Interaction of QED fields with macroscopic objects

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Introduction

In 1948 it was shown by Casimir that vacuum fluctuations of quantum fields generate an attraction between two parallel uncharged conducting planes [H.B.G. Casimir, Proc. K. Ned. Akad. Wet. **51**, 793 (1948)]

This phenomena called the Casimir effect (CE) has been well investigated with methods of modern experiments

[S.K.Lamoreaux, Phys. Rev. Lett. **78**, 5 (1997), U.Mohideen and A. Roy, Phys. Rev. Lett. **81**, 4549 (1998); A. Roy, C.-Y. Lin, and U. Mohideen, Phys. Rev. D **60**, 111101(R) (1999), B.W. Harris, F. Chen, and U. Mohideen, Phys. Rev. A **62**, 052109 (2000), G. Bressi et al., Phys. Rev. Lett. **88**, 041804 (2002)].

The CE is a manifestation of influence of fluctuations of quantum fields on the level of classical interaction of material objects.

Theoretical and experimental investigation of phenomena such a kind became very important for development of micro-mechanics and nano-technology.

Though there are many theoretical results on the CE

[K.A. Milton, J.Phys. A **37**, R209 (2004)], however the majority of them are received in framework of several models based not on the quantum electrodynamics (QED) directly.

An approach for construction of the single QED model for investigation of all peculiar properties of the CE for thin material films was proposed in [V.N.Markov, Yu.M. Pis'mak, ArXiv:hep-th/0505218, J.Phys. A: Math. Gen. **39**, 6525 (2006) (arXiv:hep-th/0606058), I.V. Fialkovsky, V.N. Markov, Yu.M. Pis'mak, Int. J. Mod. Phys. A **21**,2601 (2006), I.V. Fialkovsky, V.N.Markov, Yu.M. Pis'mak, J.Phys. A: Math. Gen. **41**, 075403 (2008)].

We consider its application for simple forms of films. We show that gauge invariance, locality and renormalizability considered as basic principles make strong restrictions for constructions of the CE models in QED, which make it possible to reveal new important features of the CE-like phenomena.

Formulation of the model

Symanzik action functional (Symanzik K 1981 Nucl. Phys. B 190 1):

$$S(\varphi) = S_V(\varphi) + S_{def}(\varphi)$$

where

$$S_D(\varphi) = \int L(\varphi(x)) d^D x, \; S_{def}(\varphi) = \int_\Gamma L_{def}(\varphi(x)) d^{D'} x,$$

and Γ is a subspace of dimension $D' < D$ in D-dimensional space.

From the principles of QED: gauge invariance, locality, renormalizability it follovs that in the model of interaction of material surface with the quantum QED fields the pure photon field contribution can be described with the action functional of the form

$$S(A) = S_0(A) + S_{def}(A)$$

Here $S_0(A)$ - is the usual free action of the photon field $A_\mu(x)$

$$S_0 = -\frac{1}{4}\int d^4x F^{\mu\nu}(x)F_{\mu\nu}(x),$$
$$F_{\mu\nu}(x) = \partial_\mu A_\nu(x) - \partial_\nu A_\mu(x),$$

and $S_{def}(A)$ is the defect action modeling the interaction of field $A_\mu(x)$ with a macroscopic inhomogeneity.

If it is a 2 surface (defect) with the form described by equation $\Phi(x) = 0$, then:

$$S_{def} = \sigma \int d^4x \epsilon^{\nu\mu\lambda\kappa} \partial_\nu \Phi(x) \delta(\Phi(x)) A_\mu(x) \partial_\lambda A_\kappa(x).$$

For the stationary defect $\partial_0 \Phi(x) = 0$ which will be considered the action $S_{def}(A)$ can be written as

$$S_{def}(A) = \sigma \int d^4x \delta(\Phi(x)) \{2i A_0(x) \vec{L}_\Phi \vec{A}(x) + \vec{\partial}\Phi[\vec{A}(x) \times \partial_0 \vec{A}(x)]\}$$

where $\vec{L}_\Phi \equiv i[\vec{\partial}\Phi \times \vec{\partial}]$ and σ is a dimensionless coupling constant.

