Non-separable two-body scattering as wave-packet dynamics: applications to atom-atom collisions in trap and other exotic systems

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Contents

- Problem: non-separable two-body scattering problem
- Approach: time-dependent wave-packet method
- Applications: ultracold atom-atom collisions in trap

  (confinement induced resonances,
  suppression of quantum scattering in trap)

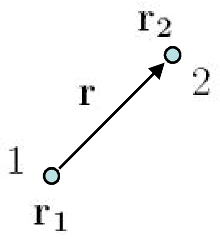
  laser-stimulated antihydrogen formation:

  $$\overline{p} + e^{+} + \hbar\omega \rightarrow \overline{H}_n + 2\hbar\omega$$

- Conclusion

ultracold atom-atom collisions

3D free space



$$V(\mathbf{r_1}, \mathbf{r_2}) = V(\mid \mathbf{r_1} - \mathbf{r_2} \mid) = V(\mid \mathbf{r} \mid)$$

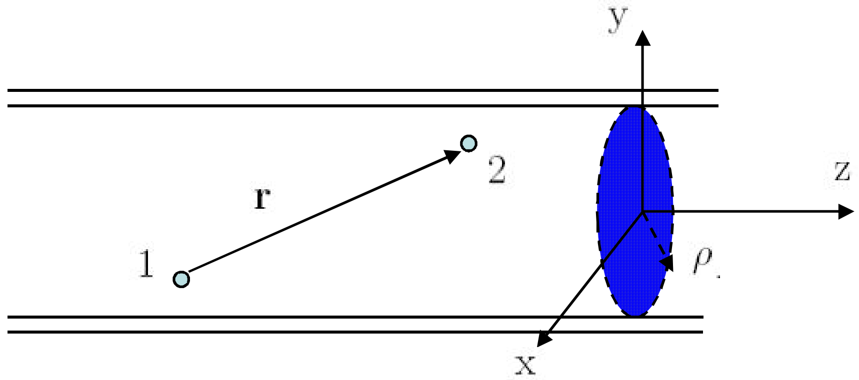
$$\psi(\mathbf{r}_1, \mathbf{r}_2) \Rightarrow \mathbf{\Psi}(\mathbf{R}_{\mathrm{cm}})\psi(\mathbf{r})$$

↓

$$R_{nl}(r) Y_{lm}(\theta, \phi)$$

$R_{nl}(r)$

confined geometry



magnetic, optical traps

quasi-1D ultracold atomic gases

$$V(\mid \mathbf{r} \mid) + \mathbf{W}(\rho_1, \rho_2) = V(r) + \frac{1}{2} m_1 \omega_1^2 \rho_1^2 + \frac{1}{2} m_2 \omega_2^2 \rho_2^2$$

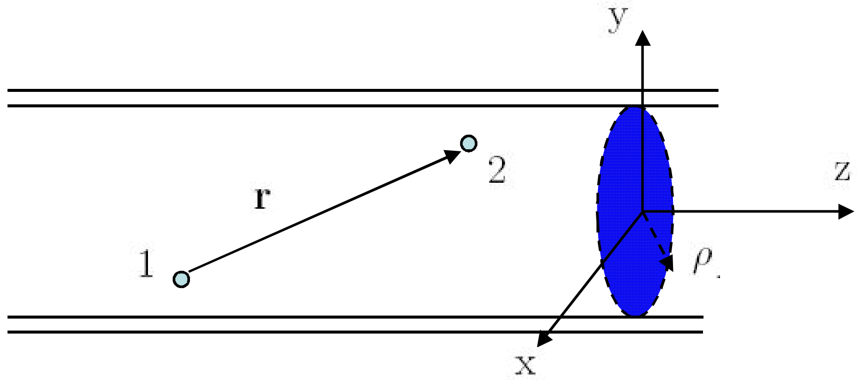
$$1 = 2 \Rightarrow \omega_1 = \omega_2 = \omega_\perp$$

confinement length $a_\perp = \sqrt{\frac{\hbar}{\mu \omega_\perp}} \geq 60nm \quad \sim 10^3 a_B$

identical particles 1 and 2

$1 = 2 \Rightarrow m_1 = m_2 \quad \omega_1 = \omega_2 = \omega_\perp$

$\Rightarrow$ no CM coupling



$$\hat{H} = \hat{H}_z + \hat{H}_\perp + \hat{V}$$

$$\hat{H}_z = -\frac{\hbar^2}{2\mu}\frac{\partial^2}{\partial z^2};$$

$$\hat{H}_\perp = -\frac{\hbar^2}{2\mu}\left[\frac{\partial^2}{\partial \rho^2} + \frac{1}{\rho}\frac{\partial}{\partial \rho}\right] + \frac{\mu}{2}\omega_\perp^2\rho^2$$

2D $\rightarrow \{z, \rho\}$ ← 5D $\rightarrow \{r, \theta, \phi, \rho_R, \phi_R\}$

$$\hat{H}(\rho_R, \boldsymbol{r}) = -\frac{1}{2M}\left(\frac{\partial^2}{\partial \rho_R^2} + \frac{1}{4\rho_R^2}\right) - \frac{1}{2M\rho_R^2}\left(\frac{\partial}{\partial \phi_R} - \frac{\partial}{\partial \phi}\right)^2 + \frac{1}{2}(m_1\omega_1^2 + m_2\omega_2^2)\rho_R^2$$
$$-\frac{1}{2\mu}\frac{\partial^2}{\partial r^2} + \frac{L^2(\theta,\phi)}{2\mu r^2} + \frac{\mu^2}{2}\left(\frac{\omega_1^2}{m_1} + \frac{\omega_2^2}{m_2}\right)\rho^2 + \mu(\omega_1^2 - \omega_2^2)\rho\rho_R cos\phi$$

$$H_{1D} = -\frac{\hbar^2}{2\mu}\frac{\partial^2}{\partial z^2} + g_{1D}\delta(z)$$

$$g_{1D} = -\frac{\hbar^2}{\mu a_{1D}} = \frac{2\hbar^2 a}{\mu a_\perp^2}\frac{1}{(1 - Ca/a_\perp)}$$

1D pseudopotential approximation

PROBLEM

1. confinement induced nonseparability of CM

5D $\rightarrow \{r, \theta, \phi, \rho_R, \phi_R\}$

$$i\hbar\frac{\partial}{\partial t}\psi'(\boldsymbol{\rho_R}, \boldsymbol{r}, t) = H'(\boldsymbol{\rho_R}, \boldsymbol{r})\,\psi'(\boldsymbol{\rho_R}, \boldsymbol{r}, t)$$

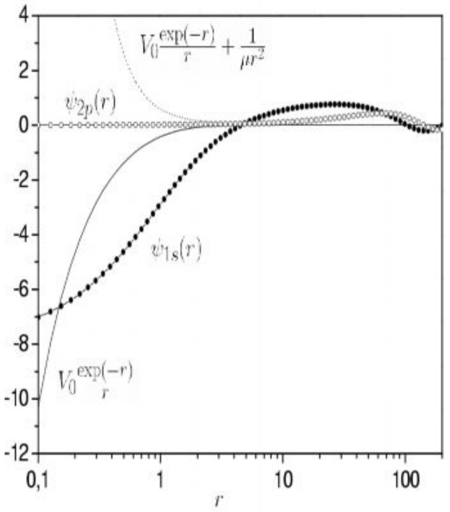
$$H'(\boldsymbol{\rho_R}, \boldsymbol{r}) = -\frac{1}{2M}(\frac{\partial^2}{\partial\rho_R^2} + \frac{1}{4\rho_R^2}) - \frac{1}{2M\rho_R^2}\frac{\partial^2}{\partial\phi_R^2} + \frac{1}{2}(m_1\omega_1^2 + m_2\omega_2^2)\rho_R^2$$

$$\frac{1}{2\mu}\frac{\partial^2}{\partial r^2} + \frac{L^2(\theta,\phi)}{2\mu r^2} + \frac{\mu^2}{2}(\frac{\omega_1^2}{m_1} + \frac{\omega_2^2}{m_2})\rho^2 + \mu(\omega_1^2 - \omega_2^2)\rho\rho_R cos(\phi - \phi_R) + V(r)$$

2. beyond the pseudopotential approach

$\omega_1 = \omega_2 = \omega_\perp \qquad m_1 \neq m_2$

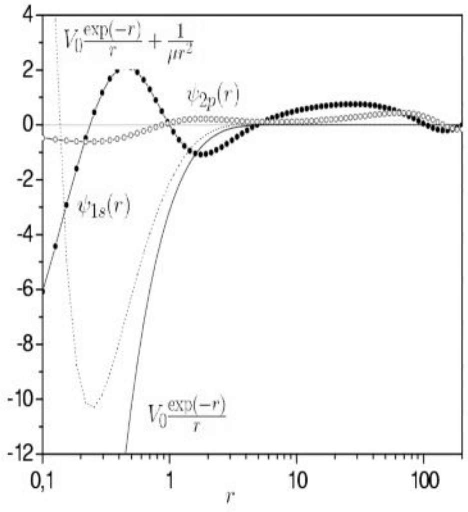
one parameter = s-wave scattering length



$V_0 = -1.136$

$\Rightarrow$

considerable s and p waves



$V_0 = -8.45$

Why wave-packet propagation method

- flexible and efficient → different few-body dynamics

harmonics generation by H-atom in elliptically polarized laser field

$H + \hbar\omega \rightarrow H + n\hbar\omega$ Melezhik, PL A230, 203 (1997)

ionization of ions moving in magnetic fields

$He^{+} + B \rightarrow He^{++} + e + B$ Melezhik & Schmelcher, PRL 84, 1870 (2000)

breakup of halo nuclei

$Be^{11} + Pb^{208} \rightarrow Be^{10} + n + Pb^{208}$ Melezhik & Baye, PR C59, 3232 (1999); PR C64, 054612 (2001)
Capel, Baye & Melezhik, PR C68, 014612 (2003)

stripping and excitation in He^{+} collisions with p

$He^{+}_{lm} + p \rightarrow He^{++} + e + p$ Melezhik, J.Cohen & Chi-Yu Hu, PR A69, 032709 (2004)
$He^{+}_{lm} + p \rightarrow He^{+}_{l'm'} + p$

- absence of problem of matching boundary conditions

- direct representation of the wave-packet time-evolution

bound $\rightarrow$ bound

bound $\rightarrow$ continuum **3D problem:** $\{r, \theta, \phi\}$

bound → bound

bound → continuum **3D problem:** $\{r, \theta, \phi\}$

continuum → continuum **5D problem:** $\{r, \theta, \phi\, \rho_R, \phi_R\}$

PROBLEM

$$i\hbar\frac{\partial}{\partial t}\psi'(\boldsymbol{\rho_R},\boldsymbol{r},t) = H'(\boldsymbol{\rho_R},\boldsymbol{r})\,\psi'(\boldsymbol{\rho_R},\boldsymbol{r},t)$$

$$H'(\boldsymbol{\rho_R},\boldsymbol{r}) = -\frac{1}{2M}(\frac{\partial^2}{\partial\rho_R^2}+\frac{1}{4\rho_R^2}) - \frac{1}{2M\rho_R^2}\frac{\partial^2}{\partial\phi_R^2} + \frac{1}{2}(m_1\omega_1^2+m_2\omega_2^2)\rho_R^2$$

$$\cdot\frac{1}{2\mu}\frac{\partial^2}{\partial r^2} + \frac{L^2(\theta,\phi)}{2\mu r^2} + \frac{\mu^2}{2}(\frac{\omega_1^2}{m_1}+\frac{\omega_2^2}{m_2})\rho^2 + \mu(\omega_1^2-\omega_2^2)\rho\rho_R cos(\phi-\phi_R) + V(r)$$

$$\psi'(\boldsymbol{\rho_R},\boldsymbol{r},t=0) = N\varphi_0'(\rho_R,\phi_R,\rho,\phi)exp\{-\frac{(z-z_0)^2}{2a_z^2}\}exp\{ikz\}$$

$$\varphi_0'(\rho_R,\phi_R,\rho,\phi) = r\sqrt{\rho_R}exp\{-\frac{\rho_R^2}{2}(\frac{1}{a_1^2}+\frac{1}{a_2^2}) - \frac{(\mu\rho)^2}{2}(\frac{1}{(m_1a_1)^2}+\frac{1}{(m_2a_2)^2})$$

$$-\frac{\rho\rho_R}{M}(\frac{m_2}{a_1^2}-\frac{m_1}{a_2^2})cos(\phi-\phi_R)\}$$

PROBLEM

$$i\hbar\frac{\partial}{\partial t}\psi'(\boldsymbol{\rho_R},\boldsymbol{r},t) = H'(\boldsymbol{\rho_R},\boldsymbol{r})\,\psi'(\boldsymbol{\rho_R},\boldsymbol{r},t)$$

$$H'(\boldsymbol{\rho_R},\boldsymbol{r}) = -\frac{1}{2M}\left(\frac{\partial^2}{\partial\rho_R^2}+\frac{1}{4\rho_R^2}\right) - \frac{1}{2M\rho_R^2}\frac{\partial^2}{\partial\phi_R^2} + \frac{1}{2}(m_1\omega_1^2+m_2\omega_2^2)\rho_R^2$$

$$-\frac{1}{2\mu}\frac{\partial^2}{\partial r^2} + \frac{L^2(\theta,\phi)}{2\mu r^2} + \frac{\mu^2}{2}\left(\frac{\omega_1^2}{m_1}+\frac{\omega_2^2}{m_2}\right)\rho^2 + \mu(\omega_1^2-\omega_2^2)\rho\rho_R cos(\phi-\phi_R) + V(r)$$

