Progress of Few-Body Calculational Methods Stimulated by μCF

—— Stau-catalyzed nuclear fusion ——

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Outline

briefly

1. Gaussian Expansion Method (GEM)
   for various 3-body problems in μCF

very
briefly

2. Examples of applications of GEM
   to various 3-, 4- and 5-body problems in physics

3. Stau-catalyzed d-d fusion and d-α fusion
   stau ($\tilde{\tau}$) = supersymmetry (SUSY) partner of tau lepton (τ)

Section 1

Gaussian Expansion Method (GEM)

for various 3-body problems in μCF

My colleagues, Kino and Hiyama, and myself have been developping a few-body calculational method, called Gaussian Expansion Method (GEM), which is reviewed in

" Gaussian Expansion Method for Few-Body Systems "

E. Hiyama, Y. Kino and M. Kamimura,

Prog. Part. Nucl. Phys. 51 (2003) 223.

This method was first proposed by myself at **μ**CF-1987 held in Leningrad.

As is explained later on,

this method, GEM, is accurately applicable both

i) to bound-state calculations such as for

$(d\ t\mu)_{11}$

ii) to reaction calculations such as for

$$(d\mu)_{1s} + t \rightarrow d + (t\mu)_{1s} + 48\,\mathrm{eV}$$

μCF-1987 in Leningrad :

I reported calculated results for the two cases.

μCF-1988 in Florida :

I applied the method to the calculations of

i) fusion rate in $(d\ t\mu)$

ii) probability of muon sticking to **α** after the fusion.

μCF-1992 in Uppsala :

Kino and myself applied GEM to (d Heμ) molecules.

We discussed competition between the partcle decay and radiative decay from the excited J=1 states of the molecule.

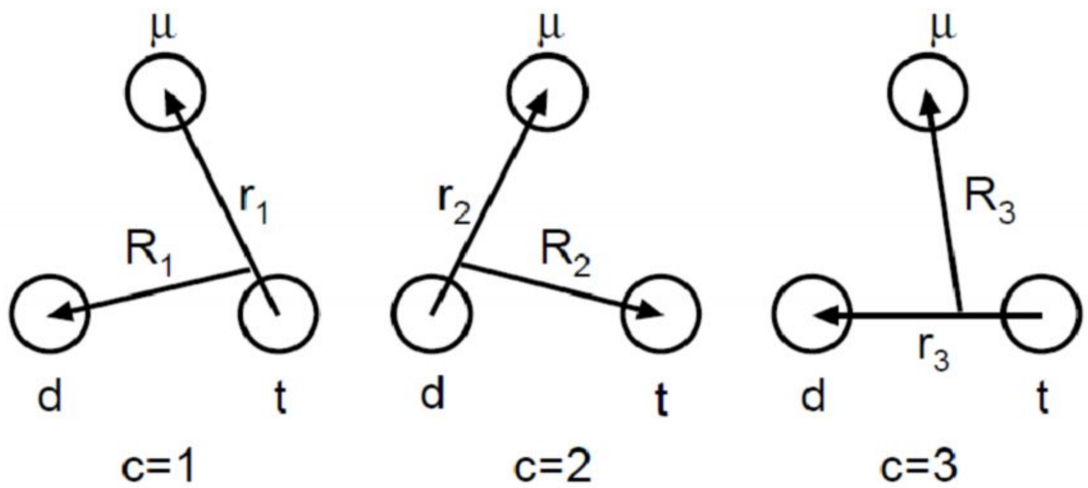
This calculation nicely explained the isotope dependence of the radiative decay seen in (d ^{3}Heμ), (d ^{4}Heμ) and (p ^{4}Heμ).

Expeimental data for the isotope dependence was just reported in the same conference by Nagamine group.

