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Collisional Stark Transitions and Induced Annihilation of Cold Antiprotonic Helium Ions

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ASACUSA EXPERIMENT - 2005:

First observation of cold, long-lived antiprotonic helium ions $(\bar{p}\,{}^4\mathrm{He}^{++})$ and $(\bar{p}\,{}^3\mathrm{He}^{++})$:
M.Hori, J.Eades, R.S.Hayano, et al. Phys. Rev. Letters **94**, 063401 (2005)

Target: $T \sim 10\,\mathrm{K}$, $\quad \rho = (0.3 \div 20) \cdot 10^{17}\,\mathrm{cm}^{-3}$
Measured values:
Decay rates of annihilation signals *vs.* target density, $\gamma_n(\rho)$, for the circular orbits $(l = n - 1)$
at $n = 28, 30, 31, 32$ in ^{4}He
and $n = 28, 29, 30, 31$ in ^{3}He .

Qualitative peculiarities of the experimental results:

- Decay rates $\gamma_n \sim \rho$ at $\rho \lesssim 10^{18}\,\mathrm{cm}^{-3}$.

- $d\gamma/d\rho$ (per-atom collisional part of the rate) increases with n and greater for ^{3}He as compare with ^{4}He.

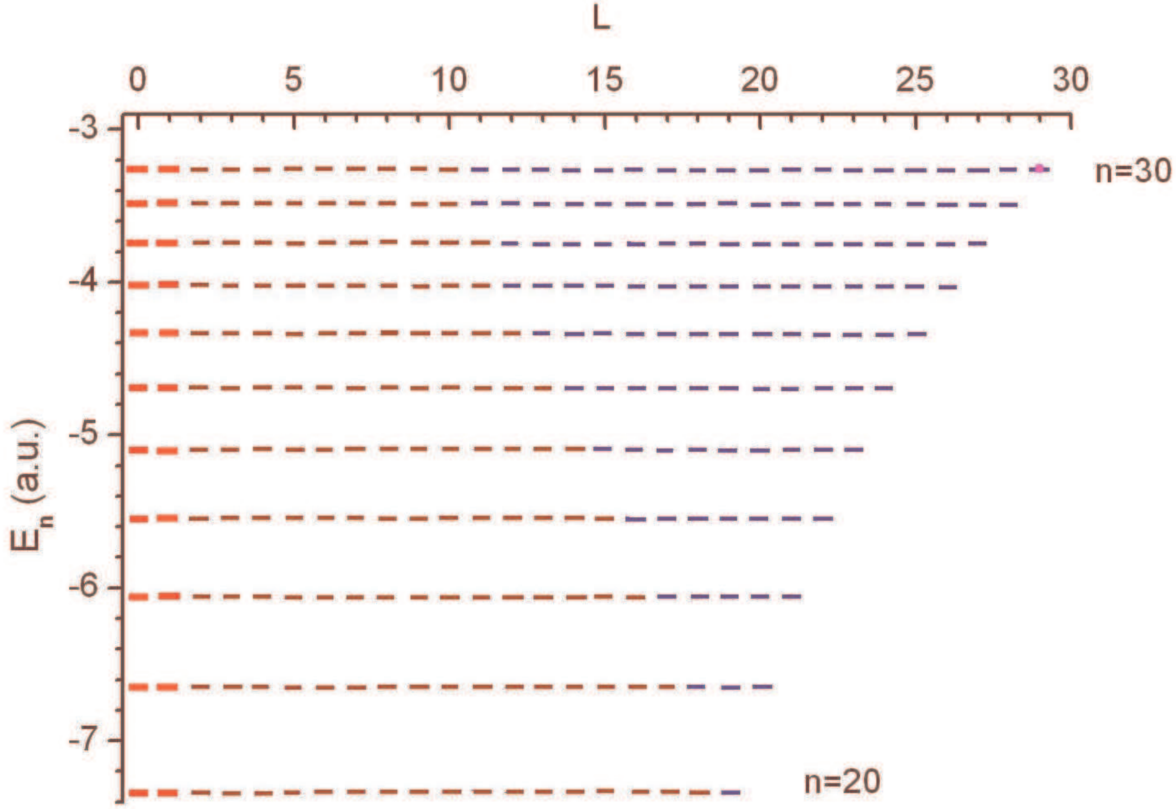
- $d\gamma/d\rho \sim (1 \div 3) \cdot 10^{-16}\,\mathrm{MHz} \cdot \mathrm{cm}^3$,
  $\therefore$ effective cross sections are huge,