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$$\left. -\frac{\rho\rho_R}{M}\left(\frac{m_2}{a_1^2}-\frac{m_1}{a_2^2}\right)cos(\phi-\phi_R)\right\}$$

frame co-rotating with the center of mass around the symmetry z-axis

unitary transformation $U = exp\{i\phi_R L_z\}$

~~ϕ_R~~

$H' \to UHU^+$ $\qquad \psi' \to U\psi$

$\phi \dashrightarrow \phi + \phi_R$

$\partial_{\phi_R} \dashrightarrow \partial_{\phi_R} - \partial_\phi$

$\Rightarrow$ 4D $(\rho_R, r, \theta, \phi)$

Computational scheme

$$i\frac{\partial}{\partial t}\Psi(t) = [H_0 + V(r)]\,\Psi(t)$$

$$\{\frac{1}{2\mu r^2}L^2(\theta,\phi) - \frac{1}{2M\rho_R^2}\frac{\partial^2}{\partial\phi^2}\}\sum_{\nu}^{\tilde{N}} Y_\nu(\Omega)(Y^{-1})_{\nu j'}\,|_{\Omega=\Omega_j} = \sum_{\nu}^{\tilde{N}} Y_{j\nu}\{-\frac{l(l+1)}{2\mu r^2} + \frac{m^2}{2M\rho_R^2}\}(Y^{-1})_{\nu j'}$$

two-dimensional DVR

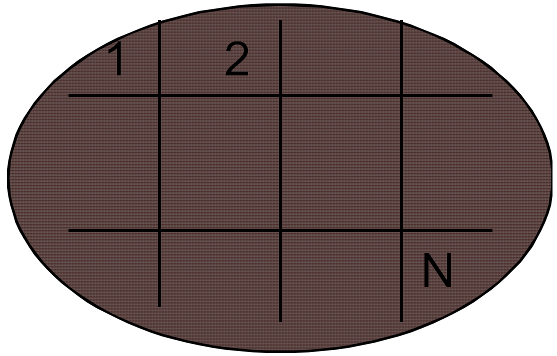
$$Y_{\nu j} = Y_\nu(\hat{\boldsymbol{r}}_j)$$

$$H^{(0)}_{jj'}(\rho_R, r) = h^{(0)}_{jj'}(\rho_R) + h^{(1)}_{jj'}(r) + W_j(\rho_R,\rho)\delta_{jj'}$$

$$h^{(0)}_{jj'}(\rho_R) = -\frac{1}{2M}(\frac{\partial^2}{\partial\rho_R^2} + \frac{1}{4\rho_R^2}) + \frac{1}{2M\rho_R^2\sqrt{\lambda_j\lambda_{j'}}}\sum_{\nu}^{N}(Y)^{-1}_{j\nu}m^2(Y^{-1})_{\nu j'} \ ,$$

$$h^{(1)}_{jj'}(r) = -\frac{1}{2\mu}\frac{\partial^2}{\partial r^2} - \frac{1}{2\mu r^2\sqrt{\lambda_j\lambda_{j'}}}\sum_{\nu}^{N}(Y)^{-1}_{j\nu}l(l+1)(Y^{-1})_{\nu j'} \ ,$$

$$W_j(\rho_R,\rho) = \frac{1}{2}(m_1\omega_1^2 + m_2\omega_2^2)\rho_R^2 + \frac{\mu^2}{2}(\frac{\omega_1^2}{m_1} + \frac{\omega_2^2}{m_2})\rho^2 + \mu(\omega_1^2 - \omega_2^2)\rho\rho_R cos\phi_j \ .$$



Computational scheme

$$i\frac{\partial}{\partial t}\Psi(t) = [H_0 + V(r)]\,\Psi(t)$$

$$\{\frac{1}{2\mu r^2}L^2(\theta,\phi) - \frac{1}{2M\rho_R^2}\frac{\partial^2}{\partial\phi^2}\}\sum_{\nu}^{\tilde N} Y_\nu(\Omega)(Y^{-1})_{\nu j'}\,|_{\Omega=\Omega_j} = \sum_{\nu}^{\tilde N} Y_{j\nu}\{-\frac{l(l+1)}{2\mu r^2} + \frac{m^2}{2M\rho_R^2}\}(Y^{-1})_{\nu j'}$$

two-dimensional DVR

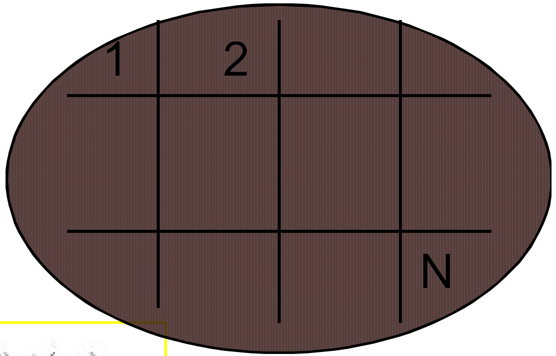
$$Y_{\nu j} = Y_\nu(\hat{\boldsymbol{r}}_j)$$

$$H^{(0)}_{jj'}(\rho_R, r) = h^{(0)}_{jj'}(\rho_R) + h^{(1)}_{jj'}(r) + W_j(\rho_R,\rho)\delta_{jj'}$$

$$h^{(0)}_{jj'}(\rho_R) = -\frac{1}{2M}(\frac{\partial^2}{\partial\rho_R^2} + \frac{1}{4\rho_R^2}) + \frac{1}{2M\rho_R^2\sqrt{\lambda_j\lambda_{j'}}}\sum_{\nu}^{N}(Y)^{-1}_{j\nu}m^2(Y^{-1})_{\nu j'} \;,$$

$$h^{(1)}_{jj'}(r) = -\frac{1}{2\mu}\frac{\partial^2}{\partial r^2} - \frac{1}{2\mu r^2\sqrt{\lambda_j\lambda_{j'}}}\sum_{\nu}^{N}(Y)^{-1}_{j\nu}l(l+1)(Y^{-1})_{\nu j'} \;,$$

$$W_j(\rho_R,\rho) = \frac{1}{2}(m_1\omega_1^2 + m_2\omega_2^2)\rho_R^2 + \frac{\mu^2}{2}(\frac{\omega_1^2}{m_1} + \frac{\omega_2^2}{m_2})\rho^2 + \mu(\omega_1^2 - \omega_2^2)\rho\rho_R cos\phi_j \;.$$



$$\psi(t_n+\Delta t) = \exp(-\frac{i}{2}\Delta t\hat W)\exp(-i\Delta t(\hat h^{(1)} + V))\exp(-i\Delta t\hat h^{(0)})\exp(-\frac{i}{2}\Delta t\hat W)\psi(t_n)$$
$$+O(\Delta t^3) \;.$$

$$\exp(-i\Delta t\hat A) \approx (1+\frac{i}{2}\Delta\hat A)^{-1}(1-\frac{i}{2}\Delta\hat A) + O(\Delta t^3)$$

$$(1+\frac{i}{2}\Delta t\hat A)\psi(t_n+\frac{\Delta t}{4}) = (1-\frac{i}{2}\Delta t\hat A)\psi(t_n)$$

V.S. Melezhik, Phys. Lett. A **330**, 203 (1997)

V.S. Melezhik, J.S. Cohen
and C.-Y. Hu, Phys. Rev. A **69**, 032709 (2004).

atom-atom interaction $V(r) = V_0 \frac{\exp\{-r\}}{r}$

$\boldsymbol{\omega_1 = \omega_2 = \omega_\perp}$ **no CM coupling**

RESULTS

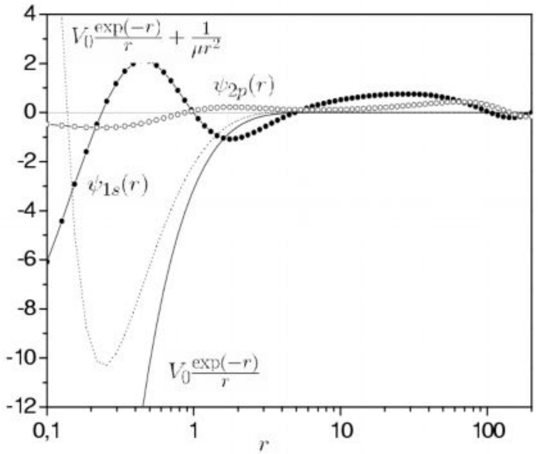
$$V(r) = V_0 \frac{e^{-r}}{r}$$

V_0	-8.45		-1.136	
$k^2/2\mu$	a_s	a_p	a_s	a_p
0.0205	4.789	-3.64	4.544	-1.19
0.0210	4.781	-3.66	4.527	-1.19
0.0220	4.764	-3.71	4.492	-1.19
0.0240	4.731	-3.80	4.426	-1.18
0.0280	4.669	-4.01	4.306	-1.17

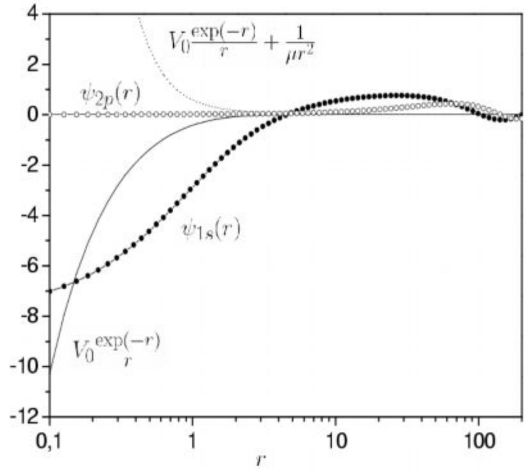
$$a_s(\varepsilon) = -\frac{\tan\delta_s(\varepsilon)}{k} \longrightarrow a_s \quad , \qquad V_p(\varepsilon) = -\frac{\tan\delta_p(\varepsilon)}{k^3} \longrightarrow V_p$$

