Stopping of Muons in Helium-3 and Deuterium

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Abstract. Stopping power ratio of helium-3 and deuterium atoms for muons slowed down in D/^{3}He gas mixture was measured using 34.0 MeV/c muon beam at PSI meson factory. We present the measurement method and the analysis of experimental data.

Collaboration Dubna-Fribourg-Cracow-PSI

MUON INDUCED PROCESSES IN HELIUM AND HELIUM-DEUTERIUM MIXTURES

Experiments performed at PSI, μ-E4 muon channel

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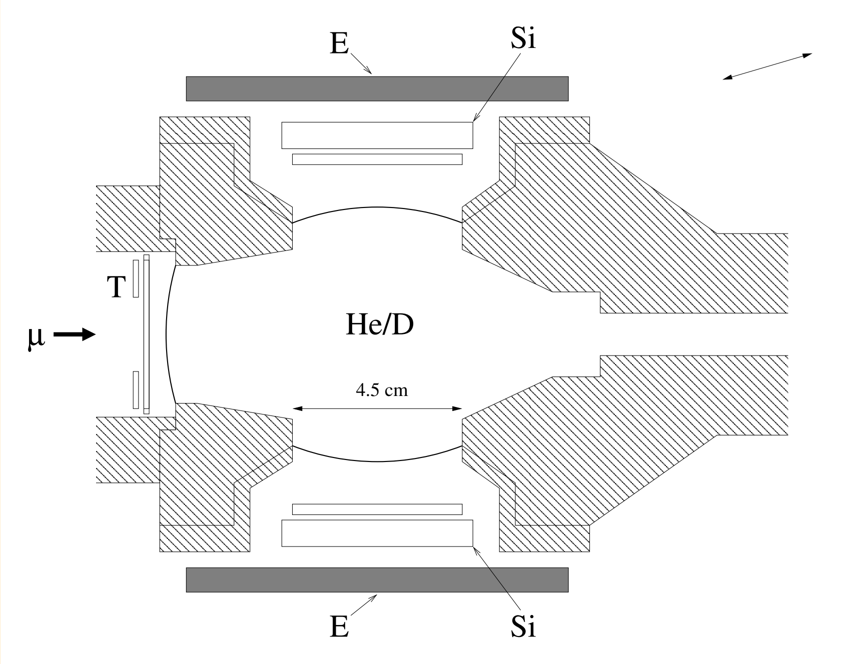
- measuring μd^3He fusion
- nuclear muon capture by ^{3}He
- various μ-atomic, μ-molecular characteristics
- muon stopping

Introduction

- Spatial distribution of muon stops gives an information on the distribution of formatted μ-atoms
- decay electrons are used as the markers of the muon stops:

$$\boxed{\mu^- \text{ slowing-down}} \longrightarrow \boxed{\text{formation of } \mu\text{-atom}} \longrightarrow \boxed{\mu\text{-decay}} \longrightarrow \boxed{e^- + \nu_\mu + \tilde{\nu}_e}$$

