Odd parity correlators in 3d

Loriano Bonora,

SISSA, Trieste

Conference RT' 2015, Dubna,

November 25-27, 2015

in coll. with

Bruno Lima De Souza,

Antonio Duarte Pereira

This talk is about:

- Conformal invariance and CFT
- (Odd parity) CFT correlators
- Regularization in momentum space
- Pure contact terms correlators
- Higher spin currents

Why are CFTs important?

- at very high energies masses should become unimportant, thus at such energies the relevant field theory should be scale invariant $\rightarrow$ conformal invariant.
- CFTs are the interface of gravity in AdS/CFT correspondence
- CFTs are relevant to strongly correlated systems
- from a theoretical point of view, CFTs can be solved (even if they are not supersymmetric and or not Lagrangian)
- CFTs say a lot about gravity

I will talk about current correlators, in particular about e.m. tensor correlators. Why are they so important?

Because the source of the e.m. tensor is the metric (or, better, the metric fluctuations). Thus the e.m. tensor correlators can be interpreted as scattering amplitudes for gravitons.

(C.Closset,D.Dumitrescu,G.Festuccia, Z.Komargodski,N.Seiberg,X.Camanho,J.Edelstein, J.Maldacena, G.Pimentel, A.Zhiboedov,…)

Conformal Invariance

The conformal group in dimension D is formed by the Poincaré group plus

- scaling transformations

$$x^\mu \to \lambda x^\mu$$

- and special conformal transformations

$$x^\mu \to \frac{x^\mu + v^\mu x^2}{1 + 2v_\mu x^\mu + v^2 x^2}$$

They form the group $SO(D,2)$.
In D=2 the group is infinite dimensional and conformal transformations are defined by holomorphic and antiholomorphic transformations

$$z = x^1 + ix^2, \qquad z \to f(z), \qquad f(z) \approx z + \epsilon(z), \qquad \epsilon(z) = \sum_n \epsilon_{-n} z^{n+1}$$

The conformal Lie algebra

The Lie algebra generators of the conformal group

$$
\begin{aligned}
P_\mu &= -i\partial_\mu \\
D &= -ix^\mu \partial_\mu \\
L_{\mu\nu} &= i(x_\mu\partial_\nu - x_\nu\partial_\mu) \\
K_\mu &= -i(2x_\mu x^\nu\partial_\nu - x^2\partial_\mu)
\end{aligned}
$$

Commutators (extra Poincaré)

$$
\begin{aligned}
[P^\mu, D] &= iP^\mu \\
[K^\mu, D] &= -iK^\mu \\
[P^\mu, K^\nu] &= 2i\eta^{\mu\nu} D + 2iL^{\mu\nu} \\
[K^\mu, K^\nu] &= 0 \\
[L^{\mu\nu}, D] &= 0 \\
[L^{\mu\nu}, K^\lambda] &= i\eta^{\lambda\mu} K^\nu - i\eta^{\lambda\nu} K^\mu
\end{aligned}
$$

Representations of the conformal Lie algebra

For a generic tensor field $O(x)$ of weight Δ

$$
\begin{aligned}
i[P_\mu, O(x)] &= \partial_\mu O(x) \\
i[L_{\mu\nu}, O(x)] &= (x_\mu \partial_\nu - x_\nu \partial_\mu) O(x) + i\Sigma_{\mu\nu} O(x) \\
i[D, O(x)] &= (\Delta + x^\mu \partial_\mu) O(x) \\
i[K_\mu, O(x)] &= (2\Delta x_\mu + 2x_\mu x^\lambda \partial_\lambda - x^2 \partial_\mu - 2ix^\lambda \Sigma_{\lambda\mu}) O(x)
\end{aligned}
$$

For the e.m. tensor, in particolar,

$$
\begin{aligned}
i[D, T_{\mu\nu}(x)] &= (d + x^\lambda \partial_\lambda) T_{\mu\nu}(x) \\
i[K_\lambda, T_{\mu\nu}] &= \left(2\Delta x_\lambda + 2x_\lambda\, x\cdot\partial - x^2 \partial_\lambda\right) T_{\mu\nu} \\
&\quad + 2\left(x^\alpha T_{\alpha\nu} \eta_{\lambda\mu} + x^\alpha T_{\mu\alpha} \eta_{\lambda\nu} - x_\mu T_{\lambda\nu} - x_\nu T_{\mu\lambda}\right)
\end{aligned}
$$

Covariance of correlators

For instance the two-point function is well-known

$$
\begin{aligned}
&\langle T_{\mu\nu}(x) T_{\rho\sigma}(y)\rangle \\
&= \frac{c/2}{(x-y)^{2d}}\left(I_{\mu\rho}(x-y) I_{\nu\sigma}(x-y) + I_{\nu\rho}(x-y) I_{\mu\sigma}(x-y) - \frac{2}{d}\eta_{\mu\nu}\eta_{\rho\sigma}\right)
\end{aligned}
$$

where

$$I_{\mu\nu}(x) = \eta_{\mu\nu} - 2\frac{x_\mu x_\nu}{x^2}$$

One can prove it satisfies

$$\left(K_\lambda(x) + K_\lambda(y)\right)\langle T_{\mu\nu}(x) T_{\rho\sigma}(y)\rangle = 0$$