$V_p \equiv a_p^3$

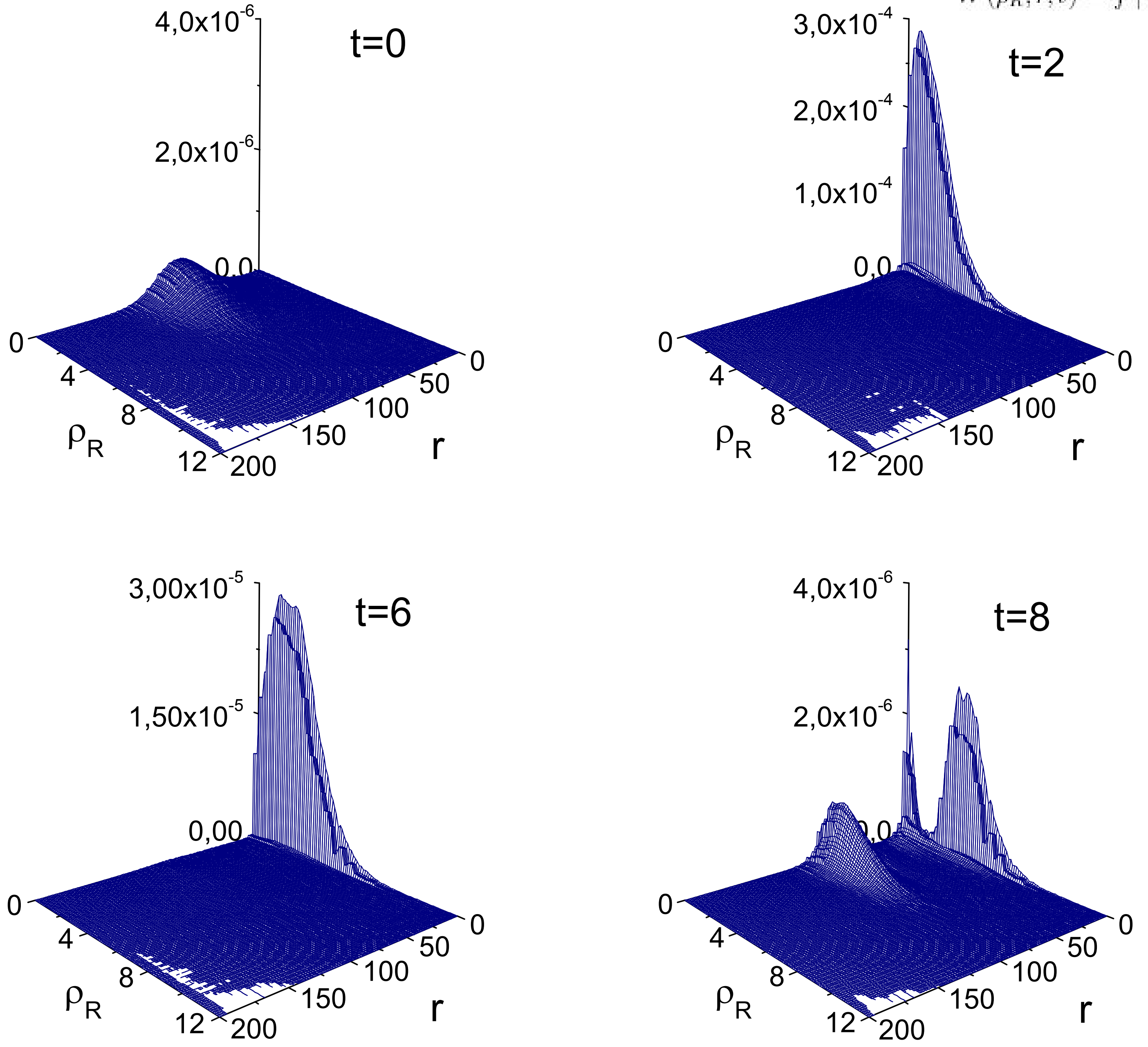
$V_0 = -8.45$



$V_0 = -1.136$

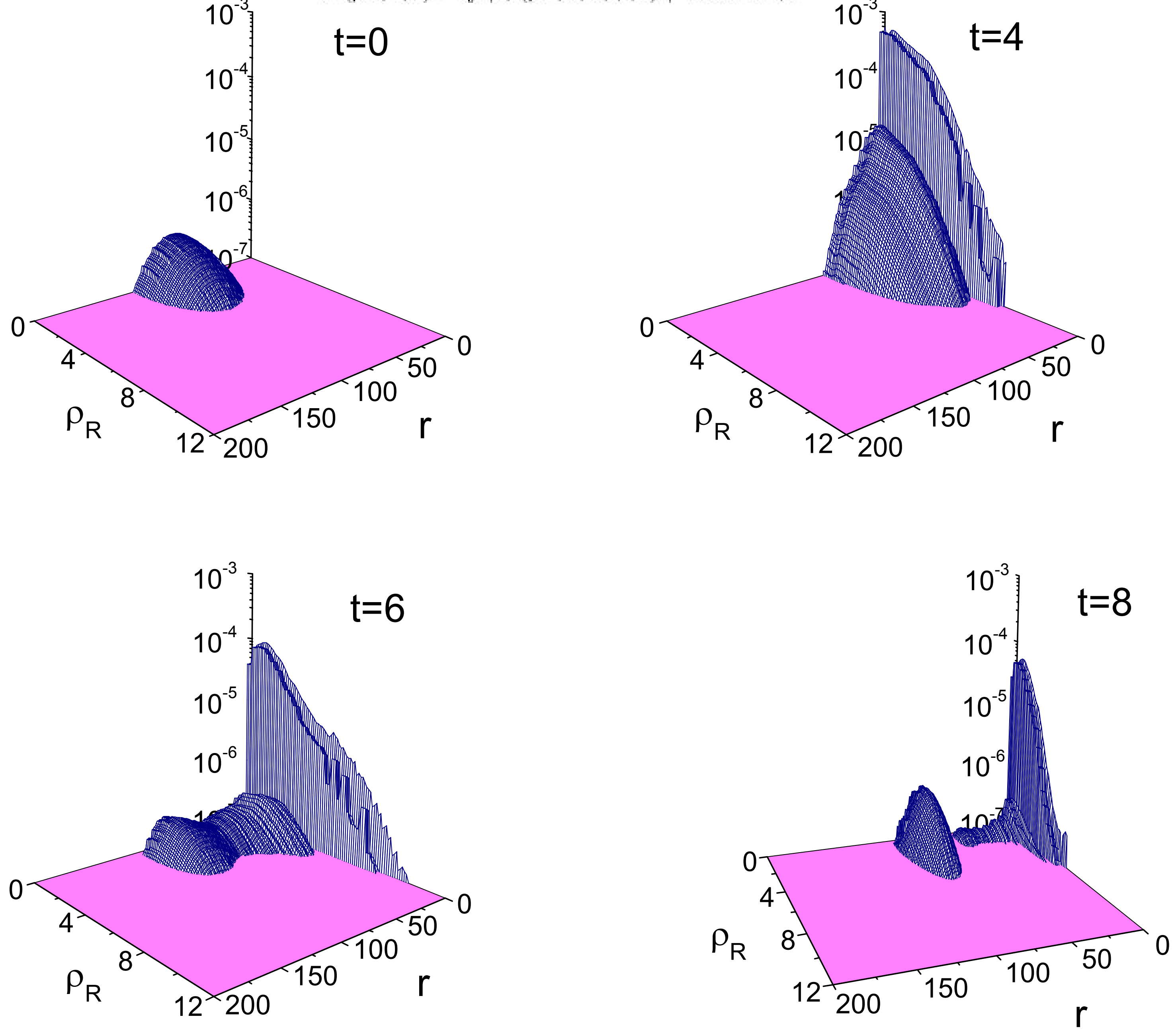


$$W(\rho_R, r, t) = \int |\psi(\rho_R, r, \theta, \phi, t)|^2 \sin\theta d\theta d\phi$$



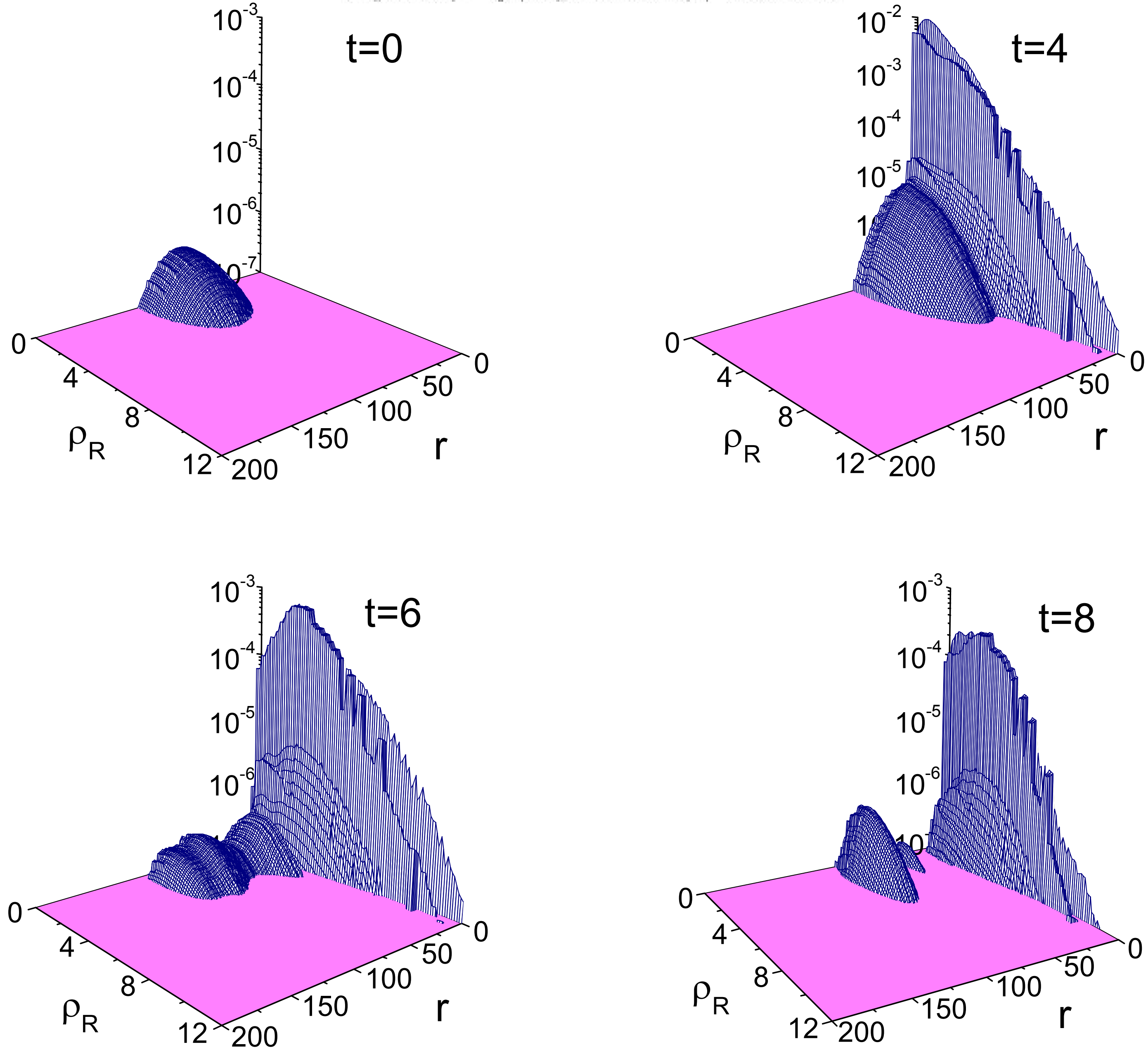
$$V_0 = -0.5 \quad (a_\perp/a_s = -3.21, \quad a_\perp/a_p = -8.01)$$

$$W(\rho_R, r, t) = \int |\, \psi(\rho_R, r, \theta, \phi, t) \,|^2 \sin\theta d\theta d\phi$$

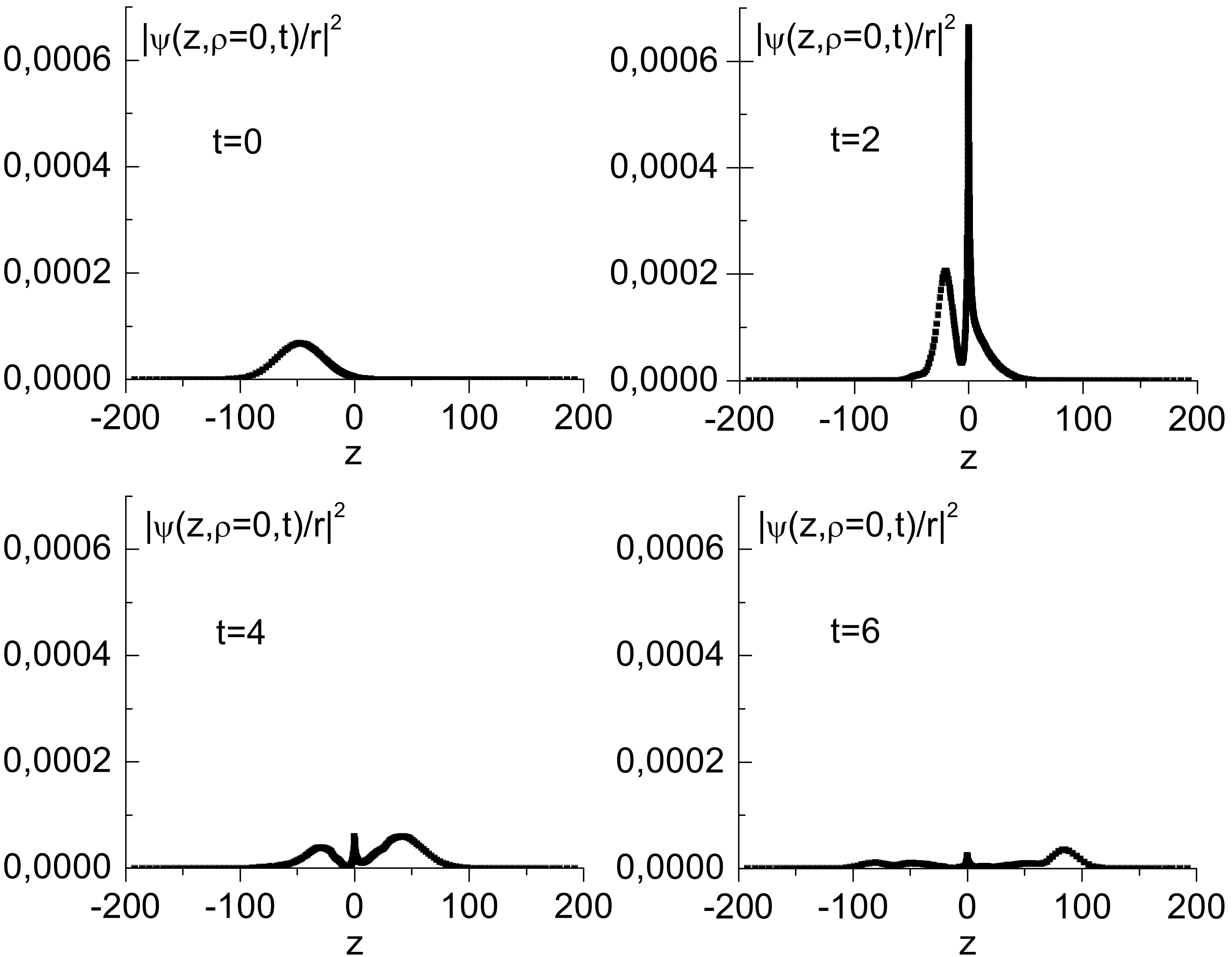


$$\mathbf{V_0 = -1.12 \quad (a_\perp/a_s = 1.47\,, \;\; a_\perp/a_p = -5.98)}$$

$$W(\rho_R, r, t) = \int |\, \psi(\rho_R, r, \theta, \phi, t)\,|^2 \sin\theta d\theta d\phi$$

