

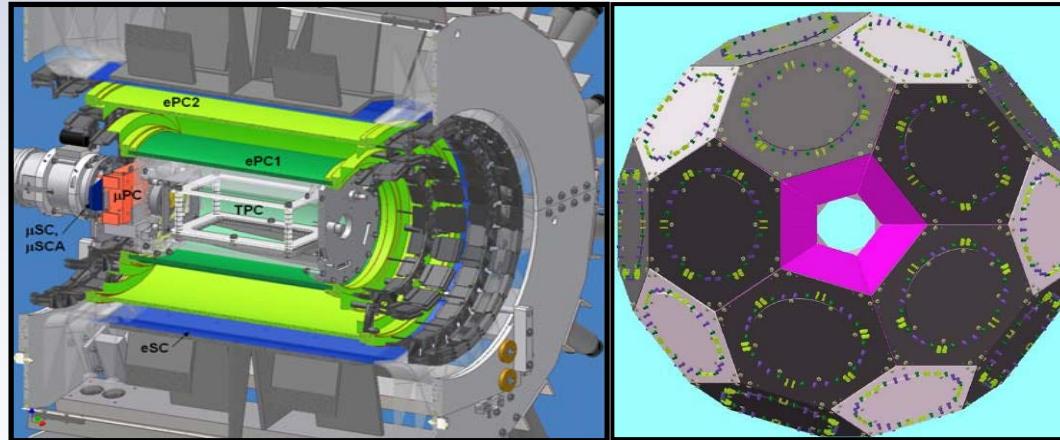
Nuclear Muon Capture in Hydrogen and its Interplay with Muon Atomic Physics

Peter Kammel



g_P

MuCap

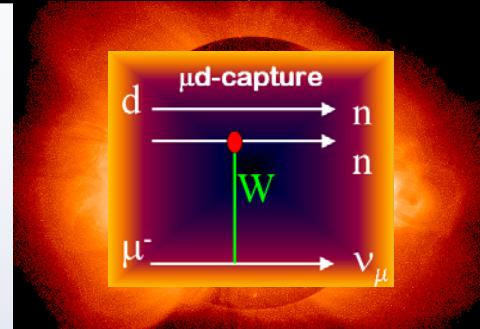


G_F

MuLan

L_{1A}

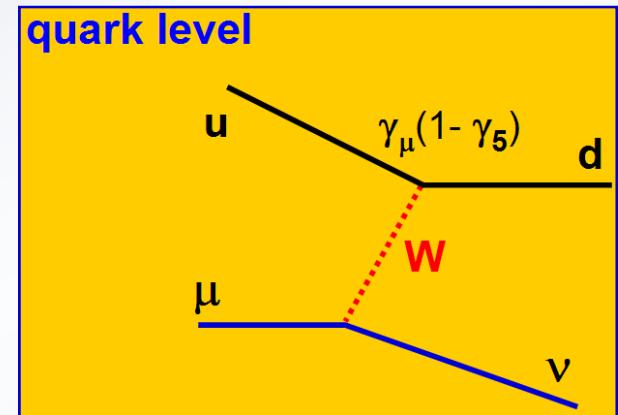
“MuSun”
project



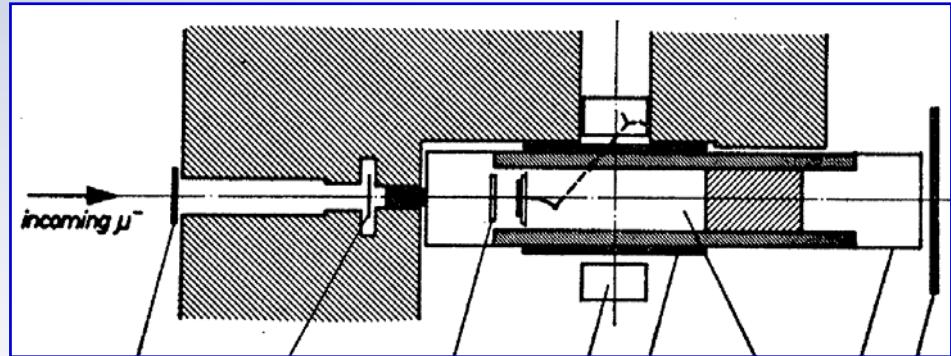
➤ Historical: V-A and μ -e Universality

$$\mu^- + p \rightarrow \nu_\mu + n$$

charged current



My Talk is Dedicated to the Pioneers



Emilio Zavattini 1927-2007

1969 Bologna-Pisa-CERN



$$\mu^- + p \rightarrow \nu_\mu + n$$

1973 Dubna group

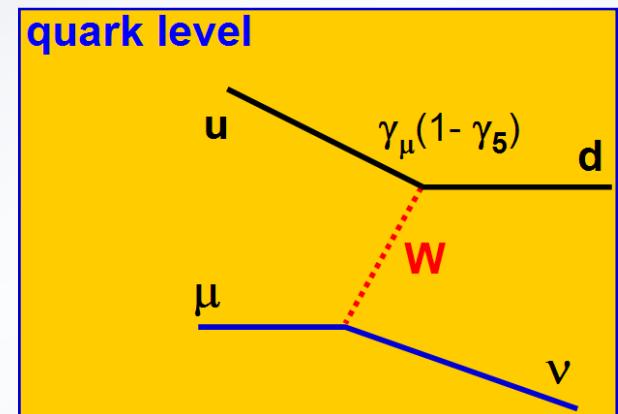




➤ Historical: V-A and μ -e Universality

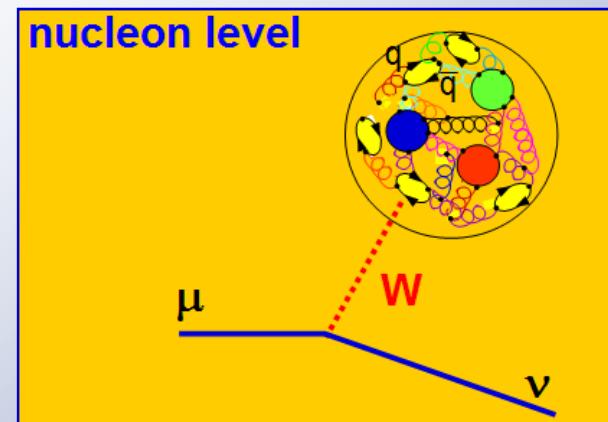
$$\mu^- + p \rightarrow \nu_\mu + n$$

charged current



➤ Today: EW current key probe for

- Understanding hadrons from fundamental QCD
- Symmetries of Standard Model
- Basic astrophysics reactions



QCD

$$\mathcal{L}_{\text{QCD}} = \bar{\psi} (i\gamma_\mu \mathcal{D}^\mu - \mathbf{m}) \psi - \frac{1}{4} \mathbf{G}_{\mu\nu} \mathbf{G}^{\mu\nu}$$



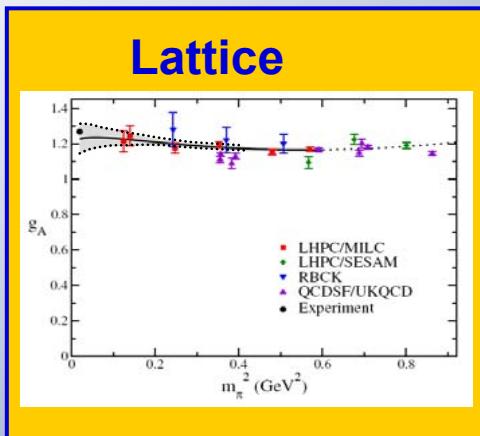
- high q^2 ($q > \text{some GeV}$) short distance $< 0.1 \text{ fm}$

**Weakly interacting quarks and gluons
asymptotic freedom**

- low q^2 ($q \ll 1 \text{ GeV}$) long distance $> 1 \text{ fm}$

**QCD has chiral symmetry
spontaneously broken
 π is Nambu-Goldstone boson, weakly interacting
chiral effective theory** \leftrightarrow **Nuclear Physics**

- **Lattice QCD: ab initio calculations**
issues: continuum transition, etc.
physical quark masses not reached



Edwards et al. *LHPC Coll (2006)*





➤ Muon Capture

$\mu^- + p \rightarrow \nu_\mu + n$ rate Λ_S at $q^2 = -0.88 m_\mu^2$

$$\mathcal{M} = \frac{-iG_F V_{ud}}{\sqrt{2}} \bar{u}(p_\nu) \gamma_\alpha (1 - \gamma_5) u(p_\mu) \bar{u}(p_f) \tau_- [V^\alpha - A^\alpha] u(p_i)$$

➤ Formfactors

Lorentz, T invariance

$$V_\alpha = g_V(q^2) \gamma_\alpha + \frac{i g_M(q^2)}{2 M_N} \sigma_{\alpha\beta} q^\beta$$

$$A_\alpha = g_A(q^2) \gamma_\alpha \gamma_5 + \frac{g_P(q^2)}{m_\mu} q_\alpha \gamma_5$$

+ second
class currents
suppressed by
isospin symm.

All form factors precisely known from
SM symmetries and data.

CVC, n beta decay

$$\frac{\delta \Lambda_S}{\Lambda_S} = 0.46\%$$

$$\frac{\delta \Lambda_S}{\Lambda_S} = 0.184 \frac{\delta g_P}{g_P} \approx 9\%$$

apart from $g_P = 8.3 \pm 50\%$





g_P determined by chiral symmetry of QCD:

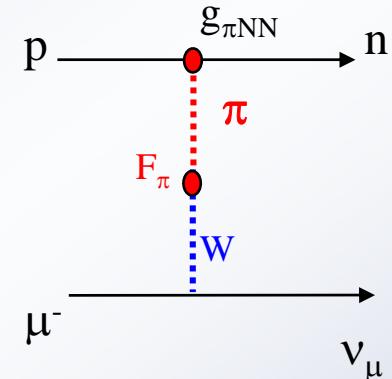
$$g_p(q^2) = \frac{2m_\mu g_{\pi NN}(q^2)F_\pi}{m_\pi^2 - q^2} - \frac{1}{3}g_a(0)m_\mu m_N r_A^2$$

$$g_P = (8.74 \pm 0.23) - (0.48 \pm 0.02) = \mathbf{8.26 \pm 0.23}$$

PCAC pole term Adler, Dothan, Wolfenstein

ChPT leading order one loop two-loop <1%

N. Kaiser Phys. Rev. C67 (2003) 027002



- g_P basic and experimentally least known EW nucleon form factor
- solid QCD prediction via HBChPT (2-3% level)
- basic test of QCD symmetries

Recent reviews:

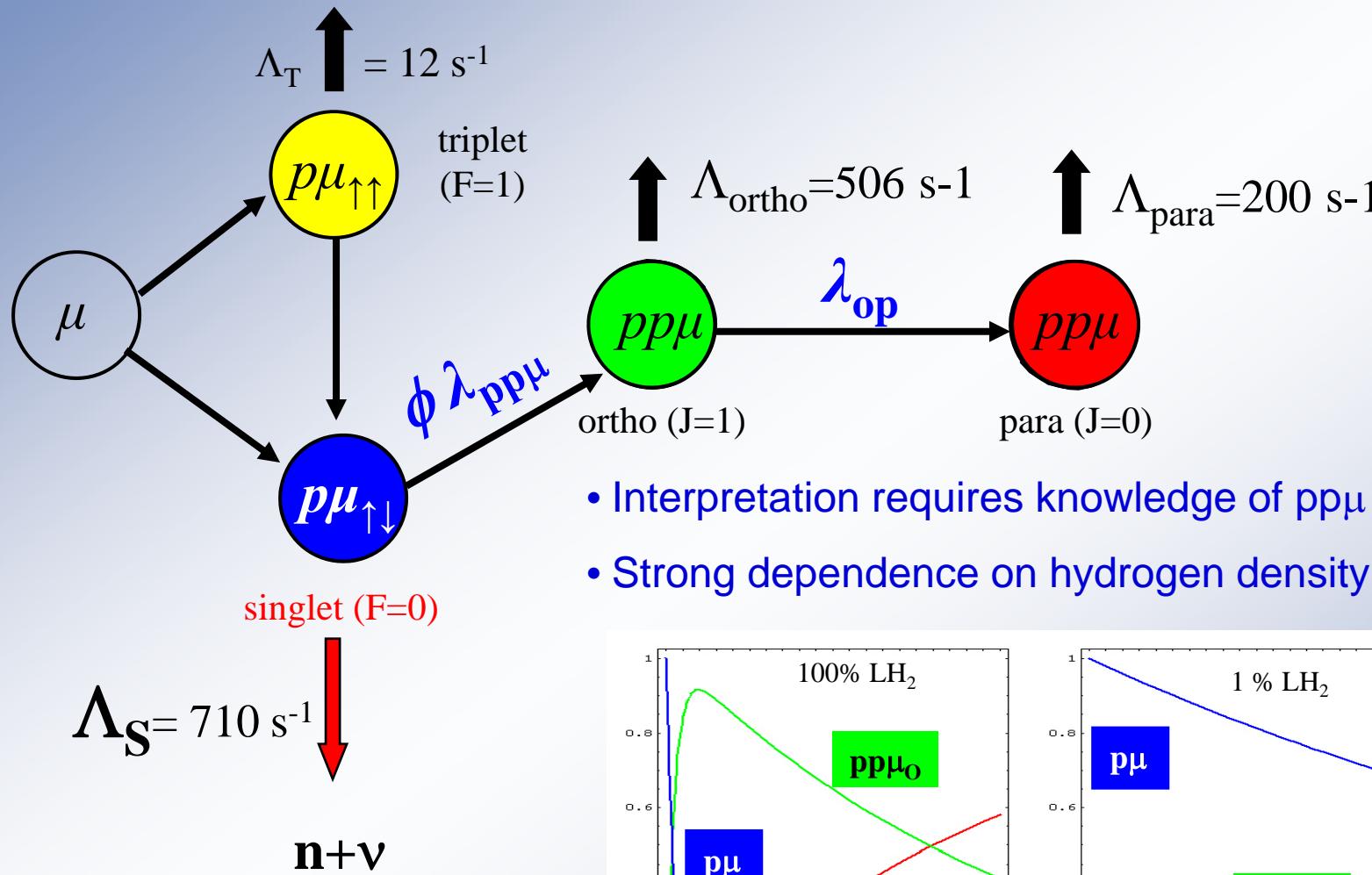
T. Gorringe, H. Fearing, Rev. Mod. Physics 76 (2004) 31

V. Bernard et al., Nucl. Part. Phys. 28 (2002), R1

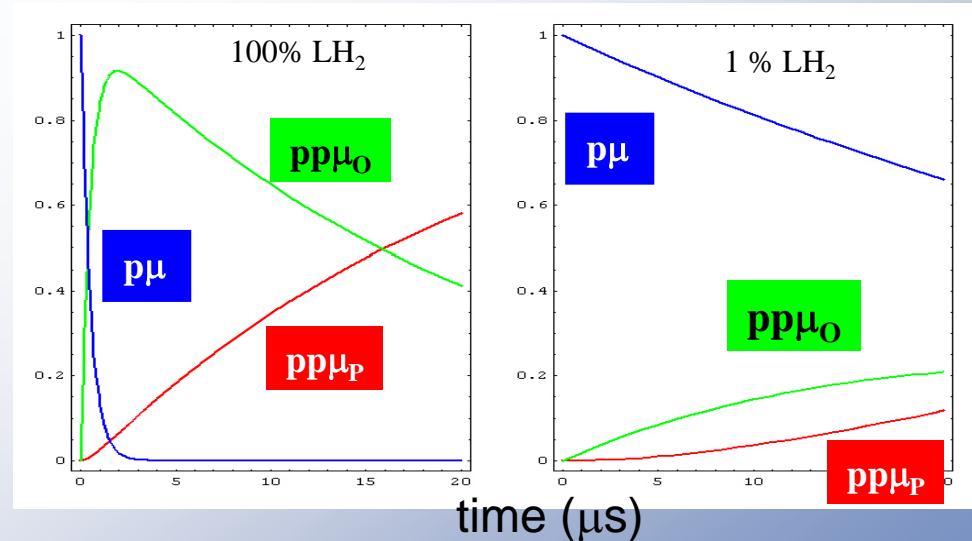


Muonic Processes Complicate Interpretation

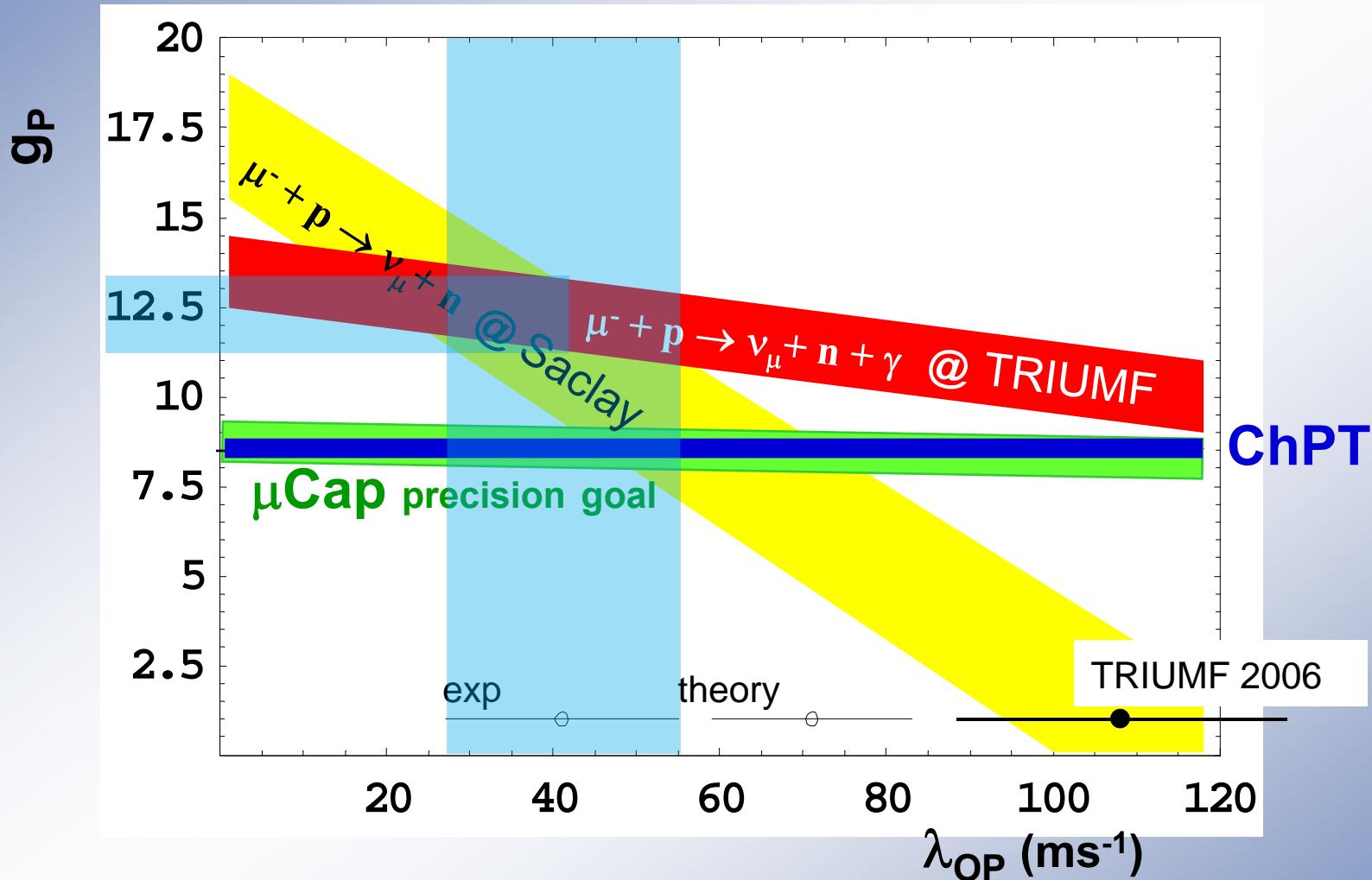
I.