Section 1.1

Brief survay of GEM

3-body Gaussian basis functions



On 3-sets of Jacobi coordinates

$$\left[\phi^{\rm G}_{n_c l_c}(\mathbf{r}_c)\,\psi^{\rm G}_{N_c L_c}(\mathbf{R}_c)\right]_{JM} \qquad (c = 1-3)$$

$$\phi^{\rm G}_{nlm}(\mathbf{r}) = \phi^{\rm G}_{nl}(r)\,Y_{lm}(\widehat{\mathbf{r}}), \qquad \psi^{\rm G}_{NLM}(\mathbf{R}) = \psi^{\rm G}_{NL}(R)\,Y_{LM}(\widehat{\mathbf{R}}),$$

$$\phi^{\rm G}_{nl}(r) = r^l\,e^{-\nu_n r^2}\,, \qquad \psi^{\rm G}_{NL}(R) = R^L\,e^{-\lambda_N R^2}.$$

Gaussian ranges:

$$\phi^{\rm G}_{nl}(r) = r^l \, e^{-\nu_n r^2} \,, \qquad \psi^{\rm G}_{NL}(R) = R^L \, e^{-\lambda_N R^2}$$

$$\nu_n = 1/r_n^2 \,, \;\; r_n = r_1 \, a^{n-1} \qquad (n = 1 - n_{\rm max}) \,,$$

$$\lambda_N = 1/R_N^2 \,, \;\; R_N = R_1 \, A^{N-1} \;\; (N = 1 - N_{\rm max}) \,.$$

Geometric progression

To take geometric progression is very suited for describing
Simultaneously both long-range asymptotic behavior
and short-range correlations:

Precisely reviewd in
" Gaussian Expansion Method for Few-Body Systems "
E. Hiyama, Y. Kino and M. Kamimura,
Prog. Part. Nucl. Phys. 51 (2003) 223.

Using the 3-body basis functions,

$$\left[\phi^{\rm G}_{n_c l_c}(\mathbf{r}_c)\,\psi^{\rm G}_{N_c L_c}(\mathbf{R}_c)\right]_{JM} \qquad (c=1-3)$$

we

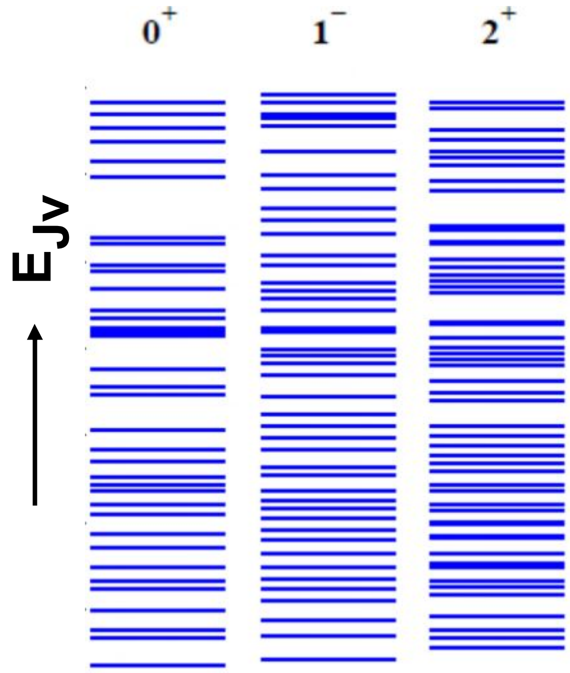
1) diagonalize the 3-body Hamiltonian,

$$\langle \Phi_{JM,\nu} |\, H \,| \Phi_{JM,\nu'} \rangle = E_{J\nu}\delta_{\nu\nu'},$$

$$(\nu, \nu' = 1 - \nu_{\rm max})$$

2) obtain 3-body eigenstates:

$$\{\Phi_{JM,\nu};\, \nu = 1 - \nu_{\rm max}\},$$



(shematic)

$\nu_{max} \sim 10^3$ **for 3-body problem**

$\sim 10^4$ **for 4-body problem**

Function space of the 3-body Gaussian basis functions spanned over the 3 sets of Jacobi coordinates is very wide.