Higher order correlators are much more complicated and may involve several tensorial structures. There is a rich literature: *H.Osborn, A.C.Petkou, M.S.Costa, J.Penedones, D.Poland, S.Rychkov, Y.Stanev, A.Zhiboedov, T.Hansen, E.Elkhidir, D.Karateev, M.Serone,...*

Singularities of correlators

All correlators are singular at coincident points, but what matters is integrability. For instance the singularity

$$\frac{1}{x^2}$$

is nonintegrable in $d = 2$, but integrable in $d = 4$ (it can be Fourier transformed).

The correct attitude is to consider such expressions as distributions. Distributions are linear functionals on specific spaces of functions (test functions). In general

- distributions are derivatives of locally integrable functions

For instance the Dirac delta function is the derivative of the step function (which is locally integrable).
So using distribution theory seems to solve all singularity problems.

EM tensor 2-pt function in 2d

The most general two-point function of the e.m. tensor in 2d has the form

$$\langle T_{\mu\nu}(x) T_{\rho\sigma}(0)\rangle = \frac{c/2}{x^4}\left(I_{\mu\rho}(x) I_{\nu\sigma}(x) + I_{\nu\rho}(x) I_{\mu\sigma}(x) - \eta_{\mu\nu}\eta_{\rho\sigma}\right)$$

Let's call it the bare or unregulated correlator. For $x \neq 0$ this 2-point function satisfies the Ward identities

$$\begin{aligned} \partial^{\mu}\langle T_{\mu\nu}(x) T_{\rho\sigma}(0)\rangle &= 0, \\ \langle T^{\mu}_{\mu}(x) T_{\rho\sigma}(0)\rangle &= 0. \end{aligned}$$

But for $x = 0$ it is ill-defined. We need to regularize it.

Differential regularization

Introduce two differential operators

$$\begin{aligned}
\mathcal{D}^{(1)}_{\mu\nu\rho\sigma} &= \partial_\mu\partial_\nu\partial_\rho\partial_\sigma - \left(\eta_{\mu\nu}\partial_\rho\partial_\sigma + \eta_{\rho\sigma}\partial_\mu\partial_\nu\right)\Box + \eta_{\mu\nu}\eta_{\rho\sigma}\Box\Box, \\
\mathcal{D}^{(2)}_{\mu\nu\rho\sigma} &= \partial_\mu\partial_\nu\partial_\rho\partial_\sigma - \frac{1}{2}\left(\eta_{\mu\rho}\partial_\nu\partial_\sigma + \eta_{\nu\rho}\partial_\mu\partial_\sigma + \eta_{\mu\sigma}\partial_\nu\partial_\rho + \eta_{\nu\sigma}\partial_\mu\partial_\rho\right)\Box \\
&\quad + \frac{1}{2}\left(\eta_{\mu\rho}\eta_{\nu\sigma} + \eta_{\nu\rho}\eta_{\mu\sigma}\right)\Box\Box.
\end{aligned}$$

Then for $x \neq 0$

$$\langle T_{\mu\nu}(x)\, T_{\rho\sigma}(0)\rangle = -\frac{c}{24}\mathcal{D}^{(1)}_{\mu\nu\rho\sigma}\left(\log\left(\mu^2 x^2\right)\right) - \frac{c}{96}\left(\mathcal{D}^{(1)}_{\mu\nu\rho\sigma} - \mathcal{D}^{(2)}_{\mu\nu\rho\sigma}\right)\left(\log\left(\mu^2 x^2\right)\right)^2 .$$

This is conserved but

$$\langle T^\mu_\mu(x)\, T_{\rho\sigma}(0)\rangle = -\frac{c}{48}\eta^{\mu\nu}\mathcal{D}^{(1)}_{\mu\nu\rho\sigma}\left(\log\left(\mu^2 x^2\right)\right) = \frac{c}{48}\left(\partial_\rho\partial_\sigma - \eta_{\rho\sigma}\Box\right)\Box\log\left(\mu^2 x^2\right).$$

Since $\Box\log\left(\mu^2 x^2\right) = 4\pi\delta^{(2)}(x)$, we get the anomalous Ward identity

$$\langle T^\mu_\mu(x)\, T_{\rho\sigma}(y)\rangle = c\frac{\pi}{12}\left(\partial_\rho\partial_\sigma - \eta_{\rho\sigma}\Box\right)\delta^{(2)}(x-y).$$

cont.

In the approximation $g^{\rho\sigma}(x) \approx \eta^{\rho\sigma} + h^{\rho\sigma}(x) + \ldots$ we have $R \approx (\partial_\mu \partial_\nu - \eta_{\mu\nu}\Box)h^{\mu\nu}$. Thus this is the lowest approximation to the covariant expression

$$\langle T^\mu_\mu \rangle = c\frac{\pi}{12}R.$$

which is the well-known expression of the trace anomaly in 2D.