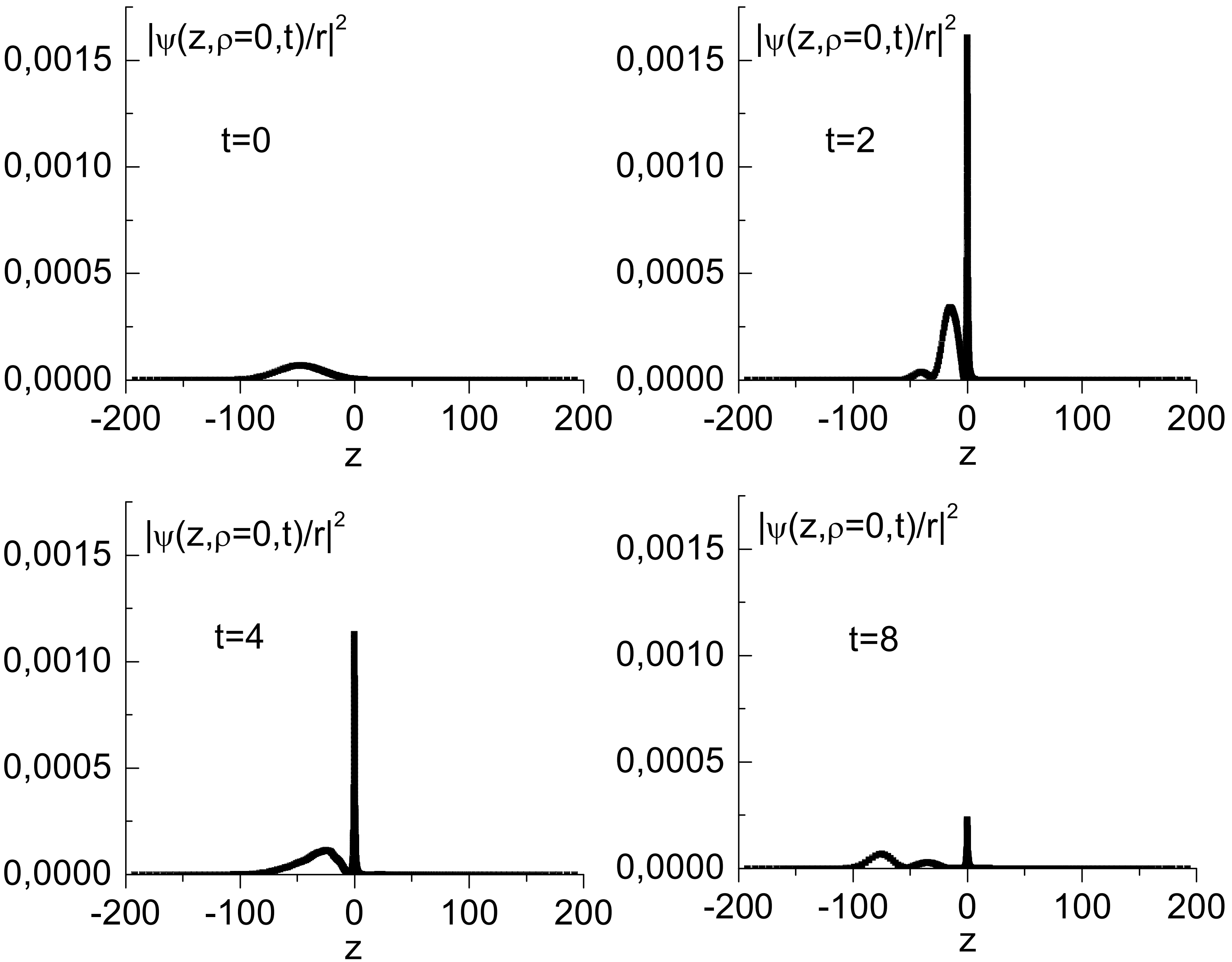


$$\mathbf{V_0 = -8.45 \quad (a_\perp/a_s = 1.46\,, \;\; a_\perp/a_p = -1.94)}$$



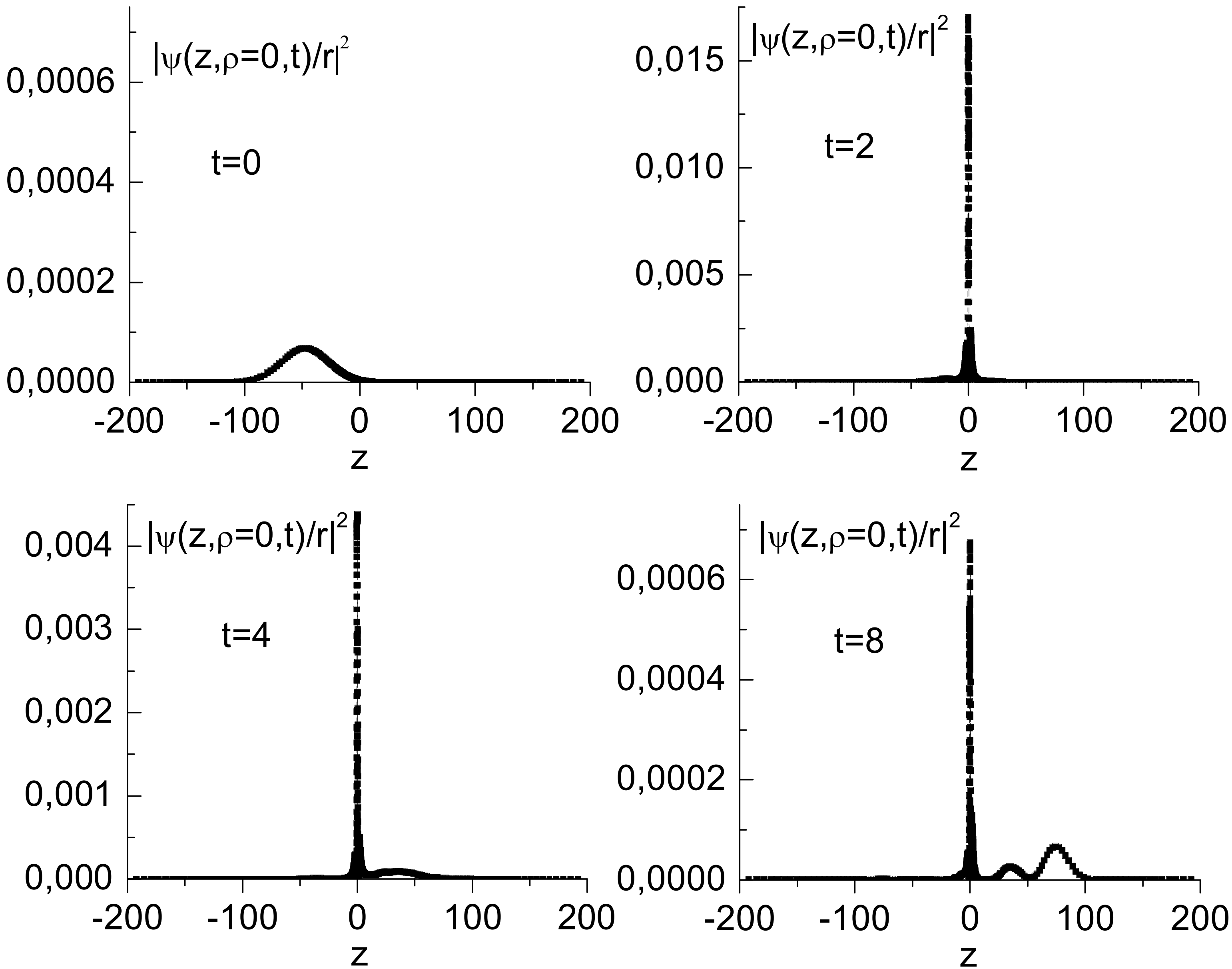
wave-packet splitting

$V_0 = -0.5 \quad (a_\perp/a_s = -3.21, \quad a_\perp/a_p = -8.01)$



$V_0 = -1.12 \quad (a_\perp/a_s = 1.47, \quad a_\perp/a_p = -5.98)$

wave-packet reflection



$V_0 = -8.45 \quad (a_\perp/a_s = 1.46\,, \quad a_\perp/a_p = -1.94)$

wave-packet transmission

1D pseudopotential approximation

"zero-range" potentials   Yu. N. Demkov and V. N. Ostrovskii,

ultracold collisions $\Rightarrow$ one parameter = s-wave scattering length a_{1D}

$$\hat{H} = \hat{H}_z + \hat{H}_\perp + \hat{V}$$

$$\hat{H}_z = -\frac{\hbar^2}{2\mu}\frac{\partial^2}{\partial z^2};$$

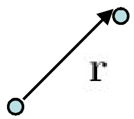
$$\hat{H}_\perp = -\frac{\hbar^2}{2\mu}\left[\frac{\partial^2}{\partial \rho^2} + \frac{1}{\rho}\frac{\partial}{\partial \rho}\right] + \frac{\mu}{2}\omega_\perp^2\rho^2$$

M. Olshanii, Phys. Rev. Lett. **81**, 938 (1998)

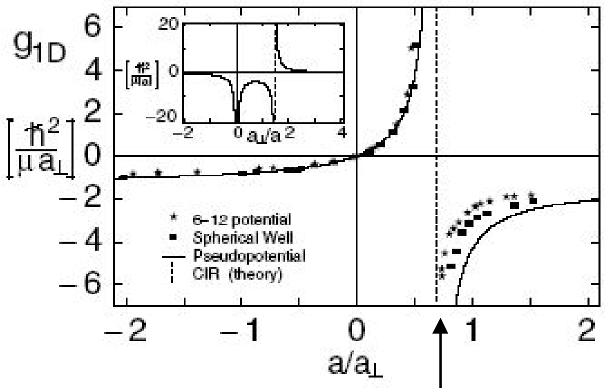
$$H_{1D} = -\frac{\hbar^2}{2\mu}\frac{\partial^2}{\partial z^2} + g_{1D}\delta(z)$$

$$g_{1D} = -\frac{\hbar^2}{\mu a_{1D}} = \frac{2\hbar^2 a}{\mu a_\perp^2}\frac{1}{(1 - Ca/a_\perp)} \rightleftarrows g_{1D} = \frac{kRe\{f_g\}}{\mu Im\{f_g\}}$$

a 3D scattering length in free space



$a_\perp$ transverse confinement length $C = 1.46...$



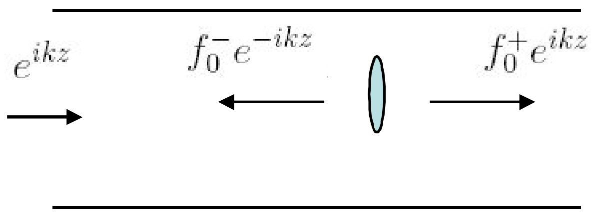
CIR

CIR "confinement induced resonance" at $a_\perp = a \times 1.46$

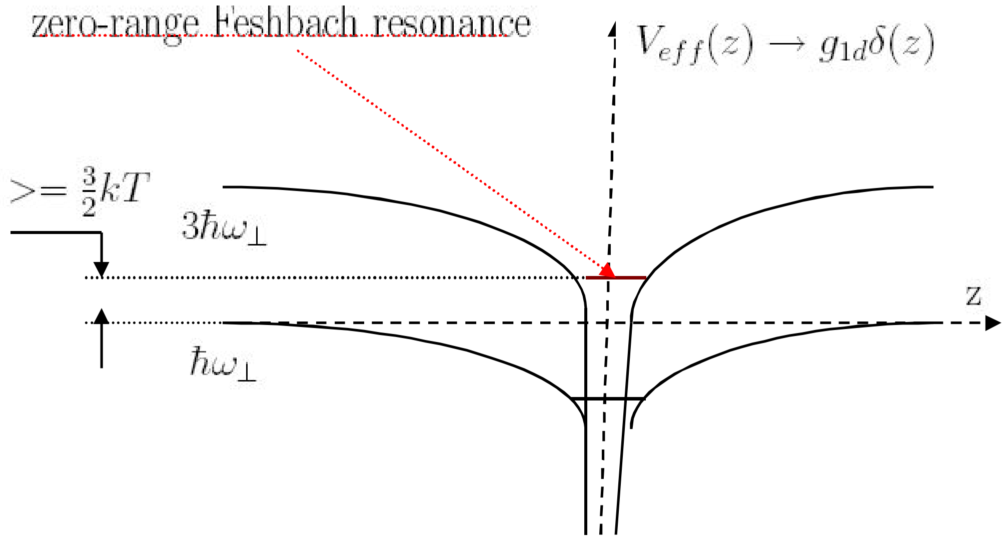
2D (z, ρ) numerical computations Bergeman, Moore, Olshanii (2003)

confinement induced resonance (CIR)

$$\psi(z,\rho) \underset{|z|\to\infty}{\longrightarrow} (e^{ikz} + f_0^{\pm} e^{ik|z|})\phi_0(\rho)$$



$$a_\perp = \sqrt{\frac{\hbar}{\mu\omega_\perp}}$$



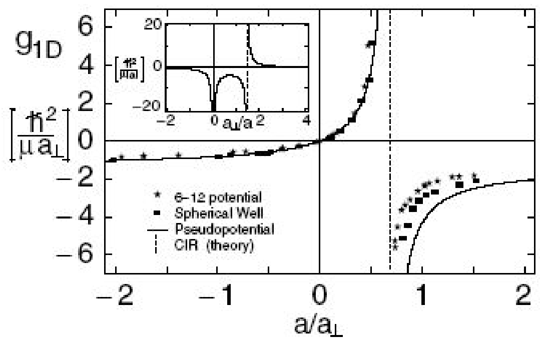
$$g_{1d} = -\frac{\hbar^2}{\mu a_{1D}} \underset{k\to 0}{=} \frac{\hbar^2 k \; Re\{f_0^+\}}{\mu \; Im\{f_0^+\}}$$

CIR: $g_{1D} \to \pm\infty \quad a_{1D} \to 0$

$$f_0^+ = -\frac{1}{1+ika_{1D}} \to -1$$

$$T = | \, 1 + f_0^+ \, |^2 \to 0 \quad !!$$

impenetrable bosons (Tonks & Girardeau)



METHOD

A. General scheme

$$i\frac{\partial}{\partial t}\psi(\rho_R,\boldsymbol{r},t) = \left[H^{(0)}(\rho_R,\boldsymbol{r}) + V(r)\right]\ \psi(\rho_R,\boldsymbol{r},t)$$

$$H^{(0)}(\rho_R,\boldsymbol{r}) = -\frac{1}{2M}(\frac{\partial^2}{\partial\rho_R^2} + \frac{1}{4\rho_R^2}) - \frac{1}{2M\rho_R^2}(\frac{\partial}{\partial\phi_R} - \frac{\partial}{\partial\phi})^2 + \frac{1}{2}(m_1\omega_1^2 + m_2\omega_2^2)\rho_R^2$$
$$-\frac{1}{2\mu}\frac{\partial^2}{\partial r^2} + \frac{L^2(\theta,\phi)}{2\mu r^2} + \frac{\mu^2}{2}(\frac{\omega_1^2}{m_1} + \frac{\omega_2^2}{m_2})\rho^2 + \mu(\omega_1^2 - \omega_2^2)\rho\rho_R cos\phi$$

$$\psi(\rho_R,\boldsymbol{r},t=0) = N\varphi_0(\rho_R,\rho,\phi)exp\{-\frac{(z-z_0)^2}{2a_z^2}\}exp\{ikz\}$$
$$= N\varphi_0(\rho_R,\rho,\phi)\chi(z-z_0)exp\{ikz\}\ ,$$

METHOD

A. General scheme

$$i\frac{\partial}{\partial t}\psi(\rho_R,\boldsymbol{r},t)=\left[H^{(0)}(\rho_R,\boldsymbol{r})+V(r)\right]\,\psi(\rho_R,\boldsymbol{r},t)$$

