muCF07, Dubna 18-21 June 2007

Importance of Fluctuations of cross section in muon-catalysed t-t fusion

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μd-t fusion

- Recent Experimental Observation: T dependence of λ_c and ω^{eff}

  cf. N. Kawamura, et al., Phys.Rev.Lett. **90**, 043401(2003)

  “Discovery of Temperature-Dependent Phenomena of Muon-Catalyzed Fusion in Solid Deuterium and Tritium Mixtures”

- due to the resonant molecular formation in

$$t\mu + \mathrm{D}_2 + \mathrm{D}_2$$

- Importance of the resonant state of $dt\mu$ mesomolecules

μt-t fusion

$$t + t + \mu \rightarrow {}^5\mathbf{He}^* + n \rightarrow \alpha + n + n + Q(11.33MeV)$$

cf. T. Matsuzaki, Phys.Lett.B 557, 176(2003)

"Evidence for strong n-α correlations in the t+t reaction proved by the neutron energy distribution of muon catalysed t-t fusion"

- No shallow bound state in $tt\mu$
- λ_c can be estimated using "in-flight" fusion model?
  $\Rightarrow$ No T dependence

- the μ cycling rate and the reaction rate

$$\lambda = \rho_{LH} < \sigma v >$$

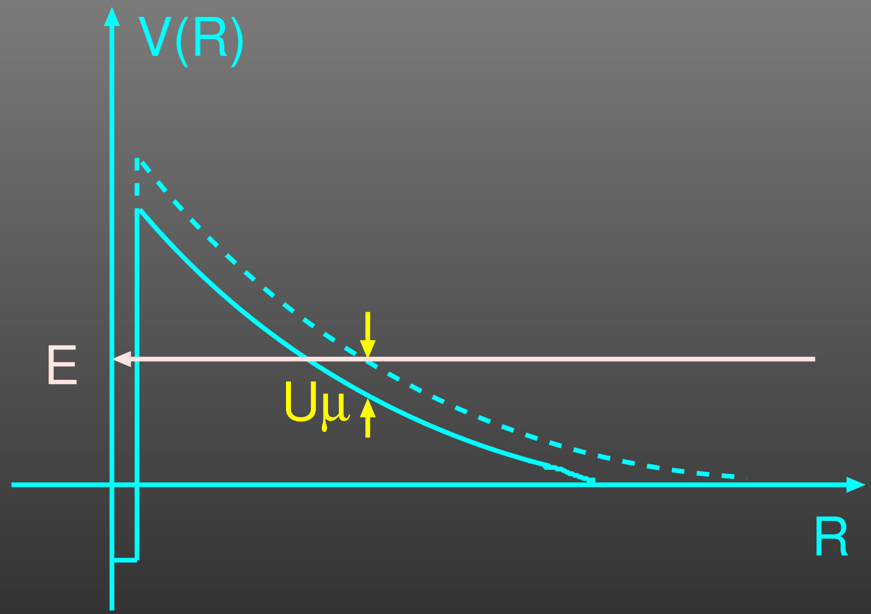
$\rho_{LH} = 4.25 \times 10^{22} cm^{-2}$
as a function of T

- Large Enhancement of the cross section by μ^-

$$\sigma(\mathsf{E}) = \mathsf{f}_\mu \sigma_0(\mathsf{E})$$

- Fluctuation of $f_\mu \Leftarrow$ Chaotic dynamics of the 3-body system

Screening Potential



U_μ : Screening Potential

$$f_\mu \equiv \frac{\sigma(E)}{\sigma_0(E)} = \frac{\sigma_0(E + U_\mu)}{\sigma_0(E)}$$

$$\sim \exp\{\pi\eta(E)\frac{U_\mu}{E}\}$$

$$U_\mu \sim \frac{E}{\pi\eta(E)} \log f$$

Constrained Molecular Dynamics (CoMD)

S.Kimura and A.Bonasera, Phys. Rev. A **72**, 014703 (2005)

Lagrange multiplier method for constraints

$$\mathcal{L} = \sum_{i} \frac{\mathbf{p}_i^2}{2m_i} - \sum_{i,j(\neq i)} U(\mathbf{r}_{ij}) + \sum_{i,j(\neq i)} \lambda_i \left(\frac{\mathbf{r}_{ij}\mathbf{p}_{ij}}{\xi \hbar} - 1 \right)$$

$$\mathbf{r}_{ij} = |\mathbf{r}_i - \mathbf{r}_j|; \;\; \mathbf{p}_{ij} = |\mathbf{p}_i - \mathbf{p}_j|$$

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Variational calculus leads Hamilton Equation with Constraint:

$$\frac{d\mathbf{r}_i}{dt} = \frac{\mathbf{p}_i}{m_i} + \frac{\lambda_i \mathbf{r}_{ij}}{\xi \hbar} \frac{\partial \mathbf{p}_{ij}}{\partial \mathbf{p}_i}$$

$$\frac{d\mathbf{p}_i}{dt} = -\nabla_{\mathbf{r}} U(\mathbf{r}_i) - \frac{\lambda_i \mathbf{p}_{ij}}{\xi \hbar} \frac{\partial \mathbf{r}_{ij}}{\partial \mathbf{r}_i}$$

Tunneling process

$$\frac{d\mathbf{r}_i}{dt} = \frac{\mathbf{p}_i}{m_i}; \qquad \frac{d\mathbf{p}_i}{dt} = -\nabla_{\mathbf{r}} U(\mathbf{r}_i)$$

Collective coordinates and momenta

$$\mathbf{R}^{coll} \equiv \mathbf{r}_P - \mathbf{r}_T; \quad \mathbf{P}^{coll} \equiv \mathbf{p}_P - \mathbf{p}_T; \quad \mathbf{F}_P^{coll} \equiv \dot{\mathbf{P}}^{coll}$$