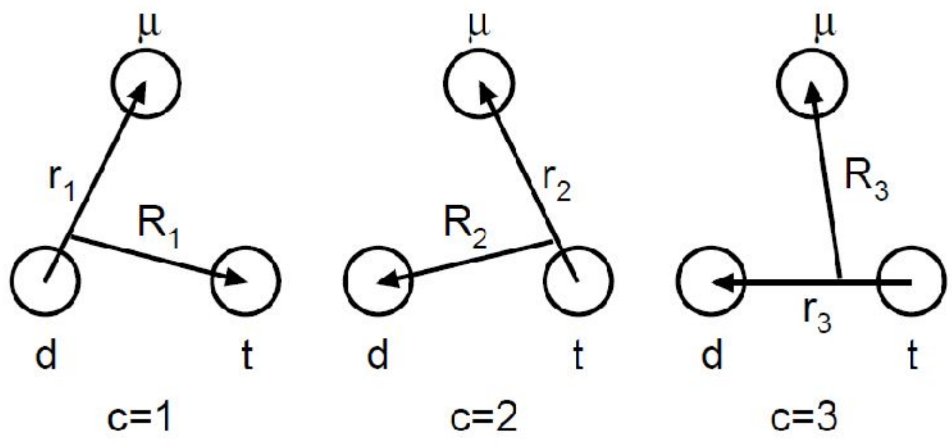
Therefore, eigenfunctions obtained above

i) are found to form a complete set in the finite spatial region,

ii) are useful to expand any asymptotically-vanishing function.

3-body reaction calculations

$$(d\mu)_{1s} + t \rightarrow d + (t\mu)_{1s} + 48\,\mathrm{eV}$$



Total wave function

$$\Psi_{JM} = \phi_{1s}^{(d\mu)}(\mathbf{r}_1)\,\underline{\chi_{JM}^{(d\mu-t)}(\mathbf{R}_1)} + \phi_{1s}^{(t\mu)}(\mathbf{r}_2)\,\underline{\chi_{JM}^{(t\mu-d)}(\mathbf{R}_2)} + \Psi_{JM}^{(\mathrm{closed})} \; .$$

$$\Psi_{JM}^{(\mathrm{closed})} = \sum_{\nu=1}^{\nu_{\max}} \underline{b_{J\nu}}\,\Phi_{JM,\nu}$$

to be solved

The third term, $\Psi_{JM}^{(\mathrm{closed})}$ **, stands for all the closed (virtually-excited) channels in the energy range of this reaction;**

This term is responsible for all the asymptotically-vanishing 3-body amplitudes that are not included in the first two scattering terms.

The term is then expanded in terms of the complete set in the finite region.

Section 2

Examples of applications of GEM to various 3-, 4- and 5-body problems in physics

Examples of application and development of GEM

1) Antiprotonic helium atom and mass of antiproton (3-body)
   (Kino)

2) 3-cluster structure of light nuclei
   (Kamimura)

3) 3- and 4-body structure of light hypernuclei (strangeness= -1, -2)
   (Hiyama)

4) 4-nucleon ground state and excited states (realistic NN force)
   (Hiyama)

5) Resonance and scattering states of 5-quark states
   (Hiyama)

6) 4-body breakup reactions induced by unstable halo nuclei
   (Hiyama, Matsumoto)

7) Stau-catalyzed nuclear fusion (3-body)
   (Kino, Kamimura) --- in PLB(2007)

....

As is recognized from this list, developments to 4- and 5-body problems has been accomplished by Hiyama.

She proposed a new type of Gaussian basis functions,

called infinitesimally-shifted Gaussian-Lobe basis functions.

$$\begin{aligned}\phi^{\rm G}_{nlm}(\mathbf{r}) &= r^l e^{-\nu r^2} Y_{lm}(\widehat{\mathbf{r}}) \\ &= \lim_{\varepsilon\to 0} \frac{1}{(\varepsilon\nu)^l} \sum_{k=1}^{k_{\rm max}} C_{lm,k}\, e^{-\nu(\mathbf{r}-\varepsilon\mathbf{D}_{lm,k})^2}\end{aligned}$$

with no $Y_{lm}(\widehat{\mathbf{r}})$

and a skilful method to take the limitting analytically

after analytical calculation of few-body Hamiltonian matrix elements.

The new type of Gaussian basis functions makes

3-, 4- and 5-body calculations extremely easier than before.

Due to this new method and many applications mentioned above,

the 2006 Yukawa Memorial Award, a very honorable award

in theoretical physics in Japan, was given to Hiyama.

It is my pleasure to see this and to note that such a development of the method started with soving difficult 3-body problems in μCF.