- experimental set-up



▷ *muon beam momentum $P_\mu = 34.0$ MeV/c, spread 1.23 FWHM*

▷ *gas targets: pure ^{3}He, D/^{3}He mixture (0.0496 He concentration)*

▷ *different gas densities (0.034 - 0.058 LHD)*

▷ *pressure 5.1 - 6.9 atm*

▷ *temperature ∼33 K*

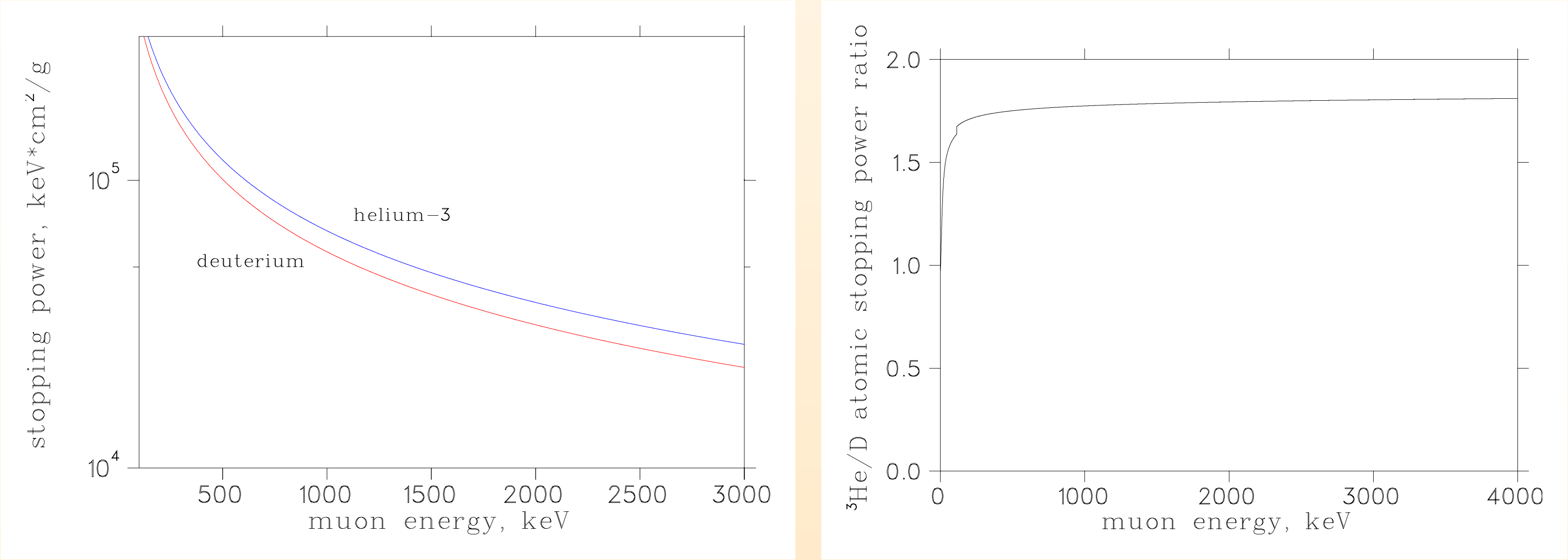
Stopping power

Per-atom stopping power [MeV·cm^2/atom]

$$S_i = \frac{1}{n_i}\left(-\frac{dE}{dx}\right)_i = A_i\left(-\frac{dE}{d\xi}\right)_i, \qquad S = \sum c_i S_i \qquad i = {}^3\mathrm{He}, \mathrm{D}$$

n_i - numeric density, A_i - atomic mass, c_i - atomic concentration, ξ - mass thickness.

Stopping power ratio: $\tilde{s} = S_{^3\mathrm{He}}/S_\mathrm{D}$

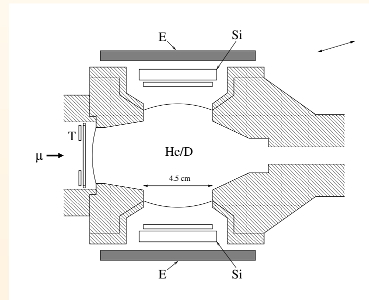


Calculated stopping powers (using data of J.F.Ziegler *et al., The Stopping...*, Pergamon Press, N.Y., 1985)

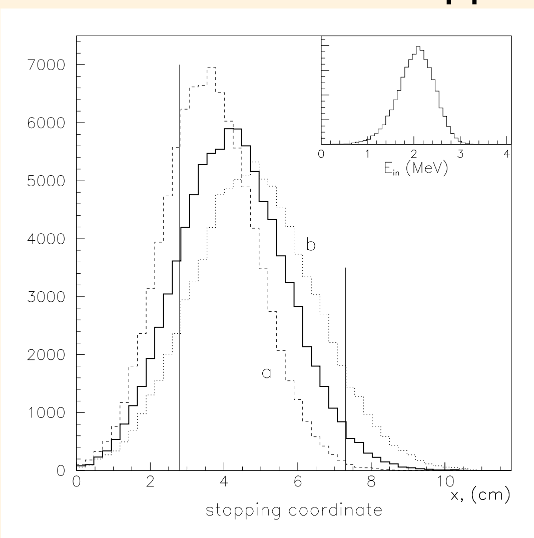
Measurement method

Muon slowing-down simulation via Monte Carlo; μ-stops distributions

- Monte Carlo code (R. Jacot-Guillarmod, Fribourg University); Ziegler's parametrisation of the stopping powers
- conditions of the experiment
  - ▷ *one target D + 0.05* ^{3}*He, fixed density* φ_{mix} *= 0.0585*
  - ▷ *a set of pure* ^{3}*He targets, different densities* φ_{He}



