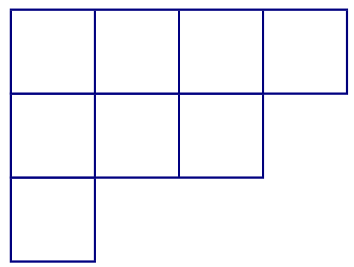
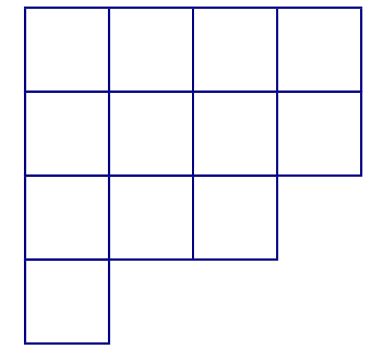
$Y_{GL(D-2)} =$  $\longrightarrow$ $\mathcal{R}_{GL(D)} =$ 

➡ *Require* $\mathcal{R}_{GL(D)}$ *to satisfy the closure and co-closure conditions*

$$d\,\mathcal{R} = 0$$

$$d^{\dagger}\,\mathcal{R} = 0$$

(w.r.t all rectangular blocks)

$\longrightarrow$ $P^2 = 0$ $\longrightarrow$ $p_{\mu} = (p_+, 0, \ldots, 0)$

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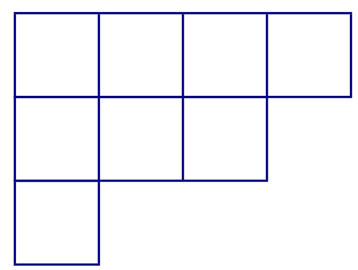
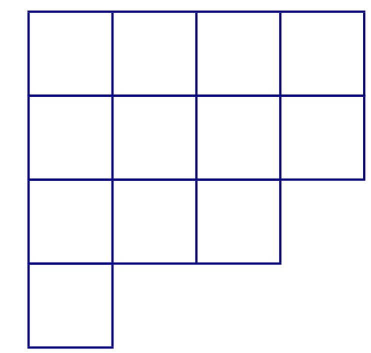
$Y_{GL(D-2)} =$  $\longrightarrow$ $\mathcal{R}_{GL(D)} =$ 

- *Require* $\mathcal{R}_{GL(D)}$ *to satisfy the closure and co-closure conditions*

$$d\,\mathcal{R} = 0 \qquad d^{\dagger}\mathcal{R} = 0 \quad \longrightarrow \quad P^2 = 0 \quad \longrightarrow \quad p_\mu = (p_+, 0, \dots, 0)$$

(w.r.t all rectangular blocks)

- *The non-vanishing components,* $\mathcal{R}_{+\,j^1_1\dots j^1_{l_1},\,\dots,\,+\,j^i_1\dots j^i_{l_i},\,\dots,\,+\,j^s_1\dots j^s_{l_s}}$,

correspond to *a multiplet of massless particles:*

branching of the GL(D-2)-irrep in terms of its O(D-2)-components.

Curvatures & wave operators for gauge potentials

High-derivative equations from curvatures

We make contact with gauge potentials solving for the closure conditions via the Generalised Poincaré Lemma:

High-derivative equations from curvatures

$\sim$

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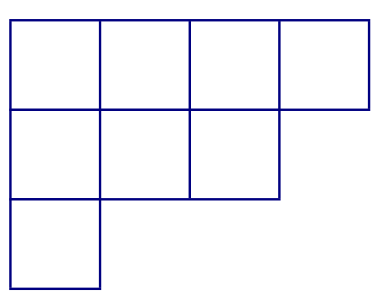
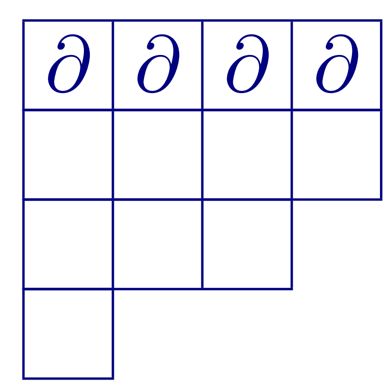
where $\mathcal{R}\,(\varphi)$ corresponds to the irrep of *GL(D)* obtained from a given tableau *Y* by adding an extra row on top of it:

$\varphi_Y =$



$\longrightarrow$

$\mathcal{R}\,(\varphi_Y) =$



High-derivative equations from curvatures

We go through the Bargmann-Wigner analysis again,
but now for high-derivative functions of gauge potentials

$$\mathcal{R}(\varphi) \equiv d^1 d^2 \cdots d^s \varphi$$

computing the divergence of $\mathcal{R}$

$$d_1 \mathcal{R}(\varphi) = d^2 \cdots d^s (\Box - d^i d_i) \varphi \sim \mathcal{O}(d) M = 0$$

where

$$M = (\Box - d^i d_i) \varphi$$

is a sort of second-order
Maxwell-like wave operator

From high- to 2nd-order equations

Problem: determine the kernel of the operator $\mathcal{O}\,(d)$

two steps:

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Getting an equation for M
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$$\longrightarrow$$

$$M = d^i d^j D_{ij}(\varphi)$$

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Show that the resulting equation
can be gauge fixed to $P^2 = 0$:

$$\Box\varphi = d^i \Lambda_i(\varphi)$$

$$\longrightarrow$$

$$\Box\varphi = 0 \qquad d^\dagger\varphi = 0$$

Same analysis for the ``standard'' BW trace conditions:

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$$T_{12}\,\mathcal{R}\,(\varphi)\;=\;d^{\,3}\,\cdots\,d^{\,s}\,\mathcal{F}\;\sim\;\hat{\mathcal{O}}\,(d)\,\mathcal{F}\;=\;0$$

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Show that the resulting equation can be gauge fixed to $P^2 = 0$:

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$$M = d^i\, d^j\, D_{ij}(\varphi)$$

still higher-derivative eqs!

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Our analysis shows that the two ``compensator'' structures

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$$M = 0 \qquad\qquad \mathcal{F} = 0$$

$$d^i d^j d_{(i} \Lambda_{j)} = 0 \qquad\qquad T_{(ij} \Lambda_{k)} = 0$$

D.F., A. Campoleoni 2013

Fronsdal-Labastida '78, '89

To summarise:

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BW trace conditions on
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BW transversality conditions on the same tensors describe multi-particle dof

_Via the Poincare' lemma
_upon partial gauge fixing
they reduce to

$$M := \Box\varphi - d^{\,i}\, d_{\,i}\, \varphi = 0$$

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Maxwell-like, N families:

(multi-particle spectrum)

$$\mathcal{L} = \frac{1}{2}\varphi\, M\, \varphi$$

$$M = (\Box - \partial^{i}\partial_{i})$$

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Fronsdal-Labastida, N families:

$$\mathcal{L} = \frac{1}{2}\varphi\left\{\mathcal{F} + \sum_{p\,=\,1}^{N}\frac{(-1)^p}{p!\,(p+1)!}\,\eta^{i_1 j_1}\,\ldots\,\eta^{i_p j_p}\,Y_{\{2^p\}}\,T_{i_1 j_1}\,\ldots\,T_{i_p j_p}\,\mathcal{F}\right\},$$

$$\mathcal{F} = \left(M + \partial^{\,i}\partial^{\,j}\,T_{ij}\right)\,\varphi$$

$$\begin{cases} T_{(\,ij}\,\Lambda_{\,k\,)} = 0 \\ T_{(\,ij}\,T_{kl\,)}\,\varphi = 0 \end{cases}$$

Reducible multiplets and tensionless strings

Massless higher spins from tensionless strings

$\sim$

Open bosonic string oscillators

$$[\alpha_k^\mu, \alpha_l^\nu] \;=\; k\,\delta_{k+l,0}\,\eta^{\mu\nu}$$

Massless higher spins from tensionless strings

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Virasoro generators and their rescaling limit:

$$L_k \ = \ \frac{1}{2} \sum_{l=-\infty}^{+\infty} \alpha^\mu_{k-l}\, \alpha_{\mu l} \ , \ \rightarrow \begin{cases} \tilde{L}_{\,k\neq 0} \ = \ \dfrac{1}{\sqrt{\alpha'}}\, L_{\,k} \\ \tilde{L}_{\,0} \ = \ \dfrac{1}{\alpha'}\, L_{\,0} \end{cases} \xrightarrow[\alpha' \to \infty]{} \begin{matrix} l_{\,k} \ = \ p_{\,\mu}\, \alpha^{\,\mu}{}_{k} \\ l_{\,0} \ = \ p_{\,\mu}\, p^{\,\mu} \end{matrix}$$