The fermion defect action can be written as

$$S_\Phi(\bar{\psi}, \psi) = \tag{2}$$

$$= \int \bar{\psi}(x)[\lambda + u^\mu \gamma_\mu + \gamma_5(\tau + v^\mu \gamma_\mu) + \omega^{\mu\nu}\sigma_{\mu\nu}]\psi(x)\delta(\Phi(x))dx$$

Here, γ_μ, $\mu = 0, 1, 2, 3$, are the Dirac matrices, $\gamma_5 = i\gamma_0\gamma_1\gamma_3\gamma_3$, $\sigma_{\mu\nu} = i(\gamma_\mu\gamma_\nu - \gamma_\nu\gamma_\mu)/2$, and λ, τ, u_μ,v_μ, $\omega^{\mu\nu} = -\omega^{\nu\mu}$, $\mu, \nu = 0, 1, 2, 3$ are 16 dimensionless parameters.

Expressions (1), (2) are the most general forms of gauge invariant actions concentrated on the defect surface being invariant in respect to reparametrization of one and not having any parameters with negative dimensions.

Casimir force

We consider defect concentrated on two parallel planes $x_3 = 0$ and $x_3 = r$. For this model, it is convenient to use a notation like $x = (x_0, x_1, x_2, x_3) = (\vec{x}, x_3)$.

Defect action (1) has the form:

$$S_{2P} = \frac{1}{2}\int (a_1\delta(x_3) + a_2\delta(x_3 - \mathrm{r}))\varepsilon^{3\mu\nu\rho}A_\mu(x)F_{\nu\rho}(x)dx.$$

It is the main point in our model formulation, and no any boundary conditions are used.

The energy density E_{2P} of defect is defined as

$$\ln G(0) = \frac{1}{2}\,\mathrm{Tr}\ln(D_{2P}D^{-1}) = -iTSE_{2P}$$

where $T = \int dx_0$ is duration of defect, and $S = \int dx_1 dx_2$, is the area of film.

It is expressed in an explicit form in terms of polylogarithm function $\mathrm{Li}_4(x)$ [V.N. Markov,Yu.M. Pis'mak, ArXiv: hep-th/0505218].

For identical films with $a_1 = a_2 = a$ it holds:

$$E_{2P} = 2E_s + E_{Cas}, E_s = \int \ln\sqrt{(1+a^2)}\frac{d\vec{k}}{(2\pi)^3},$$

$$E_{Cas} = -\frac{1}{16\pi^2 \mathrm{r}^3}\left\{\mathrm{Li}_4\left(\frac{a^2}{(a+i)^2}\right) + \mathrm{Li}_4\left(\frac{a^2}{(a-i)^2}\right)\right\}.$$

Here E_s is an infinite constant, which can be interpreted as self-energy density on the plane, and E_{Cas} is an energy density of their interaction.

The function $\mathrm{Li}_4(x)$ is defined as

$$\mathrm{Li}_4(x) = \sum_{k=1}^{\infty} \frac{x^k}{k^4} = -\frac{1}{2} \int_0^{\infty} k^2 \ln(1 - xe^{-k}) dk.$$

The force $F_{2P}(\mathrm{r}, a)$ between planes is given by

$$F_{2P}(\mathrm{r}, a) = -\frac{\partial E_{Cas}(\mathrm{r}, a)}{\partial \mathrm{r}} = -\frac{\pi^2}{240 \mathrm{r}^4} f(a).$$