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Collective coordinates and momenta

$$\mathbf{R}^{coll} \equiv \mathbf{r}_P - \mathbf{r}_T; \quad \mathbf{P}^{coll} \equiv \mathbf{p}_P - \mathbf{p}_T; \quad \mathbf{F}^{coll}_P \equiv \dot{\mathbf{P}}^{coll}$$

$$\frac{d\mathbf{r}^{\Im}_{T(P)}}{d\tau} = \frac{\mathbf{p}^{\Im}_{T(P)}}{m_{T(P)}}; \qquad \frac{d\mathbf{p}^{\Im}_{T(P)}}{d\tau} = -\nabla_{\mathbf{r}} U(\mathbf{r}^{\Im}_{T(P)}) - 2\mathbf{F}^{coll}_{T(P)}$$

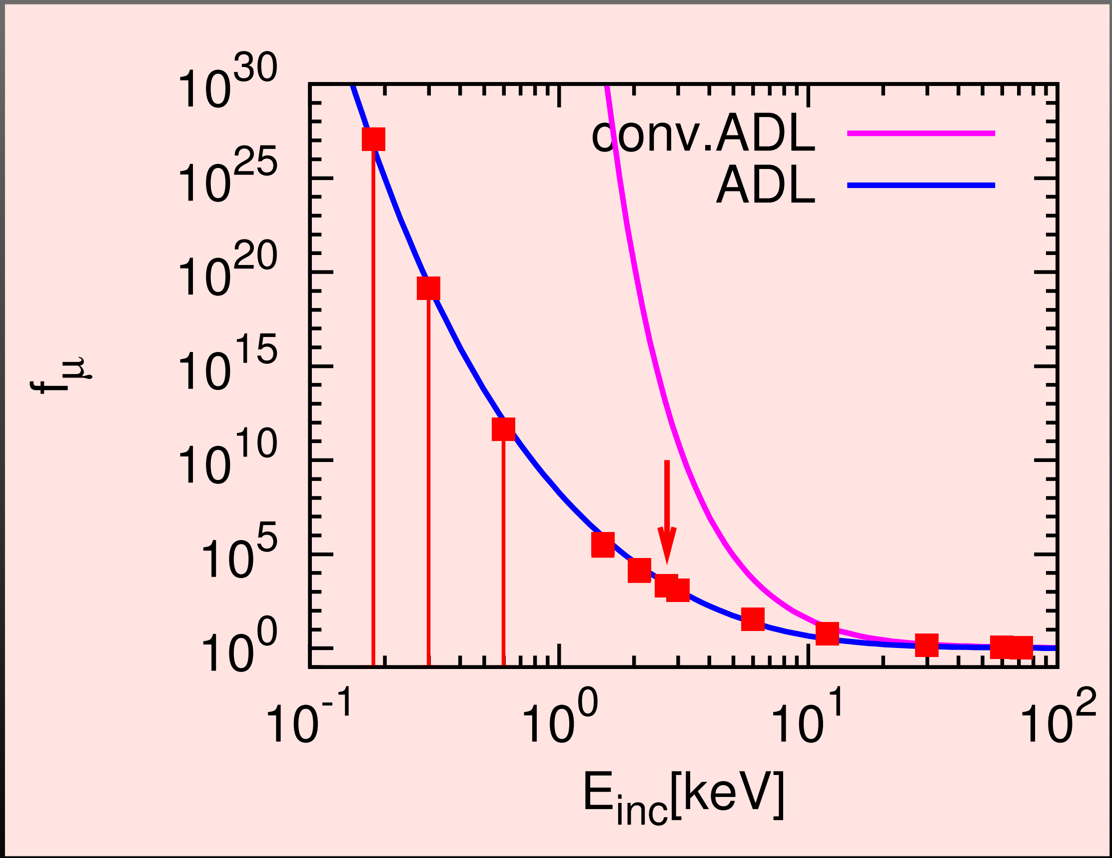
Tunneling penetrability: $\Pi(E) = \left(1 + \exp\left(2\mathcal{A}(E)/\hbar\right)\right)^{-1}$

$$\mathcal{A}(E) = \int_{r_b}^{r_a} \mathbf{P}^{coll}\, d\mathbf{R}^{coll}$$

without muon $\Rightarrow \Pi_0(E)$

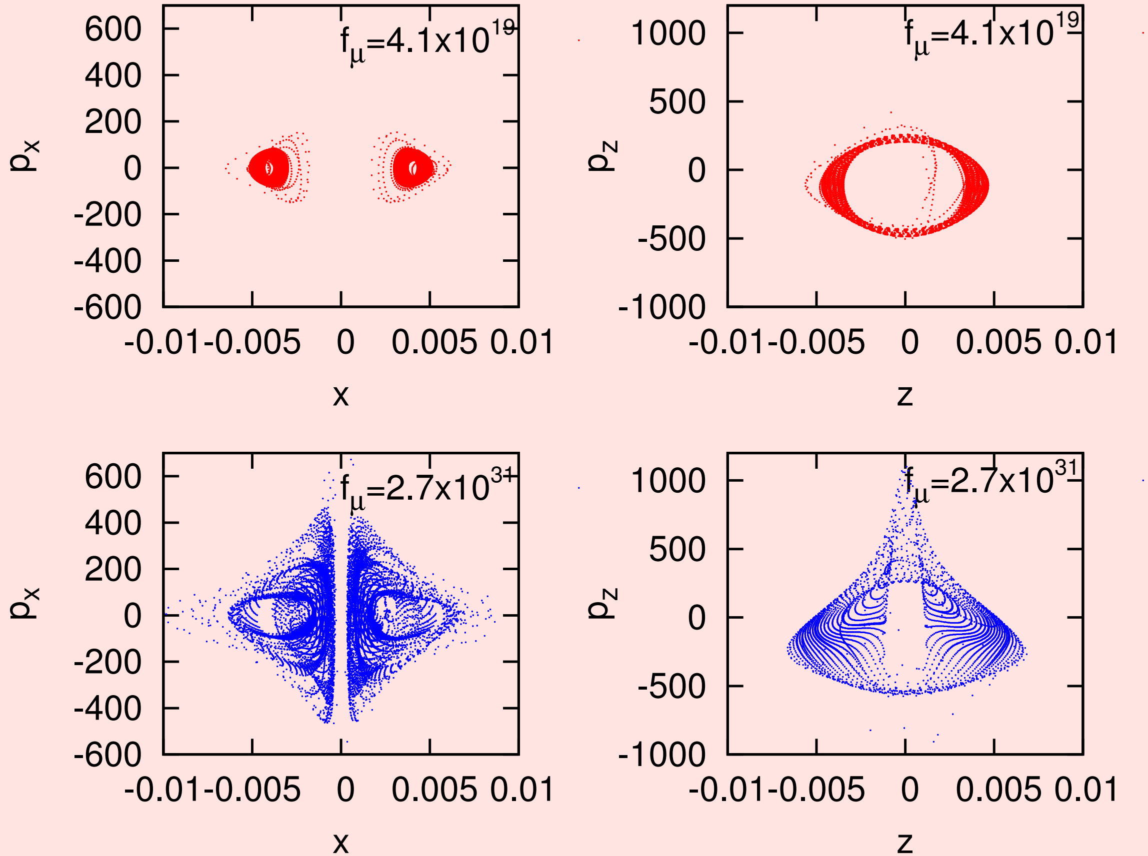
Enhancement factor: $f_\mu = \Pi(E)/\Pi_0(E)$