  $$\sigma \sim (4 \div 10) \cdot 10^{-15}\,\mathrm{cm}^2 = (140 \div 360) \cdot a_0^2,$$

  depending on n and isotope.

How to explain these results?

Full theory has consider a time-dependent cascade of the transition.



Elementary processes in the cascade:

- radiative transitions $(\bar{p}\mathrm{He}^{++})_{nl} \to (\bar{p}\mathrm{He}^{++})_{n'l'} + \gamma$,
- Stark transitions $(\bar{p}\mathrm{He}^{++})_{nl} + \mathrm{He} \to (\bar{p}\mathrm{He}^{++})_{nl'} + \mathrm{He}$,
- external Auger process (Penning ionization)

$(\bar{p}\mathrm{He}^{++})_{nl} + \mathrm{He} \to (\bar{p}\mathrm{He}^{++})_{n'l'} + \mathrm{He}^{+} + e$,

- $\bar{p}$ - annihilation,
- Coulomb (collisional) de-excitation,

etc.

Collisional Stark transitions $(\bar{p}\mathrm{He}^{++})_{nl} + \mathrm{He} \to (\bar{p}\mathrm{He}^{++})_{nl'} + \mathrm{He}$, are the most important for the first step processes: they change the inner angular momentum of the antiprotonic ion and open a possibility for other processes to be more fast.

Theoretical approach:

heavy particles (helium nuclei and antiproton) are slow ($v_{hp} \ll v_e$), $\therefore$ electronic variables can be separated out within adiabatic approximation reducing the problem to the 3 body ($\bar{p} -$ He^{++} $-$ He).

Total effective 3-body hamiltonian:

$$H = h + T + V(\mathbf{R}, \mathbf{r})$$

h and $\mathbf{r}$ are inner hamiltonian and coordinates of ($\bar{p}$He^{++}),
$T = (-1/2m)\nabla^2_{\mathbf{R}}$, $\mathbf{R}$ and m are the kinetic energy operator, relative coordinates and reduced mass of colliding subsystems,
$V(\mathbf{R}, \mathbf{r})$ is the potential energy of interaction between antiprotonic and ordinary atoms.

At $l \geqslant 2$ and large n: Eigenfunctions $\phi_{nlm}(\mathbf{r})$ and eigenvalues E_{nl} of h for the isolated ($\bar{p}$He^{++}) are hydrogen-like, with $E_{nl} = e_n \equiv -\mu Z^2/2n^2$ degenerated in l.

For ns- and np-states: $E_{nl} = e_n + \Delta E_{nl}$.
Complex shift of antiprotonic levels $\Delta E_{nl} = -\epsilon_{nl} - \mathrm{i}\Gamma_{nl}/2$ is produced, mainly, by strong $\bar{p}$-nucleus interaction, including annihilation.
n - dependence:

$$\Delta E_{ns} = \Delta E_{1s}/n^3, \quad \Delta E_{np} = \Delta E_{2p} \cdot \frac{32(n^2-1)}{3n^5}$$

For $\bar{p} -{}^4$ He:

$$\Gamma_{1s} \simeq 11\,\mathrm{keV}, \quad \Gamma_{2p} \simeq 36\,\mathrm{eV}, \quad \epsilon_{nl} \simeq 0.3\Gamma_{nl}$$

Coupled channels approach:

Basis set at a fixed n: $|j\rangle \equiv |nl, L : JM\rangle = (\phi_{nl}(\mathbf{r}) \otimes Y_L(\Omega_R))_{JM}$.