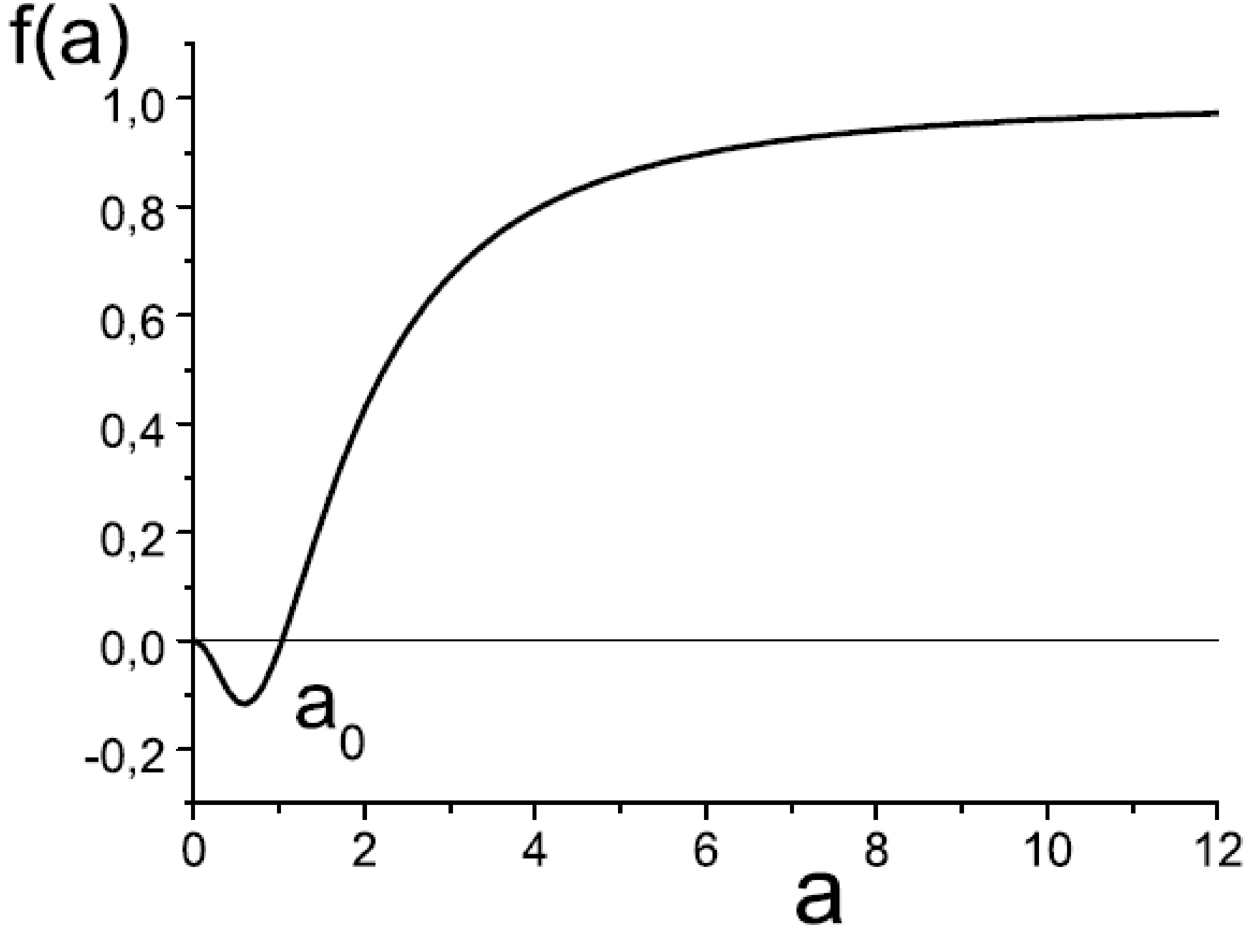

Figure 1: Function $f(a)$ determining Casimir force between parallel planes

The force F_{2P} is repulsive for $|a| < a_0$ and attractive for $|a| > a_0$, $a_0 \approx 1.03246$ (see Figure 1).

For large $|a|$ it is the same as the usual CF between perfectly conducting planes. The model predicts that the maximal magnitude of the *repulsive* F_{2P} is expected for $|a| \approx 0.6$.

For two infinitely thick parallel slabs the repulsive CF was predicted also in [O. Kenneth et al., Phys. Rev. Lett. **89**, 033001 (2002)].

Real film has a finite width, and the bulk contributions to the CF for nonperfectly conducting slabs with widths h_1, h_2 are proportional to $h_1 h_2$. Therefore it follows directly from the dimensional analysis that the bulk correction F_{bulk} to the CF is of the form $F_{bulk} \approx c F_{Cas} h_1 h_2 / r^2$ where F_{Cas} is the CF for perfectly conducting planes and c is a dimensionless constant. This estimation can be relevant for modern experiments on the CE.

Non-planar geometry

The case of cylindrical film.

$$S_{def} = a/2 \int d^4x \, \varepsilon_{\mu\nu\rho\sigma} \partial^\mu \Phi(x) A^\nu \partial^\rho A^\sigma \delta(\Phi)$$

where A — EM vector-potential, $\varepsilon_{\mu\nu\rho\sigma}$ — totaly antisymmetric tensor ($\varepsilon_{0123} = 1$), and the defect is described with equation $\Phi(x) = 0$, $x = (x_0, x_1, x_2, x_3)$.

For cylindrical shell placed along the x_3 axis, $x_1^2 + x_2^2 = R^2$ we have

$$\Phi(x) = x_1^2 + x_2^2 - R^2$$

For the sphere with radius r_0 :

$$\Phi(x) = \sqrt{x_1^2 + x_2^2 + x_3^2} - r_0,$$

$$\vec{\partial}\Phi(x) = \frac{\vec{x}}{|\vec{x}|} = \vec{n}(\vec{x}), \quad \vec{L}_\Phi = \frac{1}{|\vec{x}|} i[\vec{x} \times \vec{\partial}] = \frac{1}{|\vec{x}|}\vec{L}$$

The limit $\sigma \to \infty$ corresponds to ideal conducting surface with with conditions $n_\mu \tilde{F}^{\mu\nu}|_S = 0$.