Enhancement factor
and
Variance $\Sigma = \sqrt{\overline{f_\mu^2} - (\overline{f_\mu})^2}$



Muonic Motion and Enhancement factor as an order parameter

Small f_μ: Regular

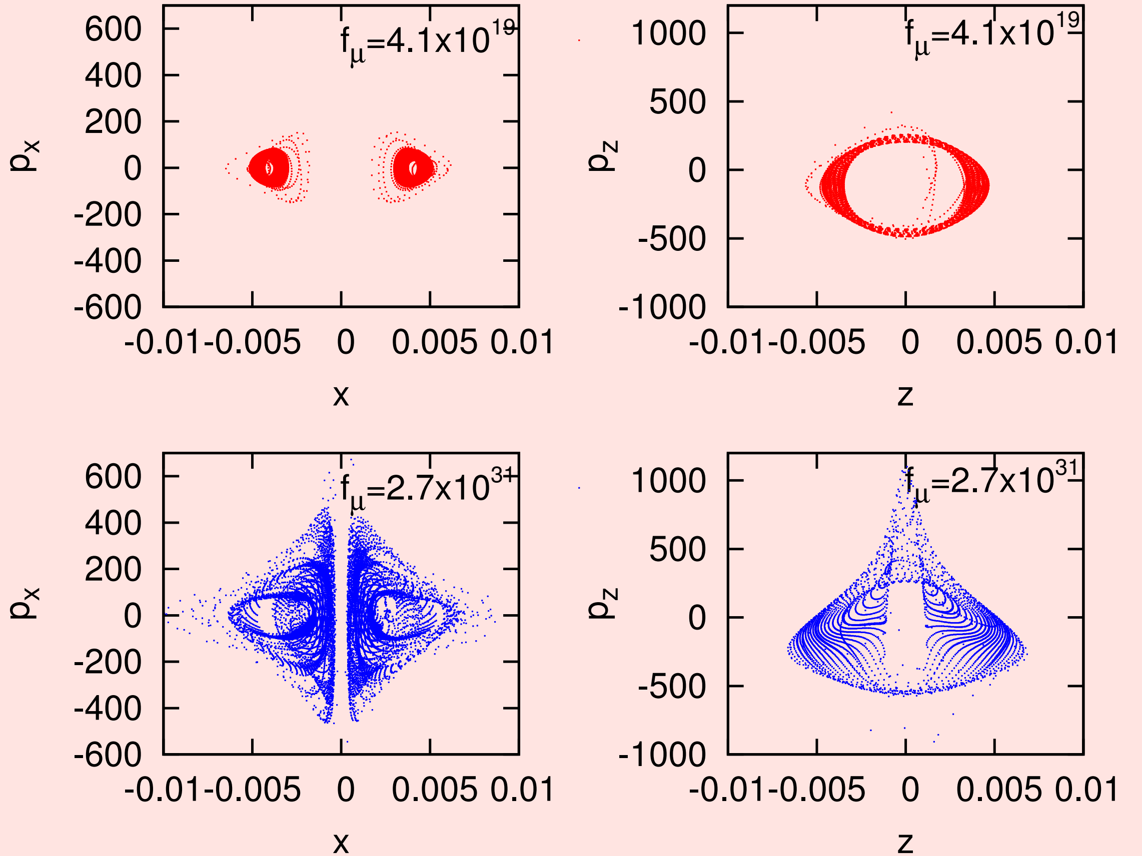


Large f_μ: Irregular(Chaotic)

Surface of Section

Muonic Motion and Enhancement factor as an order parameter

Small f_μ: Regular ($\Leftarrow$ Long TR)



Large f_μ: Irregular(Chaotic) ($\Leftarrow$ Short TR)

