Spin-Isospin Excitations and Nuclear Muon Capture

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Ordinary Muon Capture

- Nuclear muon capture

$$\mu^- + A(Z,N) \to B(Z-1, N+1) + \nu_\mu$$

- Weak currents. The nucleon current

$$J_k = t^+ \begin{cases} g_V + [g_P - g_A - g_M]\,\eta + \dfrac{g_A}{M_N}\,(\vec{p}_N, \vec{\sigma})\,, & k = 0 \\ (g_A - [g_V + g_M]\,\eta)\,\sigma_k + \dfrac{g_V}{M_N} p_{N\,k}\,, & k = 1,2,3 \end{cases}$$

$\eta = \dfrac{E_\nu}{2M_N}$. The lepton current

$$\left.\begin{array}{ll} \chi_\nu^\dagger(r)\left(1 - (\vec{\sigma}, \hat{\vec{\nu}})\right)\chi_\mu(r), & k = 0, \\ \chi_\nu^\dagger(r)\left(1 - (\vec{\sigma}, \hat{\vec{\nu}})\right)\sigma_k \chi_\mu(r), & k = 1,2,3 \end{array}\right\} \sim \exp\left[i\,(\vec{\nu}, \vec{r})\right],$$

Gamow-Teller Transitions

- Charge exchange-nuclear reaction at intermediate energies. For (p,n) reaction

$$\frac{d\sigma}{d\omega}(0^\circ) = \frac{\mu}{\pi\hbar^2}\frac{k_f}{k_i}\left[V_\tau J_\tau^2 B^-(\mathrm{F}) + V_{\sigma\tau}J_{\sigma\tau}^2 B^-(\mathrm{GT})\right].$$

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$$B^\pm(\mathrm{GT}) \equiv \frac{1}{2J_i+1}\sum_{M_i,M_f}\sum_{m=-1}^{1}\left|\langle J_f M_f|\sum_{q=1}^{A}\sigma_q^m\, t_q^\pm|J_i M_i\rangle\right|^2,$$

convention: $|n\rangle = t^+|p\rangle$.

- Ikeda's sum rule and problem of *missed* GT strength

$$\sum_k B_k^-(\mathrm{GT}) - \sum_\ell B_\ell^+(\mathrm{GT}) = 3(N-Z)$$

Missed strength and residual interactions

- Random Phase Approximation (RPA), phonon operators

$$\Omega_\rho^\dagger = \sum_{p,h} \left(\psi_{ph}^\rho a_p^\dagger a_h - \phi_{ph}^\rho a_h^\dagger a_p \right)$$

- Interaction between $1p-1h$ and $2p-2h$ configurations.
  - Second RPA

    $$\mathcal{O}_\rho^\dagger \sim \ldots + \sum_{p<p',h<h'} \left(\psi_{pp',hh'}^\rho a_p^\dagger a_{p'}^\dagger a_{h'} a_h - \phi_{pp',hh'}^\rho a_h^\dagger a_{h'}^\dagger a_{p'} a_p \right)$$

  - Fragmentation problem

    $$|JM\rho\rangle = \left(R_{\rho_1\rho} \Omega_{JM\rho_1}^\dagger + P_{\rho_1\rho_2\rho} \left[Q_{J_1\rho_1}^\dagger \otimes \Omega_{J_2\rho_2}^\dagger \right]_{JM} \right) |\rangle$$

- Strength function

$$b^{\pm}(E) = \sum_k B_k^{\pm}(\mathrm{GT})\delta(E - E_k), \quad S_m^{\pm} = \int_0^{\infty} b^{\pm}(E)E^m dE$$

- Exact results

Second RPA

$$S_0^- - S_0^+\big|_{\mathrm{SRPA}} = S_0^- - S_0^+\big|_{\mathrm{RPA}}$$
$$S_1^- + S_1^+\big|_{\mathrm{SRPA}} = S_1^- + S_1^+\big|_{\mathrm{RPA}}$$

Fragmentation problem

$$S_0^-\big|_{\mathrm{fragm.}} = S_0^-\big|_{\mathrm{RPA}}$$
$$S_1^-\big|_{\mathrm{fragm.}} = S_1^-\big|_{\mathrm{RPA}}$$

- Finally:

$$S_0^-\big|_{2p-2h} \approx S_0^-\big|_{1p-1h}$$
$$S_1^-\big|_{2p-2h} \approx S_1^-\big|_{1p-1h}$$