- Interpretation requires knowledge of $pp\mu$ population
- Strong dependence on hydrogen density ϕ



Precise Theory vs. 45 Years of Exp. Efforts I.



- no overlap theory & OMC & RMC
- large uncertainty in $\lambda_{OP} \rightarrow g_P \pm 50\% ?$

- Lifetime method

$10^{10} \mu \rightarrow e\bar{v}\bar{v}$ decays

measure τ_{μ^-} to 10 ppm,

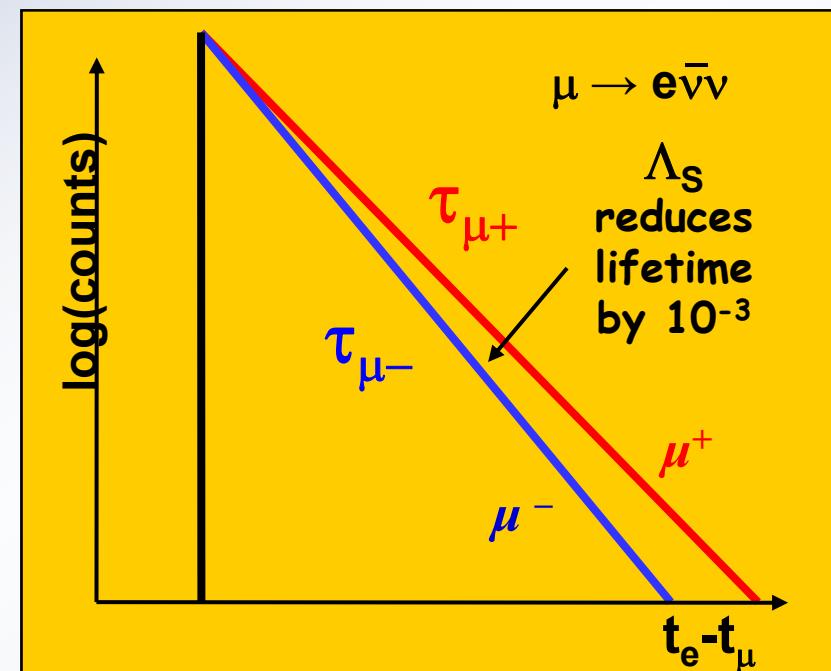
$$\rightarrow \Lambda_S = 1/\tau_{\mu^-} - 1/\tau_{\mu^+} \text{ to } 1\%$$

- Unambiguous interpretation at 1% LH_2 density

- Clean μ stop definition in active target (TPC) to avoid wall stops

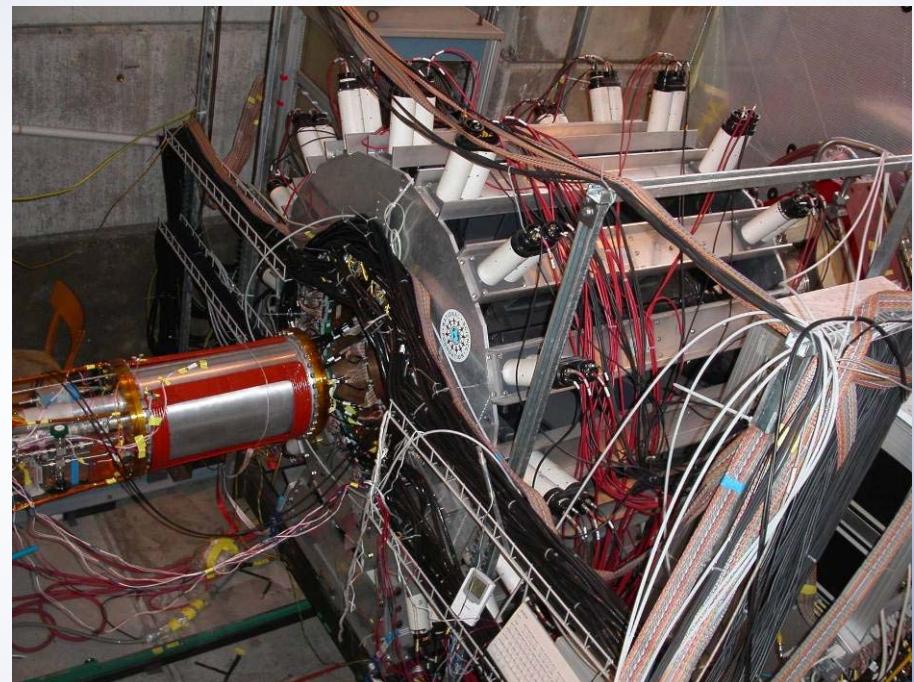
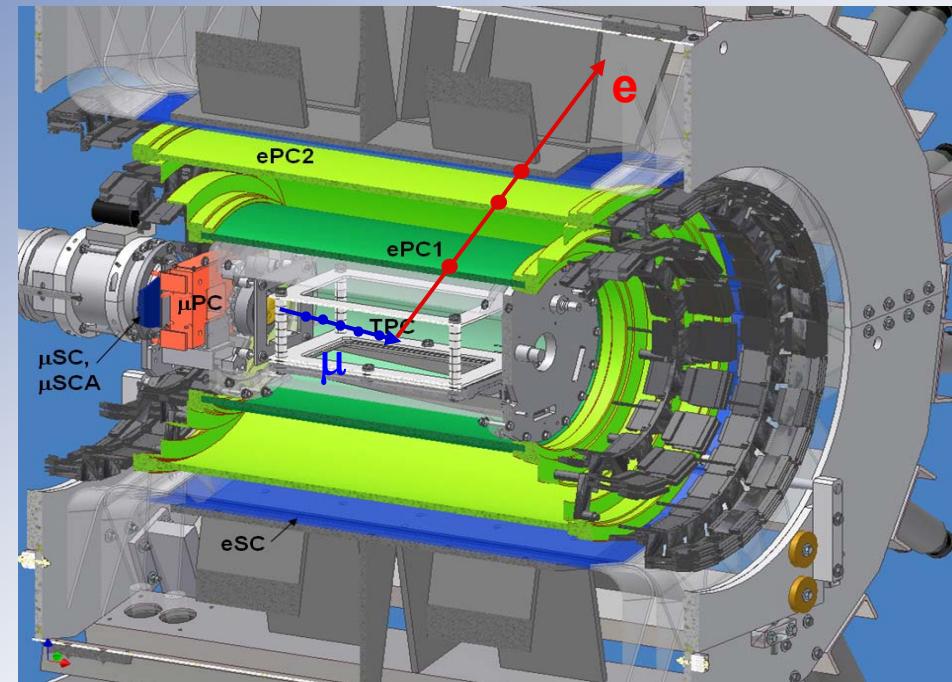
- Ultra-pure gas system and purity monitoring at 10 ppb level

- Isotopically pure “protium”

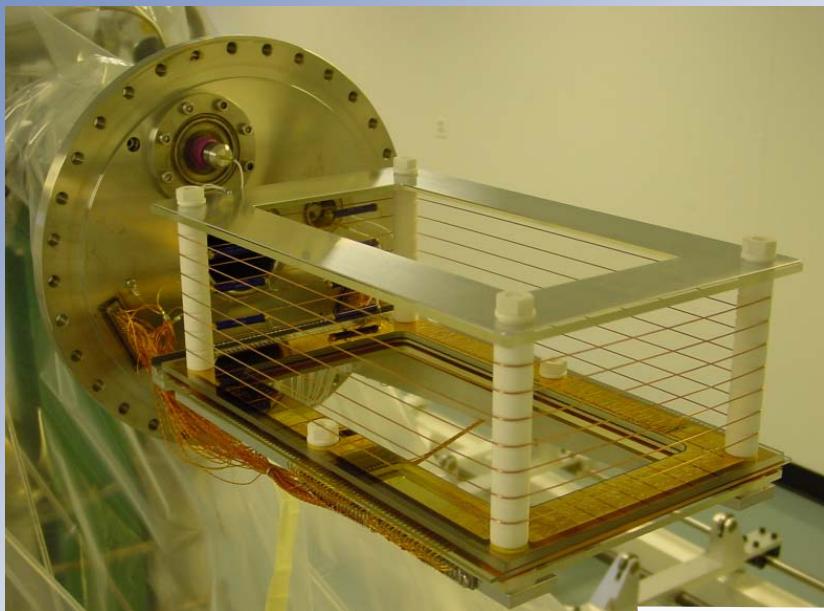


*fulfill all requirements simultaneously
unique MuCap capabilities*

MuCap Detector

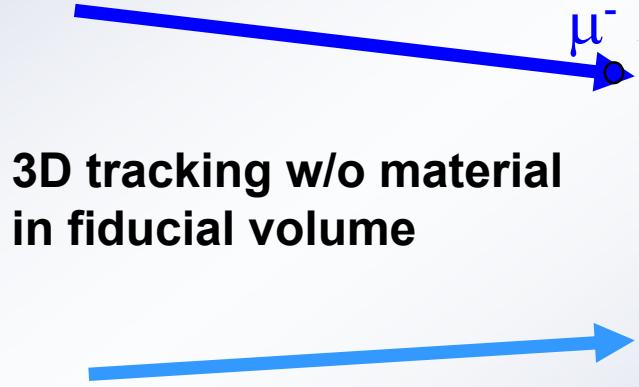


MuCap Muons stop in active TPC target

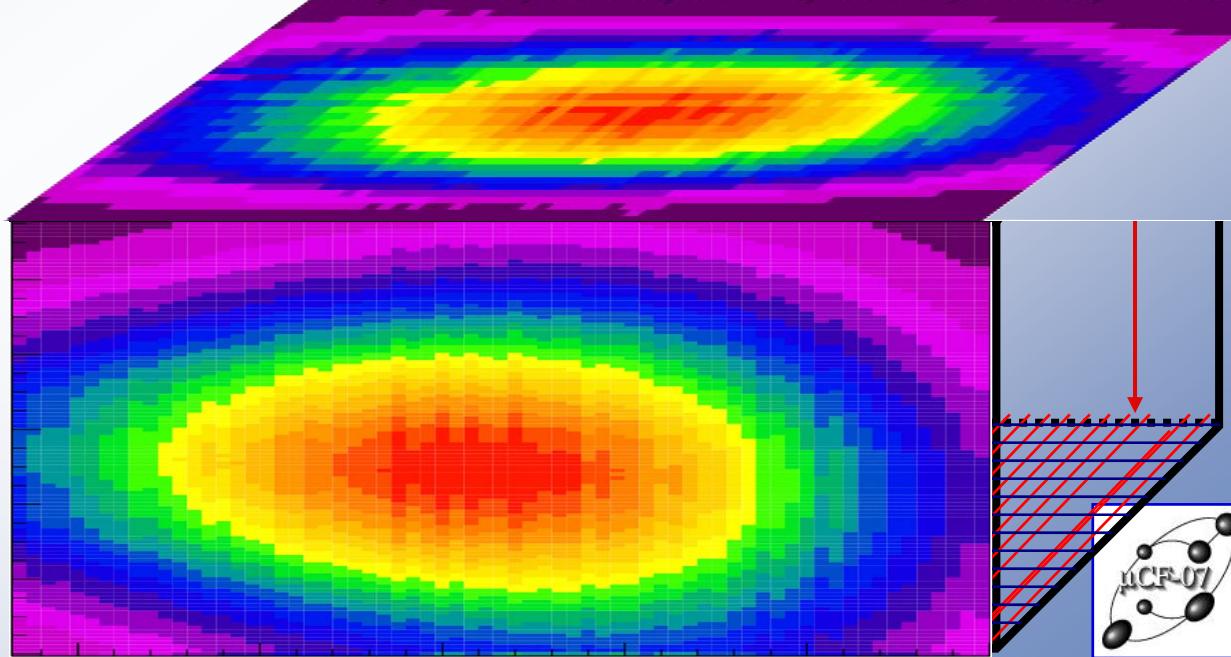


10 bar ultra-pure hydrogen, 1.16% LH₂
2.0 kV/cm drift field
~5.4 kV on 3.5 mm anode half gap
bakeable glass/ceramic materials

Observed muon stopping distribution



3D tracking w/o material
in fiducial volume



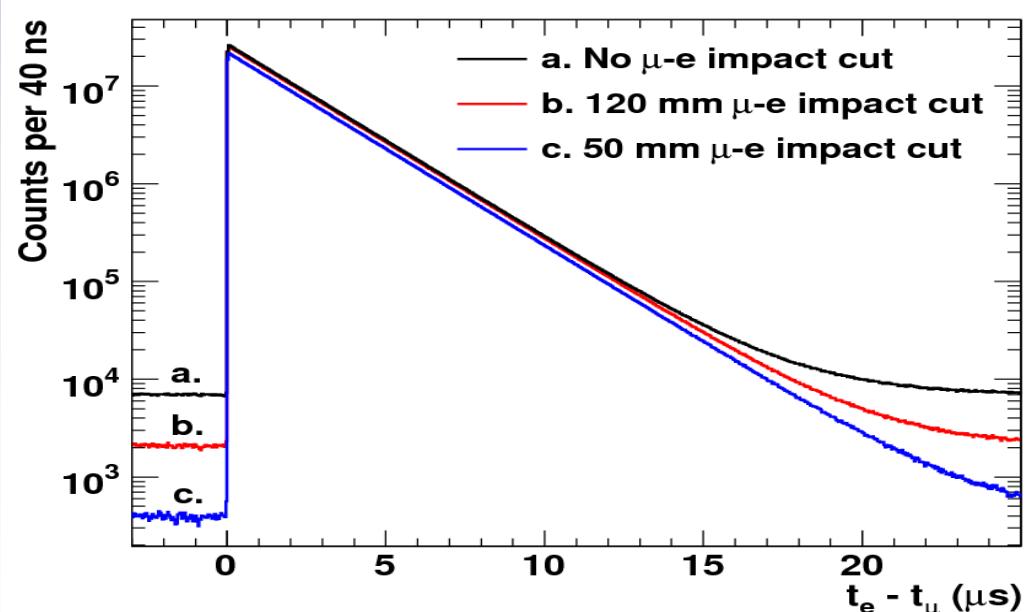


μ -e impact parameter cut

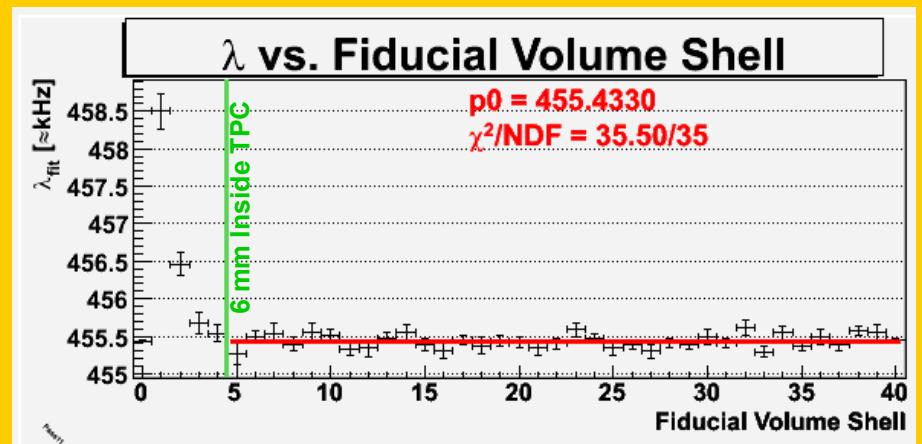
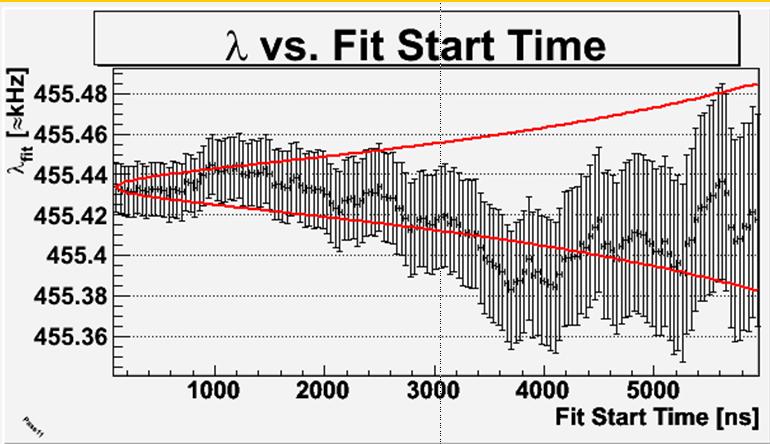
huge background suppression