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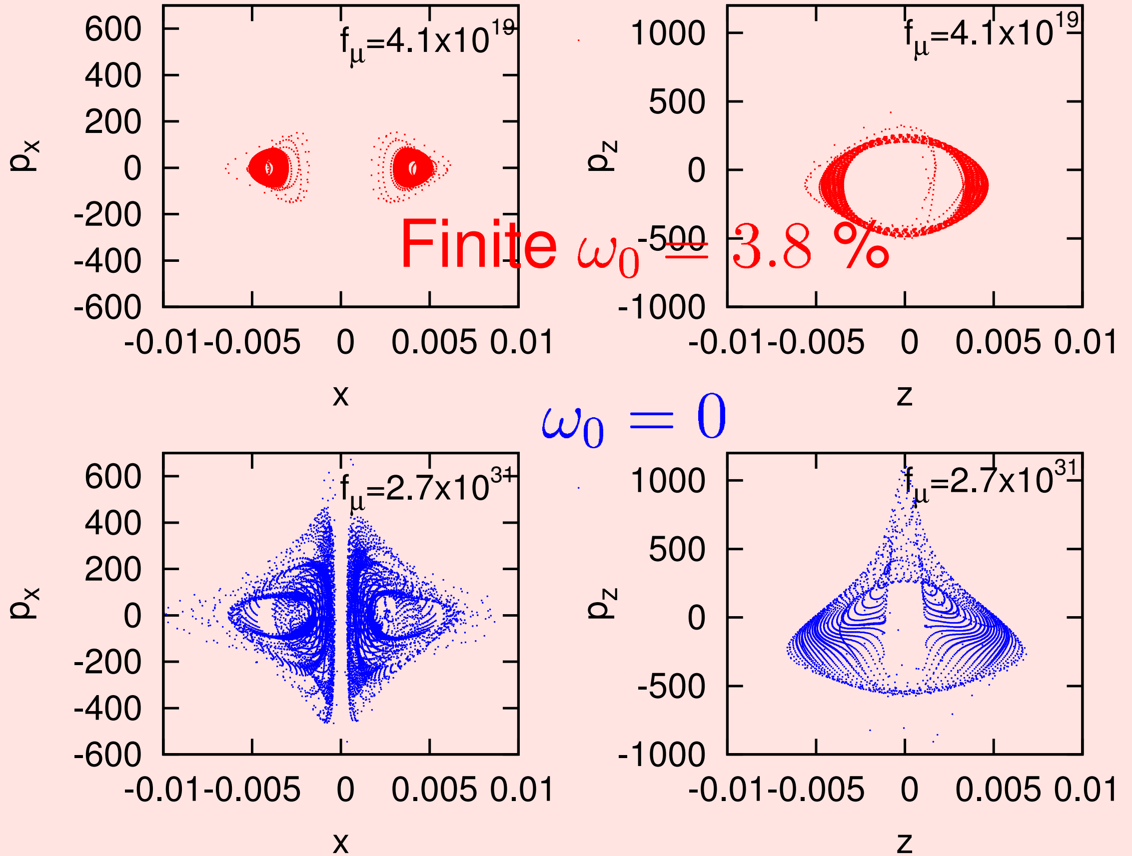

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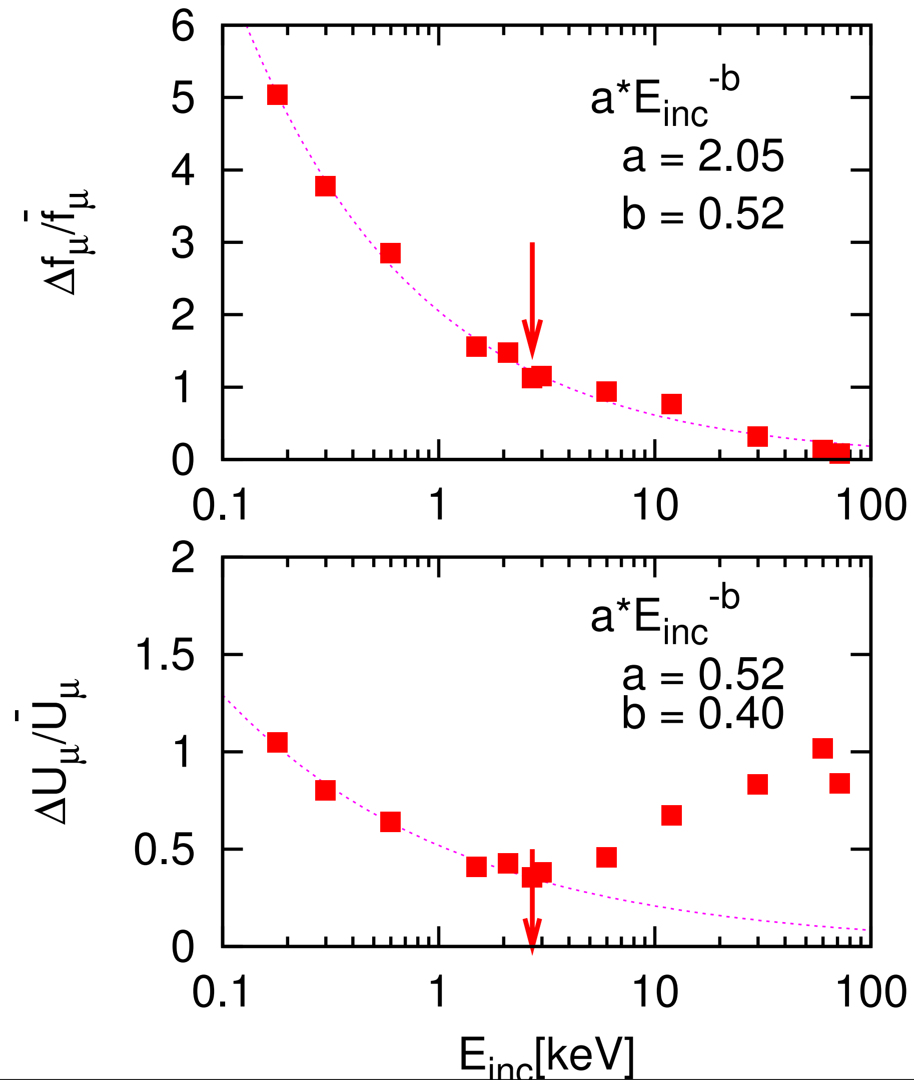

$a*E_{inc}^{-b}$
a = 2.05
b = 0.52
$\Delta f_\mu/f_\mu$
$a*E_{inc}^{-b}$
a = 0.52
b = 0.40
$\Delta U_\mu/U_\mu$
E_{inc}[keV]

Reaction Rate

$$\lambda = \rho < \sigma v >$$
$$= \rho \int \sigma_0(\mathsf{E} + \mathsf{U}_\mu) v \Psi(\mathsf{E}, \mathsf{T}) \mathsf{dE}$$

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$$= \rho \int \sigma_0(\mathsf{E} + \mathsf{U}_\mu) v \Psi(\mathsf{E}, \mathsf{T}) d\mathsf{E}$$

$$\sigma_0(\mathsf{E}) = \frac{\mathsf{S}(\mathsf{E})}{\mathsf{E}} e^{-2\pi\eta(\mathsf{E})}$$
$$\mathsf{S}(\mathsf{E}) = 0.20 - 0.32\mathsf{E} + 0.476\mathsf{E}^2 [\mathsf{MeVb}]$$
$$(\text{for the t} + \text{t reaction})$$