Regularization

To remove the ultraviolet divergencies we use the Pauli-Willars regularization:

$$S_0 \to S_{0r} = -\frac{1}{4}\int d^4x F^{\mu\nu}(x)(1+M^{-2}\partial_\lambda\partial^\lambda)F_{\mu\nu}(x)$$

$$S(A) \to S_r(A) = S_{0r} + S_{def}$$

and use for calculations the Euclidean version of the action S_E, which is obtained by replations

$$x_0 \to -ix_0, \quad \partial_0 \to i\partial_0, \quad A_0 \to iA_0, \quad a \to ia.$$

Casimir energy

For the Casimir energy E_{Cas} holds the expression

$$E_{Cas} = -\frac{1}{T} Tr \ln(D D_0^{-1})$$

where D is the propagator in the model with defect, and D_0 is the propagator for the model in homogenous space. For the spherical defect it diverges by $M \to \infty$.

Divergences and renormalization

The asymptotic of the regularized Casimir energy of the spherical defect with raduis r for large M has the form:

$$E_{Cas} = M^3 r_0^2 A(\sigma) + MB(\sigma) + \frac{F(\sigma)}{r_0} + O\left(\frac{1}{M}\right).$$

with

$$F(\sigma) = \frac{3}{64}\frac{\frac{1}{4}\sigma^2}{1+\frac{1}{4}\sigma^2} + \frac{1}{2\pi}\sum_{l=1}^{+\infty}(2l+1) \times$$

$$\times \int_0^\infty dp \quad \ln\frac{1-\sigma^2\mathcal{G}_l(p)\mathcal{R}_l(p)}{1+\frac{1}{4}\sigma^2} + \frac{\frac{\sigma^2}{4}(2l+1)^4}{\left(1+\frac{1}{4}\sigma^2\right)\left(4p^2+(2l+1)^2\right)^3}\Bigg).$$

Here the following notations are used:

$$\mathcal{G}_l(x) = I_{l+\frac{1}{2}}(x) K_{l+\frac{1}{2}}(x),$$

$$\mathcal{R}_l(x) = \left(\frac{1}{2} I_{l+\frac{1}{2}}(x) + I'_{l+\frac{1}{2}}(x)\right)\left(\frac{1}{2} K_{l+\frac{1}{2}}(x) + K'_{l+\frac{1}{2}}(x)\right).$$

with Bessel function $I_{l+\frac{1}{2}}(x)$, $K_{l+\frac{1}{2}}(x)$.

It is finite for finite M but diverges for removing of regularization $M \to \infty$. This problem is solved by the renormalization.

For $\sigma \to \infty$ we opbtaine

$$F_\infty = F(\sigma)|_{\sigma\to\infty} = \frac{3}{64} + \frac{1}{2\pi}\sum_{l=1}^{+\infty}(2l+1)\times$$
$$\times\int_0^\infty dp\left\{\ln[-4\mathcal{G}_l(p)\mathcal{R}_l(p)] + \frac{(2l+1)^4}{(4p^2+(2l+1)^2)^3}\right\}.$$

It the results for ideal connecting sphere $E_{Caz} = F_\infty/r_0$, coinciding with one obtained by Boyer.

For removing of the divergences of Casimir energy in the framework of usual multiplicative renormalization procedure one needs to add to the action the terms without photon field with Lagrangian

$$L_{cl}(x) = (Ar_0^2 + B)\delta(|\vec{x}| - r_0),$$

having two constant parameters A, B . Making renormalization of them one can cancel the divergences and obtain the finite renirmalized Casimir energy

$$E_{Cas} = 4\pi r_0^2 \alpha + \beta + \frac{F(\sigma)}{r_0}$$

with finite parameters α, β of dimension of surface energy density and energy. If $\alpha > 0$, $F(\sigma) > 0$ the function E_{Cas} has minimum with $r_0 = \sqrt[3]{F(\sigma)/8\pi\alpha}$.