Differential regularization not always

so handy...

....often more convenient to go to momentum space.

The CA in momentum space

If we Fourier transform the generators of the confomal algebra we get (a tilde represents the transformed generator and $\tilde{\partial} = \frac{\partial}{\partial k}$

$$\begin{aligned}
\tilde{P}_\mu &= -k_\mu \\
\tilde{D} &= i(d + k^\mu \tilde{\partial}_\mu) \\
\tilde{L}_{\mu\nu} &= i(k_\mu \tilde{\partial}_\nu - k_\nu \tilde{\partial}_\mu) \\
\tilde{K}_\mu &= 2d\, \tilde{\partial}_\mu + 2k_\nu \tilde{\partial}^\nu \tilde{\partial}_\mu - k_\mu \tilde{\Box}
\end{aligned}$$

Notice that $\tilde{P}_\mu$ is a multiplication operator and $\tilde{K}_\mu$ is a quadratic differential operator. The Leibniz rule does not hold for $\tilde{K}_\mu$ and $\tilde{P}_\mu$ with respect to the ordinary product. However it does hold for the convolution product:

$$\tilde{K}_\mu (\tilde{f} \star \tilde{g}) = (\tilde{K}_\mu \tilde{f}) \star \tilde{g} + \tilde{f} \star (\tilde{K}_\mu\, \tilde{g})$$

where $(\tilde{f} \star \tilde{g})(k) = \int dp\, f(k-p) g(p)$.

The CA in momentum space

Nevertheless these generators form a closed algebra under commutator

$$[\tilde{D}, \tilde{P}_\mu] = i\tilde{P}_\mu$$

$$[\tilde{D}, \tilde{K}_\mu] = i\tilde{K}_\mu$$

$$[\tilde{K}_\mu, \tilde{K}_\nu] = 0$$

$$[\tilde{K}_\mu, \tilde{P}_\nu] = i(\eta_{\mu\nu}\tilde{D} - \tilde{L}_{\mu\nu})$$

$$[\tilde{K}_\lambda, \tilde{L}_{\mu\nu}] = i(\eta_{\lambda\mu}K_\nu - \eta_{\lambda\nu}K_\mu$$

$$[\tilde{P}_\lambda, \tilde{L}_{\mu\nu}] = i(\eta_{\lambda\mu}P_\nu - \eta_{\lambda\nu}P_\mu$$

$$[\tilde{L}_{\mu\nu}, \tilde{L}_{\lambda\rho}] = i(\eta_{\nu\lambda}\tilde{L}_{\mu\rho} + \eta_{\mu\rho}\tilde{L}_{\nu\lambda} - \eta_{\mu\lambda}\tilde{L}_{\nu\rho} - \eta_{\nu\rho}\tilde{L}_{\mu\lambda}$$

Ward identities in configuration space

If b^μ is the special conformal transformation parameter the classical Ward identities for e.m. tensor correlators are

$$(b\cdot K(x) + b\cdot K(y))\langle 0|TT_{\mu\nu}(x)T_{\lambda\rho}(y)\rangle|0\rangle = 0$$

and

$$(b\cdot K(x) + b\cdot K(y) + b\cdot K(z))\langle 0|TT_{\mu\nu}(x)T_{\lambda\rho}(y)T_{\alpha\beta}(z)\rangle|0\rangle = 0$$

and so on. And if there are contact terms (possibly anomalies) they become

$$(b\cdot K(x) + b\cdot K(y))\langle 0|TT_{\mu\nu}(x)T_{\lambda\rho}(y)\rangle|0\rangle = \eta_{\mu\nu}\mathcal{A}_{\lambda\rho}(x,y)$$
$$(b\cdot K(x) + b\cdot K(y) + b\cdot K(z))\langle 0|TT_{\mu\nu}(x)T_{\lambda\rho}(y)T_{\alpha\beta}(z)\rangle|0\rangle = \eta_{\mu\nu}\mathcal{A}_{\lambda\rho\alpha\beta}(x,y,z)$$

where the RHS are (linear in b) cocycles (of SCT cohomology).

Examples of ‘bare’ correlators

The Fourier transform of the 2-pt function of a scalar field of weight Δ is $\sim (k^2)^{\Delta-\frac{d}{2}}$. Applying $\tilde{K}_\mu$

$$K_\mu(k^2)^{\Delta-\frac{d}{2}} = 0\; b\cdot k(k^2)^{\Delta-\frac{d}{2}-1}$$

The Fourier transform of 2-pt function of two currents in 3d is

$$\langle \tilde{j}_i(k)\tilde{j}_j(-k)\rangle = \frac{\delta_{ij}k^2 - k_i k_j}{|k|}$$

Working out

$$\Big(2(b\cdot\tilde{\partial}) - (b\cdot k\tilde{\Box} - 2k\cdot\tilde{\partial}\, b\cdot\tilde{\partial})\Big)\langle \tilde{j}_i(k)\tilde{j}_j(-k)\rangle$$
$$+ 2(b^l\partial_i - b_i\tilde{\partial}^l)\langle \tilde{j}_l(k)\tilde{j}_j(-k)\rangle + 2(b^l\partial_j - b_j\tilde{\partial}^l)\langle \tilde{j}_l(k)\tilde{j}_l(-k)\rangle$$

one finds out that it is 0.