Total wave function: $\Psi_i(\mathbf{R}, \mathbf{r}) = \sum_{lL} |j\rangle \psi_{ji}(R)/R$.

System of coupled-channel equations:

$$\psi''_{ji}(R) + \left[k_j^2 - L_j(L_j+1)/R^2\right] \psi_{ji}(R) = 2m \sum_k V_{jk}(R)\psi_{ki}(R)$$

$$k_j^2 = \begin{cases} 2m(E - e_n) & \text{(real) if } l \geqslant 2, \\ 2m(E - e_n + \epsilon_{nl} + \frac{\mathrm{i}}{2}\Gamma_{nl}) & \text{(complex) if } l_j \leqslant 1. \end{cases}$$

$\therefore$ Boundary conditions in the channels with l_j, $l_i \geqslant 2$ are standard, but in the channels with $l_j \leqslant 1$, $l_i \geqslant 2$ $\psi_{ji}(R) \to\sim \exp(-\mathrm{Im}(k_j)R)$, and $\psi_{ji}(R) = 0$ at $l_i \leqslant 1$.

Potential $V(\mathbf{R}, \mathbf{r})$ can be calculated by quantum-chemistry methods.
In our problem: $\langle r\rangle \sim n^2/\mu \sim 0.3$, $R_{eff} \gtrsim 1$ a.u., $\therefore$

$$V(\mathbf{R}, \mathbf{r}) \simeq V_0(R) + (\mathbf{d}\cdot\nabla_{\mathbf{R}})V_0(R) + (Q_2(\mathbf{r})\cdot C_2(\hat{\mathbf{R}}))V_0''(R) + \ldots$$

$V_0(R)$ is adiabatic potential of interaction between He atom and single positive charge of the ion, $\mathbf{d}$ and $Q_{2\mu}$ are dipole and quadruple operators of ($\bar{p}$He^{++}) that can mix degenerated nl states.

Analytical approximation of numerical potential (J.Russel, J.Cohen):

$$V_0(R) = V_M(R) + V_p(R),$$
$$V_M(R) = D_0\left(\exp[-2\beta(R-R_e)] - 2\exp[-\beta(R-R_e)]\right) \text{ (Morse)},$$
$$V_p(R) = -\frac{\alpha}{2R^4}\begin{cases} 0 & \text{at } R < R_e, \\ [1-\exp\left(-\gamma(R-R_e)^4\right)] & \text{at } R > R_e, \end{cases}$$
(polarization long-range interaction)

Parameters: $D_0 = 0.075$, $R_e = 1.46$, $\beta = 1.65$, $\alpha = 1.383$, $\gamma = 0.005$ a.u.