Cilindrical film

The result for CE has the form

$$E = E_{div} + E_{fin}$$

$$E_{div} = M^3 R f_3(a) + \frac{M}{R} f_1(a),$$

$$E_{fin} = \frac{E_{cas}(a)}{4\pi R^2} + O(1/M)$$

[I.V. Fialkovsky, V.N.Markov, Yu.M. Pis'mak, J.Phys. A: Math. Gen. **41**, 075403 (2008)]

Electromagnetic fields generated by simplest fermion defects

We consider the fermionic defect of the form

$$S_{\lambda q}(\bar{\psi}, \psi) \equiv \int \bar{\psi}(\vec{x}, 0)(\lambda + \hat{q})\psi(\vec{x}, 0)d\vec{x}$$

Here $\bar{\psi}$, ψ are the Dirac spinor fields, λ is a constant parameter, q is a fixed 4-vectors, $\hat{q} = q_\mu \gamma^\mu$ (γ^μ are the Dirac gamma-matrices), and we used the short hand notation for the 4-vector: $x = (x_0, x_1, x_2, x_3) = (\vec{x}, x_3)$.

Vector $q = (\vec{q}, 0)$ and scalar λ describe the interaction of current and density of Dirac field with material defect. Namely, the zero component of vector $\vec{q}$ defines a surface charge density and space like components of vector $\vec{q}$ parallel to the defect plane describe the surface current.

The scalar λ defect can be interpreted as a surface mass term.

The interaction of vacuum fluctuations of the Dirac field with the background generates quantum corrections to usual classical effects.

Asymptotics of the generated by the defect electromagnetic fields for large and small x_3 are the following

[I.V. Fialkovsky,V.N. Markov,Yu.M. Pis'mak, Int. J. Mod. Phys. A **21**,2601 (2006)].

If $q = (\kappa, 0, 0, 0) = q^{(1)}$, the defect generates pure electric field E_3

$$E_3 \underset{x_3 \to 0}{\approx} \frac{em^2}{8\pi^2\omega^2}\left[(1+\omega^2)\mathrm{Arctg}(\omega) - \omega\right]\left(\frac{1}{m^2 x_3^2} - 2\right).$$

$$E_3 \underset{x_3 \to \infty}{\approx} -\frac{em^2}{4\pi^2\omega^2}\left[\mathrm{Arctg}(\omega) - \omega\right],\ \omega = \frac{4\kappa}{4-\kappa^2}.$$

For $q = (0, \kappa, 0, 0) \equiv q^{(2)}$ the field is pure magnetic

$$H_2 \underset{x_3 \to\ 0}{\approx} -\frac{em^2}{16\pi^2\omega'^2}\left[(1+\omega'^2)\ln\frac{1+\omega'}{1-\omega'} - 2\omega'\right]\left(\frac{1}{m^2 x_3^2} - 2\right).$$

$$H_2 \underset{x_3 \to \infty}{\approx} \frac{em^2}{8\pi^2\omega^{2'}}\left[\ln\frac{1+\omega'}{1-\omega'} - 2\omega'\right],\ \omega' = \frac{4\kappa}{4+\kappa^2}.$$

For $q = (0, \kappa, 0, 0) \equiv q^{(2)}$ the field is pure magnetic

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$$H_2 \underset{x_3 \to \infty}{\approx} \frac{em^2}{8\pi^2\omega^{2'}}\left[\ln\frac{1+\omega'}{1-\omega'} - 2\omega'\right], \ \omega' = \frac{4\kappa}{4+\kappa^2}.$$

For $q = (\kappa, \kappa, 0, 0) \equiv q^{(3)}$, $E_1 = E_2 = H_1 = H_3 = 0$, and asymptotics of the fields E_3, H_2 are of the form

$$E_3 \underset{x_3 \to 0}{\approx} H_2 \underset{x_3 \to 0}{\approx} -\frac{e\kappa m^2}{12\pi^2}\left(\frac{1}{m^2 x_3^2} - 2\right),$$

$$E_3 \underset{x_3 \to \infty}{\approx} H_2 \underset{x_3 \to \infty}{\approx} -\frac{e\kappa m^2}{12\pi^2}.$$

Casimir-Polder effect

Casimir-Polder effect was predicted theoretically in 1948 (H.B.G.Casimir and D.Polder, Phys.Rev. 73, 360 (1948)). Casimir and Polder found the energy of a neutral point atom in its ground state in the presence of a perfectly conducting infinite plate. In the case of a perfectly conducting plate one can say that the interaction of a fluctuating dipole with the electric field of its image yields the Casimir-Polder potential.

Model

In our model the interaction of the plane surface $x_3 = 0$ with a quantum electromagnetic field A_μ is described by the action:

$$S_{def}(A) = a \int \epsilon^{\alpha\beta\gamma 3} A_\alpha(x) \partial_\beta A_\gamma(x) \delta(x_3) dx.$$

We will use latin indices for the components of 4- tensors with numbers $0, 1, 2$, also the following notations:

$$P^{lm}(\vec{k}) = g^{lm} - k^l k^m / \vec{k}^2,$$

$$L^{lm}(\vec{k}) = \epsilon^{lmn3} k_n / |\vec{k}|, \ \vec{k}^2 = k_0^2 - k_1^2 - k_2^2,$$

where $|\vec{k}| = \sqrt{\vec{k}^2}$, and g - metric tensor.