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$$\mid\psi(t\to+\infty)>\underset{z\to+\infty}{\longrightarrow}\mid\psi(t)^+>=(1+f^+(k))N\varphi_0(\rho_R,\rho,\phi)\tilde{\chi}(z-(z_0+vt))exp\{ikz\}$$
$$\underset{z\to-\infty}{\longrightarrow}\mid\psi(t)^->=f^-(k)N\varphi_0(\rho_R,\rho,\phi)\tilde{\chi}(-z-(z_0+vt))exp\{-ikz\}\ ,$$

where

$$f^\pm(k)=f_r^\pm(k)+if_i^\pm(k)\qquad \tilde{\chi}(z-(z_0+vt))=\frac{D_z(t)}{a_z^2}^{-1/4}\exp\{-\frac{(z-(z_0+vt))^2}{4D_z(t)}(1-i\frac{t}{\mu a_z^2})+i\varepsilon t\}$$

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$$\mid \psi^{(0)}(t\to+\infty) > \underset{z\to+\infty}{\longrightarrow} \mid \psi^{(0)+} >= N\varphi_0(\rho_R,\rho,\phi)\tilde{\chi}(z-(z_0+vt))\exp\{ikz\}$$
$$\underset{z\to-\infty}{\longrightarrow} \mid \psi^{(0)-} >= 0$$

METHOD

A. General scheme

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$$\underset{z\to-\infty}{\longrightarrow}\mid\psi^{(0)-}>=0$$

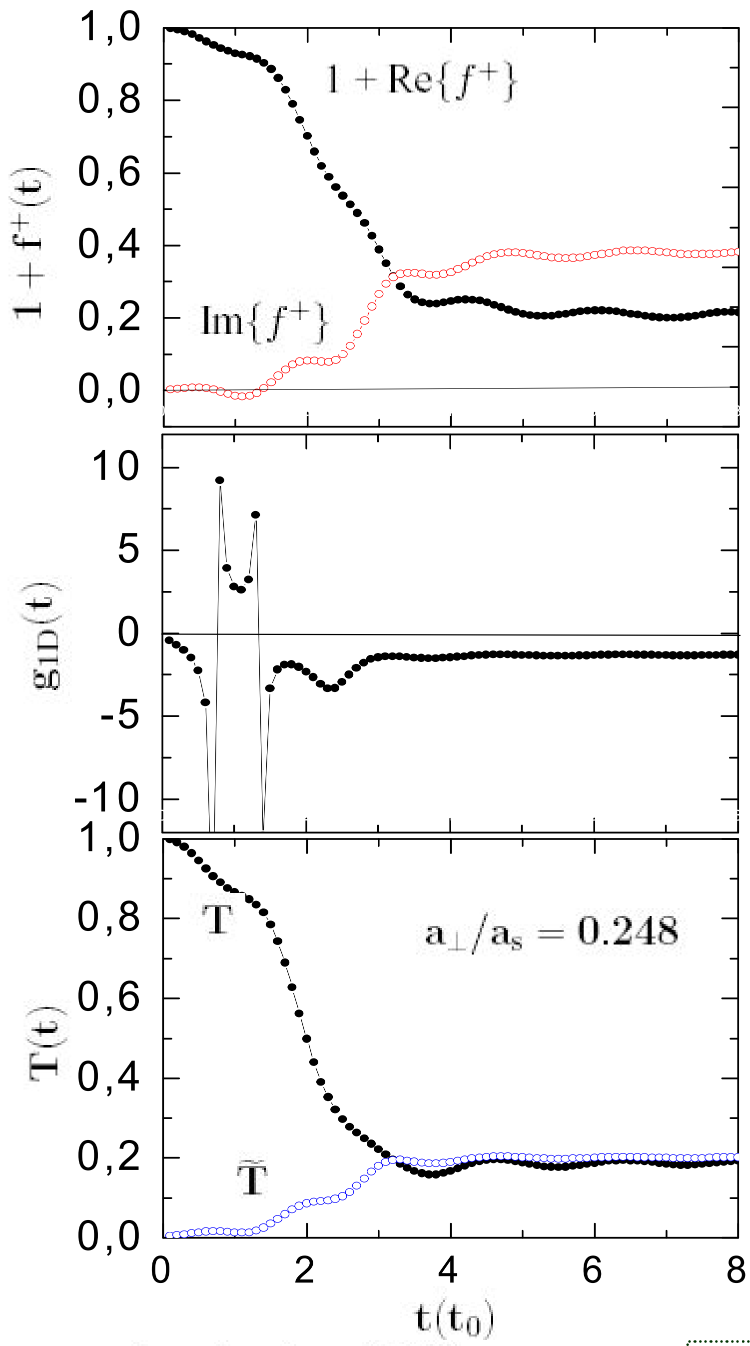
$$<\psi^{(0)}(t)\mid\psi(t)>\underset{t\to+\infty}{\longrightarrow}(1+f^+)$$

$$g_{1D}=\underset{k\to0}{lim}\frac{k}{\mu}\frac{f_{g,r}(k)}{f_{g,i}(k)}=\underset{k\to0}{lim}\frac{k}{\mu}\frac{f_r^+(k)}{f_i^+(k)}=\underset{k\to0}{lim}\frac{k}{\mu\tan[\delta(k)]}=-\frac{1}{\mu a}$$

$$T=\mid 1+f^+\mid^2$$

$$g_{1d} = -\frac{\hbar^2}{\mu a_{1D}} \underset{k \to 0}{=} \frac{\hbar^2 k \; Re\{f_0^+\}}{\mu \; Im\{f_0^+\}}$$

$$T = | \, 1 + f_0^+ \, |^2$$



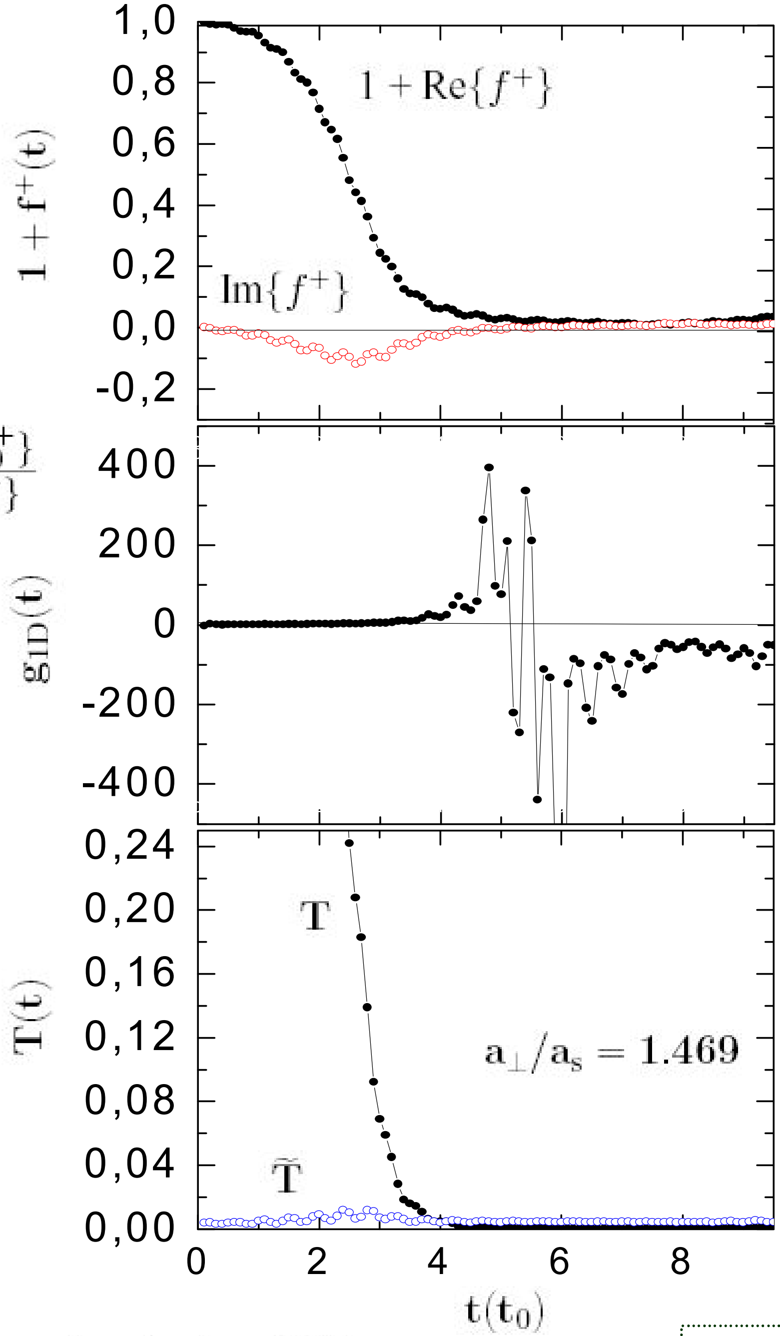
pure s-wave atom-atom scattering in free (3D) space

$V_0 = -1.136$

$$g_{1d} = -\frac{\hbar^2}{\mu a_{1D}} \underset{k \to 0}{=} \frac{\hbar^2 k \; Re\{f_0^+\}}{\mu \; Im\{f_0^+\}}$$

$$g_{1D} \to \pm\infty$$

$$T = \mid 1 + f_0^+ \mid^2 \to 0$$



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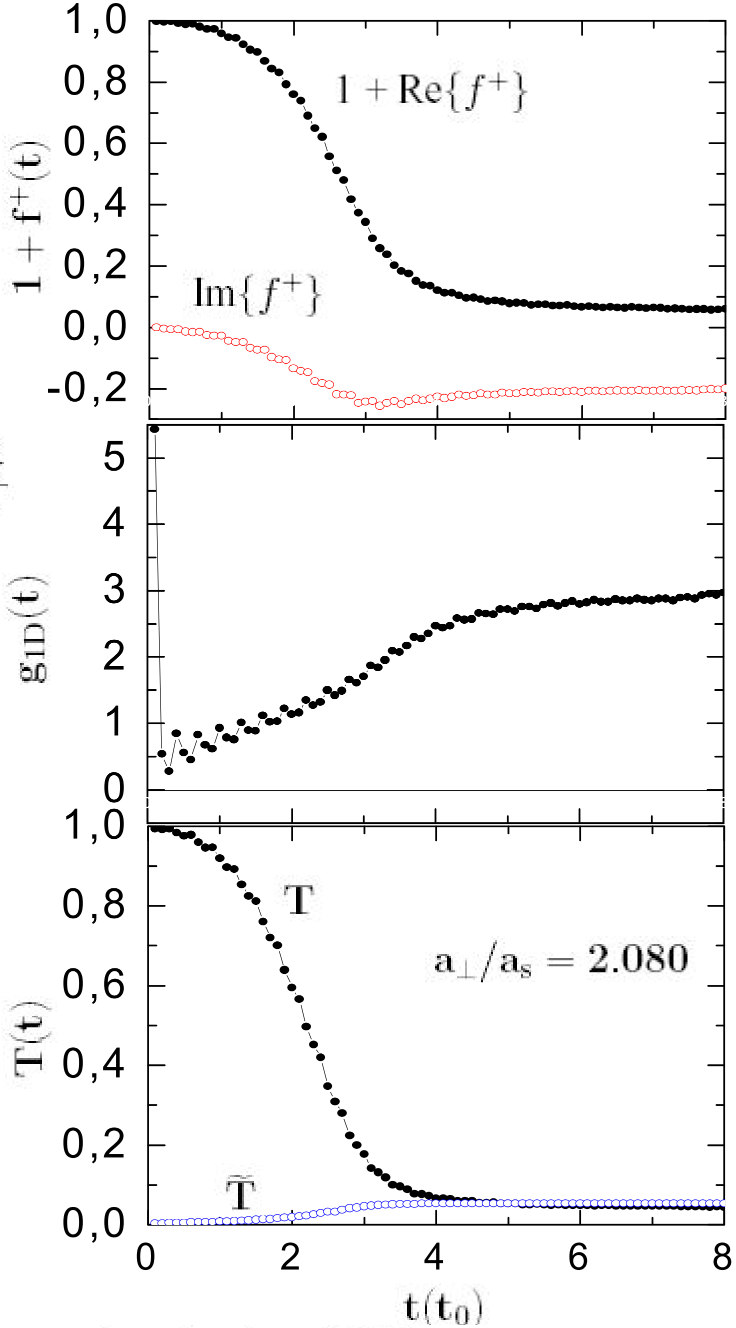