diffusion (deuterium) monitoring

*blinded master
clock frequency*



variety of consistency checks



MuCap Unique Capabilities: Impurities

rare impurity capture $\mu Z \rightarrow (Z-1) + n + \nu$

$$\Lambda_Z (\text{C, N, O}) \sim (40-100) \times \Lambda_s$$

~10 ppb purity required

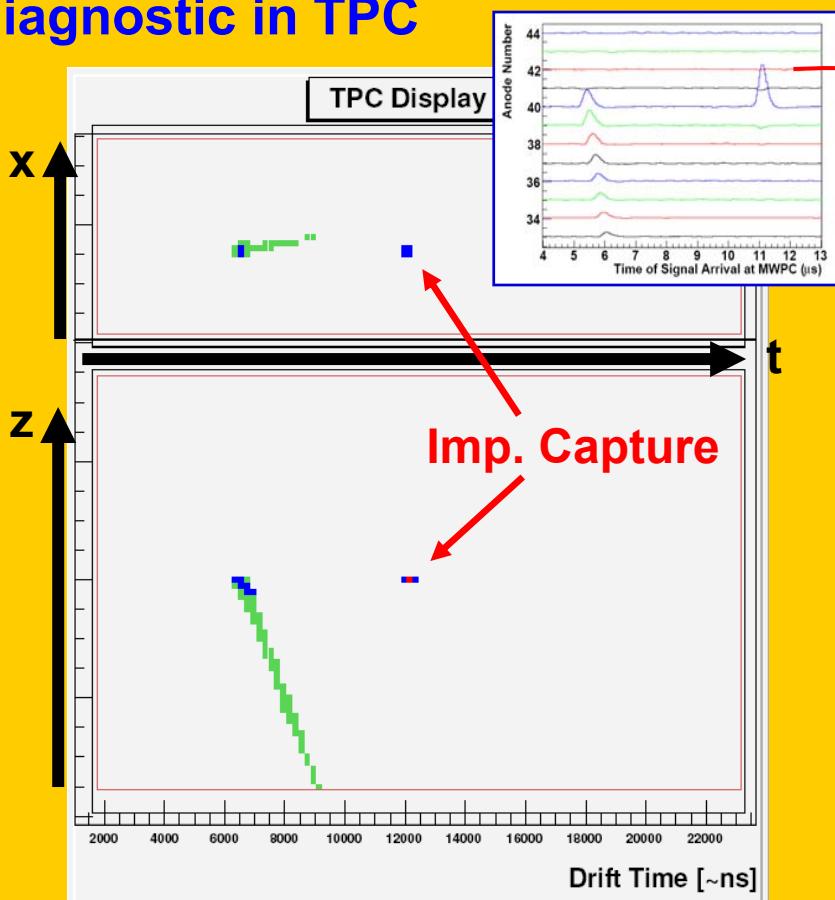
Hardware

Circulating Hydrogen Ultrahigh Purification System

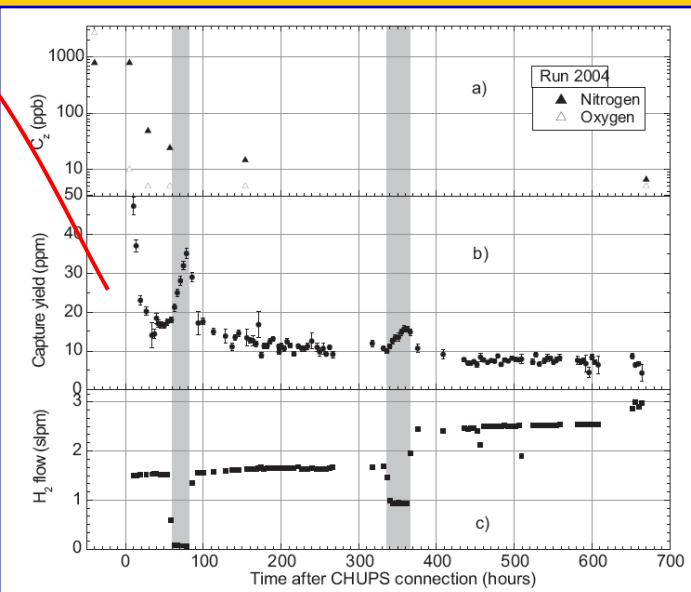
Gas chromatography

CRDF 2002, 2005

Diagnostic in TPC



Results



- $c_N, c_O < 7 \text{ ppb}$, $c_{H_2O} \sim 18-30 \text{ ppb}$
- correction based on observed capture yield

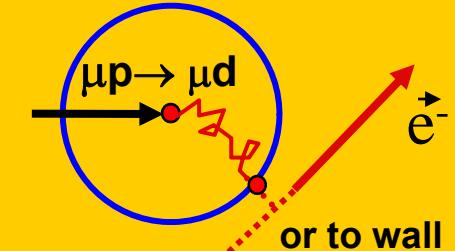
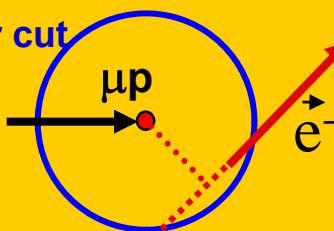
MuCap Unique Capabilities: μp , μd diffusion



large diffusion range of μd

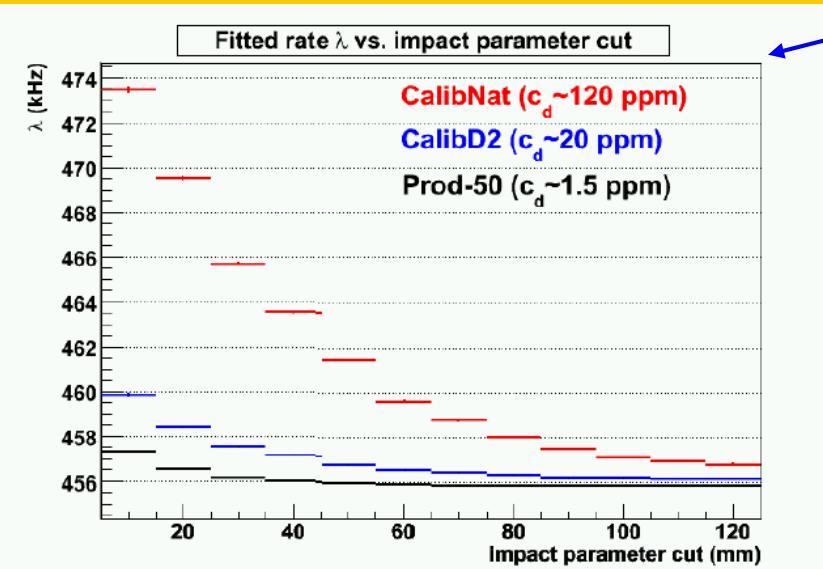
< 1 ppm isotopic purity required

μ -e impact par cut



Diagnostic:

- λ vs. μ -e vertex cut



- AMS, ETH Zurich

Results

- Directly from data

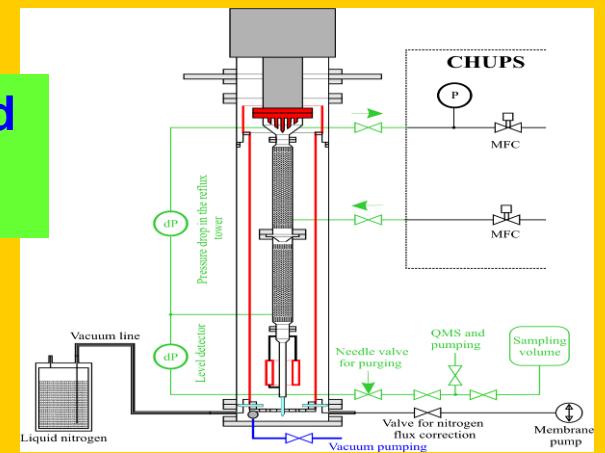
$$c_d = 1.49 \pm 0.12 \text{ ppm}$$

- AMS (2006)

$$c_d = 1.44 \pm 0.15 \text{ ppm}$$

On-site isotopic purifier 2006 (PNPI, CRDF)

World Record
 $c_d < 0.1$ ppm





V.A. Andreev,¹ T.I. Banks,² T.A. Case,² D.B. Chitwood,³ S.M. Clayton,³ K.M. Crowe,² J. Deutsch,⁴ J. Egger,⁵
S.J. Freedman,² V.A. Ganzha,¹ T. Gorringe,⁶ F.E. Gray,² D.W. Hertzog,³ M. Hildebrandt,⁵ P. Kammel,³
B. Kiburg,³ S. Knaack,³ P.A. Kravtsov,¹ A.G. Krivshich,¹ B. Lauss,² K.L. Lynch,⁷ E.M. Maev,¹ O.E. Maev,¹
F. Mulhauser,^{3,5} C.S. Özben,³ C. Petitjean,⁵ G.E. Petrov,¹ R. Prieels,⁴ G.N. Schapkin,¹ G.G. Semenchuk,¹
M.A. Soroka,¹ V. Tishchenko,⁶ A.A. Vasilyev,¹ A.A. Vorobyov,¹ M.E. Vznuzdaev,¹ and P. Winter³

(MuCap Collaboration)

¹Petersburg Nuclear Physics Institute, Gatchina 188350, Russia

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⁴Université Catholique de Louvain, B-1348, Louvain-la-Neuve, Belgium

⁵Paul Scherrer Institute, CH-5232 Villigen PSI, Switzerland

⁶University of Kentucky, Lexington, KY 40506, USA

⁷Boston University, Boston, MA 02215, USA

(Dated: April 16, 2007)

[arXiv:0704.2072v1](https://arxiv.org/abs/0704.2072v1) [nucl-ex]

accepted PRL

$$\Lambda_S^{\text{MuCap}} = 725.0 \pm 13.7_{\text{stat}} \pm 10.7_{\text{sys}} \text{ s}^{-1}$$

Average of HBChPT calculations of Λ_S :

$$(687.4 \text{ s}^{-1} + 695 \text{ s}^{-1})/2 = 691.2 \text{ s}^{-1}$$

Apply new rad. correction (2.8%):

$$(1 + 0.028)691.2 \text{ s}^{-1} = 710.6 \text{ s}^{-1}$$

further sub percent theory required

$$\Lambda_S^{\text{theory}} = 710.6 \text{ s}^{-1}$$

[arXiv:0704.3968v1](https://arxiv.org/abs/0704.3968v1) [hep-ph]

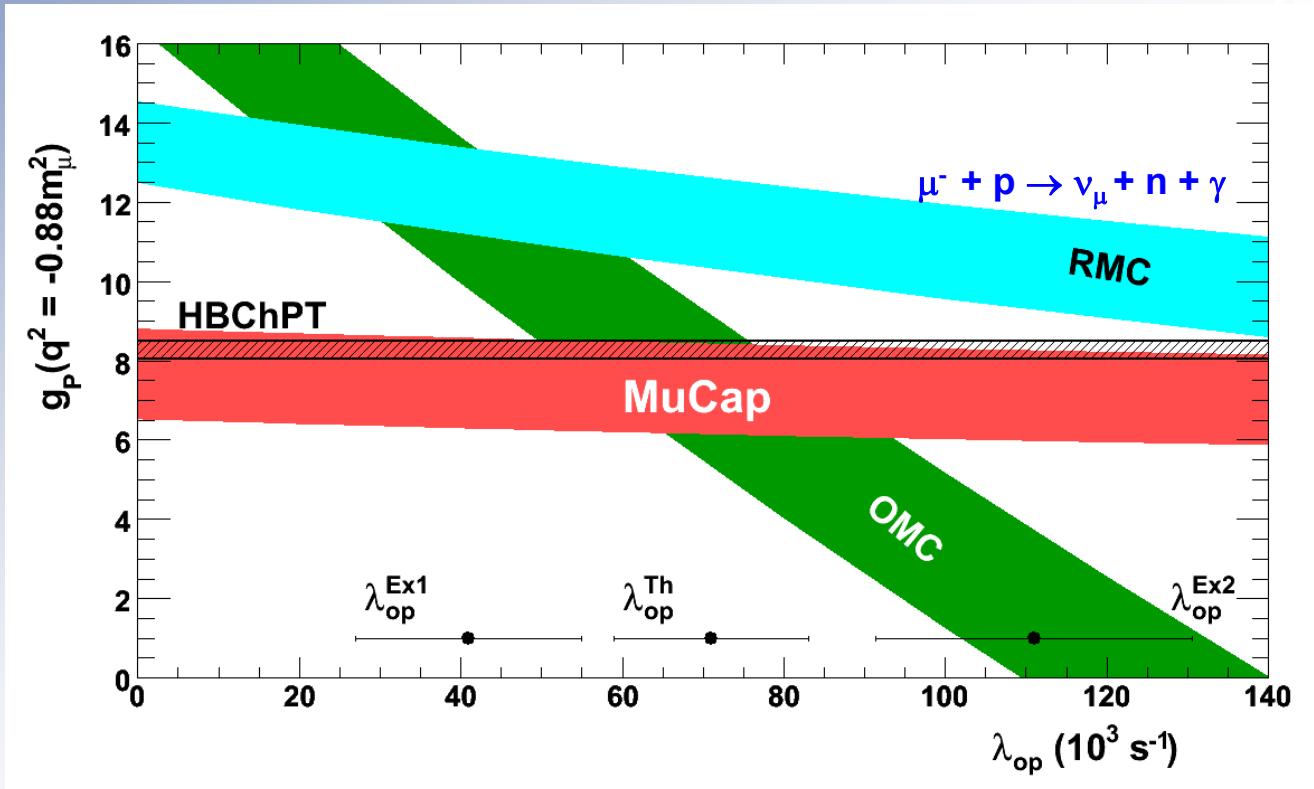
Czarnecki, Marciano, Sirlin

$$g_P = 7.3 \pm 1.1$$

(MuCap 2007)



g_P Landscape after MuCap 06



Before MuCap experiments inconclusive and mutually inconsistent

MuCap

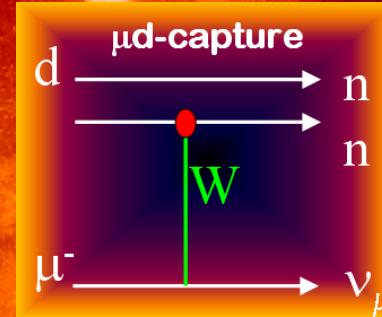
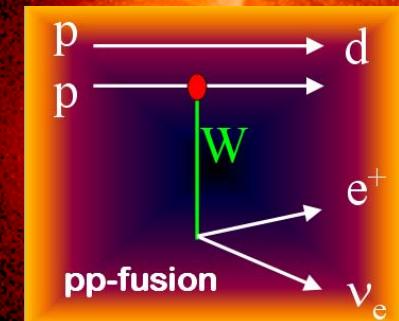
- MuCap result nearly model independent
First precise and unambiguous result
- Consistent with chiral prediction
Does not confirm radiative muon capture (RMC) discrepancy
- Final result ('06 and '07 data) will reduce error twofold



“Calibrating the Sun” via Muon Capture on the Deuteron



“MuSun”



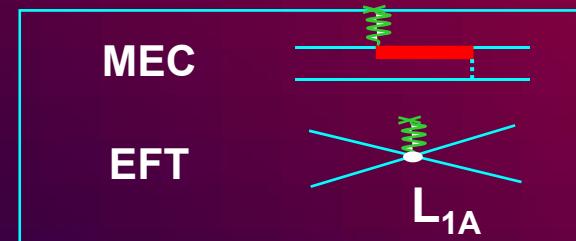
model-independent connection via EFT & L_{1A}

Goal

total μd capture rate to 1% precision

Motivation

- first precise measurement of basic EW reaction in 2N system, **benchmark measurement with 10x higher precision**
- impact on fundamental astrophysics reactions (SNO, pp)
- comparison of modern high precision calculations
- high precision feasible by μ Cap technique and careful optimization