How do regularized correlators look like?

Regularized correlators contain:

- non-local terms (bare correlators)
- local terms (delta functions or derivative thereof in configuration space)
- partially local (a mixture)

Contact terms are the local ones. They appear as:

- delta function-like objects in configuration space
- polynomials of the momenta in momentum space.

What do contact terms represent?

Contact terms are

- anomalies in even dimensions
- action terms in odd dimensions

In momentum space they show up as polynomials of the external momenta, so they are particularly simple to analyse.

Program:

- Solve the conformal WI in general (Bzowski, Mc Fadden, Skenderis)
- Find all possible contact terms
- Find all the odd parity contact terms

Search for odd-parity contact terms

A systematic search can be done

- by means of a Mathematica code to solve WI's (B. Lima de Souza)
- by resorting to calculable models.

In the following I would like to concentrate on a simple model, a free massive fermion model in 3d.

Such a model is not conformal! So we will take the IR and UV limit of correlators in order to find conformal covariant ones.

Free massive fermion model in 3d

Action

$$S = \int d^3x \, \left[i\bar{\psi}\gamma^\mu D_\mu \psi - m\bar{\psi}\psi \right], \quad D_\mu = \partial_\mu + A_\mu$$

where $A_\mu = A_\mu^a(x) T^a$ and T^a are the generators of a gauge algebra. The generators are antihermitean, $[T^a, T^b] = f^{abc} T^c$, with normalization $\mathrm{tr}(T^a T^b) = \mathrm{n}\delta^{ab}$. The current

$$J_\mu^a(x) = \bar{\psi}\gamma_\mu T^a \psi$$

is (classically) covariantly conserved on shell

$$(DJ)^a = (\partial^\mu \delta^{ac} + f^{abc} A^{b\mu}) J_\mu^c = 0$$

(see also Dunne,Babu,Das, Panigrahi)

Free massive fermion model in 3d (cont.)

The effective action is given by

$$W[A] = \sum_{n=1}^{\infty} \frac{i^{n+1}}{n!} \int \prod_{i=1}^{n} d^3x_i A^{a_1\mu_1}(x_1)\dots A^{a_n\mu_n}(x_n)\langle 0|\mathcal{T} J^{a_1}_{\mu_1}(x_1)\dots J^{a_n}_{\mu_n}(x_n)|0\rangle$$

We will consider 2-pt and 3-pt current correlators,

$$\langle 0|\mathcal{T} J^a_\mu(x) J^b_\nu(y)|0\rangle, \quad \text{and} \quad \langle 0|\mathcal{T} J^a_\mu(x) J^b_\nu(y) J^b_\lambda(z)|0\rangle \tag{1}$$

whose Fourier transform are $\tilde{J}^{ab}_{\mu\nu}(k)$ and $\tilde{J}^{abc}_{\mu\nu\lambda}(k_1, k_2)$. The one-loop conservation law in momentum space is

$$\begin{aligned} &k^\mu \tilde{J}^{ab}_{\mu\nu}(k) = 0 \\ &-iq^\mu \tilde{J}^{abc}_{\mu\nu\lambda}(k_1, k_2) + f^{abd} \tilde{J}^{dc}_{\nu\lambda}(k_2) + f^{acd} \tilde{J}^{db}_{\lambda\nu}(k_1) = 0 \end{aligned}$$

where $q = k_1 + k_2$.

Free massive fermion model:2-pt

The 2-pt function is

$$\tilde{J}^{ab(odd)}_{\mu\nu}(k) \quad = \quad \frac{\mathrm{n}}{2\pi}\delta^{ab}\epsilon_{\mu\nu\sigma}k^\sigma\,\frac{m}{k}\arctan\frac{k}{2m}$$

where $k = \sqrt{k^2} = \sqrt{E}$. The IR and UV limit correspond to $\frac{m}{\sqrt{E}} \to \infty$ and 0, respectively. We get

$$\tilde{J}^{ab(odd)}_{\mu\nu}(k) = \frac{\mathrm{n}}{2\pi}\delta^{ab}\epsilon_{\mu\nu\sigma}k^\sigma \begin{cases} \frac{1}{2} & \mathrm{IR} \\ \frac{\pi}{2}\frac{m}{k} & \mathrm{UV} \end{cases}$$

Fourier anti-transforming and substituting in $W(A)$ one gets

$$\int d^3x\epsilon^{\mu\nu\lambda}\, A^a_\mu\partial_\nu A^a_\lambda$$

Free massive fermion model:3-pt

The 3-pt function is more complicated

$$\tilde{J}^{1,abc}_{\mu\nu\lambda}(k_1,k_2) = i\int \frac{d^3p}{(2\pi)^3}\mathrm{Tr}\left(\gamma_\mu T^a \frac{1}{\not{p}-m}\gamma_\nu T^b \frac{1}{\not{p}-\not{k}_1-m}\gamma_\lambda T^c \frac{1}{\not{p}-\not{q}-m}\right)$$