- calculated stopping distributions for D/^{3}He target and ^{3}He target, example:



solid line: D/^{3}He mixture, $\varphi_{mix} = 0.0585$

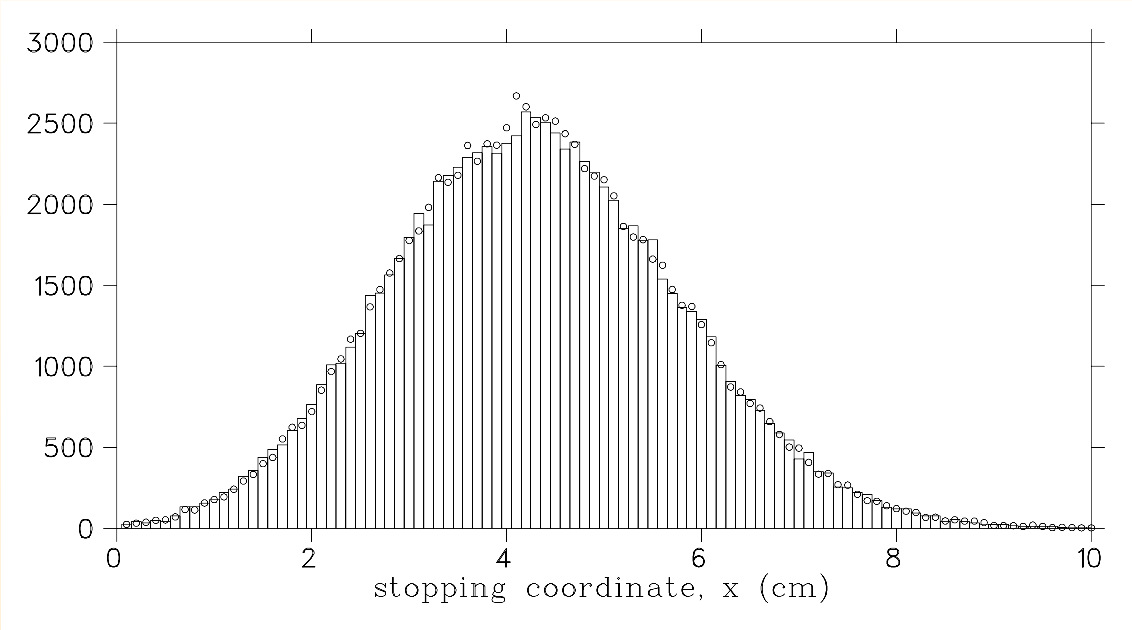
dotted,dashed lines: pure ^{3}He, densities:

(a) $\varphi_{He} = 0.0380$,
(b) $\varphi_{He} = 0.0320$

in the insert: energy spectrum of incident muons (E_{in})

Measurement method

Equivalence of the stopping distributions by Monte Carlo simulation



Histo: D/^{3}He mixture.

Circles: pure ^{3}He with equivalent density $\tilde{\varphi}_{He}$.

$\chi^2/df = 0.92$

Similar equivalency is also achieved in the plane perpendicular to the moun beam direction

Measurement method

The range of muon

- $$r_\mu = \int_0^{E_{in}} \frac{1}{-\left(\frac{dE}{dx}\right)} dE = \frac{N_a}{\varphi\, n_o} \int_0^{E_{in}} \frac{1}{\sum c_i S_i} dE \qquad (1)$$
- Keeping constant the beam momentum and changing the gas density in a given type of the target (^{3}He) one can obtain the same μ-stopping distribution as for another fixed-density target (D+^{3}He mixture)
- Such equivalence of muon ranges (for the whole initial energy spectrum of muons) is a key for the stopping power ratio measurement

Equivalence of the stopping distributions

- When the muon ranges are equal for both media then taking into account the similarity of the individual stopping powers of helium-3 and deuterium one obtains from (1) a formula for the mean stopping powers ratio:
- $$< \tilde{s} >= \tilde{s}(\overline{E}) = S_{^3\mathrm{He}}/S_\mathrm{D} = \frac{c_\mathrm{D}\varphi_{mix}}{\tilde{\varphi}_\mathrm{He} - c_\mathrm{He}\varphi_{mix}}. \qquad (2)$$
- $\tilde{\varphi}_\mathrm{He}$, φ_{mix} - equivalent densities of the pure ^{3}He target and the mixture D/^{3}He target.
- Equation (2) gives the receipe for the measurement of $< \tilde{s} >$ -value