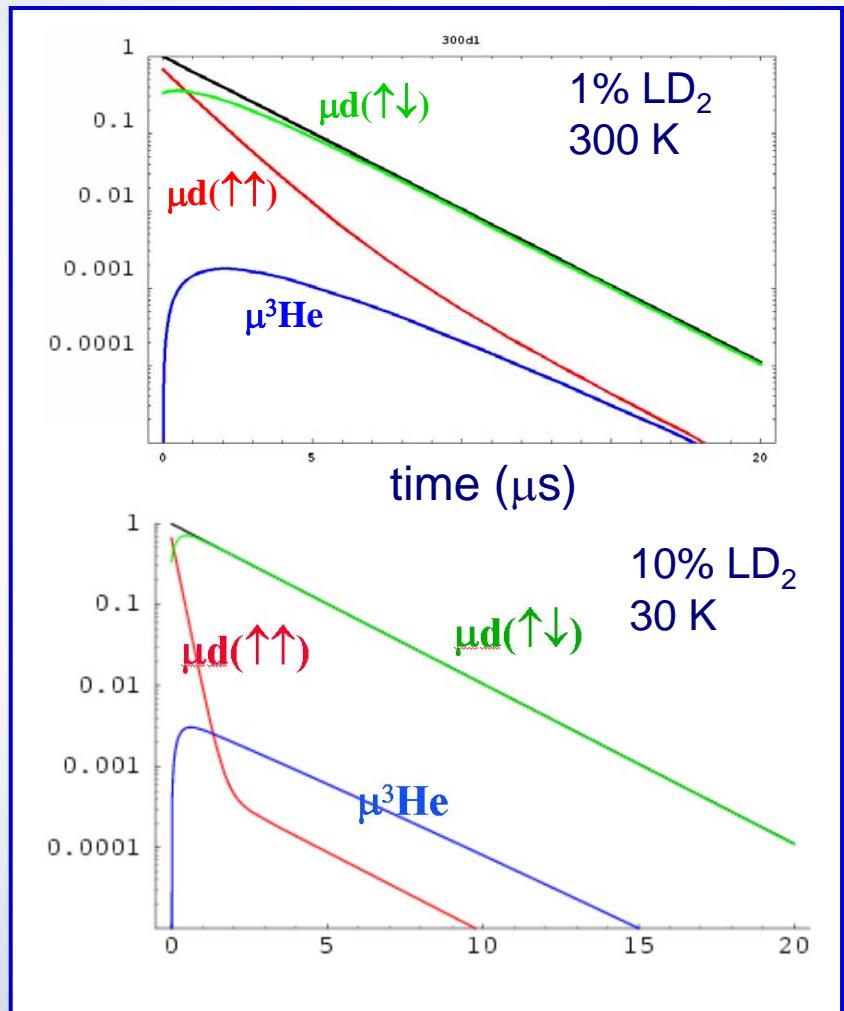
measurement of absolute rate to <1%

μSun I: μCap technique, 1% LD₂, 300 K,
measure time spectra of capture neutrons
monitor populations with fusion and
capture reactions

First measurement of polarization
observables in $\mu+d$ capture?

μSun II: new cryo TPC

Kinetics requires optimized target
conditions: T<50 K, >5% LD₂ density





■ Fundamental electroweak parameters

G_F	α	M_Z
9 ppm	0.0007 ppm	23 ppm

■ G_F

Implicit to all EW precision physics

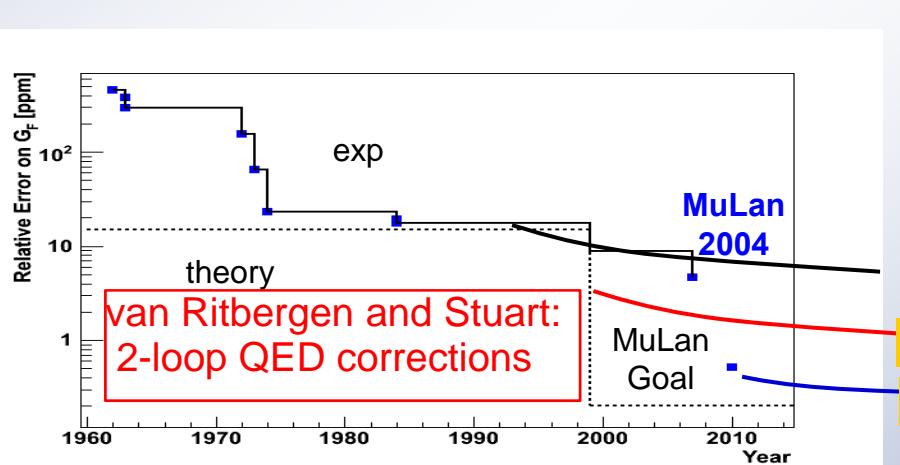
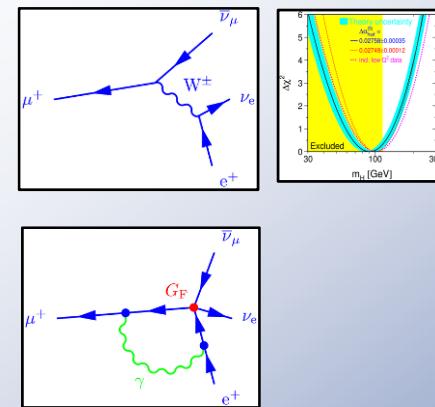
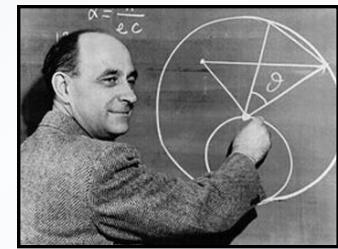
Uniquely related to muon decay

Precision $G_F \leftrightarrow \tau$ relation no longer theory limited

$$\frac{G_F}{\sqrt{2}} = \frac{g^2}{8M_W^2} (1 + \Delta r(m_t, m_H, \dots))$$

$$\frac{1}{\tau_{\mu^+}} = \frac{G_F^2 m_\mu^5}{192\pi^3} (1 + q)$$

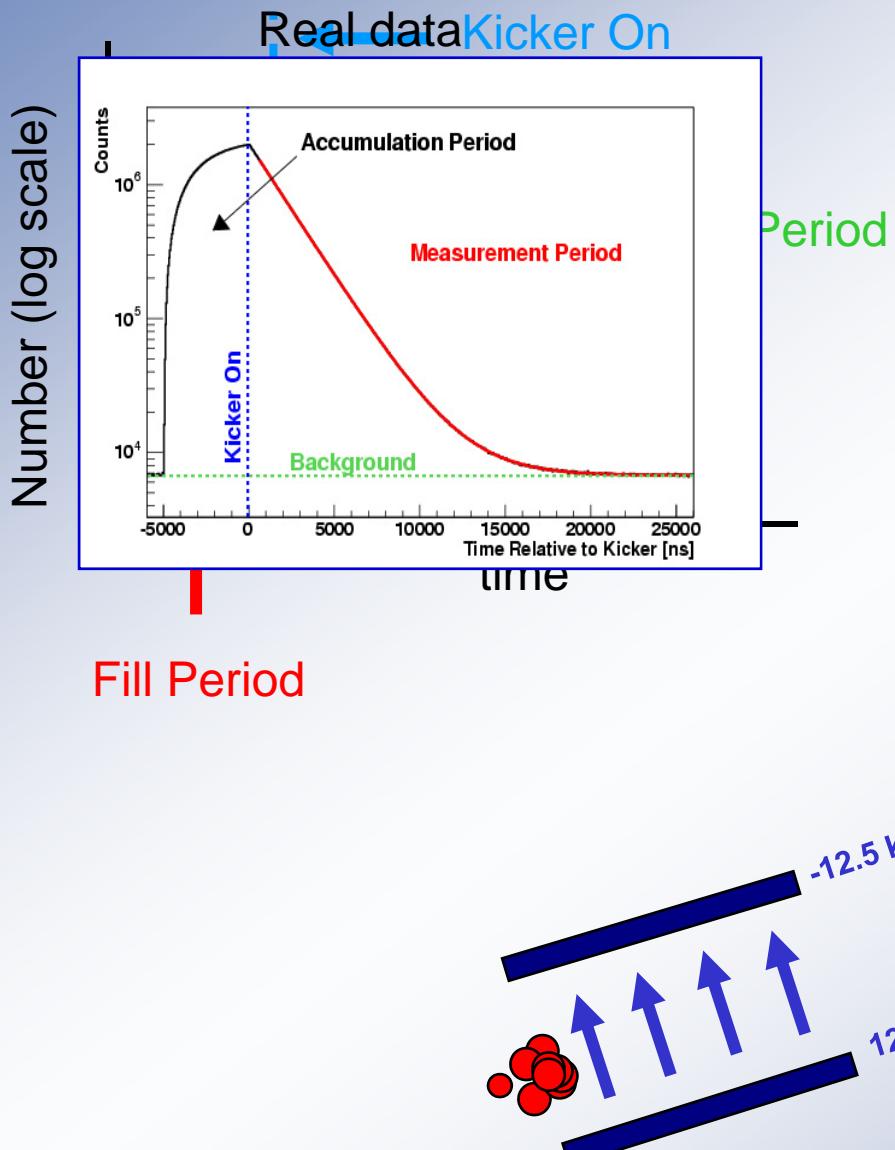
QED



$$\frac{\delta G_F}{G_F} = \frac{1}{2} \sqrt{\left(\frac{\delta \tau_\mu}{\tau_\mu} \right)^2 + \left(5 \frac{\delta m_\mu}{m_\mu} \right)^2 + \left(\frac{\delta \text{theory}}{\text{theory}} \right)^2}$$

17 ppm 18 ppm 90 ppb 30 ppm

MuLan Experiment



“Early-to-late” changes

- Instrumental shifts

Gain or threshold

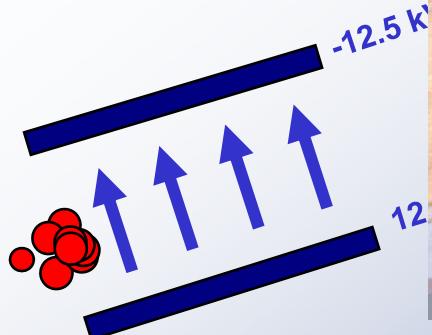
Time response

- Effective acceptance

Residual polarization or precession

Pileup

- Missing events



D.B. Chitwood,¹ T.I. Banks,² M.J. Barnes,³ S. Battu,⁴ R.M. Carey,⁵ S. Cheekatmallla,⁴ S.M. Clayton,¹ J. Crnkovic,¹
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 R. McNabb,¹ J.P. Miller,⁵ F. Mulhauser,¹ C.J.G. Onderwater,^{1,7} C.S. Özben,¹ Q. Peng,⁵ C.C. Polly,¹ S. Rath,⁴
 B.L. Roberts,⁵ V. Tishchenko,⁴ G.D. Wait,³ J. Wasserman,⁵ D.M. Webber,¹ P. Winter,¹ and P.A. Żołnierczuk⁴

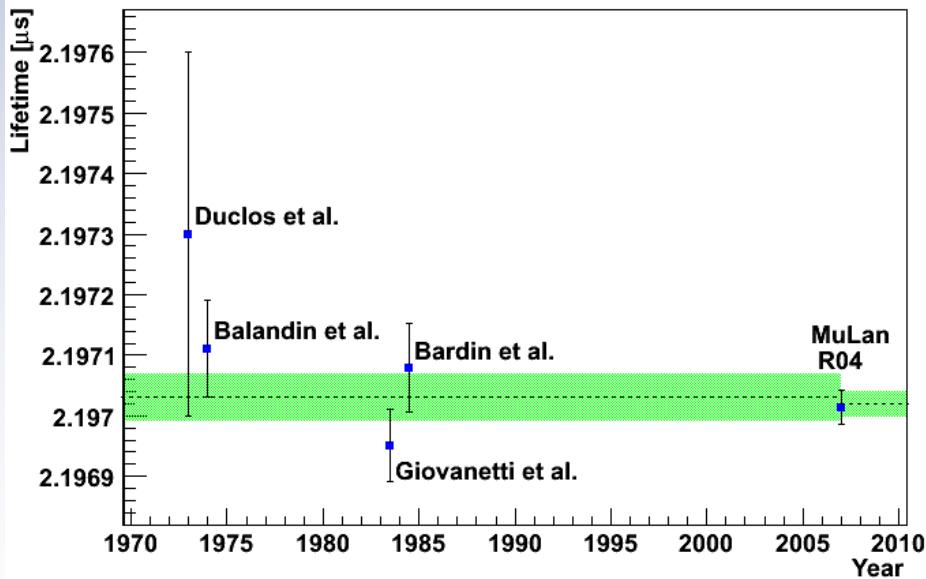
(MuLan Collaboration)

¹Department of Physics, University of Illinois at Urbana-Champaign, Urbana, IL 61801, USA²Department of Physics, University of California, Berkeley, CA 94720, USA³TRIUMF, Vancouver, BC, V6T 2A3, Canada⁴Department of Physics and Astronomy, University of Kentucky, Lexington, KY 40506, USA⁵Department of Physics, Boston University, Boston, MA 02215, USA⁶Department of Physics, James Madison University, Harrisonburg, VA 22807, USA⁷Kernfysisch Versneller Instituut, Groningen University, NL 9747 AA Groningen, The Netherlands

The mean life of the positive muon has been measured to a precision of 11 ppm using a low-energy, pulsed muon beam stopped in a ferromagnetic target, which was surrounded by a scintillator detector array. The result, $\tau_\mu = 2.197\,013(24) \mu\text{s}$, is in excellent agreement with the previous world average. The new world average $\tau_\mu = 2.197\,019(21) \mu\text{s}$ determines the Fermi constant $G_F = 1.166\,371(6) \times 10^{-5} \text{ GeV}^{-2}$ (5 ppm). Additionally, the precision measurement of the positive muon lifetime is needed to determine the nucleon pseudoscalar coupling g_F .

6/5/07 accepted PRL

arXiv:0704.1981v1 [hep-ex]



$$\tau_\mu(\text{MuLan}) = 2.197\,013(21)(11) \mu\text{s} \text{ (11 ppm)}$$

$$\tau_\mu(\text{World}) = 2.197\,019(21) \mu\text{s} \text{ (9.6 ppm)}$$

$$G_F = 1.166\,371(6) \times 10^{-5} \text{ GeV}^{-2} \text{ (5 ppm)}$$



Summary I: Relevant MCF issues



MuCap:

- Th/Exp: ortho-para rate $\lambda_{OP}(\phi)$
- Exp: Precision measurement of formation rate $p\bar{p}\mu$ -ortho planned
- Th: $p\bar{p}\mu$ -para formation suppressed, $S_{tot}=1/2$ assumed
- Th/Exp: new experimental info on μp and μd scattering, theory cross sections and simulations
- Exp: measurements of μZ transfer and Auger effect
- Th: cross section for μ^+ diffusion

MuSun:

- Exp: $d\mu$ hyperfine transition at 300K
- Th/Exp: time evolution of $d\mu$ polarization
- Exp: $d\mu$ polarization observables in muon decay and capture
- Th/Exp: precision measurement $dd \rightarrow {}^4He + \gamma ?$



Summary II: Weak Interactions



MuLan:

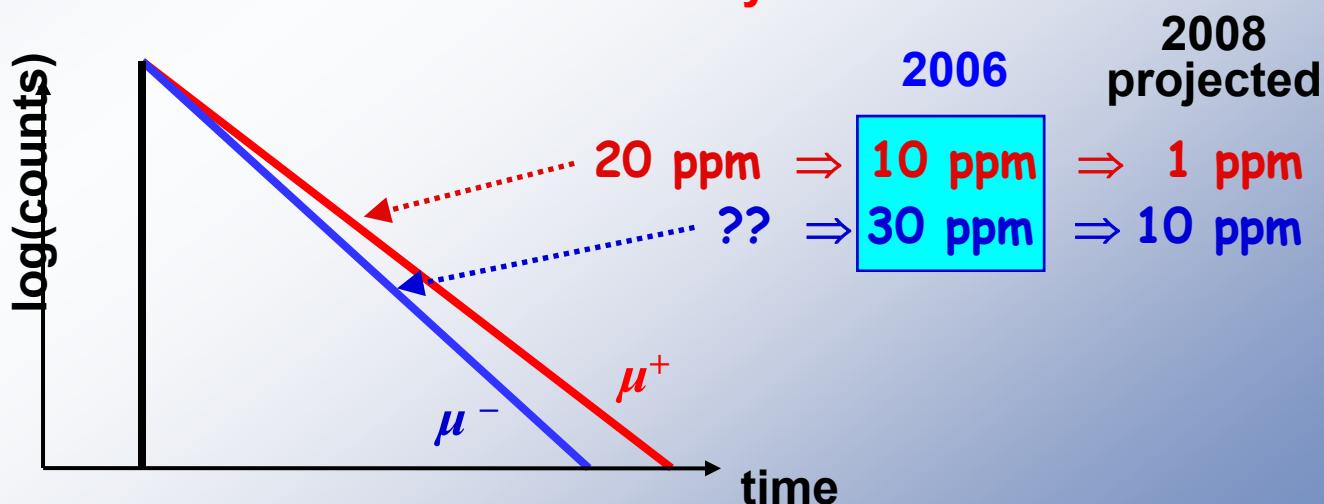
- First G_F update in 23 years – 2.5x improvement, no surprise in result
- Factor 10 additional improvement on the way

MuCap:

- First precise g_P measurement with clear interpretation
- Consistent with ChPT expectation, does not support RMC puzzle
- Factor 2-3 additional improvement on the way

MuSun

- muon-deuteron capture, needs g_P as input
- New benchmark in EW reactions in 2N system