$$T = | 1 + f_0^+ |^2$$

pure s-wave atom-atom scattering in free (3D) space

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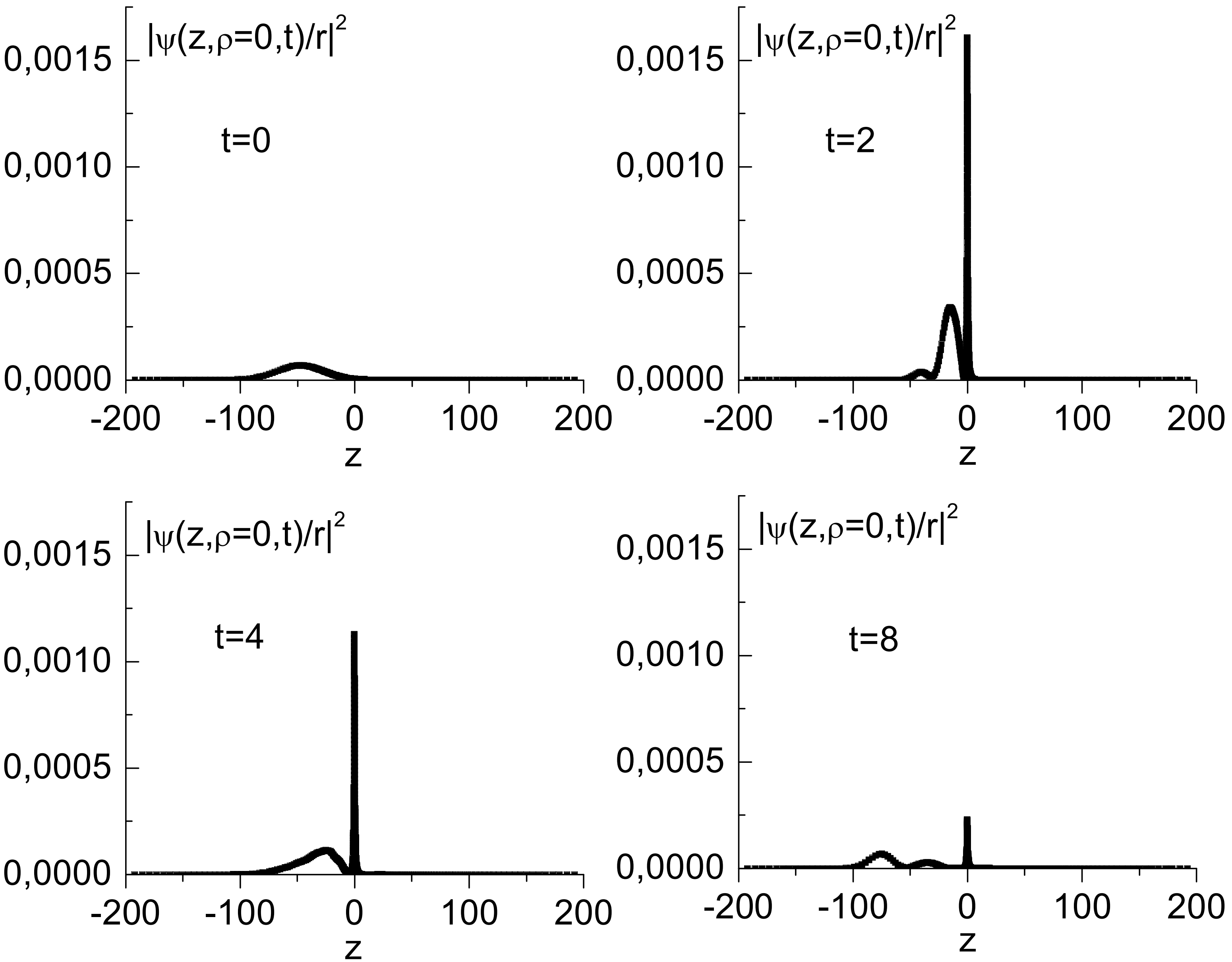
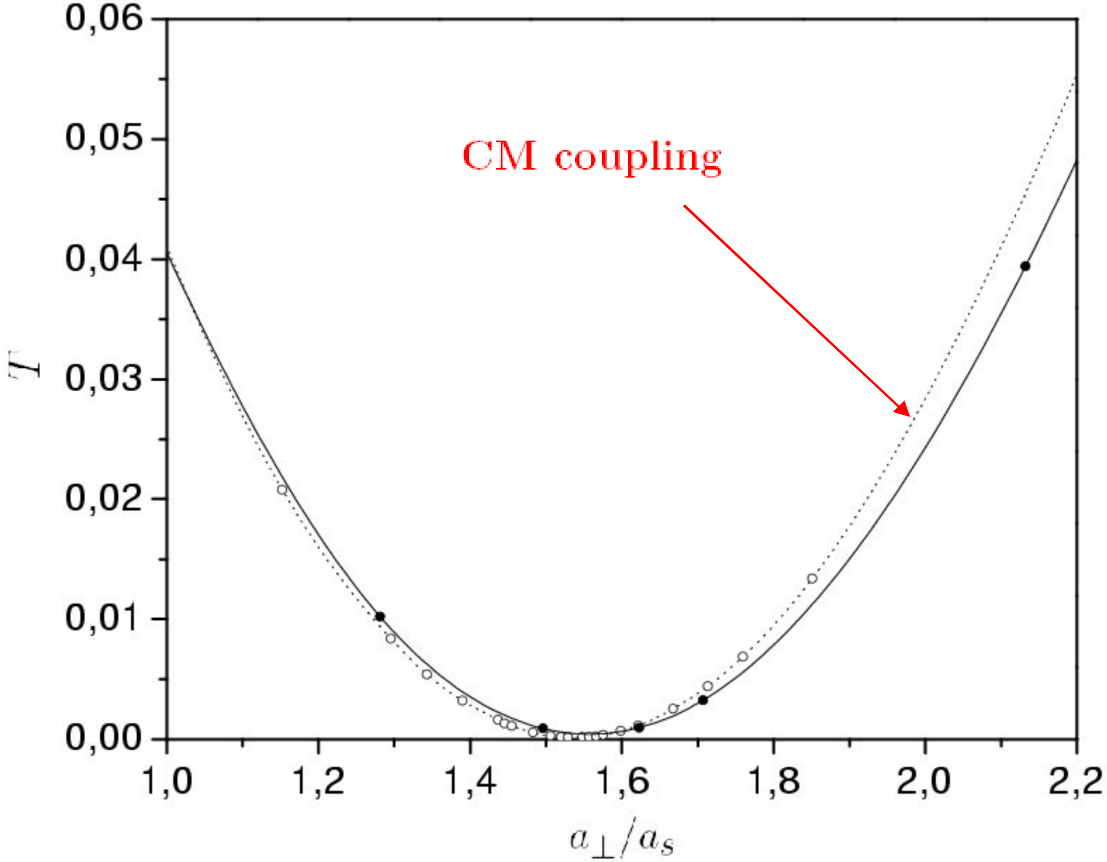


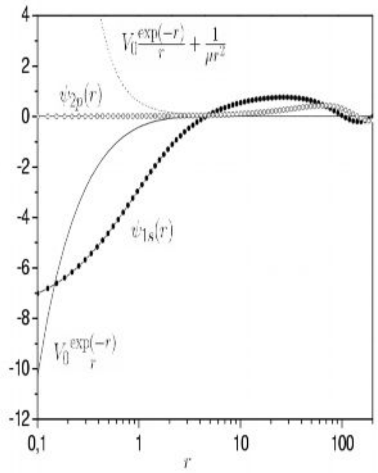
$V_0 = -1.12 \quad (a_\perp/a_s = 1.47, \quad a_\perp/a_p = -5.98)$

wave-packet reflection

confinement induced resonance (CIR)

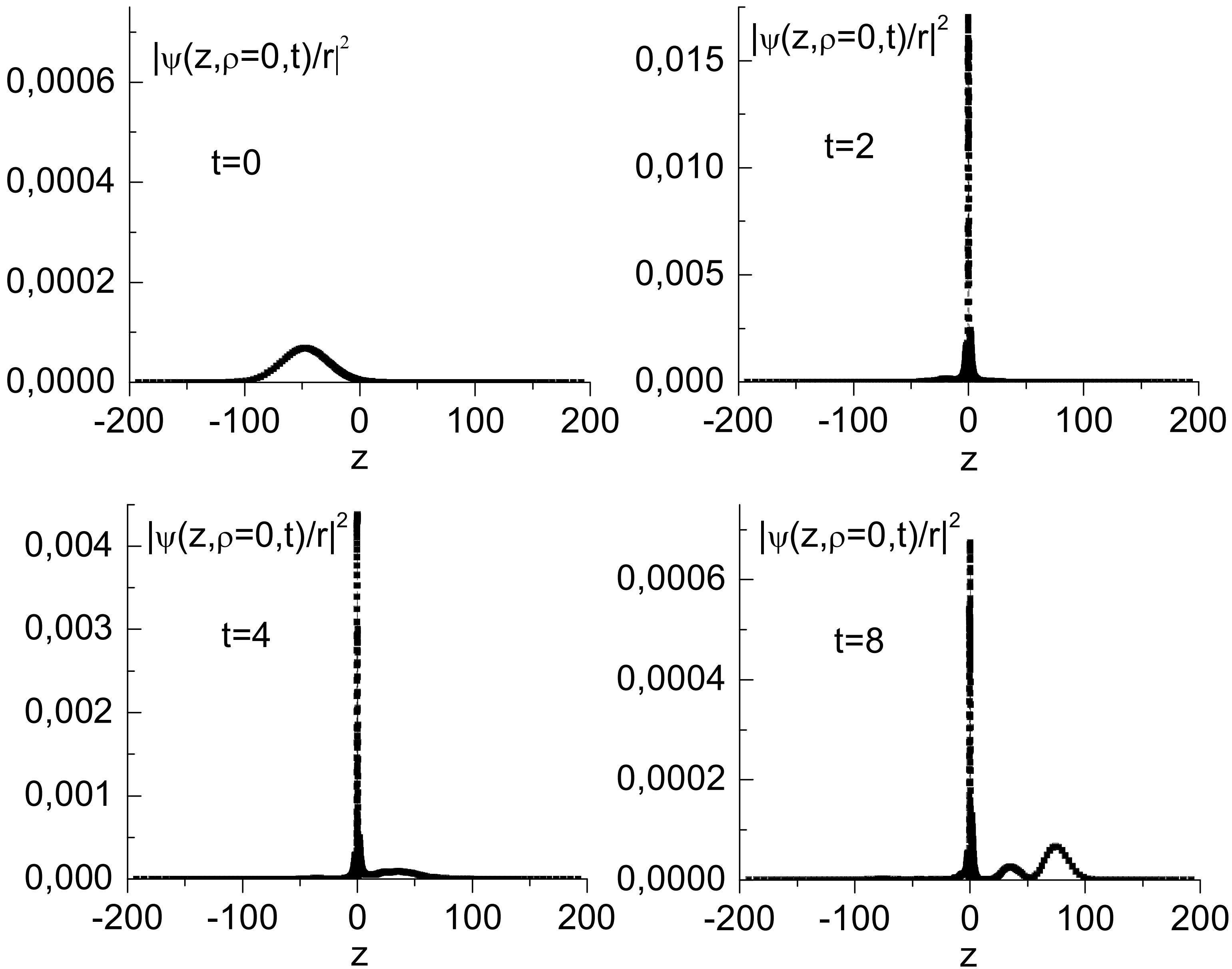


$V_0 = -1.136$



$$T = |\, 1 + f_0^+ \,|^2 \longrightarrow 0$$

pure s-wave atom-atom scattering in free (3D) space



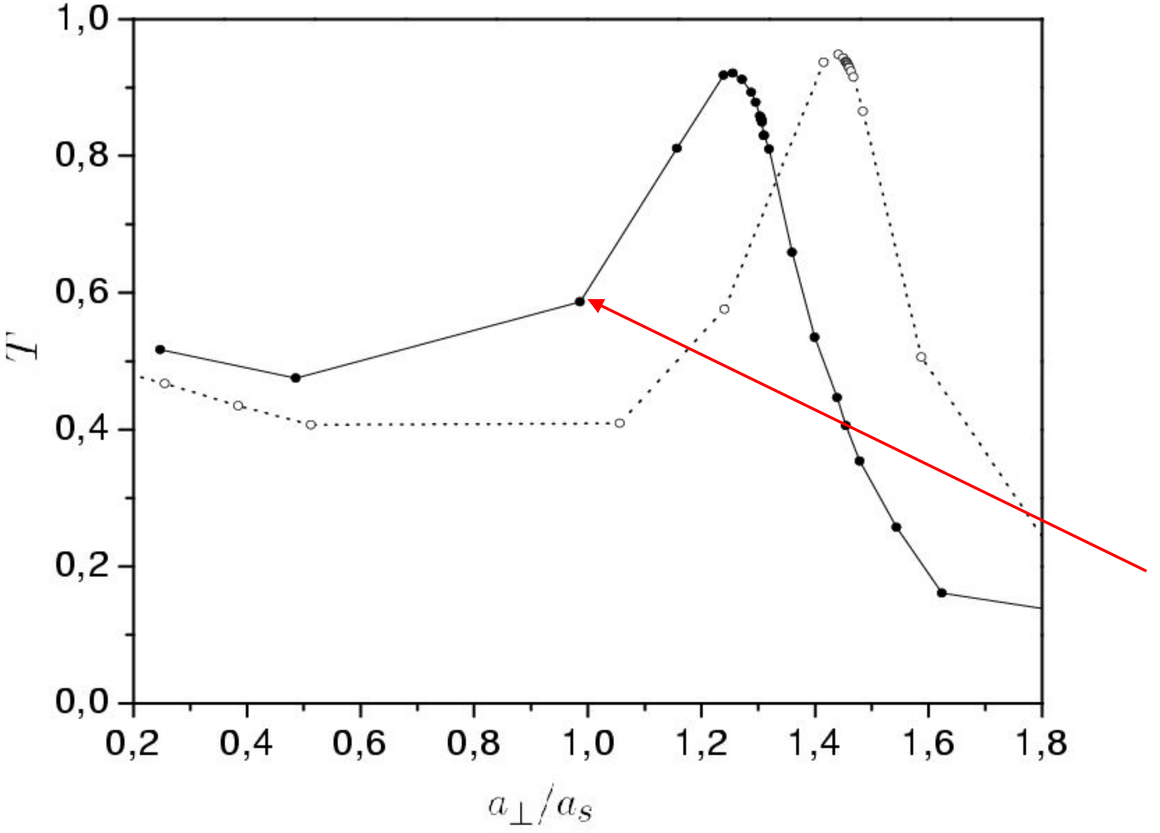
$V_0 = -8.45 \quad (a_\perp/a_s = 1.46\,, \quad a_\perp/a_p = -1.94)$