The result is a generalized Lauricella function (Boos,Davydychev). In the IR we find

$$\tilde{J}^{1,abc(odd)}_{\mu\nu\lambda}(k_1,k_2) \approx i\frac{\mathrm{n}}{32\pi}\sum_{n=0}^{\infty}\left(\frac{\sqrt{E}}{m}\right)^{2n} f^{abc}\tilde{I}^{(2n)}_{\mu\nu\lambda}(k_1,k_2)$$

and, in particular,

$$I^{(0)}_{\mu\nu\lambda}(k_1,k_2) = -6\epsilon_{\mu\nu\lambda}$$

which corresponds to the action term

$$\sim \int d^3x\, \epsilon^{\mu\nu\lambda} f^{abc} A^a_\mu A^b_\nu A^c_\lambda$$

Laurricella hypergeometric function

Basic integral

$$J_3(\alpha,\beta,\gamma;m) = \int \frac{d^dp}{(2\pi)^d} \frac{1}{(p^2-m^2)^\alpha((p-k_1)^2-m^2)^\beta((p-q)^2-m^2)^\gamma}$$

This can be transformed into

$$\begin{aligned} J_3(\alpha,\beta,\gamma;m) &= \frac{i^{1-d}}{(4\pi)^{\frac{d}{2}}}(-m^2)^{\frac{d}{2}-\alpha-\beta-\gamma}\frac{\Gamma(\alpha+\beta+\gamma-\frac{d}{2})}{\Gamma(\alpha+\beta+\gamma)} \\ & \quad \Phi_3\left[\begin{matrix}\alpha+\beta+\gamma-\frac{d}{2},\alpha,\beta,\gamma \\ \alpha+\beta+\gamma\end{matrix}\middle| \frac{k_1^2}{m^2},\frac{q^2}{m^2},\frac{k_2^2}{m^2}\right]\end{aligned}$$

where Φ_3 is a generalized Lauricella function $\Big((a)_n = \Gamma(a+n)/\Gamma(n)\Big)$:

$$\begin{aligned} &\Phi_3\left[\begin{matrix}a_1,a_2,a_3,a_4 \\ c\end{matrix}\middle| z_1,z_2,z_3\right] \\ &= \sum_{j_1=0}^{\infty}\sum_{j_2=0}^{\infty}\sum_{j_2=0}^{\infty}\frac{z_1^{j_1}}{j_1!}\frac{z_2^{j_2}}{j_2!}\frac{z_3^{j_3}}{j_3!}\frac{(a_1)_{j_1+j_2+j_3}(a_2)_{j_1+j_2}(a_3)_{j_1+j_3}(a_4)_{j_2+j_3}}{(c)_{2j_1+2j_2+2j_3}}\end{aligned}$$

Free massive fermion model:3-pt (cnt.)

Putting things together we find the effective CS action

$$CS = \frac{\kappa}{4\pi}\int d^3x \mathrm{Tr}\left(A \wedge dA + \frac{2}{3} A \wedge A \wedge A\right)$$

- In the IR $\kappa = 1$, so the CS action is invariant also under large gauge transformation.
- - In the UV things are more complicated. Eventually we get the same action with $\kappa = \pi \frac{m}{k}$. So, the UV limit is 0.
  - If ψ carries a flavour index $i = 1, \dots, N$, the previous result is multiplied by N, and $\kappa = \pi N \frac{m}{k}$. So we can consider the scaling limit $N \to \infty$, $\frac{m}{k} \to 0$ and κ fixed and finite.

Important! Both 2-pt and 3-pt correlator satisfy the WI's of CFT (and they are pure contact term)!

A note on conservation

For the 3-pt correlator of the massive theory we have

$$
\begin{aligned}
q^{\mu}\,\tilde{J}^{abc}_{\mu\nu\lambda}(k_1,k_2) \quad = \quad & -\frac{i}{4\pi} f^{abc}\epsilon_{\lambda\nu\sigma}k_1^{\sigma}\,\frac{2m}{k_1}\mathrm{arccot}\left(\frac{2m}{k_1}\right) \\
& -\frac{i}{4\pi} f^{abc}\epsilon_{\lambda\nu\sigma}k_2^{\sigma}\,\frac{2m}{k_2}\mathrm{arccot}\left(\frac{2m}{k_2}\right) \neq 0
\end{aligned}
$$

but

$$
\begin{aligned}
& -iq^{\mu}\,\tilde{J}^{(odd)abc}_{\mu\nu\lambda}(k_1,k_2) + f^{abd}\,\tilde{J}^{(odd)dc}_{\nu\lambda}(k_2) + f^{acd}\,\tilde{J}^{(odd)db}_{\lambda\nu}(k_1) \\
& = -\frac{1}{4\pi} f^{abc}\epsilon_{\lambda\nu\sigma}\left(k_1^{\sigma}\,\frac{2m}{k_1}\mathrm{arccot}\left(\frac{2m}{k_1}\right) + k_2^{\sigma}\,\frac{2m}{k_2}\mathrm{arccot}\left(\frac{2m}{k_2}\right)\right) \\
& +\frac{1}{4\pi} f^{abc}\epsilon_{\lambda\nu\sigma}\left(k_1^{\sigma}\,\frac{2m}{k_1}\mathrm{arccot}\left(\frac{2m}{k_1}\right) + k_2^{\sigma}\,\frac{2m}{k_2}\mathrm{arccot}\left(\frac{2m}{k_2}\right)\right) = 0
\end{aligned}
$$

In the IR and UV limit the last equality is not conserved and, to preserve covariance, one has to subtract counterterms.