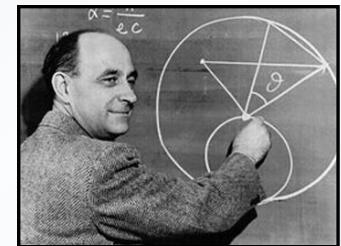
MuLan





■ Fundamental electroweak parameters

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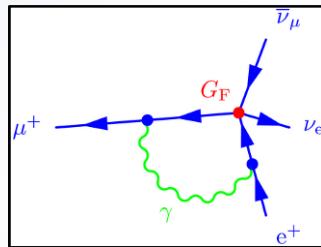


■ G_F

Uniquely defined by muon decay

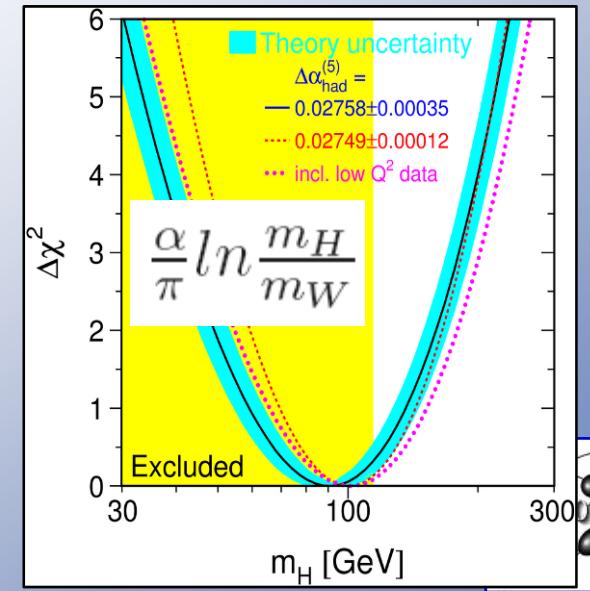
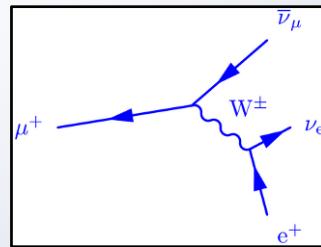
$$\frac{1}{\tau_{\mu^+}} = \frac{G_F^2 m_\mu^5}{192\pi^3} (1 + q)$$

QED

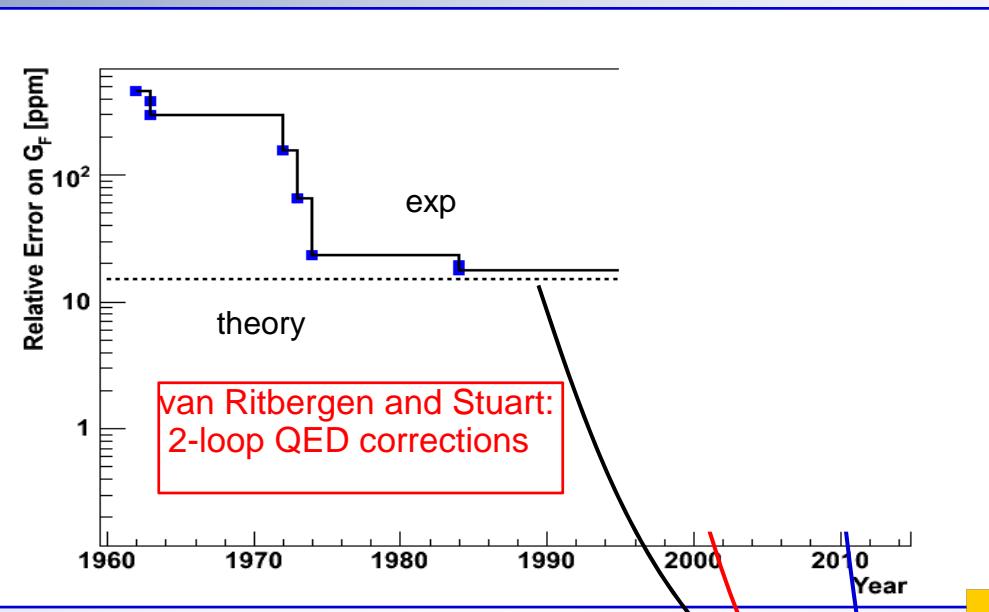


Implicit to all EW precision physics

$$\frac{G_F}{\sqrt{2}} = \frac{g^2}{8M_W^2} (1 + \Delta r(m_t, m_H, \dots))$$



Extraction of G_F from τ_μ not theory limited



δv_μ from δm^2

$$\frac{\delta G_F}{G_F} = \frac{1}{2} \sqrt{\left(\frac{\delta \tau_\mu}{\tau_\mu} \right)^2 + \left(5 \frac{\delta m_\mu}{m_\mu} \right)^2 + \left(\frac{\delta \text{theory}}{\text{theory}} \right)^2}$$

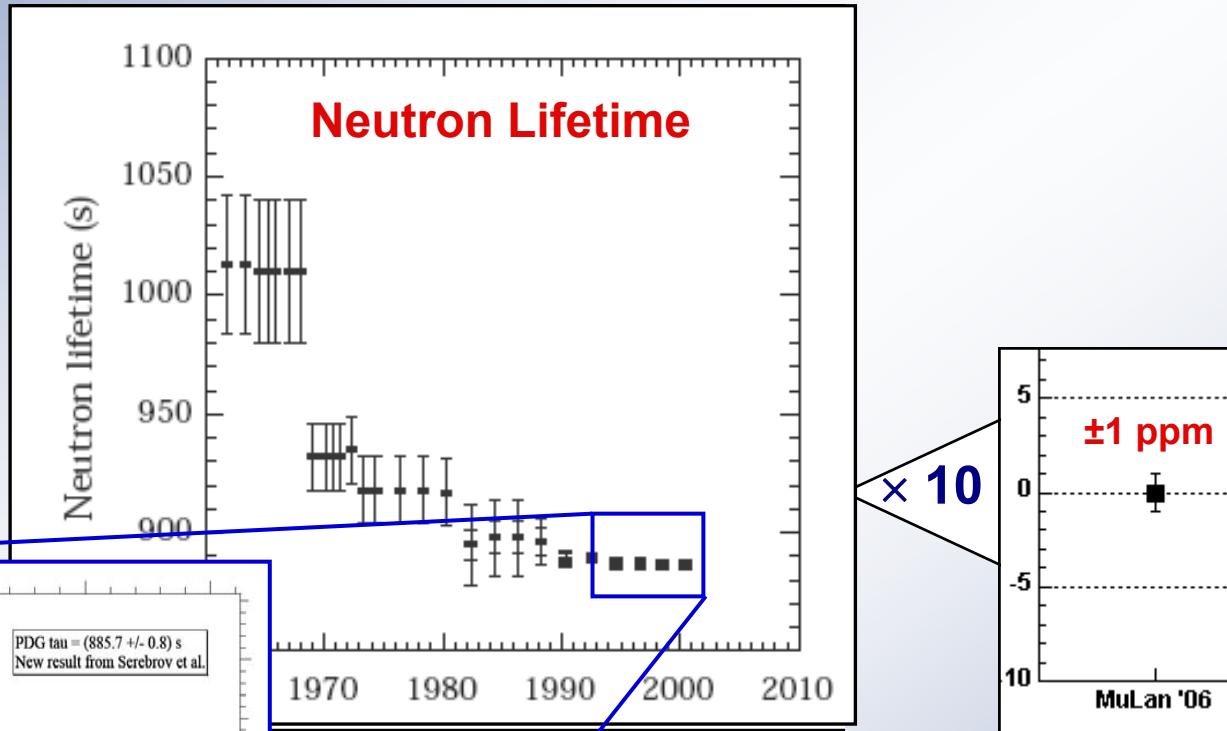
17 ppm 18 ppm 90 ppb 30 ppm

Present $\delta\tau_\mu/\tau_\mu$ is 18 ppm and best single measurement is 27 ppm



Lessons from History

MuLan



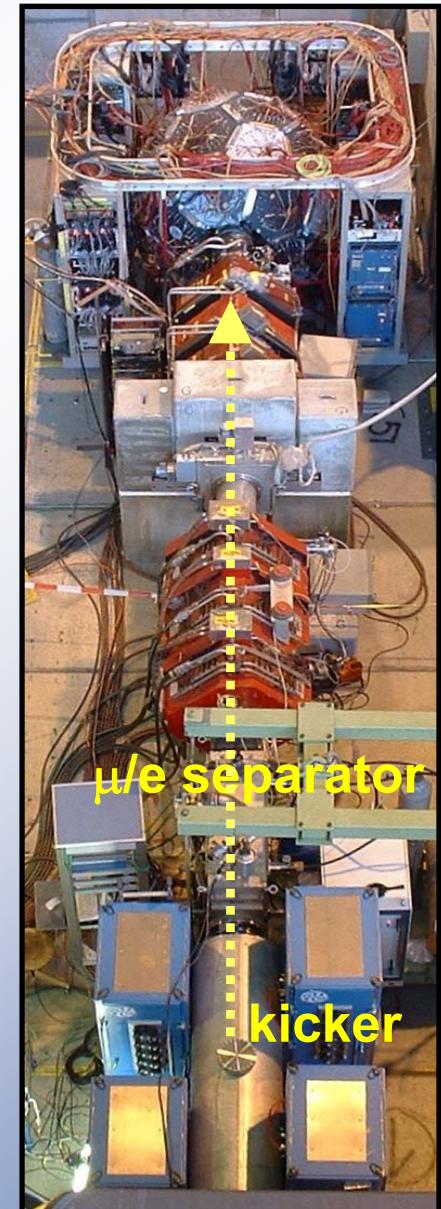
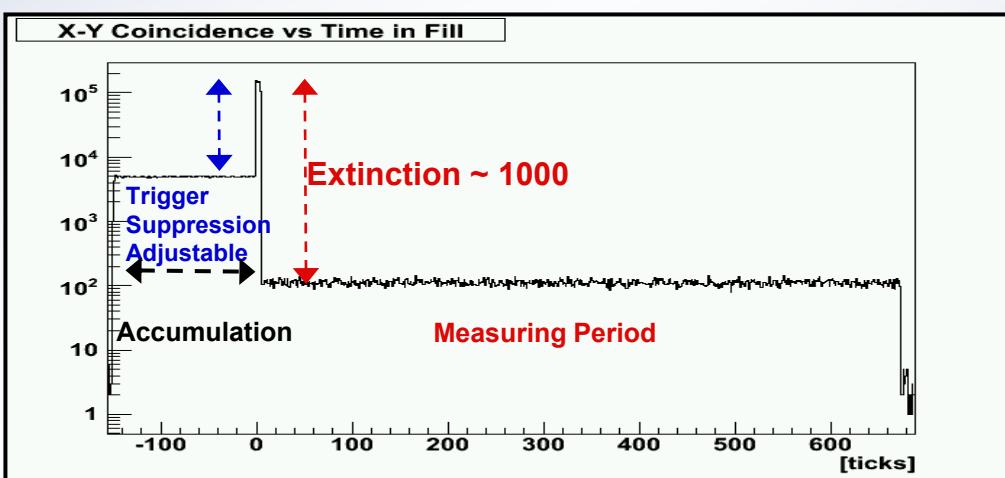
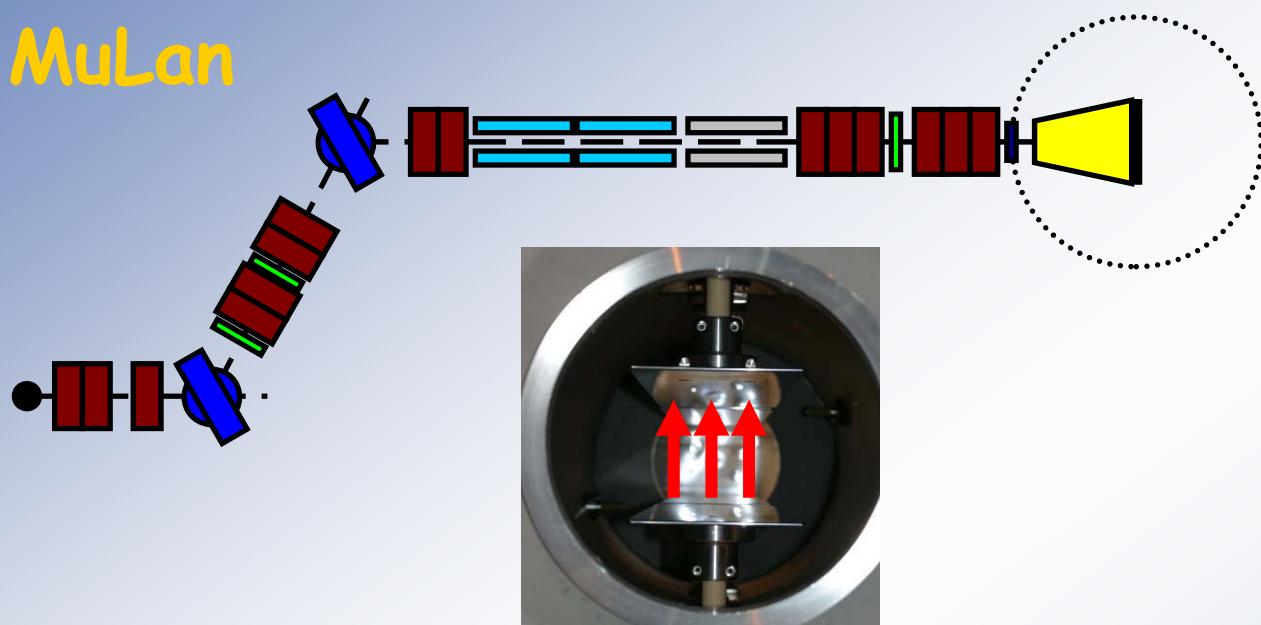
Psychology of making these measurements has blind analyses, many parameters stored, analyses, etc.

from D. Herzog



Create a time-structured “surface” muon beam with flux of roughly $10^7 \mu^+ \text{ Hz}$ @ 28 MeV/c, (~ 4 MeV)

MuLan



Systematics

“Early-to-late” changes

- Instrumental shifts
 - Gain or threshold
 - Time response
 - Kicker and accidentals



Source	Size (ppm)
Extinction stability	3.5
Errant muon stops	2.0
Dead time correction	2.0
Gain stability	1.8
MTDC response	1.0
Repeated events (+1 ppm shift)	1.0
Multiple hit timing shifts	0.8
Queuing loss	0.7
Total	5.2

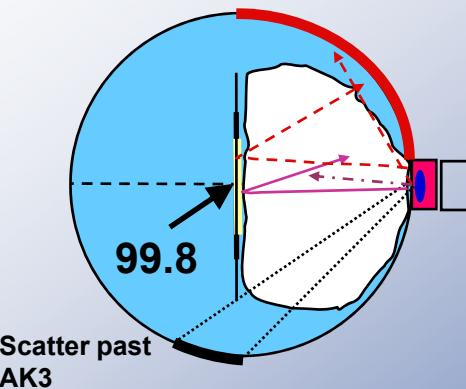
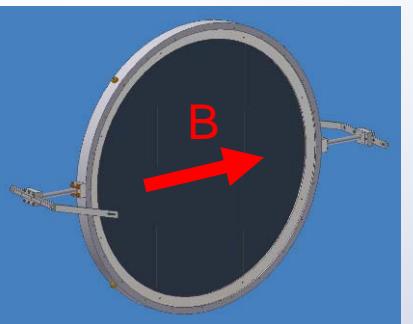
500 MHz WFD in 2005

- Effective acceptance
 - Residual polarization or precession

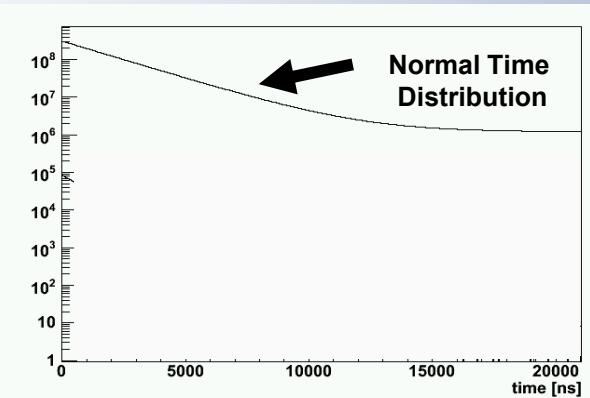
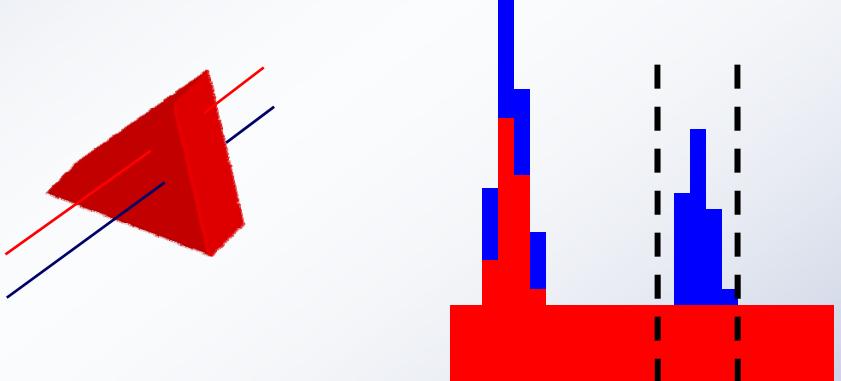
target: Arnokrome III (AK-3) internal ~4000 G

symmetric detector

stray muons studied



- Pileup leads to missed events



Systematics

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- Instrumental shifts
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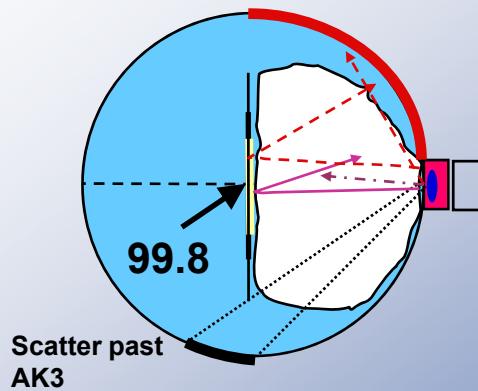
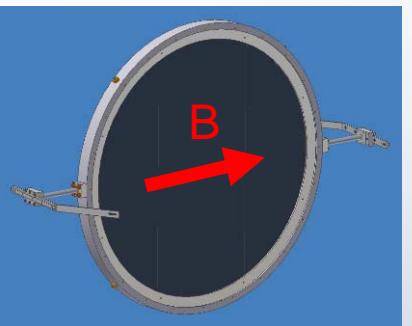
500 MHz WFD in 2005