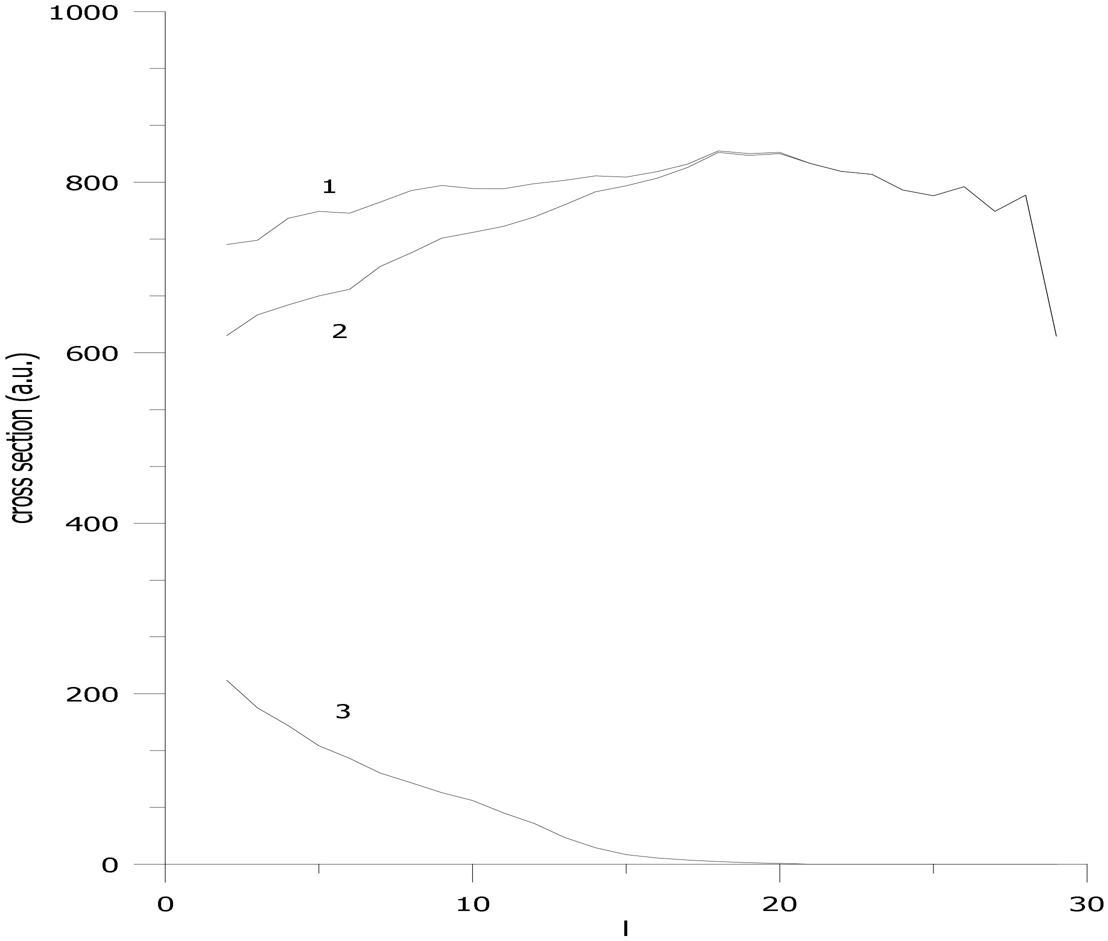
RESULTS

Relative importance of different terms in interaction:

Total cross sections of the Stark transitions from the circular state with $n = 30$ at $E = 10$ K.