Experiment

Experiment conditions

Run	Target	Temp. [K]	Presssure [atm]	φ [LHD]	C_{He} [%]	N_μ [10^9]
1	3He	32.9	6.92	0.0363	100	1.3625
2			6.85	0.0359		0.7043
3			6.78	0.0355		0.7507
4			6.43	0.0337		0.4136
5	D/^{3}He	32.8	5.11	0.0585	4.96	8.875

R-ratio measurement

Run	Target	φ [LHD]	N_μ [10^9]	N_e [10^6]	R [10^{-3}]
1	3He	0.0363	1.3625	0.5302 (14)	0.3891 (10)
2		0.0359	0.7043	0.2765 (10)	0.3926 (14)
3		0.0355	0.7507	0.2975 (10)	0.3963 (14)
4		0.0337	0.4136	0.1657 (8)	0.4007 (18)
5	D/^{3}He	0.0585	8.875	3.4635 (35)	0.3903 (4)

- *$P_\mu = 34.0$ MeV/c was chosen such to stop all entering muons inside the D/^{3}He target.*
- *electron time spectra analysis*
- $$R(\varphi) = \frac{N_e}{N_\mu}$$

$$R_{mix}(\varphi = 0.0585) = R_{He}(\varphi_{He}) \quad \longrightarrow$$ equivalent ^{3}He density $\tilde{\varphi}_{He}$

Experiment

When $\tilde{\varphi}_{He}$ is found (i.e. the muon stops numbers[1] are equal for both target gases, ^{3}He and D/^{3}He) it has been also observed experimentally that

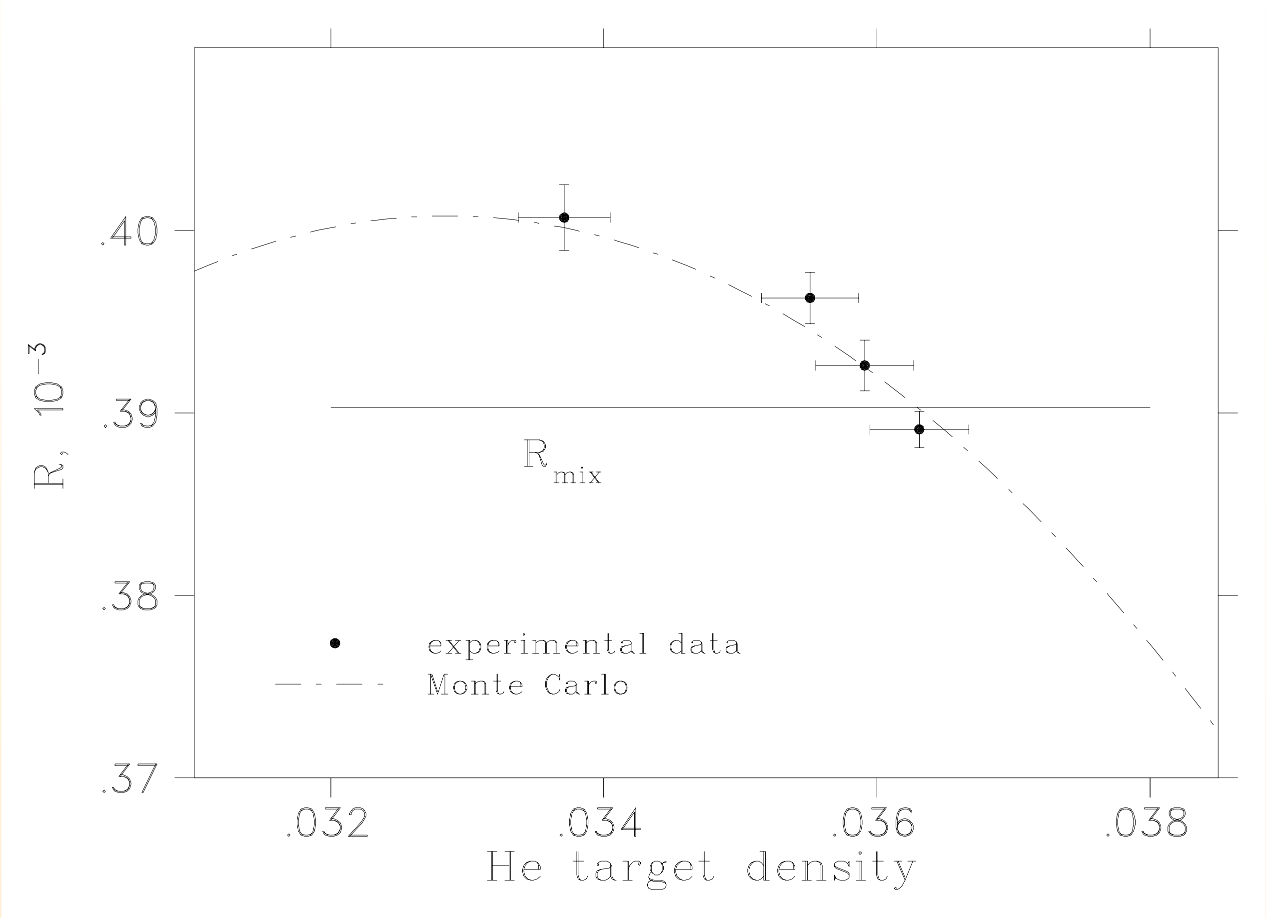
- R_{Al} *(the number of stops in the target walls),*
- R_{Au} *(the number of stops in the entrance gold ring)*