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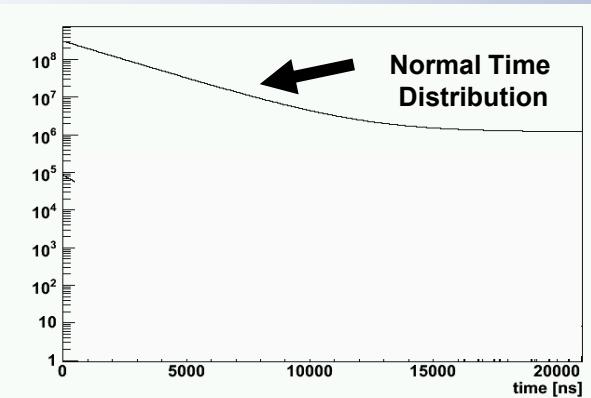
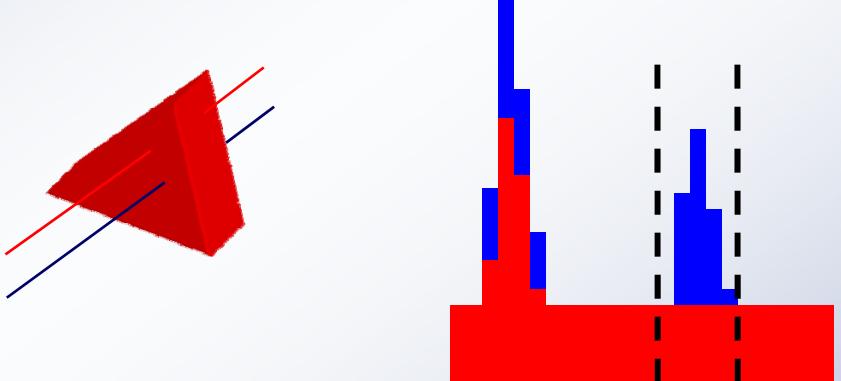
target: Arnokrome III (AK-3) internal ~4000 G

symmetric detector

stray muons studied



- Pileup leads to missed events





G_F and new physics

W. Marciano 1999

- RC O(α^2)

$$\text{R.C.} = \frac{\alpha}{2\pi} \left(\frac{25}{4} - \pi^2 \right) \left[1 + \frac{\alpha}{\pi} \left(\frac{2}{3} \ln \frac{m_\mu}{m_e} - 3.7 \right) + \left(\frac{\alpha}{\pi} \right)^2 \left(\frac{4}{9} \ln^2 \frac{m_\mu}{m_e} - 2.0 \ln \frac{m_\mu}{m_e} + C \right) + \dots \right]$$

- Input

$\alpha(0)$	= 1/137.035999710(96)	0.7 ppb
$\alpha(M_Z)$	= 1/127.918(18)	140 ppm
G_F	= 1.166 37(1) x 10 ⁻⁵ GeV ⁻²	9 ppm
M_Z	= 91.1876(21) GeV	23 ppm
m_t	= 172.7(2.9) (0.6) GeV	
M_W	= 80.392(39) GeV	
$\sin^2 \theta_W$	= 0.23100(22) GeV	

- Interesting physics reach

But

- other GF determinations 100x poorer

$$g_{2_0}^e = g_{2_0}^\mu = g_{2_0}^\tau$$

$$\sin^2 \theta_W^0 = \frac{e_0^2}{g_{2_0}^2} = 1 - (m_W^0/m_Z^0)^2$$



Note: Experimental limits on η (non SM) are largest uncertainty of Fermi constant

I.

G_F depends on η

Danneberg et al,
PRL 94 021802 (2005)

$$G_F \approx G_F^{V-A} \left(1 - 2\eta \frac{m_e}{m_\mu} \right), \quad (7)$$

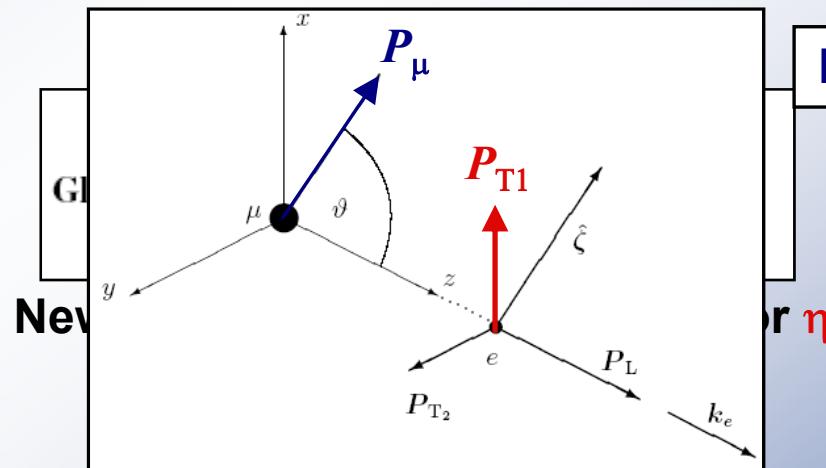
where m_e/m_μ is the mass ratio of electron and muon.

$$\eta = (-2.1 \pm 7.0 \pm 1.0) \times 10^{-3},$$

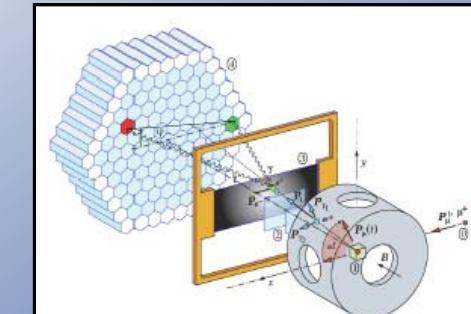


70 ppm
uncertainty
on G_F

Access to η through transverse polarization measurement of outgoing positron



Fetscher expt. PSI



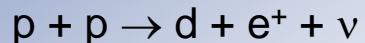
MuSun



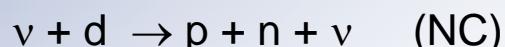
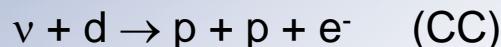
■ Reactions



basic solar fusion reaction



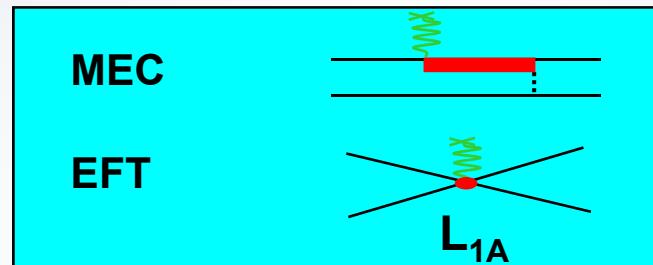
key reactions for SNO



...

■ Theory SNPA – EFT (HBChPT, π EFT, hybrid)

- 1B NN description accurate
- 2B not well constrained by theory

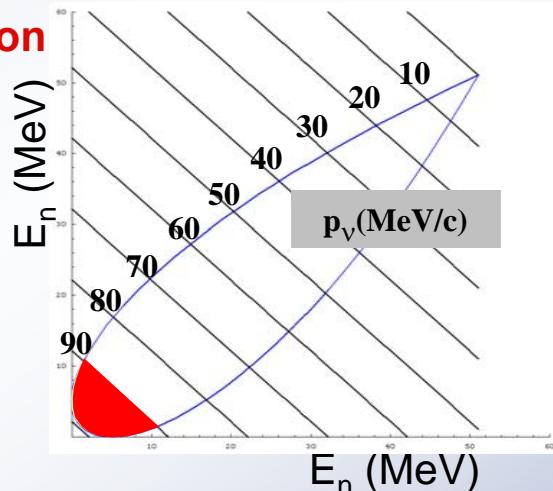


EFT: Class of axial current reactions related by single parameter L_{1A}

→ **Quest for L_{1A}**

■ Precision $\mu+d$ experiment (PK, Chen)

best determination
of L_{1A} from
2N system



theory:

precise enough?

reaction soft enough for L_{1A} ?

Ando, Park, Kubodera, Myhrer (2002)

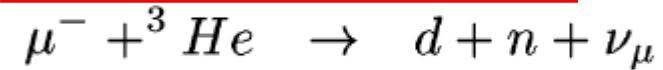
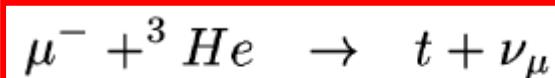
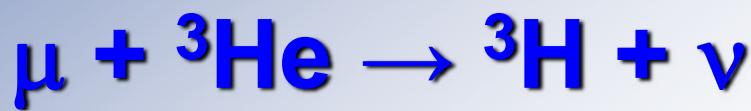
Chen, Inoue, Ji, Li (2005)

experiment:

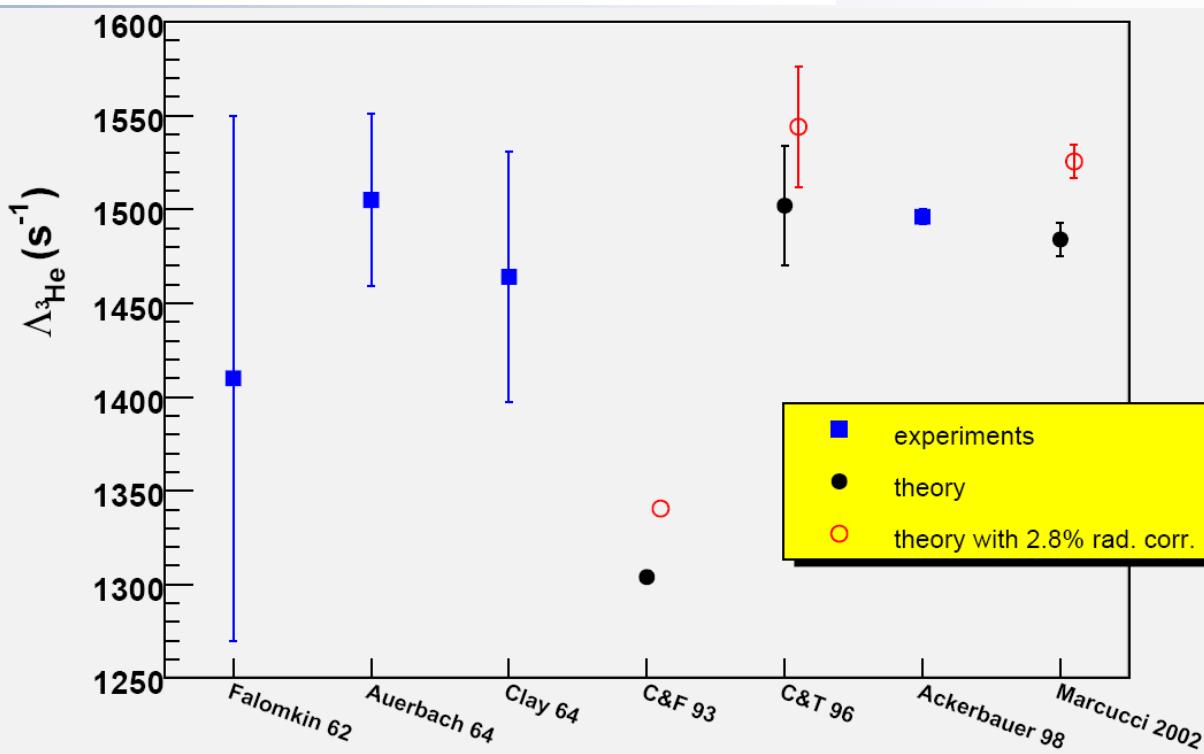
1% precision possible ?

MuCap technique





authors	$\Lambda_{\text{stat}} (\text{s}^{-1})$	comment
Congleton & Fearing	1304	IA
Congleton & Truhlik	1502 ± 32	IA + MEC
Ackerbauer et al	1496.0 ± 4.0	${}^3\text{He TPC}$
Marcucci et al.	1484 ± 8	IA+MEC, T beta constraint



PCAC $g_P = 8.12$
Exp & CT 8.53 ± 1.54
Exp & M 7.55 ± 0.42
Exp & M, rad 9.48 ± 0.42

needs check



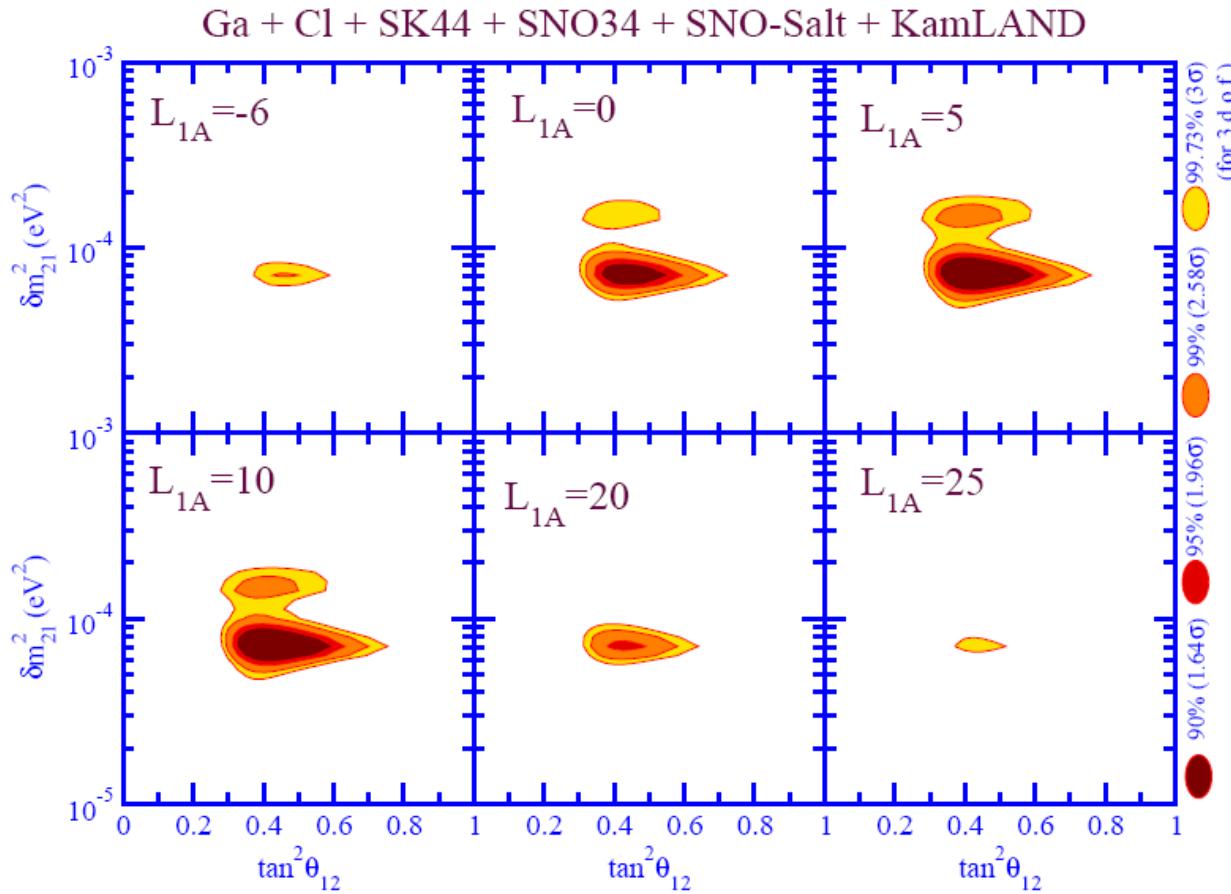
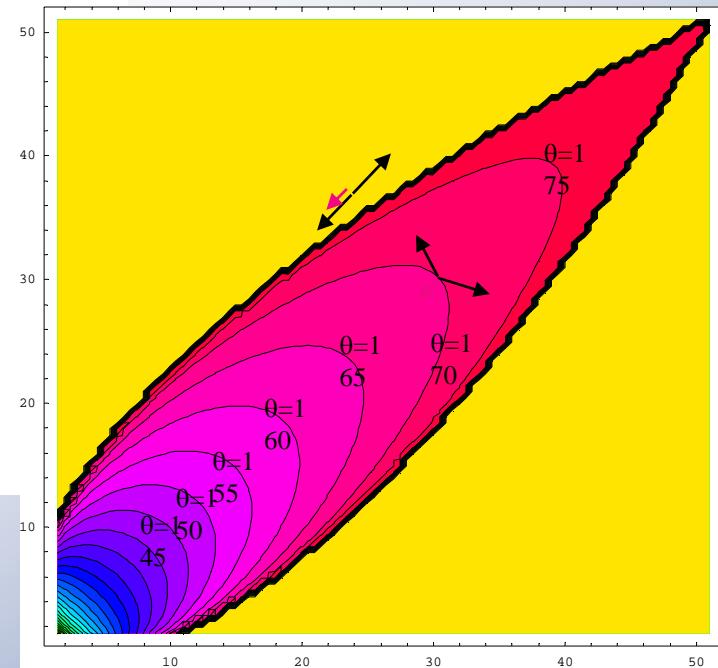
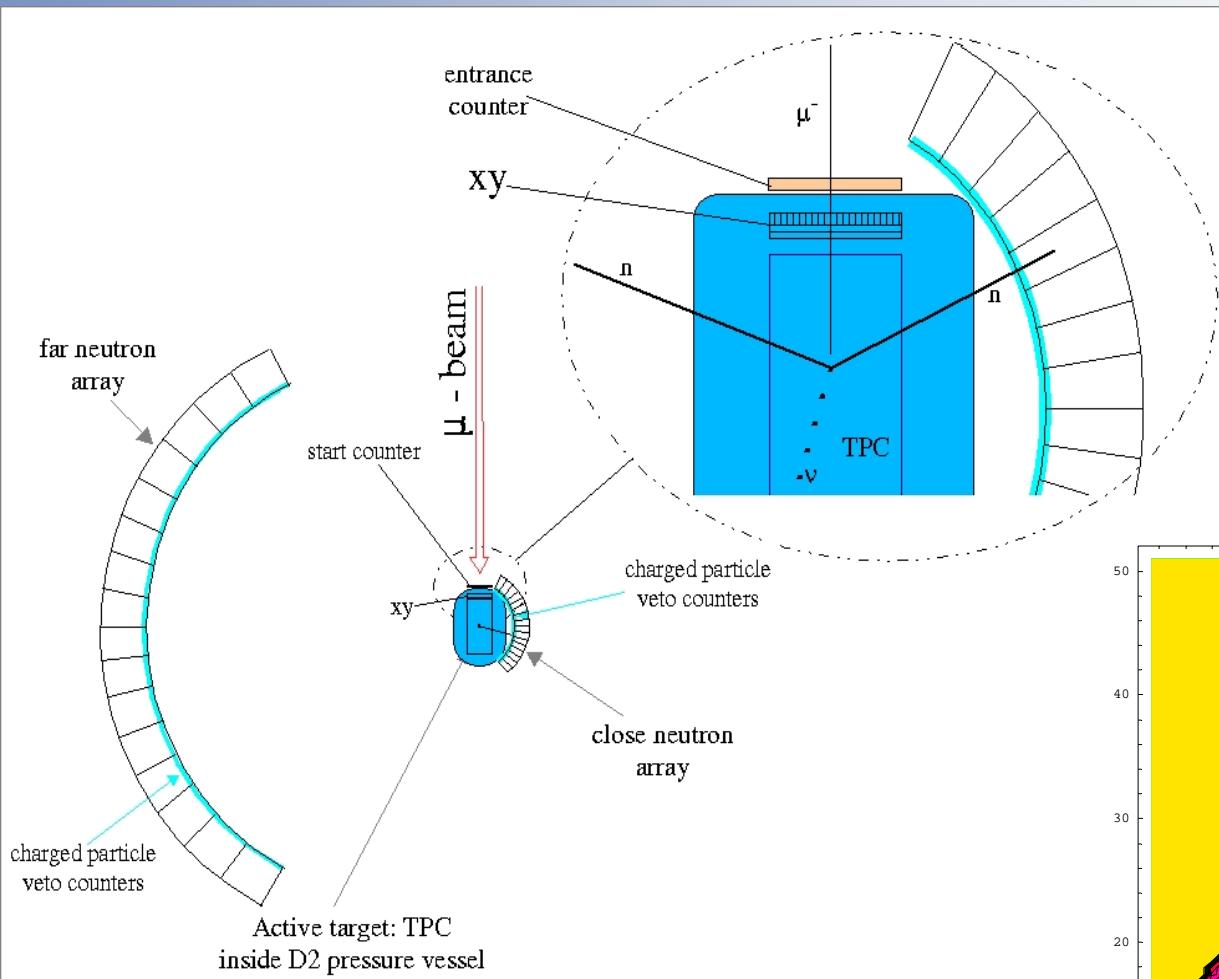