The atom is modeled as a localized electric dipole at the point $(x_1, x_2, x_3) = (0, 0, l)$, which is described by the current $J_\mu(x)$:

$$J_0(x) = \sum_{i=1}^{3} p_i(t)\partial^i \delta(x_1)\delta(x_2)\delta(x_3 - l),$$

$$J_i(x) = -\dot{p}_i(t)\delta(x_1)\delta(x_2)\delta(x_3 - l), \ \ i = 1, 2, 3.$$

The condition of the current conservation holds:

$$\partial_\mu J^\mu = 0,$$

$p_i(t)$ is a function with a zero average and the pair correlator

$$\langle p_j(t_1) p_k(t_2) \rangle = -i \int_{-\infty}^{+\infty} \frac{e^{-i\omega(t_1 - t_2)}}{2\pi} \alpha_{jk}(\omega) d\omega,$$

where $\alpha_{jk}(\omega)$ for $\omega > 0$ coincides with the atomic polarizability.

The aim is to calculate the interaction energy E of the atom with a plane, and we will use the following representation for the energy:

$$E = \frac{i}{T} \left\langle \left\{ \ln \int \exp\left(iS(A) + JA\right) DA - \ln \int \exp\left(iS(A)\right) DA \right\}_{(a)} \right\rangle,$$

$\{\cdots\}_{(a)}$ means that the $a = 0$ value of the a-dependent function has to be subtracted: $\{f(a)\}_{(a)} \equiv f(a) - f(0)$.

The ground state energy of a neutral atom in the presence of a plane with Chern-Simons interaction is obtained in the form:

$$E = -\frac{1}{64\pi^2 l^3}\frac{a^2}{1+a^2}\Bigg(\int_0^{+\infty} d\omega e^{-2\omega l} 2(1+2\omega l)\alpha_{33}(i\omega)$$

$$+\int_0^{+\infty} d\omega e^{-2\omega l}(1+2\omega l+4\omega^2 l^2)\big(\alpha_{11}(i\omega)+\alpha_{22}(i\omega)\big)\Bigg)$$

$$+\frac{1}{64\pi^2 l^2}\frac{a}{1+a^2}\int_0^{+\infty} d\omega e^{-2\omega l} 2\omega\big(1+2\omega l\big)\big(\alpha_{12}(i\omega)-\alpha_{21}(i\omega)\big)$$

Consider the system with a nonzero $\alpha^A_{jk}(\omega)$ and assume for simplicity the one mode model of the atomic polarizability with a characteristic frequency ω_{10}. Then $\alpha^A_{12}(\omega) = i\omega C_2/(2(\omega_{10}^2 - \omega^2))$, where C_2 is a real constant. In the limit of large separations $\omega_{10} l \gg 1$ we obtain :

$$E|_{\omega_{01} l \gg 1} = -\frac{a^2}{1+a^2}\frac{\alpha_{11}(0)+\alpha_{22}(0)+\alpha_{33}(0)}{32\pi^2 l^4} - \frac{a}{1+a^2}\frac{C_2}{32\pi^2\omega_{10}^2 l^5}$$

At large enough separations the first term in $E|_{\omega_{01} l \gg 1}$ always dominates. Assuming for simplicity $\alpha_{11}(0) = \alpha_{22}(0) = \alpha_{33}(0) = C_1/(3\,\omega_{10})$, C_1 is a positive constant, one can see from () that if the condition $\frac{|a|C_1}{|C_2|} < 1$ holds then for separations $l \lesssim \frac{|C_2|}{|a|C_1\omega_{10}}$ the term with off-diagonal elements of the atomic polarizability (the second term in $E|_{\omega_{01} l \gg 1}$ dominates.

Interaction of film with classical charge and current

The classical charge and the wire with current near defect plane are modeled by appropriately chosen 4-current J in (3).

The mean vector potential $\mathcal{A}_\mu$ generated by J and the plane $x_3 = 0$, with $a_1 = a$ can be calculated as

$$\mathcal{A}^\mu = -i \frac{\delta G(J)}{\delta J_\mu}\Big|_{a_1=a, a_2=0} = i D_{2P}^{\mu\nu} J_\nu \big|_{a_1=a, a_2=0}. \qquad (5)$$

Using notations $\mathcal{F}_{ik} = \partial_i \mathcal{A}_k - \partial_k \mathcal{A}_i$, one can present electric and magnetic fields as $\vec{E} = (\mathcal{F}_{01}, \mathcal{F}_{02}, \mathcal{F}_{03})$, $\vec{H} = (\mathcal{F}_{23}, \mathcal{F}_{31}, \mathcal{F}_{12})$.