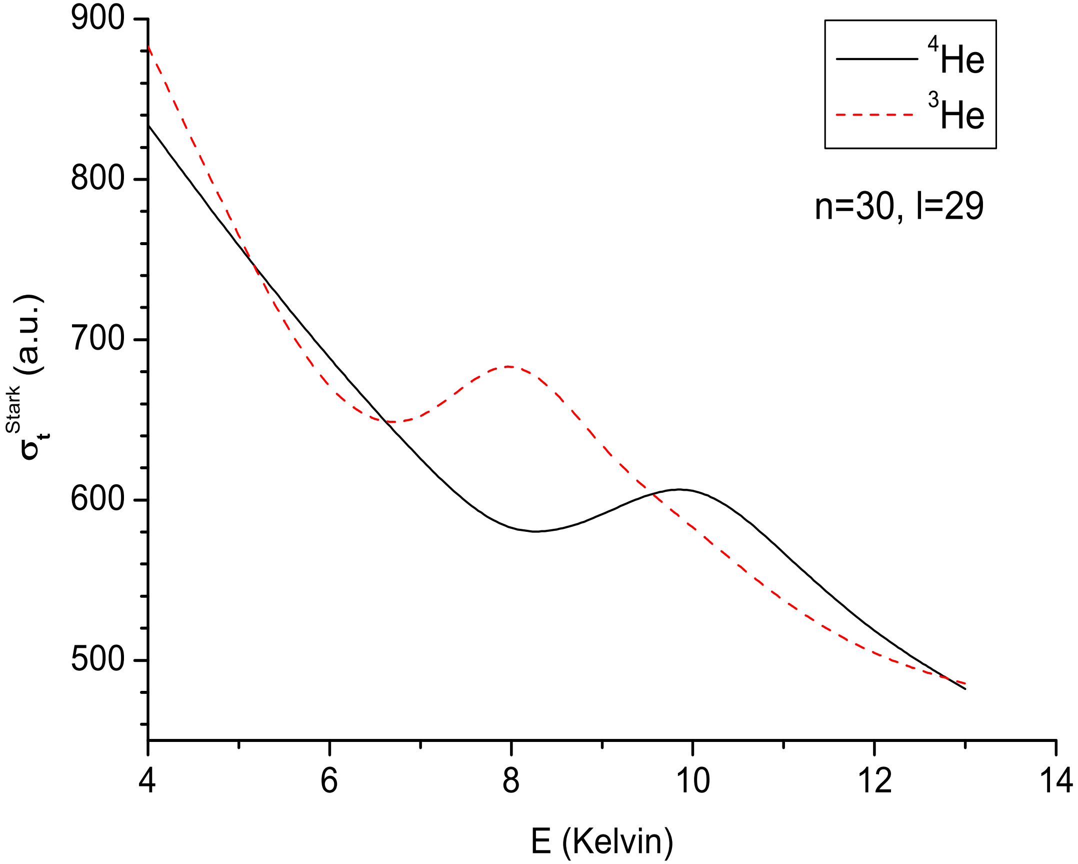
Elastic int. V_0	V_M	$V_M + V_p$		
Inelastic int.	dipole	dip.	dip. + quadruple	dip. + annih.
σ_{Stark}	116.9	619.6	620.4	619.6

Dependence of total Stark and induced annihilation cross section on initial l-state ($n = 30$, $E = 10$ K)



1 - σ_{St} without annihilation, 2 - σ_{St} with account for annihilation, 3 - σ_{annih}