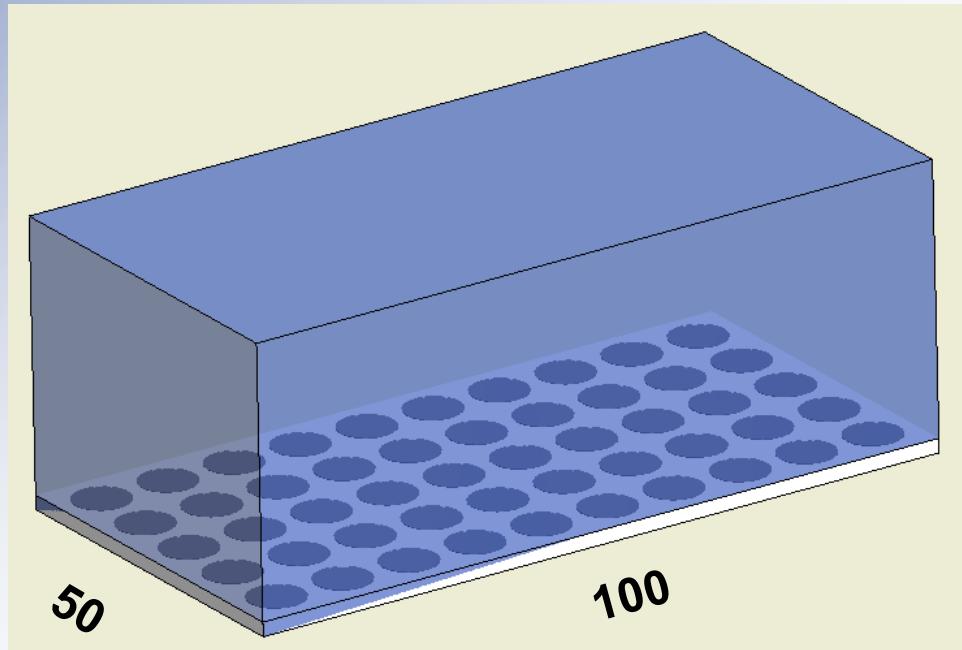
Fig. 3. The change in the allowed region of the mixing parameter space using combined solar neutrino data and KamLAND as a function of L_{1A} . In the calculations leading to this figure the neutrino mixing angle θ_{13} is taken to be zero. The shaded areas corresponds to 90 % , 95 % , 99 % , and 99.73 % confidence levels.

Concept stage II



TPC details

40 mm



**GEM
Microstructure ?**

	grid cath			grid an			gas	cath
	mm	kV	kV/mm	mm	kV	kV/mm		mm
He TPC	14.3	40	2.80	0.8	3.5	4.38	120bar He	8
D TPC	40	100	2.50	0.8	4	5.00	50-100bar D	10

Recombination ?

Signal (Gain?)

Summary physics motivation



precision measurement of total μd capture rate to 1% provides

- first precise measurement of charged-current reaction in 2N system
- first precise 2N experimental information relevant for *absolute* solar neutrino cross sections and flux.
- comparison of EFT/SNPA approach for space-like axial two-body current and 2N vs 3N constraints.
- a_{nn} information ? (*needs study*)

systematic measurement of μd capture Dalitz Plot provides

- information on time-like axial two-body currents
- reduced rate Λ' , accessible to EFT
- complementary g_P sensitivity, if MECs sufficiently under control (*needs study*)

first measurement of μd capture asymmetry and hfs effects provides

- complementary g_p info (*needs study*)



MuCap



$$\begin{aligned} L_{\text{QCD}} = & -\frac{1}{4} F_{\mu\nu}^{(a)} F^{(a)\mu\nu} + i \sum_q \bar{\psi}_q^i \gamma^\mu (D_\mu)_{ij} \psi_q^j \\ & - \sum_q m_q \bar{\psi}_q^i \psi_{qi} , \\ F_{\mu\nu}^{(a)} = & \partial_\mu A_\nu^a - \partial_\nu A_\mu^a - g_s f_{abc} A_\mu^b A_\nu^c , \\ (D_\mu)_{ij} = & \delta_{ij} \partial_\mu + i g_s \sum_a \frac{\lambda_{i,j}^a}{2} A_\mu^a , \end{aligned}$$



Additional slides start here



Λ_S sensitivity to g

$$\frac{\Delta \Lambda_S}{\Lambda_S} = 0.466 \frac{\Delta g_V}{g_V} + 0.151 \frac{\Delta g_M}{g_M} + 1.567 \frac{\Delta g_A}{g_A} - 0.184 \frac{\Delta g_P}{g_P}$$

$\delta \Lambda_S$ with present δg

$$\frac{\Delta \Lambda_S}{\Lambda_S} (\%) = \sqrt{0.0239^2 + 0.0104^2 + 0.504^2 + 9.2^2}$$

$$\frac{\delta \Lambda}{\Lambda} = 0.466 \frac{\delta g_V}{g_V} + 0.151 \frac{\delta g_M}{g_M} + 1.567 \frac{\delta g_A}{g_A} - 0.184 \frac{\delta g_P}{g_P} + 0.0238 \delta g_{S,T}$$

$$\frac{\delta \Lambda}{\Lambda} [\%] = 0.024 \quad 0.01 \quad 0.38 \quad 3.7 \quad 0.24$$

assuming optimistic
20% g_P error

assuming $g_T < 0.1$

error from $V_{ud} = 0.16 \%$



45 Years of Experiments to Determine g_P

■ $\mu^- + p \rightarrow \nu_\mu + n$ OMC BR~ 10^{-3}

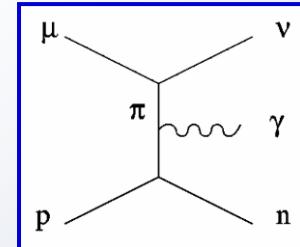
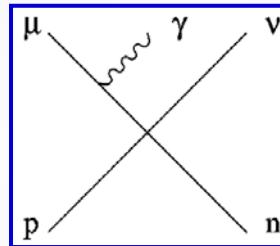
8 experiments, typical precision 10-15%, Saclay 4%

■ $\mu^- + p \rightarrow \nu_\mu + n + \gamma$ RMC BR~ 10^{-8} , E>60 MeV

279±25 events

$BR_\gamma(k>60\text{MeV}) = (2.10\pm0.21)\times10^{-8}$

Wright et al. (1998)



...

■ $\mu^- + {}^3\text{He} \rightarrow \nu_\mu + {}^3\text{H}$

	authors	$\Lambda_{\text{stat}} (\text{s}^{-1})$	comment	
theory 1993	<i>Congleton & Fearing</i>	1304	1B	
theory 1996	<i>Congleton & Truhlik</i>	1502 ± 32	1B + 2B	
exp 1998	<i>Ackerbauer et al.</i>	1496.0 ± 4.0	${}^3\text{He TPC}$	$g_P(-0.954m_\mu^2) = 8.53 \pm 1.54$
theory 2002	<i>Marcucci et al.</i>	1484 ± 8	1B+2B, T beta constraint	

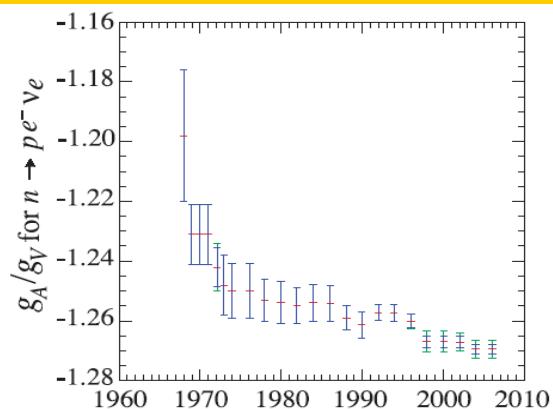
rad. corrections?

■ Beta Decay Correlations



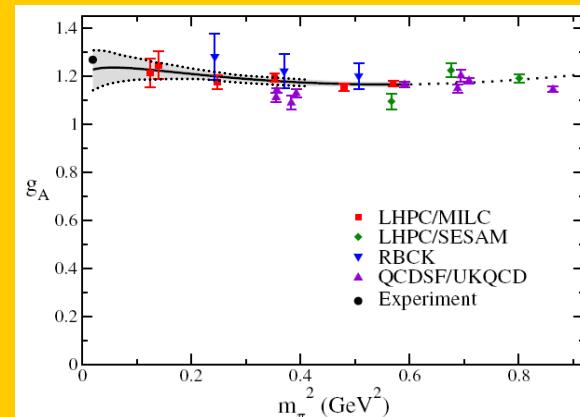


Exp. History



PDG 2006

Lattice QCD



Edwards et al. LHP Coll (2006)

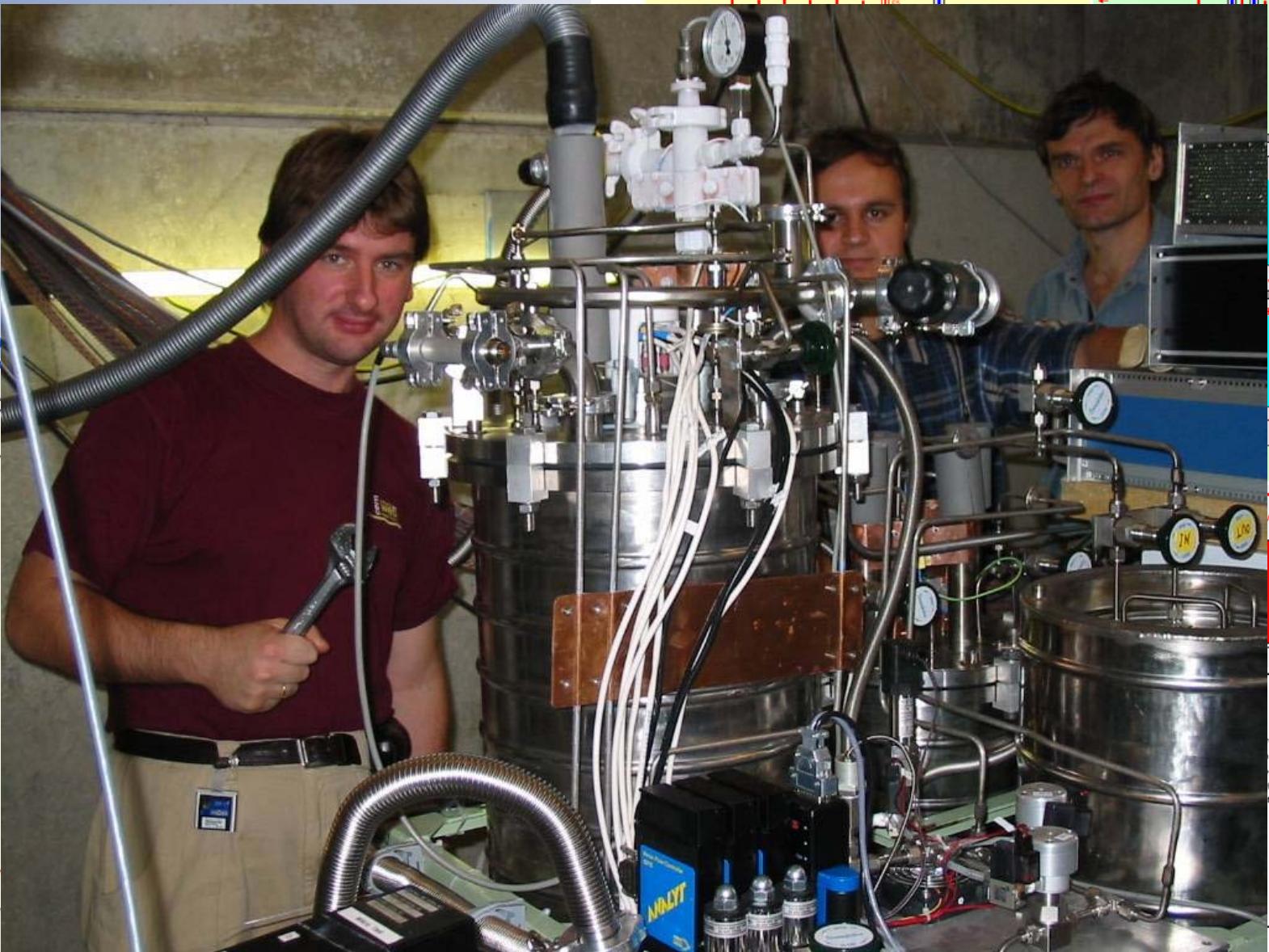
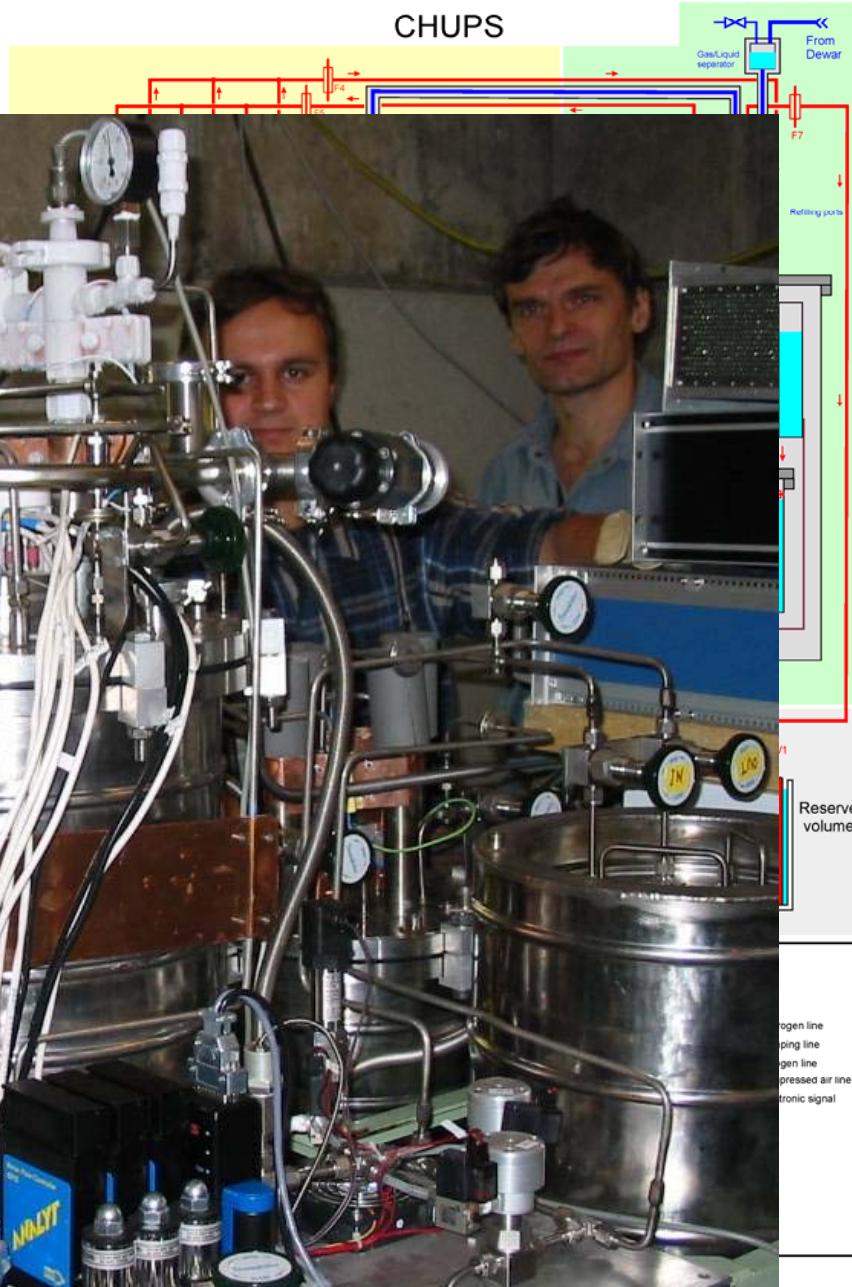
$$g_A(q^2) = g_A(0)(1 + \frac{1}{6} \langle r_A^2 \rangle q^2)$$