not change if D/^{3}He is replaced by ^{3}He($\tilde{\varphi}$).

It's an additional argument for the spatial equivalency of muon stops distributions in ^{3}He($\tilde{\varphi}$) and D/^{3}He) targets.

[1] per incident muon

Analysis



$$\tilde{\varphi}_{He} = 0.0363 \pm 0.0005 \rightarrow \boxed{<\tilde{s}>= S_{^3\mathrm{He}}/S_{\mathrm{D}} = 1.66 \pm 0.04}$$ [2]

[2] V.M. Bystritsky *et al.*, Eur.Phys.J. D, **42** (2007) 79

Mean ratio of the stopping powers

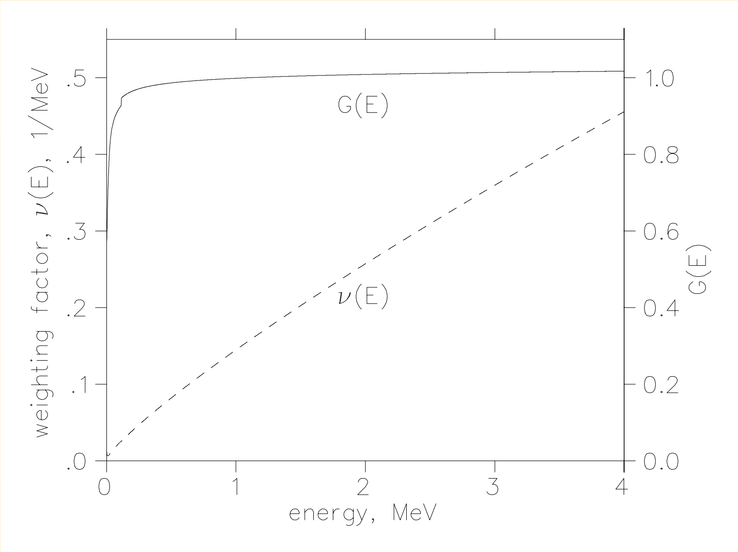
Equality of ranges $\int_0^{E_{in}} \frac{dE}{-\left(\frac{dE}{dx}\right)_{mix}} = \int_0^{E_{in}} \frac{dE}{-\left(\frac{dE}{dx}\right)_{He}}$ can be rewritten as

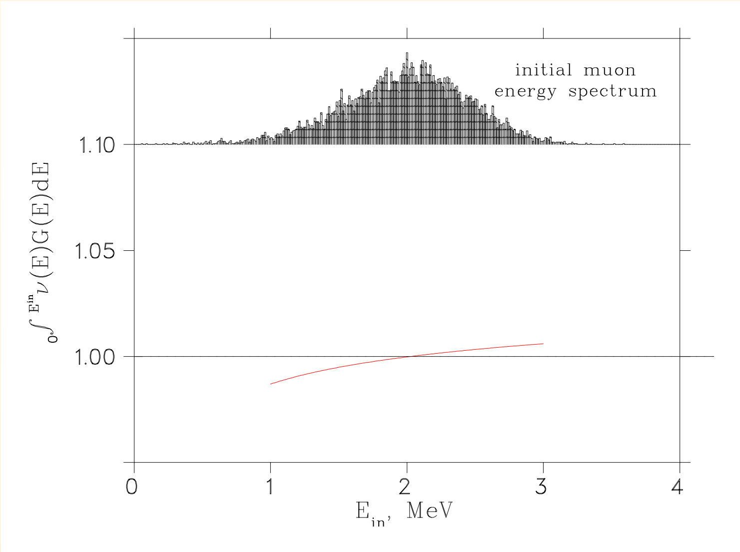
$\int_0^{E_{in}} G(E)\, \nu(E)\, dE = 1,$