S. Winkler et al., J. Phys. G **18**(1991) L147

Reaction Rate

$$\begin{aligned}\lambda &= \rho < \sigma v > \\ &= \rho \int \sigma_0(\mathsf{E}+\mathsf{U}_\mu) v \Psi(\mathsf{E},\mathsf{T}) \mathsf{N}(\mathsf{U}_\mu) \mathsf{dU}_\mu \mathsf{dE}\end{aligned}$$

$$\begin{aligned}\sigma_0(\mathsf{E}) &= \frac{\mathsf{S(E)}}{\mathsf{E}} e^{-2\pi\eta(\mathsf{E})} \\ \mathsf{S(E)} &= 0.20 - 0.32\mathsf{E} + 0.476\mathsf{E}^2[\mathsf{MeVb}] \\ & (\text{for the t + t reaction})\end{aligned}$$

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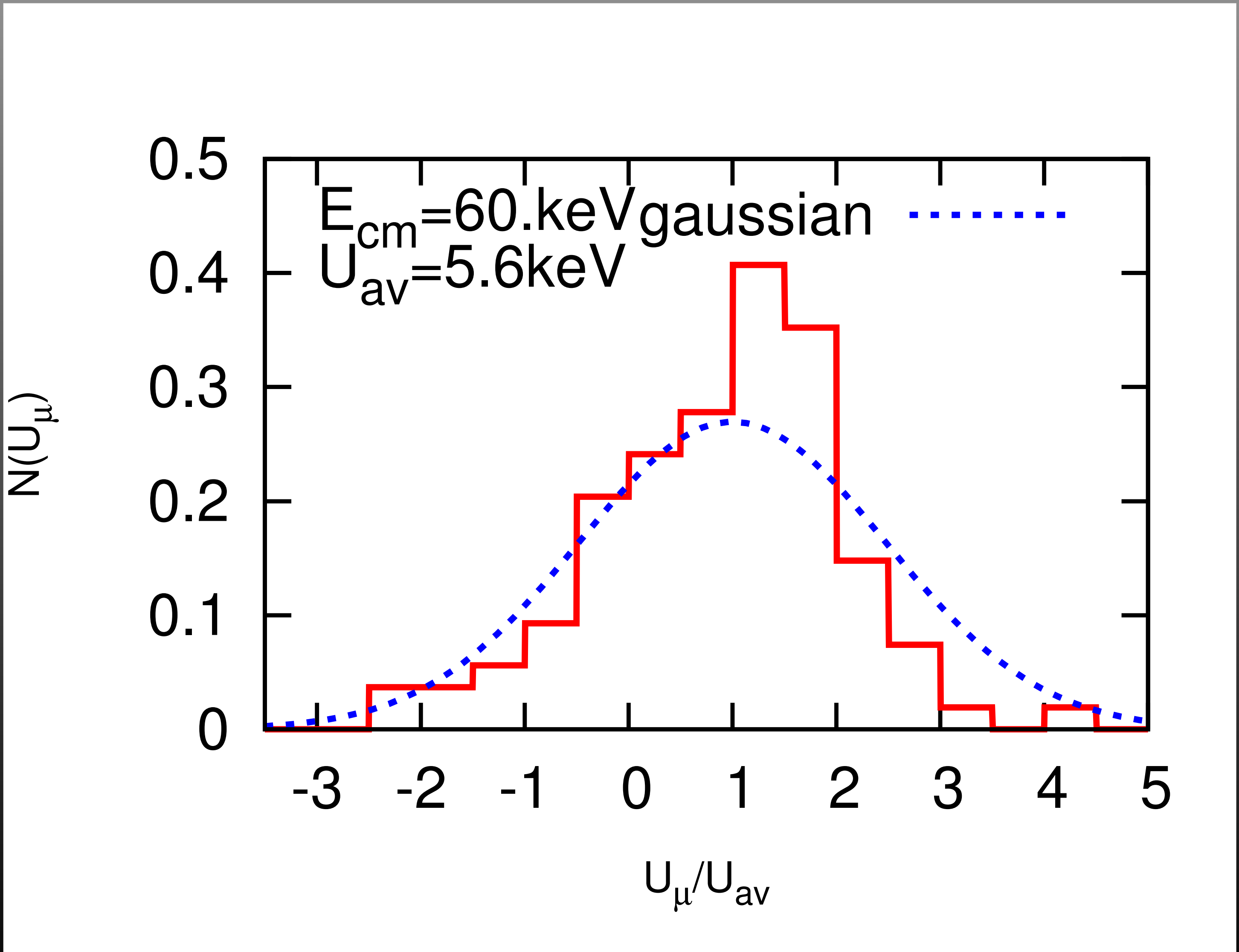
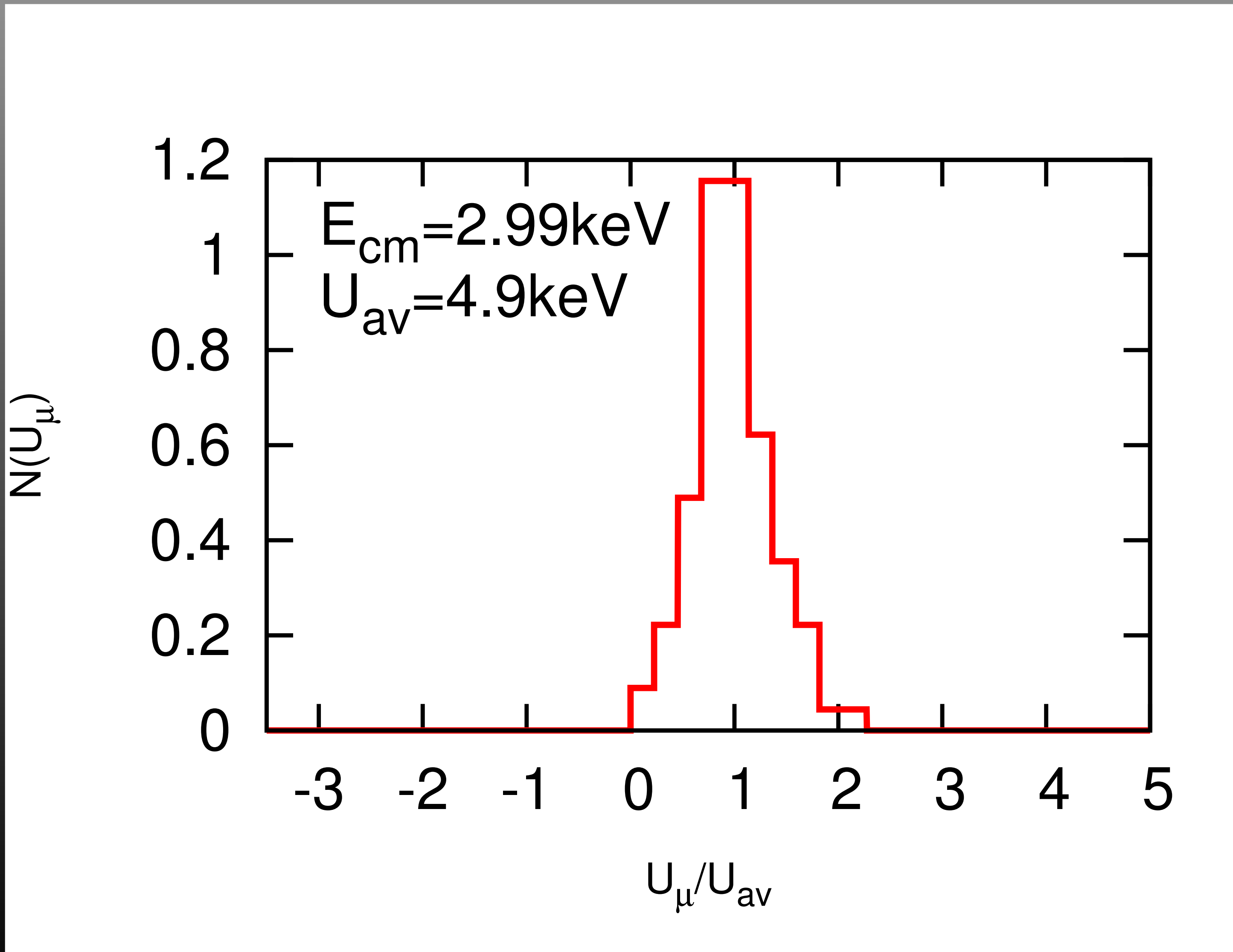
Distributions of U_μ