E.m. tensor correlators

Next come the e.m. tensor correlator. It is naturally coupled to the metric. The action in the massive model is

$$S = \int d^3x\, e \left[i\bar{\psi} E^{\mu}_{a}\gamma^a \nabla_\mu \psi - m\bar{\psi}\psi\right], \quad \nabla_\mu = \partial_\mu + \frac{1}{2}\omega_{\mu bc}\Sigma^{bc}, \quad \Sigma^{bc} = \frac{1}{4}\left[\gamma^b, \gamma^c\right].$$

The mass term breaks parity!

The energy-momentum tensor

$$T^{\mu\nu} = \frac{i}{4}\bar{\psi}\left(E^{\mu}_{a}\gamma^a \overleftrightarrow{\nabla}^{\nu} + \mu \leftrightarrow \nu\right)\psi.$$

is covariantly conserved (on shell): $\nabla_\mu T^{\mu\nu} = 0$.

At quantum level (the Fourier trasform of) the 2-pt correlator is

$$\tilde{T}^{(odd)}_{\mu\nu\lambda\rho}(k) = \frac{m}{256\pi}\epsilon_{\sigma\nu\rho}\, k^\sigma \left[2m\left(\eta_{\mu\lambda} - \frac{k_\mu k_\lambda}{k^2}\right) + \left(\eta_{\mu\lambda} + \frac{k_\mu k_\lambda}{k^2}\right)\frac{k^2 + 4m^2}{|k|}\arctan\frac{|k|}{2m}\right] +$$

In the effective action the e.m. tensor couples to $h_{\mu\nu}$, where $g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu} + \ldots$

Gravitational CS

In the IR and UV limit this corresponds to the action term

$$S_{\rm eff}^{\rm P-odd} = \frac{\kappa}{192\pi} \int d^3x \, \epsilon_{\sigma\nu\rho} \, h^{\mu\nu} \, \partial^\sigma \left(\partial_\mu \partial_\lambda - \eta_{\mu\lambda} \Box\right) h^{\lambda\rho} \tag{1}$$

This is nothing but the lowest order expansion in $h_{\mu\nu}$ of the gravitational Chern-Simons action in 3d.

$$CS \quad = -\frac{\kappa}{96\pi} \int d^3x \, \epsilon^{\mu\nu\lambda} \left(\partial_\mu \omega_\nu^{ab} \omega_{\lambda ba} + \tfrac{2}{3} \omega_{\mu a}{}^b \omega_{\nu b}{}^c \omega_{\lambda c}{}^a\right)$$

- In the IR limit we find $\kappa = 1$ (the action is well defined)
- In the UV limit $\kappa = \frac{3}{2}\pi \frac{m}{|k|}$. So again the limit vanishes unless we consider N flavours, in which case we can take the scaling limit that leaves $\lambda = N \frac{m}{|k|}$ fixed.