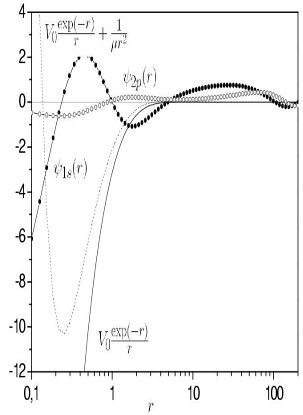
wave-packet transmission

confinement induced resonance (CIR)

s- and p-wave considerable in atom-atom scattering in free (3D) space

$V_0 = -8.45$



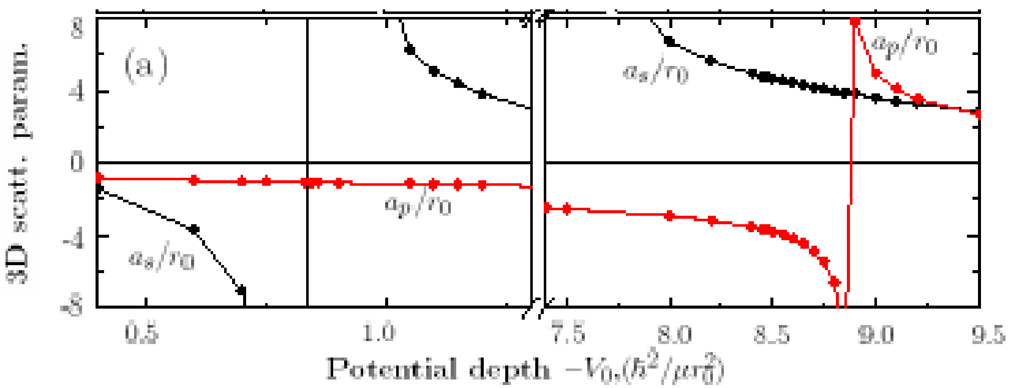


$\omega_1 = 1.35\omega_2$

$m_1/m_2 = 40/87 \quad (^{40}K : ^{87}Rb)$

$$T = | 1 + f_0^+ |^2 = | 1 + f_{0g}^+ + f_{0u}^+ |^2 \rightarrow | 1 - 1 - 1 |^2 \rightarrow 1$$

Suppression of Quantum Scattering in Strongly Confined Systems

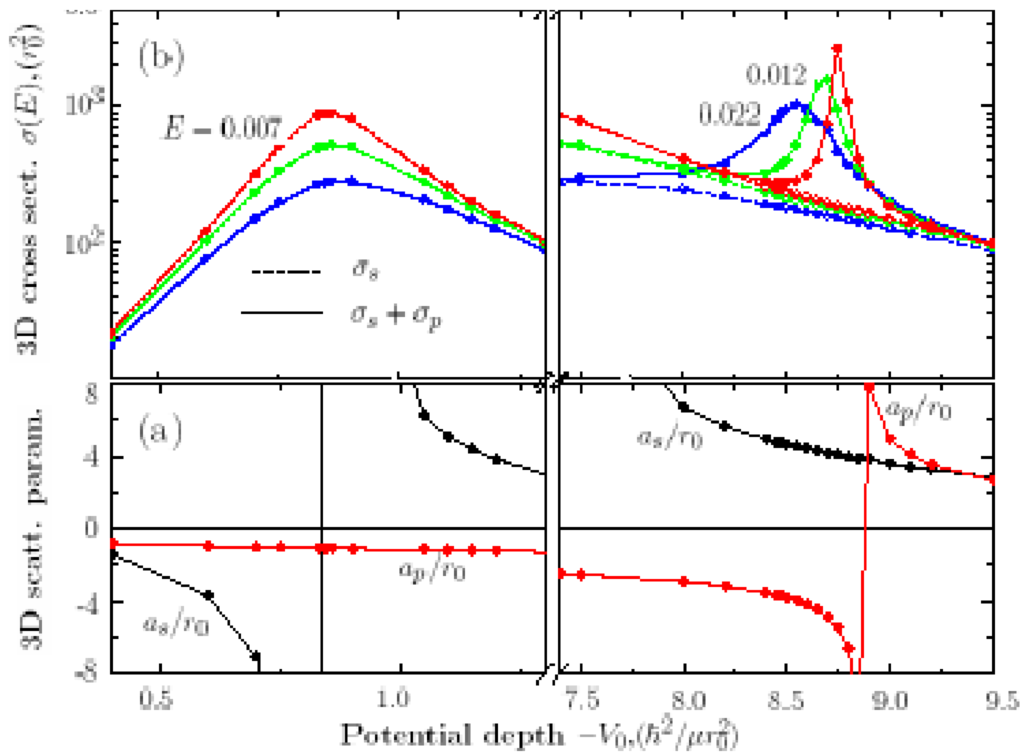


free motion

atom-atom interaction $V(r) = V_0 \frac{\exp\{-r\}}{r}$

J.I.Kim, V.S.Melezhik and P.Schmelcher, PRL (2006)

Suppression of Quantum Scattering in Strongly Confined Systems



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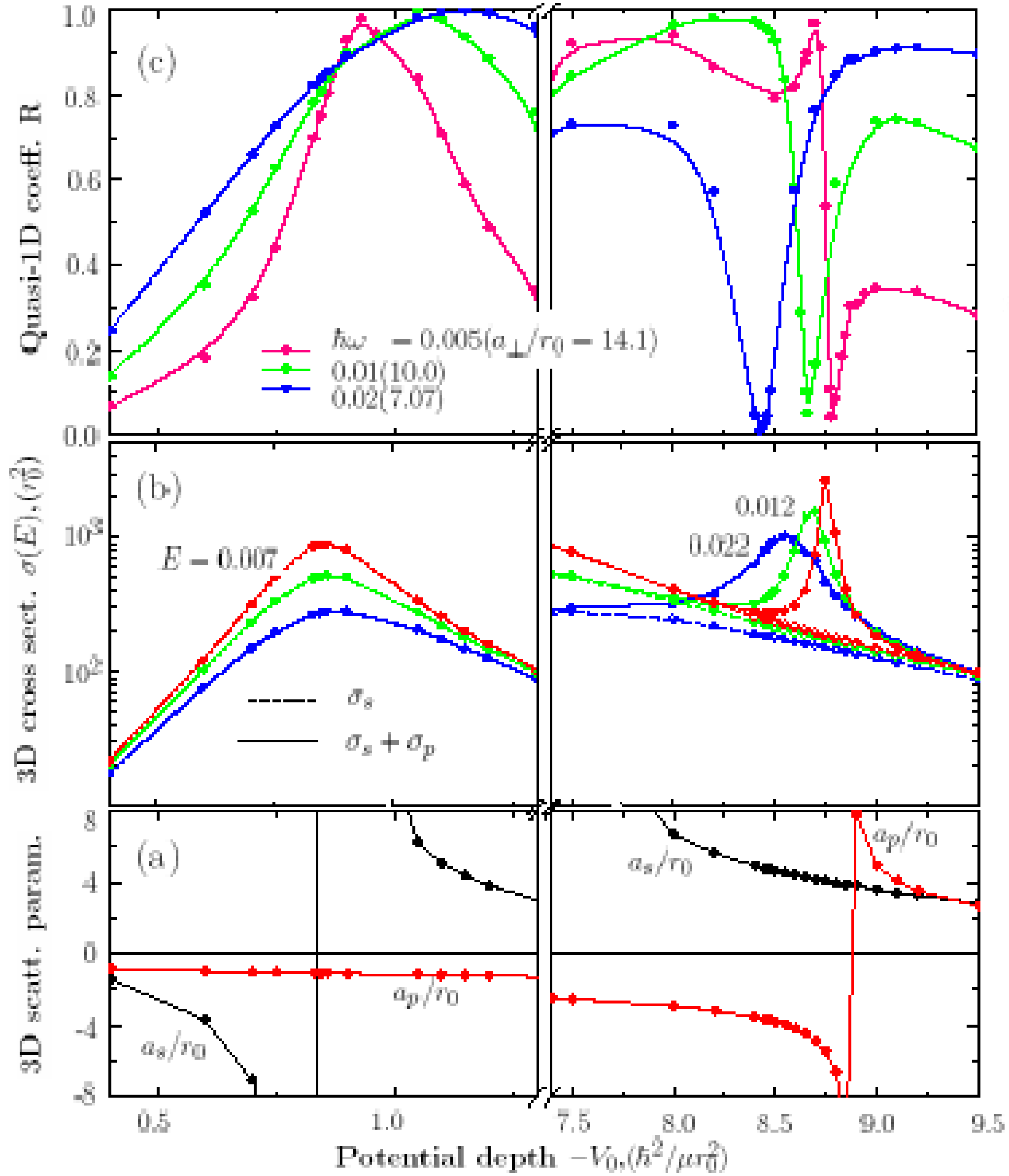


(1D) confinement

reflection coefficient

$$R = 1 - T = 1 - |1 + f^+_{0g} + f^+_{0u}|^2 \to 1 - |1 - 1 - 1|^2 \to 0$$

$$E = \hbar\omega_\perp + \varepsilon$$

$$a_\perp = \sqrt{\frac{\hbar}{\mu\omega_\perp}}$$

free motion

atom-atom interaction $V(r) = V_0 \frac{\exp\{-r\}}{r}$

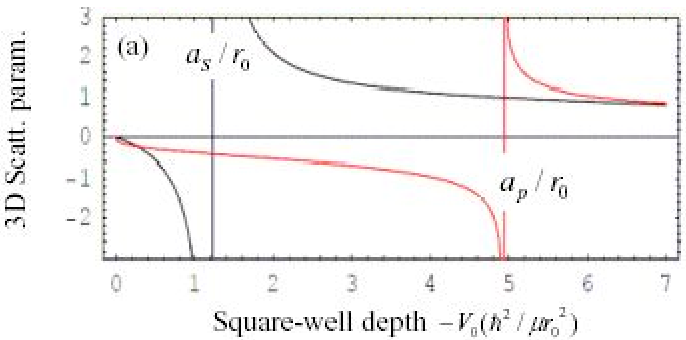
J.I.Kim, V.S.Melezhik and P.Schmelcher, PRL (2006)

Suppression of Quantum Scattering in Strongly Confined Systems

is qualitatively confirmed in simpler though solvable model:

square-well potential (depth V_0

and radius r_0)



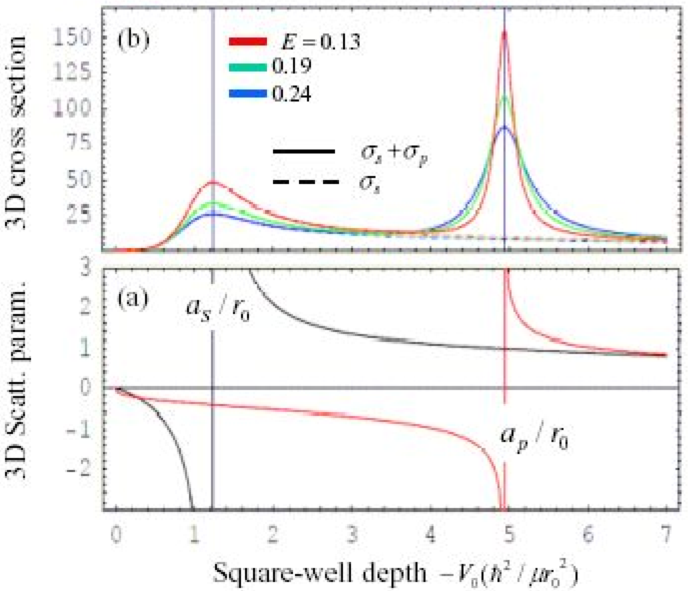
Suppression of Quantum Scattering in Strongly Confined Systems

is qualitatively confirmed in simpler though solvable model:

- - - - - - - - - - σ_s

———————— $\sigma_s + \sigma_p$

square-well potential (depth V_0 and radius r_0)

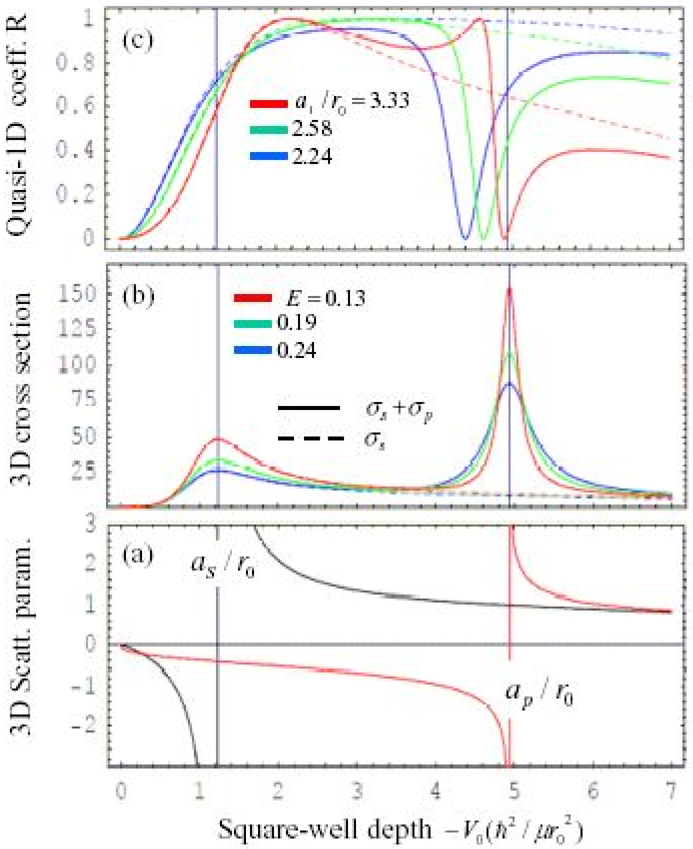


Suppression of Quantum Scattering in Strongly Confined Systems

is qualitatively confirmed in simpler though solvable model:

Kim, Melezhik & Schmelcher (in press)

Kim, Schmiedmayer & Schmelcher, PR A72, 042711 (2005)