where

$\nu(E) = \frac{1/S_{He}}{\int_0^{E_{in}} 1/S_{He}\, dE}$ is a weight function, $\int_0^{E_{in}} \nu(E)\, dE = 1,$

and $G(E) = \frac{\tilde{\varphi}_{He}}{\varphi_{mix}(\tilde{s}^{-1}(E)c_D + c_{He})}$ (energy dependent via $\tilde{s}(E)$).





finaly, $1 = \overline{G} \approx G(\overline{A(E)}) \approx G(A(\overline{E}))$ gives the formula for $< \tilde{s} >$.

Atomic capture in a binary mixture

- Atomic capture probability: $w_D = \frac{1}{1+Ac}, \qquad w_{He} = \frac{Ac}{1+Ac},$ (*)

  where $A = \frac{\sigma(He)}{\sigma(D)}$ - per-atom capture ratio (reduced ratio), $c = c_{He}/c_D$ - ratio of atomic concentrations

- "A" is usually energy dependent; $\overline{A} = \frac{\overline{\sigma(He)}}{\overline{\sigma(D)}}$ is more useful for experiments.

- The question of averaging.

- No simple relationship exists between the stopping power and the primary atomic capture; slowing down "is working" in MeV - keV region, capture occurs at low energies ($\sim< 100$ eV).

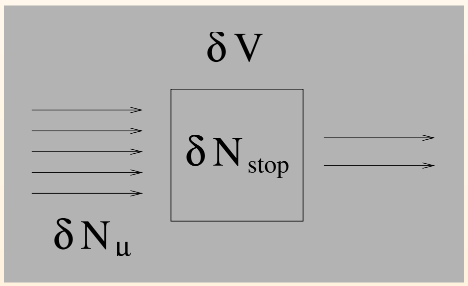
- Petrukhin's phenomenological model[3]: expressions (*) fit well the experimental data for H/He mixture when stopping power ratio $\tilde{s}$ is used instead of A

$$w_D = \frac{1}{1+\tilde{s}c}, \qquad w_{He} = \frac{\tilde{s}c}{1+\tilde{s}c}.$$

[3] V.I.Petrukhin and V.M.Suvorov, Zh. Eksp. Theor. Fiz. **70** (1976) 1145

reduced capture ratio

Let through a volume δV located in any given point in the target pass δN_μ muons



$\delta N_{stop} = \delta N_\mu n_o \varphi \overline{\sigma} \delta x$

- $\delta N^{He}_{stop} = \delta N_\mu n_o \varphi_{He} \overline{\sigma}_{He} \delta x$

  $\delta N^{mix}_{stop} = \delta N_\mu n_o \varphi_{mix} (c_D \overline{\sigma}_D + c_{He} \overline{\sigma}_{He}) \delta x$

- for the same stopping distributions in D/^{3}He and in ^{3}He targets:

  $\delta N^{He}_{stop}(\tilde{\varphi}_{He}) = \delta N^{mix}_{stop}(\varphi_{mix})$ for any volume element

- from this $\tilde{\varphi}_{He} \overline{A} = \varphi_{mix}(c_D + \overline{A} c_{He})$

  or $\overline{A} = \frac{\overline{\sigma}(He)}{\overline{\sigma}(D)} = \frac{c_D \varphi_{mix}}{\tilde{\varphi}_{He} - c_{He} \varphi_{mix}}$

- In general, the quantity $\overline{A}$ can be dependent on concentrations c_D, c_{He}. For a week dependency of the muon energy distribution on mixture composition at low energies, the $\overline{A}$-value is practically constant and can be interpreted as the mean reduced probability ratio for the muon capture by helium-3 and deuterium atoms.
- Above consideration can justify Petrukhin's model for the H/He mixture.