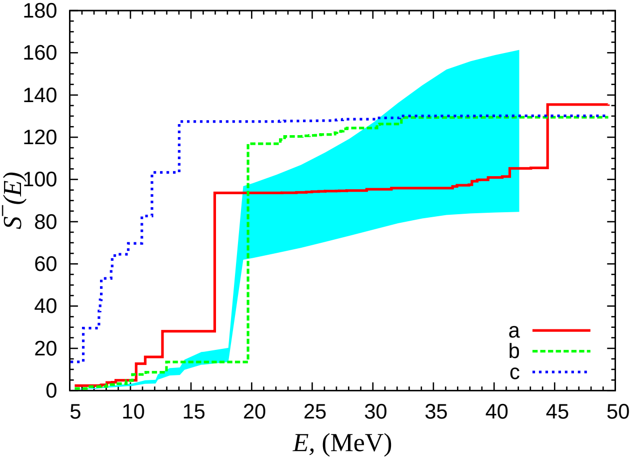
Average excitation energies: $\langle E \rangle|_{2p-2h} \approx \langle E \rangle|_{1p-1h}$

The variants of residual interactions.

	The residual interaction		
	Mixes $\Delta N = 0$ and $\Delta N \geq 2$ configurations a	Not mixes $\Delta N = 0$ and $\Delta N \geq 2$ configurations b	Without residual interaction c
S_0^+	6.55	0.49	1.15
S_0^-	135.55	129.49	130.15
$S_0^- - S_0^+$	129.00	129.00	129.00

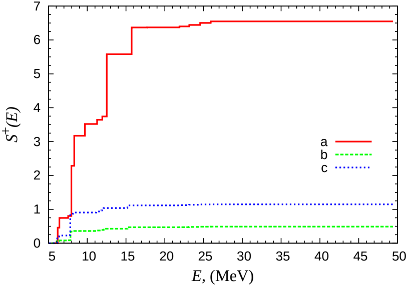
σt^- strength, as in $^{208}\mathrm{Pb}(p,n)^{208}\mathrm{Bi}$ reaction.

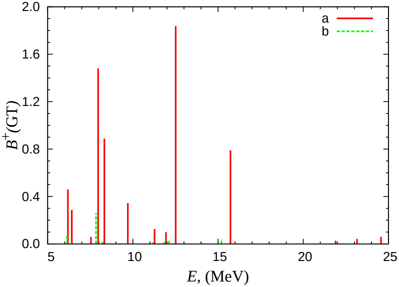
$$S^-(E) = \sum_{k:E_k \le E} B_k^-(\mathrm{GT}) = \int_0^E b^-(t)dt$$



Experiment: B. S. Flanders *et al.*, Phys. Rev. C **40,** 1985 (1989)

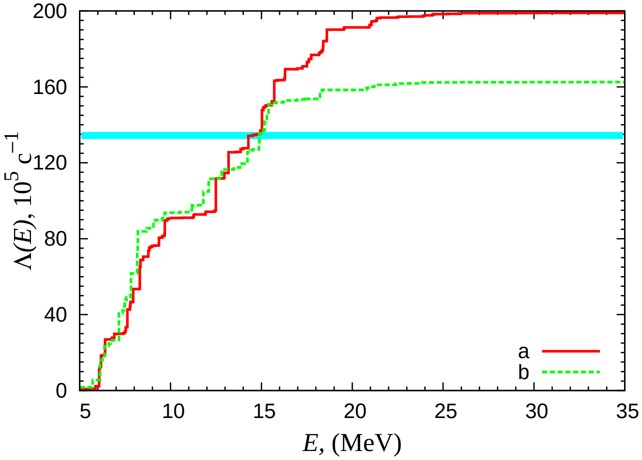
σt^{+} strength, as in $^{208}\mathrm{Pb}(n,p)^{208}\mathrm{Tl}$ reaction.