- - - - - - - - σ_s

———— $\sigma_s + \sigma_p$

square-well potential (depth V_0 and radius r_0)

$$E = \hbar\omega_\perp + \varepsilon$$

$$a_\perp = \sqrt{\frac{\hbar}{\mu\omega_\perp}}$$

$$f_{0g} = -(1 + i\cot\delta_g)^{-1} \dashrightarrow \approx -1.$$

$$\cot\delta_g = -[a_\perp/a_s - (C^2 - a_\perp^2 k_0^2)^{1/2}]a_\perp k_0/2$$

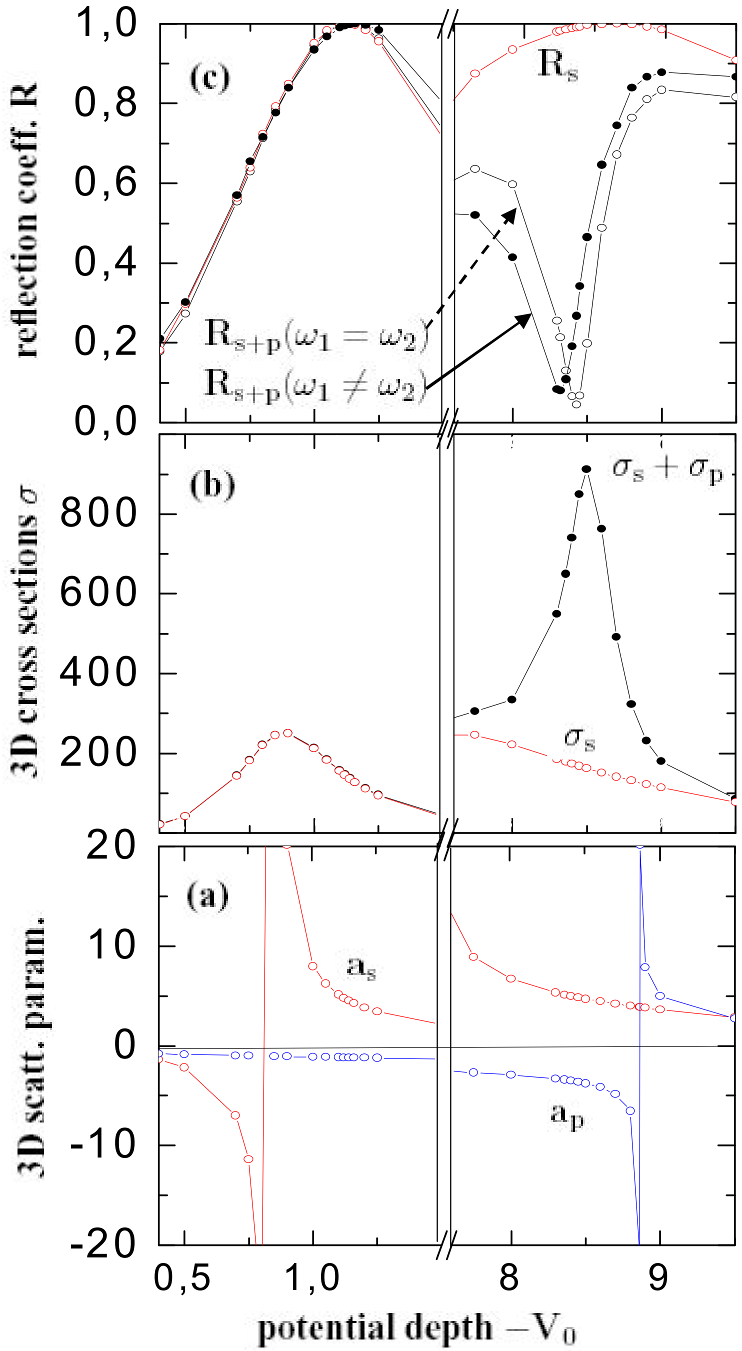
$$f_{0u} = -(1 + i\cot\delta_u)^{-1} \dashrightarrow \approx -1.$$

$$\cot\delta_u = -[a_\perp^3/V_p + (C^2 - a_\perp^2 k_0^2)^{3/2}]/(6a_\perp k_0) \qquad C = 2$$

$$a_\perp = 2a_s = -2a_p$$

$$T = |\,1 + f_0^+\,|^2 = |\,1 + f_{0g}^+ + f_{0u}^+\,|^2 \to |\,1 - 1 - 1\,|^2 \to 1$$

$$R = 1 - T \to 0$$



(c)
reflection coeff. R
1,0
0,8
0,6
0,4
0,2
0,0
R_s
$R_{s+p}(\omega_1 = \omega_2)$
$R_{s+p}(\omega_1 \neq \omega_2)$
(b)
3D cross sections σ
800
600
400
200
$\sigma_s + \sigma_p$
σ_s
(a)
3D scatt. param.
20
10
0
-10
-20
a_s
a_p
0,5
1,0
8
9
potential depth $-V_0$

Conclusion

- new approach for treating few-body processes

suppression of quantum scattering in strongly confined systems

- might be useful in "interacting" quasi-1D ultracold atomic gases
- guided atom interferometry
- electron scattering by impurities of quantum wires

Conclusion

- wave-packet propagation method for atom-atom collisions in harmonic traps (beyond zero-range potential, confinement induced CM nonseparability)
- confinement induced suppression of quantum scattering

next step:

- **inelastic collisions (transverse excitations)**
- **anharmonic effects**
- **anisotropy:** $\frac{1}{2}\mu\{\omega_1^2 x^2 + \omega_2^2 y^2\}$
- ...

laser-stimulated formation of $\overline{H}$

two-body radiative capture (spontaneous)

$\overline{p} + e^+ \rightarrow \overline{H}_n + \hbar\omega$

$$\frac{mV_0^2}{2} + \frac{1}{2n^2} = \hbar\omega$$

laser-induced capture **A.Wolf (1983-1997)**

$$\overline{p} + e^+ + \hbar\omega \rightarrow \overline{H}_n + 2\hbar\omega$$

enhancement factor

$G_n = \frac{\sigma_n^{ind}}{\sigma^{spon}}$

σ_n^{ind} - laser-induced transition to a specific level n of $\overline{H}_n$

σ^{spont} - spontaneous transitions summed over all possible n

experiments

- proton-electron recombination

  Heidelberg (1991) (heavy ion storage ring)

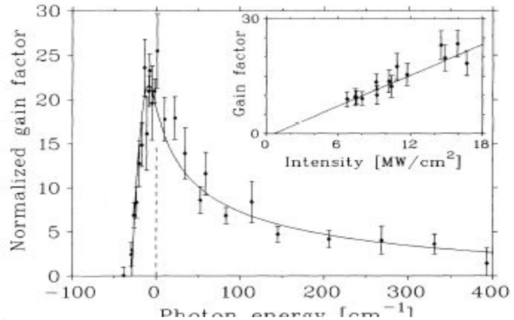
  laser $I = 20 MW/cm^2$ pulse $\sim 20 ns$

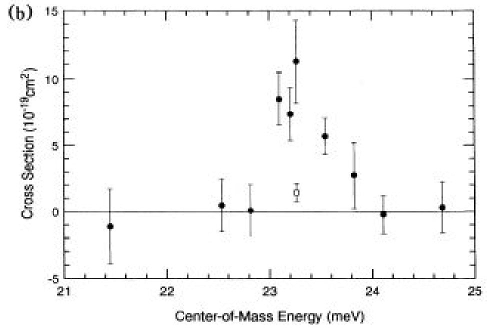
  $\lambda = 450.46 nm \Rightarrow$ continuum $\rightarrow$ bound (n=2)

  Western Ontario (1991) (merged-electron-ion- beam)

  CO_2 continuously operating laser $I \sim 20 kW/cm^2$

  $\lambda \approx 10.5 \mu m \Rightarrow$ continuum $\rightarrow$ bound (n=11,12)





- laser-induced formation of $\bar{H}$

  ATHENA-collaboration (PRL 97 (2006))

  CO_2 continuously operating laser $I \sim 1 kW/cm^2$

  $\lambda \approx 10.5 \mu m \Rightarrow$ continuum $\rightarrow$ bound (n=11)

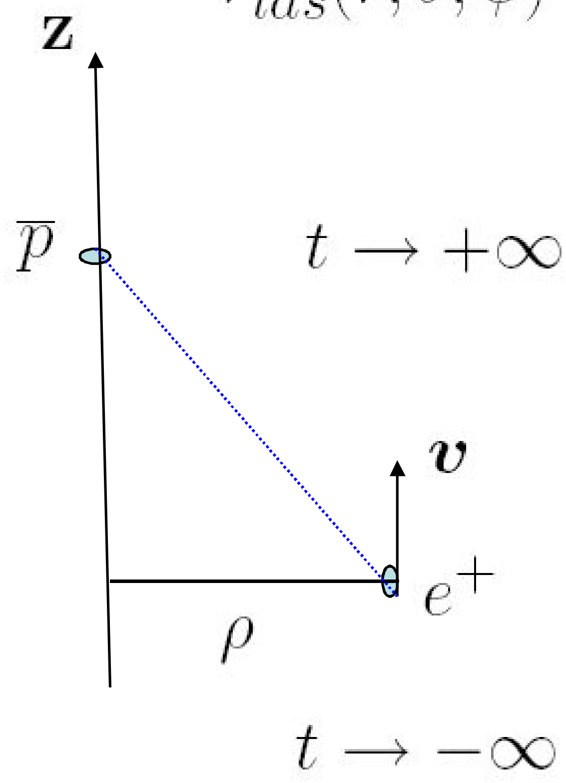
small laser effect on $\bar{H}$ formation

likely the three-body capture is the dominant mechanism (?)

dependence of $\overline{H}$ formation on laser polarization?

$$\boldsymbol{E} = \boldsymbol{E}_z \cos \omega t + (\boldsymbol{E}_y \sin \omega t)\ \gamma$$

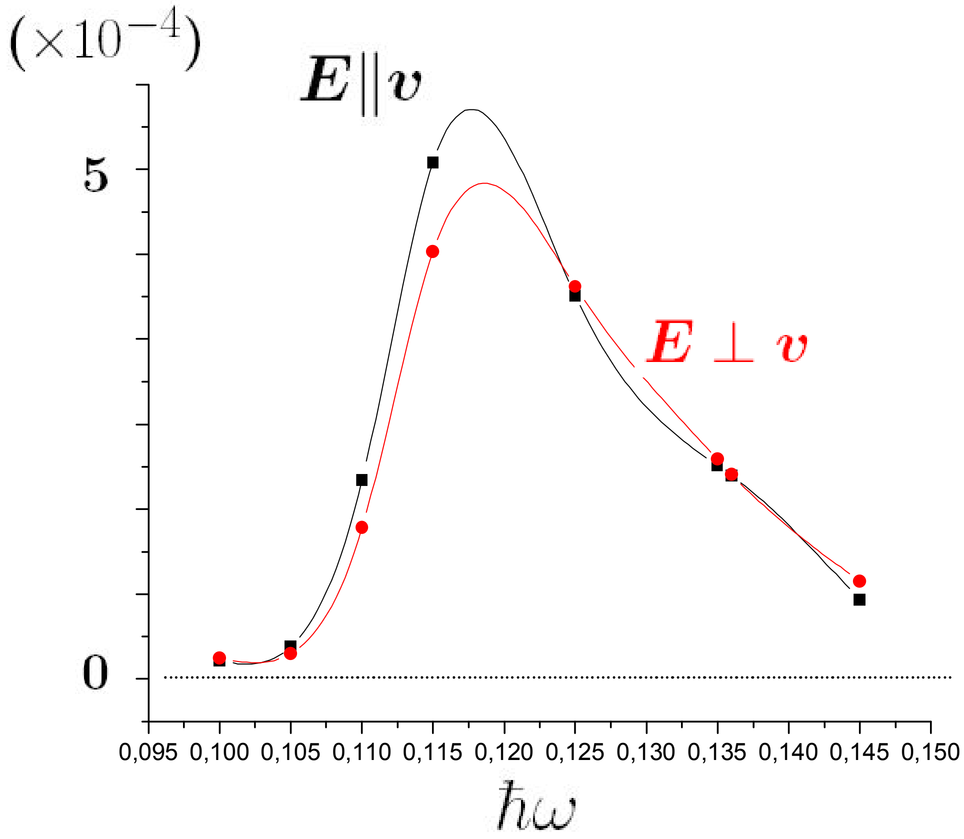
$$V_{las}(r, \theta, \phi) = (\boldsymbol{E} \cdot \boldsymbol{r}) = E_z z + E_y y$$



linear polarization $\gamma = 0$

$\boldsymbol{E} \| \boldsymbol{v}$ $\boldsymbol{E} \perp \boldsymbol{v}$

$\overline{H}_n$ formation probability: n=2



$$\Sigma_l \, |< \psi(t \to +\infty \mid \phi_{nl} >|^2$$

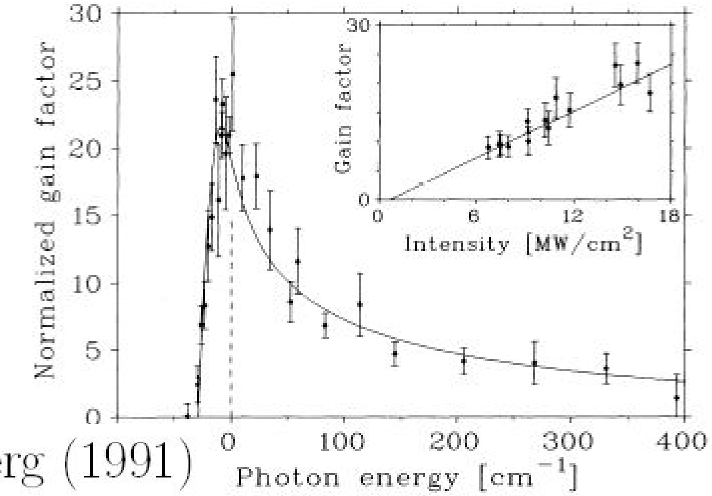
$$I \Rightarrow E^2 = (10^{-3})^2(a.u.) = 3.51 \times 10^{10} W/cm^2$$

$$\hbar\omega \sim \frac{1}{2 \cdot 2^2} = 0.125 \Rightarrow \lambda \sim 450.46nm$$

$$\rho = 1(a.u.)$$

$$\frac{mV}{2} = 10^{-2}(a.u.) = 0.27eV$$

Heidelberg (1991)



Conclusion

- **dependence of $\overline{H}_n$ formation on γ**
- **Rydberg levels $n = 10 \sim 20$ (?)**
- **pulse shape (?)**