Distributions of U_μ



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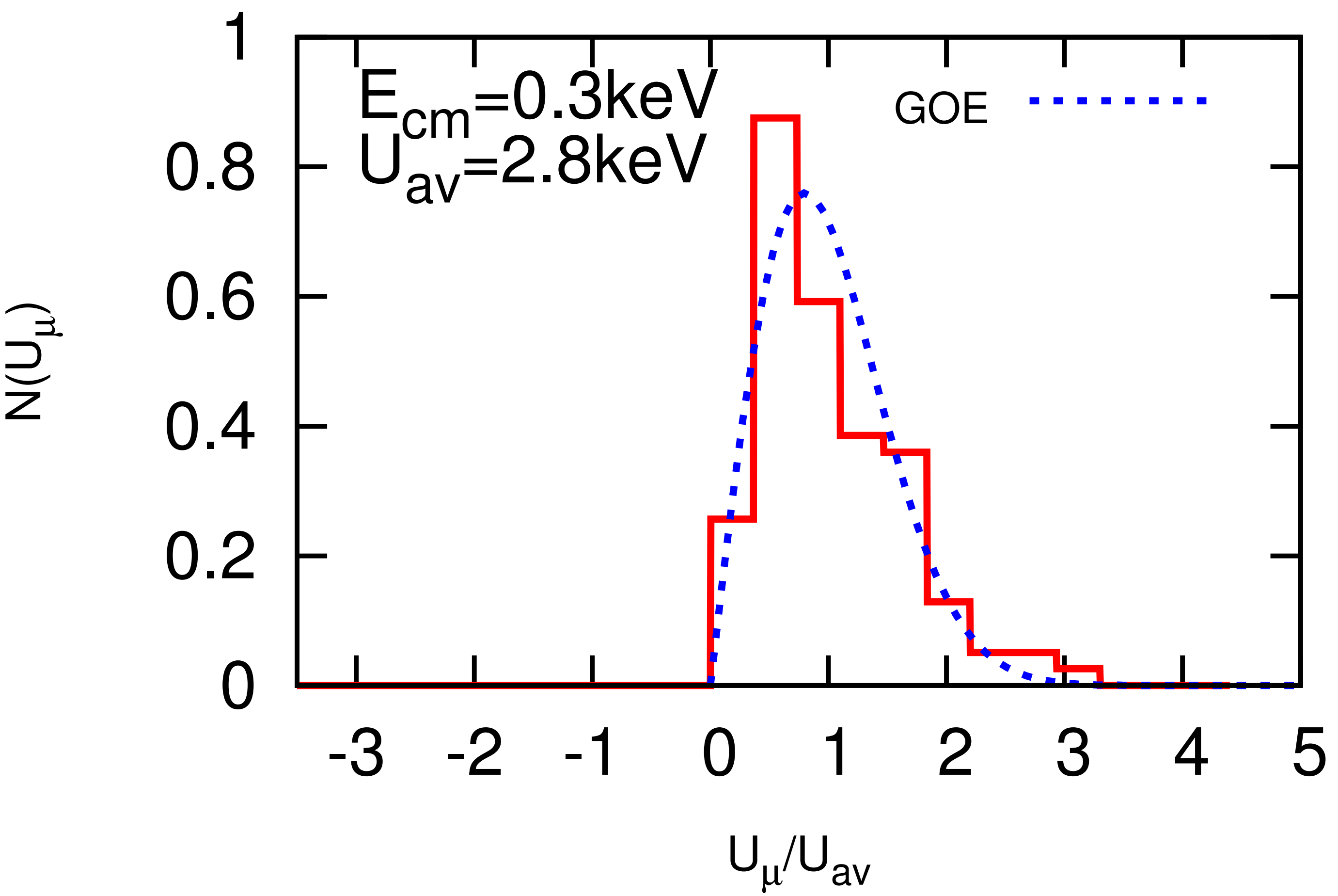