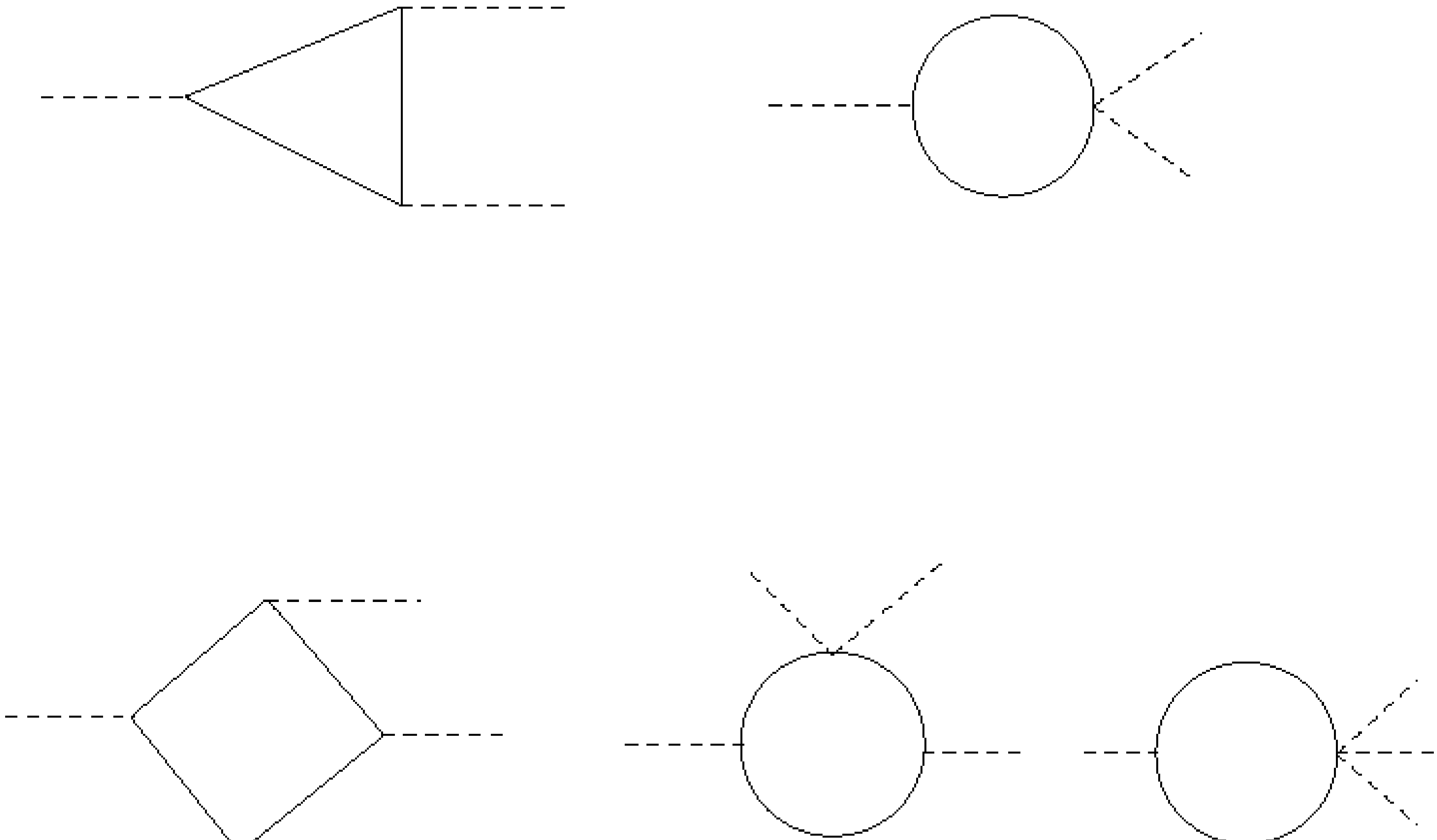
Gravitational CS (cnt.)

The next order in the CS action is

$$\begin{aligned} CS^{(3)} \quad = \quad & \frac{\kappa}{384\pi} \int d^3x \, \epsilon^{\mu\nu\lambda} \Big(2\partial_a h_{\nu b} \partial_\lambda h^b_\sigma \partial_\mu h^{\sigma a} - 2\partial_a h^b_\mu \partial^c h_{b\nu} \partial^a h_{c\lambda} - \frac{2}{3} \partial_a h^b_\mu \partial_b h^c_\nu \partial_c h^a_\lambda \\ & -2\partial_\mu \partial^b h^a_\nu (h^c_a \partial_c h_{b\lambda} + h^c_b \partial_c h_{a\lambda}) + \partial_\mu \partial^b h^a_\nu (h^c_\lambda \partial_a h_{bc} - h^c_\lambda \partial_b h_{ac}) \\ & + \Big(\partial_\mu \partial^b h^a_\nu \partial_b h^c_\lambda h_{ca} - \partial_\mu \partial^b h^a_\nu \partial_a h^c_\lambda h_{cb} \Big) - h^\rho_\lambda h^a_\rho \partial_\mu \left(\Box h_{a\nu} - \partial_a \partial_b h^b_\nu \right) \Big) \end{aligned}$$

which is generated by the 3-pt e.m. tensor correlator.

(work in progress, almost finished)



Higher spin currents

In the massive fermion model in 3d we have other conserved currents. The next after the em tensor is the third order current

$$J_{\mu_1\mu_2\mu_3} = \bar{\psi}\gamma_{(\mu_1}\partial_{\mu_2}\partial_{\mu_3)}\psi - \frac{5}{3}\partial_{(\mu_1}\bar{\psi}\gamma_{\mu_2}\partial_{\mu_3)}\psi + \frac{1}{3}\eta_{(\mu_1\mu_2}\partial^\sigma\bar{\psi}\gamma_{\mu_3)}\partial_\sigma\psi - \frac{m^2}{3}\eta_{(\mu_1\mu_2}\bar{\psi}\gamma_{\mu_3)}\psi$$

This is conserved (on-shell). We consider the external source $B^{\mu\nu\lambda}$ and couple it to the theory via the action term

$$\int d^3x J_{\mu\nu\lambda}B^{\mu\nu\lambda}$$

Due to current conservation this coupling is invariant under the (infinitesimal) transformations

$$\delta B_{\mu\nu\lambda} = \partial_{(\mu}\Lambda_{\nu\lambda)}$$

In the limit $m \to 0$ we have also invariance under the transformation

$$\delta B_{\mu\nu\lambda} = \Lambda_{(\mu}\eta_{\nu\lambda)}$$

which induces the tracelessness of $J_{\mu\nu\lambda}$ in any couple of indices.