For charge e at the point $(x_1, x_2, x_3) = (0, 0, l)$, $l > 0$ the corresponding classical 4-current is

$$J_\mu(x) = 4\pi e\delta(x_1)\delta(x_2)\delta(x_3 - l)\delta_{0\mu}$$

In virtue of (5) the mean vector potential $\mathcal{A}^\mu(x)$ is independent on x_0 and the electric field in considered system is defined by potential

$$\mathcal{A}_0(x_1, x_2, x_3) = \frac{e}{\rho_-} - \frac{a^2}{a^2+1}\frac{e}{\rho_+}.$$

where $\rho_+ \equiv \sqrt{x_1^2 + x_2^2 + (|x_3| + l)^2}$, $\rho_- \equiv \sqrt{x_1^2 + x_2^2 + (x_3 - l)^2}$.

The electric field $\vec{E} = (E_1, E_2, E_3)$ is of the form

$$E_1 = \frac{ex_1}{\rho_-^3} - \frac{a^2}{a^2+1}\frac{ex_1}{\rho_+^3}, \;\; E_2 = \frac{ex_2}{\rho_-^3} - \frac{a^2}{a^2+1}\frac{ex_2}{\rho_+^3},$$

$$E_3 = \frac{e(x_3 - l)}{\rho_-^3} - \frac{a^2\epsilon(x_3)}{a^2+1}\frac{e(|x_3| + l)}{\rho_+^3}.$$

Here, $\epsilon(x_3) \equiv x_3/|x_3|$.

We see that for $x_3 > 0$ the field $\vec{E}$ coincides with field generated in usual classical electrostatic by charge e placed on distance l from infinitely thick slab with dielectric constant $\epsilon = 2a^2 + 1$.

Because $\mathcal{A}^\mu(x) \neq 0$ for $\mu = 1, 2, 3$, the defect generate also a magnetic field $\vec{H} = (H_1, H_2, H_3)$:

$$H_1 = \frac{eax_1}{(a^2+1)\rho_+^3},\ H_2 = \frac{eax_2}{(a^2+1)\rho_+^3},\ H_3 = \frac{ea(|x_3|+l)}{(a^2+1)\rho_+^3}.$$

It is an anomalous field which doesn't arise in classical electrostatics. Its direction depends on sign of a.

A current with density j flowing in the wire along the x_1-axis is modeled by

$$J_\mu(x) = 4\pi j\delta(x_3 - l)\delta(x_2)\delta_{\mu 1}$$

For magnetic field from (5) one obtains in region $x_3 > 0$ the usual results of classical electrodynamics for the current parallel to infinitely thick slab with permeability $\mu = (2a^2+1)^{-1}$. There is also an anomalous electric field $\vec{E} = (0, E_2, E_3)$:

$$E_2 = \frac{2ja}{a^2+1}\frac{x_2}{\tau^2}, \;\; E_3 = \frac{2ja}{a^2+1}\frac{|x_3|+l}{\tau^2}$$

where $\tau = (x_2^2 + (|x_3|+l)^2)^{\frac{1}{2}}$.

Problem of scattering

The model of the photon field interacting with the two-dimensional material surface, given by equation $\Phi(x) = 0$, is characterized by the action functional

$$S(A) = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + S_{\phi}(A)$$

where

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\mu A_\nu,$$

$$S_\phi(A) = \frac{a}{2} \int \varepsilon^{\lambda\mu\nu\rho} \partial_\lambda \Phi(x) A_\mu(x) F_{\nu\rho}(x) \delta(\Phi(x)) dx.$$

For the plane defect $\Phi(x) = x_3$ the Euler-Lagrange equations of the model are written as modified Maxwell's equations:

$$\frac{\delta S(A)}{\delta A_\nu} = \partial_\mu F^{\mu\nu} + a\varepsilon^{3\nu\sigma\rho} F_{\sigma\rho}\delta(x_3) = 0.$$

We solve them using the Fourier transform over coordinates x_0, x_1, x_2 for the vector-potential A_μ:

$$A_\mu(x) = \frac{1}{(2\pi)^{\frac{3}{2}}} \int e^{i\overline{px}} A_\mu(x_3, \overline{p}) d\overline{p},$$

$$A_\mu(x_3, \overline{p}) = \frac{1}{(2\pi)^{\frac{3}{2}}} \int e^{-i\overline{px}} A_\mu(x) d\overline{x}.$$

Here and later we use the notation $\overline{p}$ for vector $\overline{p} = (p_0, p_1, p_2)$, $\overline{p}^2 = p_0^2 - p_1^2 - p_2^2$, $\overline{px} = p_0x_0 - p_1x_1 - p_2x_2$.