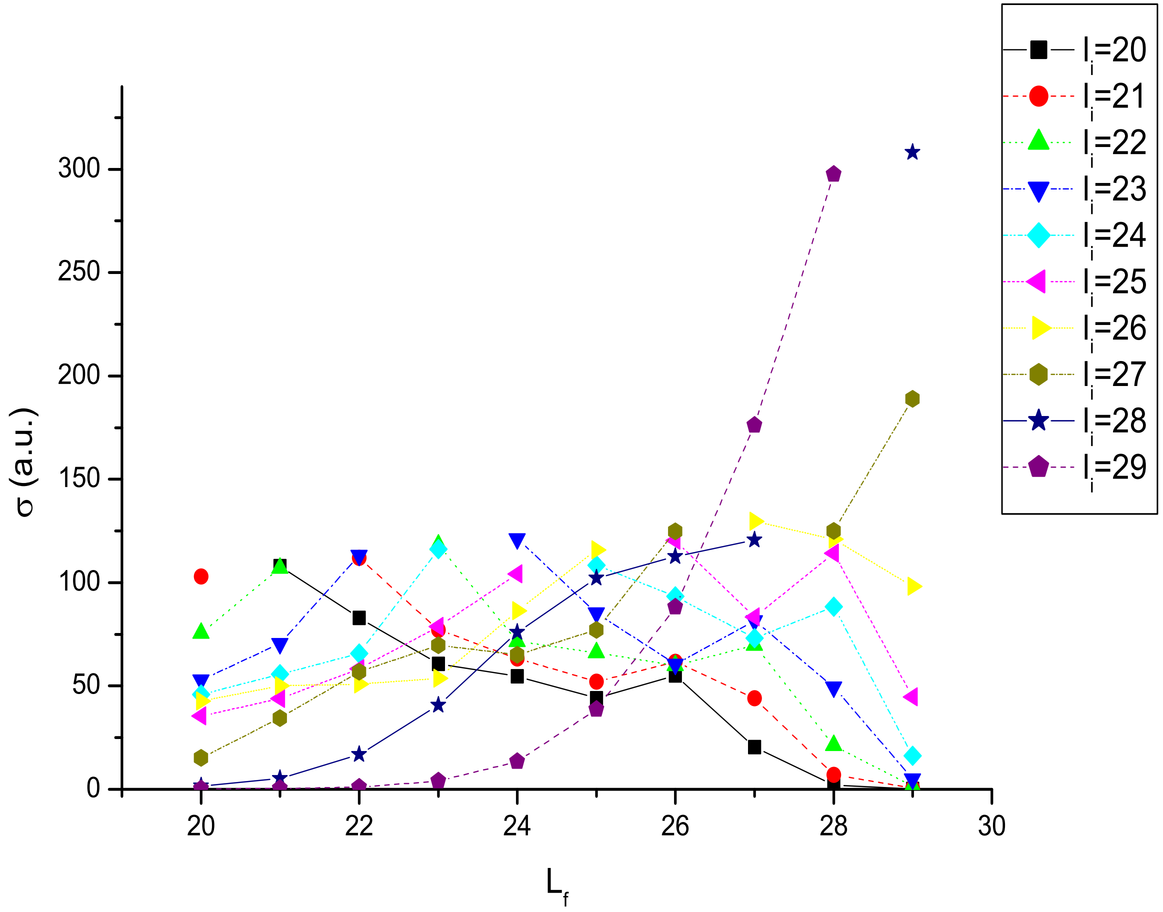
Energy dependence of the Stark cross section for the circular orbit with $n = 30$



Principal quantum number n and isotope dependence of the Stark cross section for the circular orbit

isotope	n				
	28	29	30	31	32
^{4}He	573.5	598.2	619.6	636.5	649.0
^{3}He	539.3	566.5	583.4	597.0	605.1

Partial Stark cross section $\sigma_{St}(nl \rightarrow nl')$ for ^{4}He at $E = 10$ K



Per-atom rates of Stark transitions from circular orbits, averaged over thermal motion, $\kappa = \langle v\sigma \rangle$
(units: $v_a a_0^3 = 6.126 \times 10^{-9}\,\mathrm{cm}^3/\mathrm{s} = 6.126 \times 10^{-15}\,\mathrm{MHz} \cdot \mathrm{cm}^3$)

isotope	n			
	28	30	32	units
^{4}He	0.0615	0.0678	0.0729	a.u.
	3.77	4.15	4.46	$10^{-16}\,\mathrm{MHz} \cdot \mathrm{cm}^3$
^{3}He	0.0728	0.0786	0.0818	a.u.
	4.46	4.82	5.01	$10^{-16}\,\mathrm{MHz} \cdot \mathrm{cm}^3$

Experiment: $d\gamma/d\rho \sim (1 \div 3) \cdot 10^{-16}\,\mathrm{MHz} \cdot \mathrm{cm}^3$

CONCLUSION

We have considered collisional Stark transitions and induced annihilation of antiproton in the high states ($n \sim 30$) of antiprotonic helium ion at very low energy (~ 10 K).

The processes are considered in the framework of quantum coupled-channels method taking into account all the states with different l at given n (~ 30). The most important contribution to the processes comes from the long-range polarization interaction. Admixtures of the s- and p-states to the states with higher l during collisions induce the effective annihilation cross sections for the initial l up to 15, but don't affect the Stark cross sections for the initial states nearly to circular orbits.

Total rates of the Stark transitions from the circular orbits with $n = 28 \div 32$, averaged over the thermal motion, are compatible with the ASACUSA data. The dependence on n as well as isotope effect are also qualitatively agree with the experiment.