2-pt correlators

We can construct an effective action for $B_{\mu\nu\lambda}$ via

$$W[B] = \sum_{n=1}^{\infty} \frac{i^{n+1}}{n!} \int \prod_{i=1}^{n} dx_i B^{\mu_i\nu_i\lambda_i}(x_i) \langle 0 | \mathcal{T} \left\{ J_{\mu_1\nu_1\lambda_1}(x_1) \dots J_{\mu_n\nu_n\lambda_n}(x_n) \right\} |0\rangle$$

by computing the n-pt fucntions. For instance the (odd-parity) 2-pt correlator (after subtractions) in the IR is

$$\tilde{T}^{(odd,IR)}_{\mu_1\mu_2\mu_3\nu_1\nu_2\nu_3}(k) = \frac{1}{8\pi} \epsilon_{\mu_1\nu_1\sigma} k^\sigma \Big[\frac{1}{30} k^4 \eta_{\mu_2\mu_3}\eta_{\nu_2\nu_3} - \frac{1}{15} k^4 \eta_{\mu_2\nu_2}\eta_{\mu_3\nu_3}$$
$$- \frac{1}{30} k^2 \left(k_{\nu_2}k_{\nu_3}\eta_{\mu_2\mu_3} + k_{\mu_2}k_{\mu_3}\eta_{\nu_2\nu_3} \right) + \frac{2}{15} k^2 k_{\mu_2}k_{\nu_2}\eta_{\mu_3\nu_3} - \frac{1}{30} k_{\mu_2}k_{\mu_3}k_{\nu_2}k_{\nu_3} \Big]$$

which is conserved and traceless. In the UV limit (after subtractions) we get

$$\tilde{T}^{(odd,UV)}_{\mu_1\mu_2\mu_3\nu_1\nu_2\nu_3}(k) = \frac{1}{8}\frac{m}{|k|} \epsilon_{\mu_1\nu_1\sigma} k^\sigma \Big[\frac{1}{12} k_{\mu_2}k_{\mu_3}k_{\nu_2}k_{\nu_3} - \frac{1}{3} k^2 k_{\mu_3}k_{\nu_3}\eta_{\mu_2\nu_2}$$
$$+ \frac{k^2}{12} \left(k_{\nu_2}k_{\nu_3}\eta_{\mu_2\mu_3} + k_{\mu_2}k_{\mu_3}\eta_{\nu_2\nu_3} \right) + \frac{1}{6} k^4 \eta_{\mu_2\nu_2}\eta_{\mu_3\nu_3} - \frac{1}{12} k^4 \eta_{\mu_2\mu_3}\eta_{\nu_2\nu_3} \Big]$$

which is also conserved and traceless.

Effective action

They correspond to the effective action term

$$\sim \int d^3x \quad \epsilon_{\mu_1\nu_1\sigma}\Big[\partial^\sigma B^{\mu_1\mu_2\mu_3}\partial_{\mu_2}\partial_{\mu_3}\partial_{\nu_2}\partial_{\nu_3}B^{\nu_1\nu_2\nu_3} - 4\partial^\sigma B^{\mu_1\mu_2\mu_3}\Box\partial_{\mu_3}\partial_{\nu_3}B^{\nu_1\nu_3}{}_{\mu_2}$$
$$+2\partial^\sigma B^{\mu_1\lambda}{}_{\lambda}\Box\partial_{\nu_2}\partial_{\nu_3}B^{\nu_1\nu_2\nu_3} + 2\partial^\sigma B^{\mu_1\mu_2\mu_3}\Box^2 B^{\nu_1}{}_{\mu_2\mu_3}$$
$$-\partial^\sigma B^{\mu_1\lambda}{}_{\lambda}\Box^2 B^{\nu_1\rho}{}_{\rho}\Big]$$

Same construction for higher order currents.

Possible generalizations

- Covariantize with respect to a gauge field

$$J^{(n)}_{\mu\ldots}(x) \longrightarrow J^{(n)a}_{\mu\ldots}(x) = \bar{\psi}\gamma_\mu T^a \ldots \psi$$

coupled to $B^a_{\mu\ldots}$ with gauge transformations

$$\delta B^a_{\mu\ldots} = (D\Lambda)^a_{\mu\ldots}, \qquad D = d + A$$

- Covariantize wrt the metric. Replace everywhere ∂ by ∇ and consider diffeomorphisms.

Conclusion and prospects

The massive fermion model in 3d is an example of pure contact terms correlators:

- they are local;
- they correspond to terms of (gauge, gravity, more general) CS action;
- they are characterized by a more complex conservation law;
- no free field theory generates them.

Conclusion and prospects (cnt.)

Ongoing program and questions:

- find all the CFT correlators, more specifically the odd-parity and local ones;
- in the odd-dimensional case, recognize the limiting (CS?) UV and IR theories;
- understand the nature of the parameter m in the framework of AdS/CFT;
- study the problem of off-shell gravity amplitudes (scheme in/dependence)

THANKS