It follows from $A_\mu(x) = A_\mu^*(x)$ that $A^*(x_3, \overline{p}) = A(x_3, -\overline{p})$. Using this relation we can obtain an integral representation

$$A_\mu(x) = \frac{2\Re}{(2\pi)^{\frac{3}{2}}} \int \theta(p_0) \left[e^{i\overline{px}} A_\mu(x_3, \overline{p}) \right] d\overline{p}$$

where $\Re$ denotes the real part.

We make calculations in the temporal gauge $A_0 = 0$, where electric and magnetic fields $\vec{E}$, $\vec{H}$ are expressed through the vector-potential $A = (0, \vec{A})$ by relations $\vec{E} = \partial_0 \vec{A}$, $\vec{H} = \vec{\partial} \times \vec{A}$

Scattering on plane

For the wave falling on the plane from the half space with negative coordinate x_3 we should have in a half space $x_3 > 0$ only the transmitted wave, moving from the plane $x_3 = 0$ in positive direction of the third axis.

For the intensities $I_{in},\ I_r,\ I_{tr}$ of the incident, reflected and transmitted waves we have

$$I_{in} = \frac{|\vec{\mathcal{A}}_{in}(\overline{p})|^2}{2\pi^3},\ \vec{I}_r = \frac{|\vec{\mathcal{A}}_r(\overline{p}))|^2}{2\pi^3},\ \vec{I}_{tr} = \frac{|\vec{\mathcal{A}}_{tr}(\overline{p}))|^2}{2\pi^3}$$

Therefore

$$I_r = \frac{a^2}{1+a^2} I_{in},\ I_{tr} = \frac{1}{1+a^2} I_{in}.$$

Hence, the reflection $K_r \equiv I_r/I_{in}$ and transmission coefficients $K_{tr} \equiv I_{tr}/I_{in}$ for flat waves scattering on the plane does not depend on the frequency and incidence angle and can be expressed through the characterizing the scattering material coupling constant a :

$$K_r = \frac{a^2}{1+a^2},\ K_{tr} = \frac{1}{1+a^2}.$$

Consider the movement of waves along the axis x_3. In this case $p_1 = p_2 = 0, \rho = p_0$ and

$$\vec{E}_{in} = p_0^3(-\beta_{in},\ \alpha_{in},\ 0),\ \vec{E}_{tr} = \frac{1}{1+a^2}\vec{E}_{in} + \frac{a}{1+a^2}\vec{Q},$$

$$\vec{Q} \equiv p_0^3(\alpha_{in},\ \beta_{in},\ 0),$$

$$\vec{E}_r = \frac{ap_0^3}{1+a^2}(-\beta_r a - \alpha_r,\ \alpha_r a - \beta_r, 0),$$

and replacing in $\vec{E}_r$ sign of x_3 on the opposite we obtain

$$T\vec{E}_r = \frac{a^2}{1+a^2}\vec{E}_{in} - \frac{a}{1+a^2}\vec{Q}.$$

We see that by the scattering of waves moving perpendicular to the plane, apart from the usual for process of scattering waves, there are waves with electric field rotated by an angle $\pi/2$ ($\vec{E}_{in}\vec{Q} = 0$).

For the case of a monochromatic plane wave with arbitrary polarization the vectors of electric and magnetic fields of the reflected and transmitted waves, and also the transmission and reflection coefficients are obtained in an explicit form. The transmission and reflection coefficients are expressed through coupling constant a and do not depend on the wave frequency and fall angle. For waves propagating in the orthogonal to the plane direction by small a, the electric field vector of the reflected wave is rotated on the close to $\pi/2$ angle with respect to its direction for the case of a perfectly conducting plane ($a \to \infty$).

Conclusion

In the proposed model the interaction of photon fields with sharp boundary is presented by Chern-Simons potential with one dimensionless coupling constant.

It brooks space parity and time inversion symmetries.

All the effects of interaction of boundary with QED fields can by described in the framework of one model.

In the limit of infinite coupling constant one obtains the known results of models with boundary conditions.

By finite value of coupling constants the model predicts unusual effects which could be important for micro-mechanics, nano-electronics, constructing of new materials, interpretations of the astronomical dates.