Reaction Rate

$$\lambda = \rho \int \sigma_0(\mathsf{E} + \mathsf{U}_\mu) v \Psi(\mathsf{E}, \mathsf{T}) \; \mathsf{N}(\mathsf{U}_\mu) \; d\mathsf{E} d\mathsf{U}_\mu$$

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$$\lambda = \rho \int \sigma_0(\mathsf{E} + \mathsf{U}_\mu) v \Psi(\mathsf{E}, \mathsf{T}) \, \mathsf{N}(\mathsf{U}_\mu) \, \mathsf{dE} \mathsf{dU}_\mu$$

$$\mathsf{N}(\mathsf{U}_\mu) = \frac{1}{\sqrt{2\pi}\Delta \mathsf{U}_\mu} \exp\left(-\frac{(\mathsf{U}_\mu - \bar{\mathsf{U}}_\mu)^2}{2\Delta \mathsf{U}_\mu^2}\right) \quad (\mathsf{E} > \mathsf{I.E.})$$

$$= \frac{\pi}{2} \left(\frac{\mathsf{U}_\mu}{\bar{\mathsf{U}}_\mu}\right) \exp\left(-\frac{\pi}{4}\left(\frac{\mathsf{U}_\mu}{\bar{\mathsf{U}}_\mu}\right)^2\right) \quad (\mathsf{E} < \mathsf{I.E.})$$

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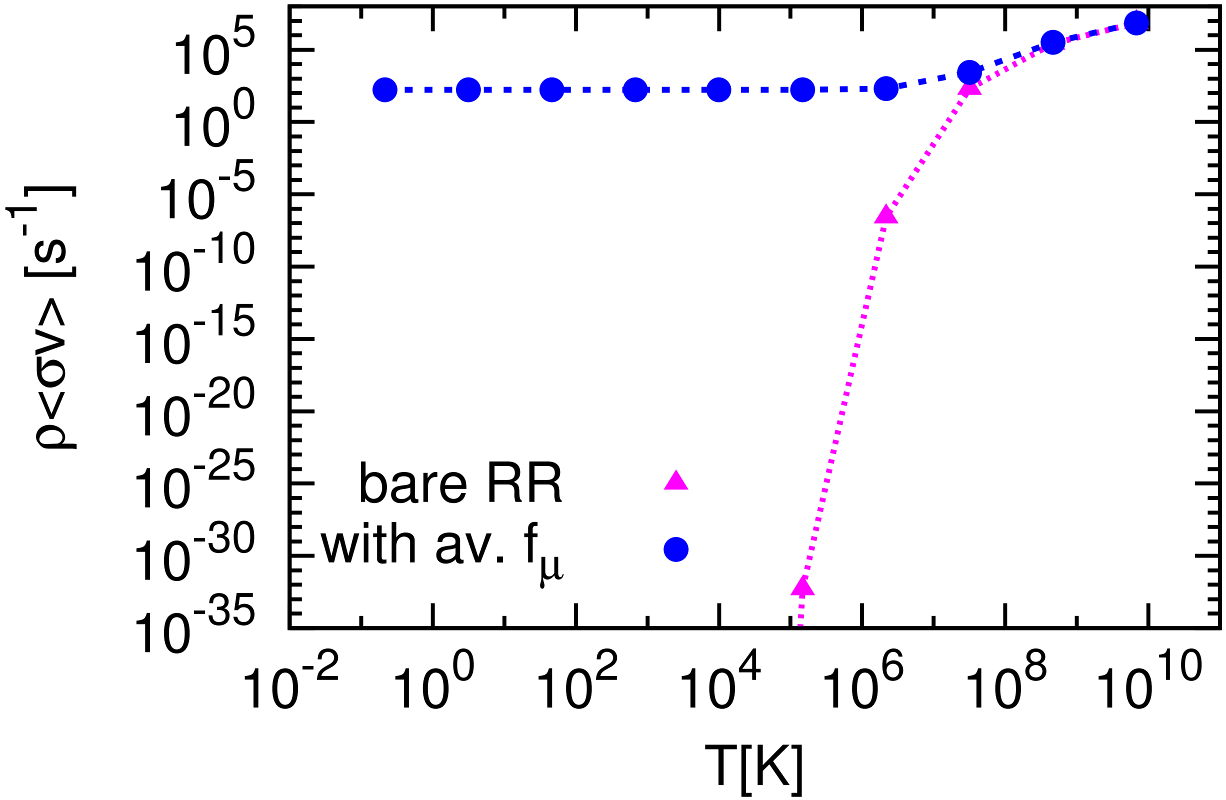
Reaction Rate of the t+t