$$g_A(0) = 1.2695 \pm 0.0029$$

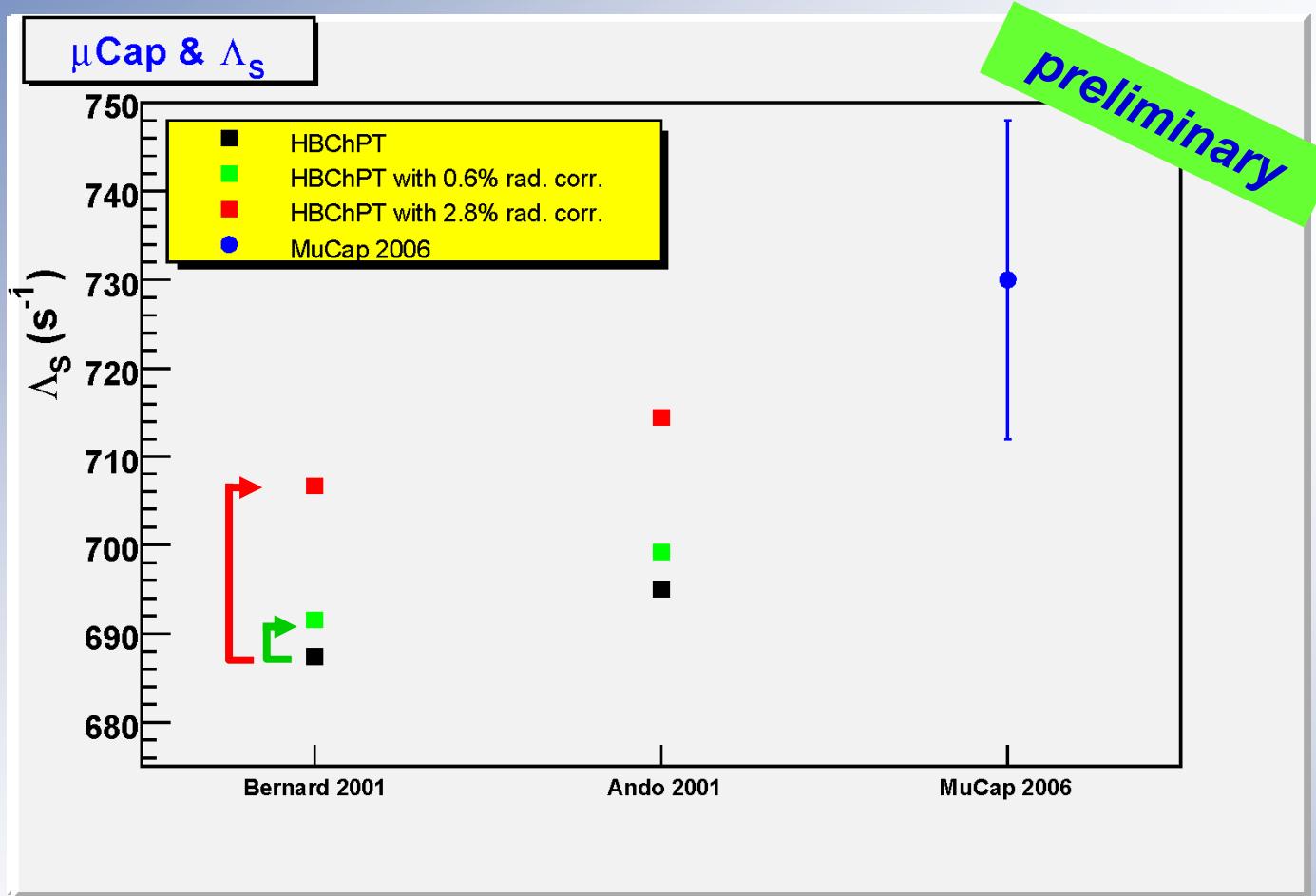
$$g_A(-0.88m_\mu^2) = 1.245 \pm 0.004$$

introduces 0.46% uncertainty to Λ_S (theory)

CHUPS



μ Cap and Λ_s calculations



rad. corrections

- *Goldman (1972)*
- *Czarnecki Marciano Sirlin (2006)*
private comm. preliminary

MuCap agrees within $\sim 1\sigma$ with Λ_s theory

Thorough theory studies needed for next MuCap 1% stage !



Theory and Sensitivities



PCAC:

$$q^2=0$$

GT relation:

$$g_{\pi NN}(0) F_\pi = M g_A(0)$$

$$q^2 < 0$$

$$g_p(q^2) = 2 m M / (m_\pi^2 - q^2) g_A(0)$$

$$g_p = 8.7$$

author	year	g_p	Λ_S	Λ_T	comment
Primakoff	1959		664(20)	11.9(7)	smaller g_A
Opat	1964		634	13.3	smaller g_A
Bernard et al	1994	8.44(23)			
Fearing et al	1997	8.21(9)			
Govaerts et al	2000	8.475(76)	688.4(38)	12.01(12)	
Bernard et al	2000/1		687.4 (711*)	12.9	NNLO, small scale
Ando et al	2001		695 (722*)	11.9	NNLO

Sensitivity of capture rate:

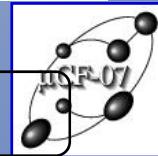
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$$\frac{\delta \Lambda}{\Lambda} [\%] = 0.024 \quad 0.01 \quad 0.38 \quad 3.7 \quad 0.24$$

assuming optimistic
20% g_P error

assuming $g_T < 0.1$

error from $V_{ud} = 0.16 \%$



Parameters



TABLE I. Numerical values of the parameters and derived quantities used in the text and in our evaluations of rates for comparison with experiment.

Symbol	Description	Value	Reference
F_π	pion decay constant	92.4 ± 0.3 MeV	Particle Data Group (2000)
$g_{\pi NN}(m_\pi^2)$	pion nucleon coupling	13.05 ± 0.08	de Swart <i>et al.</i> (1997)
$G_F V_{ud}$	Fermi constant for β decay	1.13548×10^{-5} GeV $^{-2}$	Particle Data Group (2000)
$g_a(0)$	axial coupling from β decay	1.2670 ± 0.0035	Particle Data Group (2000)
r_A^2	rms radius squared for g_a	0.44 ± 0.02 fm 2	Liesenfeld <i>et al.</i> (1999)
g_p^{PCAC}	PCAC value, $g_p(-0.88m_\mu^2)$	$6.87 g_a(0) = 8.70$	Eq. (5), leading term only
	PCAC value, NLO constant term included	$6.50 g_a(0) = 8.23$	Eq. (5), including NLO correction
$\Lambda_{p\mu p}$	$p\mu p$ molecular formation rate	2.5×10^6 s $^{-1}$	average, Wright <i>et al.</i> (1998)
$\Lambda_{p\mu p}^{ortho}/\Lambda_{p\mu p}^{para}$	ratio of ortho to para molecular formation	240:1	Faifman and Men'shikov (1999)
Λ_{op}	ortho to para transition rate	$4.1 \pm 1.4 \times 10^4$ s $^{-1}$	Bardin <i>et al.</i> (1981a)
$2\gamma^{ortho}$	ortho-molecular overlap factor	1.009 ± 0.001	Bakalov <i>et al.</i> (1982)
$2\gamma^{para}$	para-molecular overlap factor	1.143 ± 0.001	Bakalov <i>et al.</i> (1982)
$g_m(0)$	weak magnetism coupling, $\kappa_p - \kappa_n$	3.705 89	Particle Data Group (2000)
r_m^2	rms radius squared for g_m	0.80 fm 2	Mergell <i>et al.</i> (1996)
r_v^2	rms radius squared for g_v	0.59 fm 2	Mergell <i>et al.</i> (1996)



Nucleon charged current at $q^2 = -0.88$

$$m_\mu^2$$



$$\mathbf{J}_\alpha = \mathbf{V}_\alpha \cdot \mathbf{A}_\alpha$$

$$\mathbf{V}_\alpha = g_V(q^2) \gamma_\alpha + ig_M(q^2)/2M \sigma_{\alpha\beta} q^\beta + g_S(q^2)/m q_\alpha$$

$$\mathbf{A}_\alpha = g_A(q^2) \gamma_\alpha \gamma_5 + g_P(q^2) \mathbf{q}_\alpha / m \gamma_5 + ig_T(q^2)/2M \sigma_{\alpha\beta} q^\beta \gamma_5$$

nucleon weak formfactors g_V, g_M, g_A

- determined by SM symmetries and data
- contribute <0.4% uncertainty to Λ_S

$$g_V = 0.9755(5)$$

$$g_M = 3.5821(25)$$

$$g_A = 1.245(3)$$

remains

$$g_P = ?$$

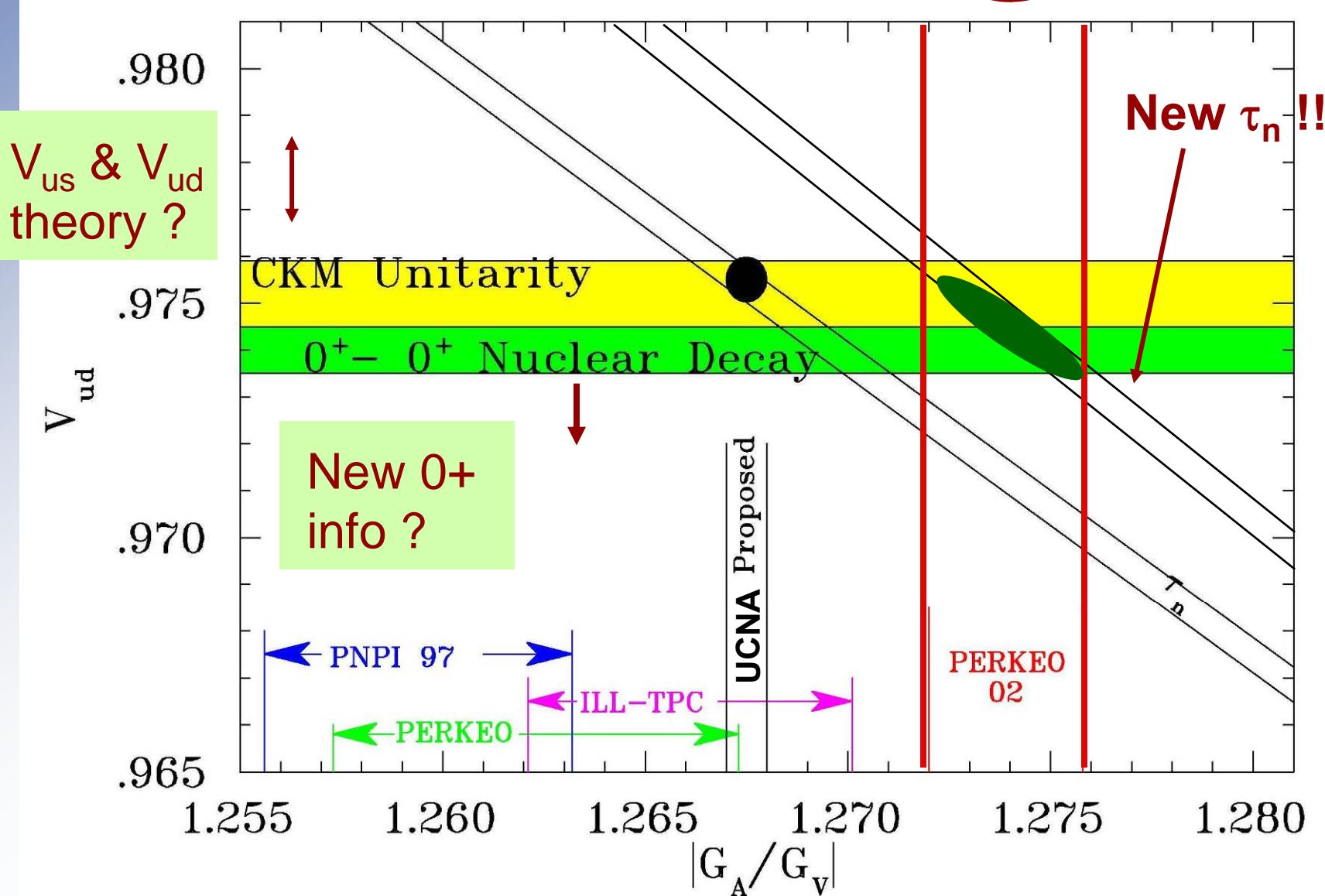
- Vector current in SM determined via CVC
 $g_V(0)=1, g(q^2)=1+q^2 r_V^2/6, r_V^2=0.59 \text{ fm}^2$
 $g_M(0)=\mu_p-\mu_n-1=3.70589, r_M^2=0.80 \text{ fm}^2$
 q^2 dependence from e scatt.
- Axial vector FF from experiment
 $g_A(0)=1.2670(35), r_A^2=0.42 \pm 0.04 \text{ fm}^2$
 q^2 dependence from quasi-elastic ν scattering,
 π e-production
- 2nd class FF g_S, g_T forbidden by G symmetry, e.g.
 $g_T/g_A=-0.15 \pm 0.15$ (exp),
 -0.0152 ± 0.0053 (QCD sum rule, up-down mass difference)
- error from $V_{ud} = 0.16 \%$



Neutron



CKM Summary: New V_{us} & τ_n ?



neutron (J. Nico, CIPANP 06)



$$dW \propto (g_V^2 + 3g_A^2)F(E_e)[1 + a\frac{\vec{p}_e \cdot \vec{p}_\nu}{E_e E_\nu} + \vec{\sigma}_n \cdot (A\frac{\vec{p}_e}{E_e} + B\frac{\vec{p}_\nu}{E_\nu} + D\frac{\vec{p}_e \times \vec{p}_\nu}{E_e E_\nu})]$$

Jackson, Treiman, Wyld, *Nucl. Phys.* **4**, 206 (1957)

Lifetime

$$\tau = \frac{1}{f(1 + \delta_R)} \frac{K/\ln 2}{(1 + \Delta_R^V)(g_V^2 + 3g_A^2)} = (885.7 \pm 0.8) \text{ s}$$

Coupling ratio

$$\lambda = \frac{|g_A|}{|g_V|} e^{i\phi} = (-1.2695 \pm 0.0029)$$

Electron-antineutrino asymmetry

$$a = \frac{1 - |\lambda|^2}{1 + 3|\lambda|^2} = (-0.103 \pm 0.004)$$

Spin-antineutrino asymmetry

$$B = 2 \frac{|\lambda|^2 - |\lambda| \cos \phi}{1 + 3|\lambda|^2} = (0.983 \pm 0.004)$$

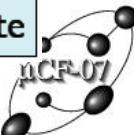
Spin-electron asymmetry

$$A = -2 \frac{|\lambda|^2 + |\lambda| \cos \phi}{1 + 3|\lambda|^2} = (-0.1173 \pm 0.0013)$$

Triple correlation

$$D = 2 \frac{|\lambda| \sin \phi}{1 + 3|\lambda|^2} = (-4 \pm 6) \times 10^{-4}$$

PDG, 2005 update



neutron