Total rates of muon capture $^{208}\mathrm{Pb}(\mu,\nu)^{208}\mathrm{Tl}$

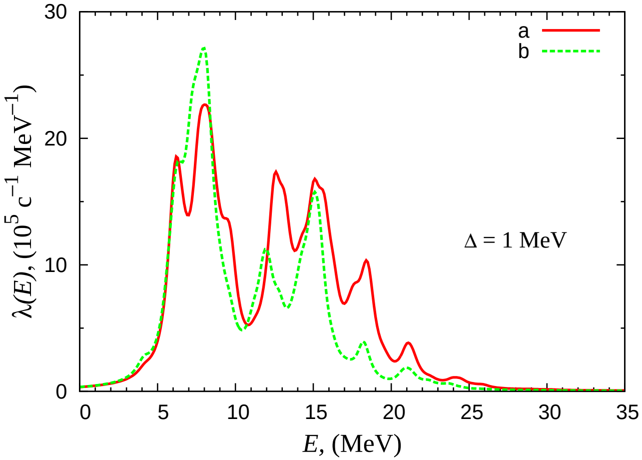
$$\Lambda(E) = \sum_{k:E_k \le E} \Lambda_k$$



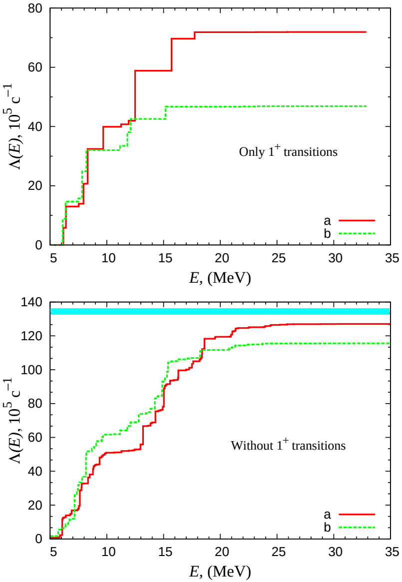
Experiment: T. Suzuki, D. F. Measday, and J. P. Roalsvig, Phys. Rev. C **35,** 2212 (1987)

Rates of muon capture $^{208}\mathrm{Pb}(\mu,\nu)^{208}\mathrm{Tl}$

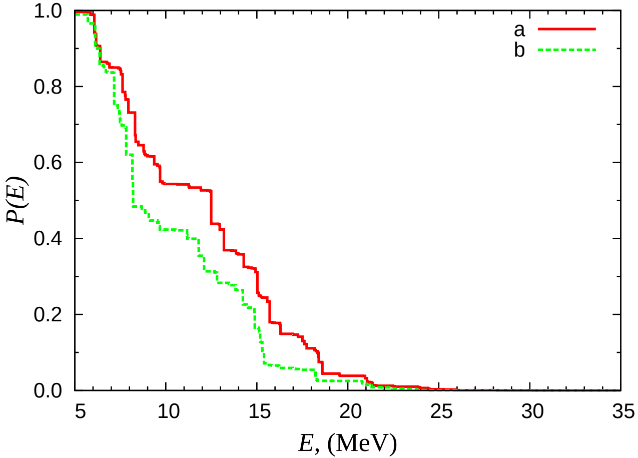
$$\lambda(E) = \sum_k \rho(E - E_k)\Lambda_k, \quad \rho(E - E_k) = \frac{\Delta}{2\pi}\frac{1}{(E - E_k)^2 + \Delta^2/4}.$$



Rates of muon capture $^{208}\mathrm{Pb}(\mu,\nu)^{208}\mathrm{Tl}$



$$P(E) = 1 - \sum_{k:E_k \le E} \Lambda_k / \Lambda_{\text{tot}}$$



Estimations for the probability of neutron emission in $^{208}\mathrm{Pb}(\mu, \nu\, kn)^{208-k}\mathrm{Tl}$.

Interaction or $S_{k\,n}$	Probability of emission k neutrons			
	$k = 0$	$k = 1$	$k = 2$	$k \geq 3$
a	38	44	17	1
b	55	38	6	1
$S_{k\,n}$, (MeV)		9.305	16.147	22.650
Experiment on $^{209}\mathrm{Bi}$	5	47	29	19
$S_{k\,n}$, (MeV)		5.092	12.459	19.197

Experiment: D. F. Measday, T. J. Stocki, and H. Tan, Phys. Rev. C **75**, 045501 (2007).

Conclusions

1. The calculated pattern of neutron multiplicity in reaction $^{208}\mathrm{Pb}(\mu, \nu\, kn)^{208-k}\mathrm{Tl}$ depends on the nuclear spin-isospin residual interaction in particle-hole channel.
2. The experimental pattern will be a strong test for nuclear spin-isospin residual interaction, and therefore for the effect of missed Gamow-Teller strength.