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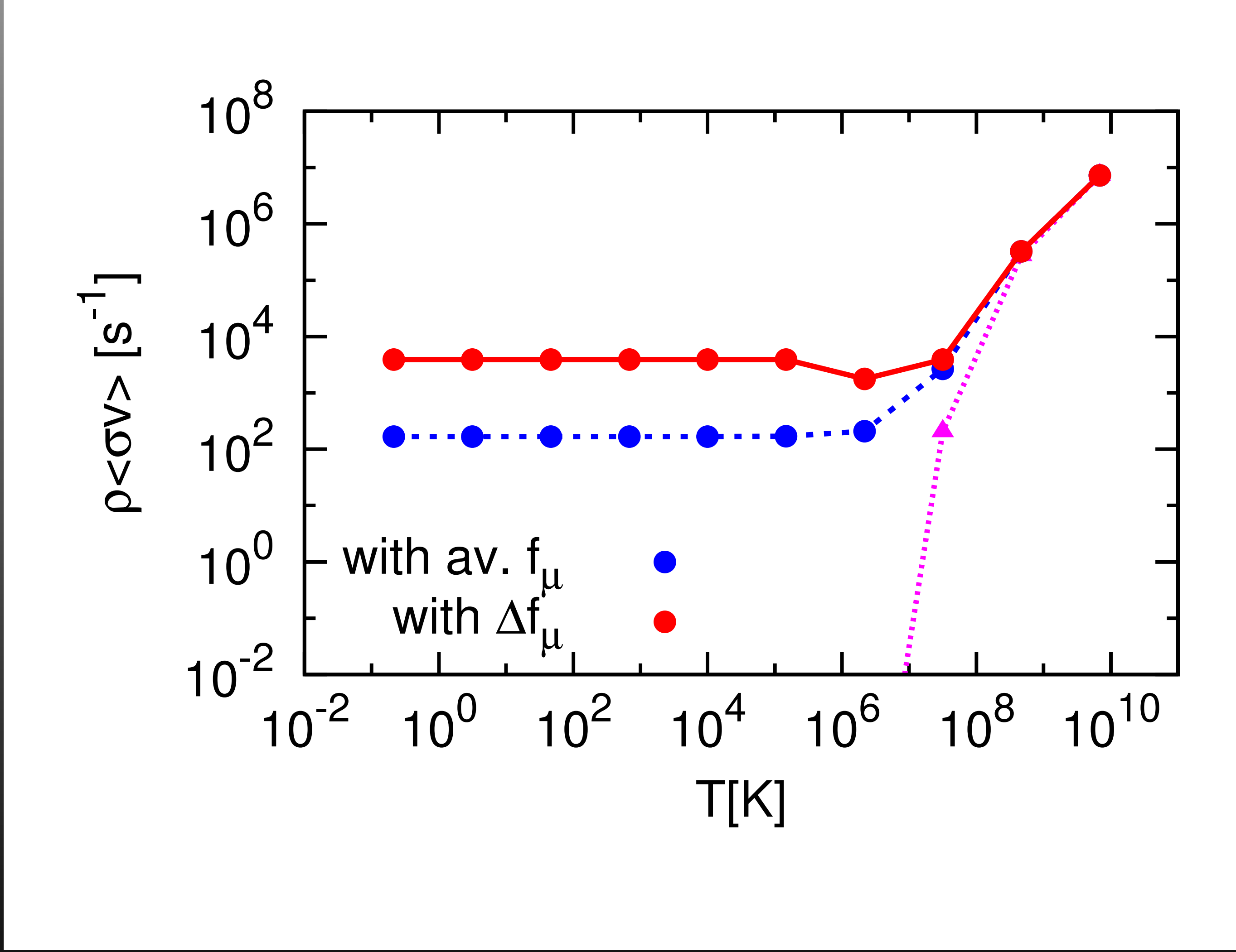
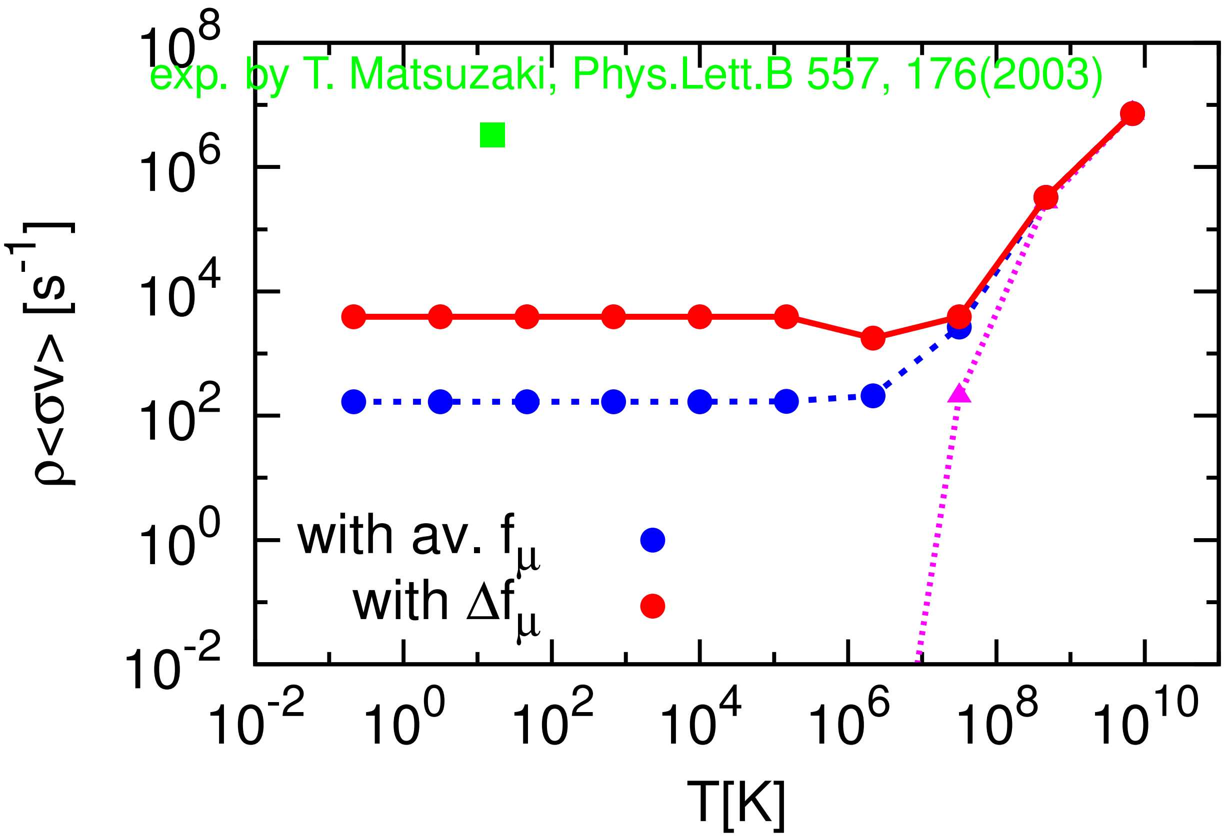


Reaction Rate of the t+t



Conclusions

Importance of Fluctuations of cross section in muon-catalysed t-t fusion

- Reaction rate and μ cycling rate
- Numerical simulation by the CoMD
- A characteristic change of the slope of $\Delta \mathrm{U}_\mu / \bar{\mathrm{U}}_\mu$ at the ionization energy of the μ molecule.
- Fluctuation of σ contributes to enhance the RR
- Obtained RR has no T dependence in the low T region, as expected. However it is smaller than exp. μ cycling rate