``tensionless" limit

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Algebra with no central charge $\Longrightarrow$ identically nilpotent BRST charge $\mathcal{Q}$

same charge from tensionless limit of open string BRST charge, after rescaling of ghosts

Massless higher spins from tensionless strings

$\sim$

$$\mathcal{L} = \frac{1}{2} \langle \psi | \, Q \, | \psi \rangle \xrightarrow[\alpha' \to \infty]{} $$

decomposes in diagonal blocks

Massless higher spins from tensionless strings

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for ``diagonal blocks'' associated to symmetric, rank-s tensors $\varphi_{\mu_1 \cdots \mu_s}$, (states generated by powers of α^{μ}_{-1}) the corresponding Lagrangian is

$$\mathcal{L}_{triplet} \;=\; \frac{1}{2}\,\varphi\,\Box\,\varphi - \frac{1}{2}\,s\,C^{2} - \binom{s}{2}\,D\,\Box\,D + s\,\partial\cdot\varphi\,C + 2\binom{s}{2}\,D\,\partial\cdot C$$

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equations of motion		*gauge transformations*
$\Box \varphi = \partial C$	$\varphi \to$ spin s	$\delta \varphi = \partial \Lambda$
$C = \partial \cdot \varphi - \partial D$	$C \to$ spin $s-1$	$\delta C = \Box \Lambda$
$\Box D = \partial \cdot C$	$D \to$ spin $s-2$	$\delta D = \partial \cdot \Lambda$

Massless higher spins from tensionless strings

➙ the field C is purely auxiliary (no kinetic term) and can be directly integrated away from the Lagrangian

Massless higher spins from tensionless strings

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the eom for the physical field from the tensionless string

$$M\,\varphi = 2\partial^2\mathcal{D}$$

are just the Maxwell-like equations with a ``compensator''

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Bengtsson, Ouvry-Stern '86 Henneaux-Teitelboim '88
D.F.-Sagnotti '02, Sagnotti-Tsulaia '03
Fotopoulos-Tsulaia '08 . . .

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$\longrightarrow$

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are just the Maxwell-like equations with a ``compensator''

[also valid for mixed-symmetry fields]

Maxwell-like geometric Lagrangians

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how does the Lagrangian would look in terms of the physical field only?

Maxwell-like geometric Lagrangians

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how does the Lagrangian would look in terms of the physical field only?

Integrating over the fields C and D we find

$$\mathcal{L}_{eff}(\varphi) = \frac{1}{2}\varphi(\Box - \partial\partial\cdot)\varphi + \frac{1}{2}\binom{s}{2}\partial\cdot\partial\cdot\varphi(\Box + \frac{1}{2}\partial\partial\cdot)^{-1}\partial\cdot\partial\cdot\varphi$$

Maxwell-like geometric Lagrangians

The inverse of the operator $\mathcal{O} = \Box + \frac{1}{2}\,\partial\,\partial\cdot$ on rank-*k* tensors is

$$\mathcal{O}^{-1}_{(k)} = \frac{1}{\Box}\{1 + \sum_{m=1}^{k}(-1)^m \frac{m!}{2^m \prod_{l=1}^{m}(1+\frac{l}{2})}\frac{\partial^m}{\Box^m}\partial\cdot^m\}$$

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Maxwell-like geometric Lagrangians

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and the resulting Lagrangian is

$$\mathcal{L}_{eff}(\varphi) = \frac{(-1)^s}{2(s+1)}\mathcal{R}^{(s)}_{\mu_1\cdots\mu_s,\,\nu_1\cdots\nu_s}\frac{1}{\Box^{s-1}}\mathcal{R}^{(s)\,\mu_1\cdots\mu_s,\,\nu_1\cdots\nu_s}$$

Lagrangians ~ *squares of curvatures*

Conclusions

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$$\mathcal{R}^{\alpha}{}_{\alpha\,\mu_3\ldots\mu_s\,,\,\nu_1\ldots\nu_s} = 0$$

``Ricci = 0" provides the backbone of gauge theories...

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reducible, multi-particle theories

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when the focus is on *single-particle interactions*

Alternative option:

reducible, multi-particle theories

``Maxwell = 0'' seems to provide the proper model to this end

$$\partial^{\alpha}\mathcal{R}_{\alpha\,\mu_2\ldots\mu_s\,,\,\nu_1\ldots\nu_s} \;=\; 0$$

Why?

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~

Exploit an alternative basis of field variables

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- *SFT makes use of this very basis and it is full of such couplings.*

what are their actual role and meaning?

in progress…