Section 3

Stau-catalyzed d- d fusion and d-α fusion

stau (scalar tau) particle ($\tilde{\tau}$)

1) Supersymmetry (SUSY) particle (lepton)
beyond the standard model (not discovered yet)

2) the scalar partner (boson) of the tau lepton (fermion)

3) ・ the lightest SUSY particle = gravitino
(cadidate of the dark matter,
the fermion partner of the graviton),

・ the next lightest SUSY particle (NLSP) = stau

4) stau mass ~ a few 100 GeV

lifetime ~ seconds to years

$$\tilde{\tau} \to \tau \tilde{\chi}^0,$$
$$\tilde{\tau} \to \pi \nu_\tau \tilde{\chi}^0,$$
$$\tilde{\tau} \to l \nu_l \nu_\tau \tilde{\chi}^0.$$

5) charged lepton (usually written as X^- , X^+ , X^0)

6) Coulombic interaction and weak interaction

The **stau** particle is expected to be discovered

at LHC (Linear Hadron Colider) in CERN

at the early stage after the first beam (2007) (before Higgs particle?).

Therefore, many theorists in the elementary particle physics

are eagerly making many predictions about the stau particle.

Six months ago, Kino and myself were asked to help

three of them who are studying **stau**-catalyzed nuclear fusion.

particle
physicists

K. Hamaguchi, T. Hatsuda and T. Yanagida

(University of Tokyo)

hadron physicist

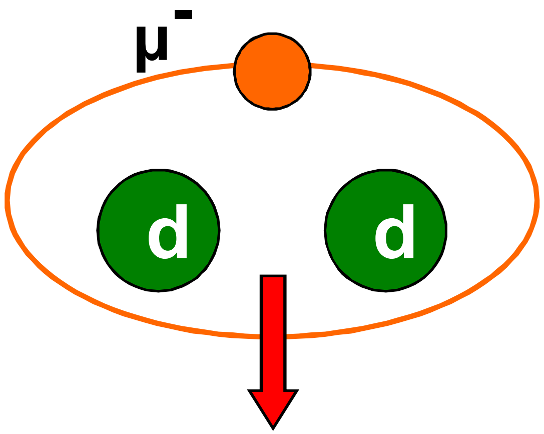
Here, I introduce you two examples of their study.

Section 3.1

stau-catalyzed d-d fusion

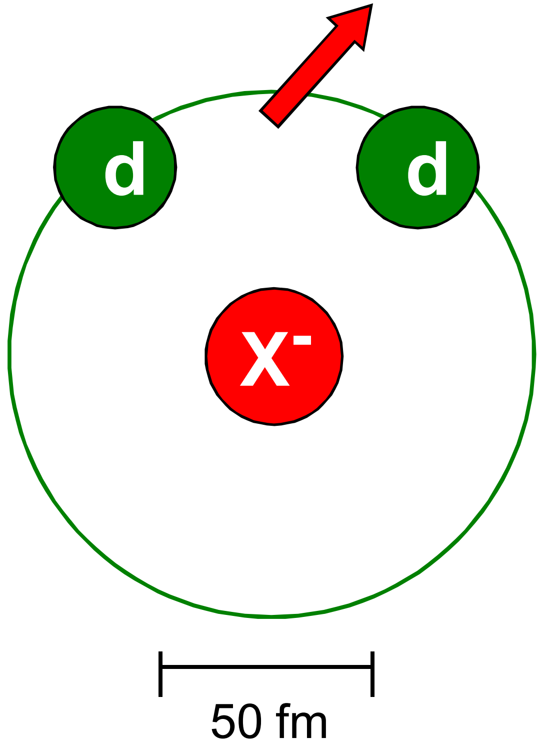
stau (X^-) particle

- Long-lived, negatively-charged , heavy lepton
- Coulombic interaction (and weak interaction)



muon-catalyzed **d-d** fusion

stau-catalyzed **d-d** fusion



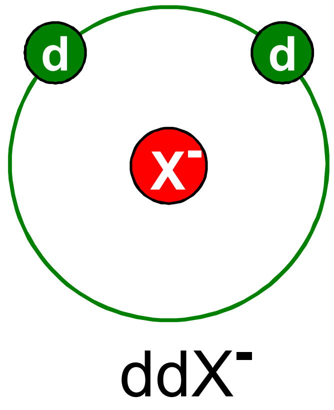
extremely exotic atom !

arXiv: hep-ph/0607256 (July, 2006)

" Stau-Catalyzed Nuclear Fusion "

K. Hamaguchi, T. Hatsuda and T. Yanagida

(University of Tokyo)

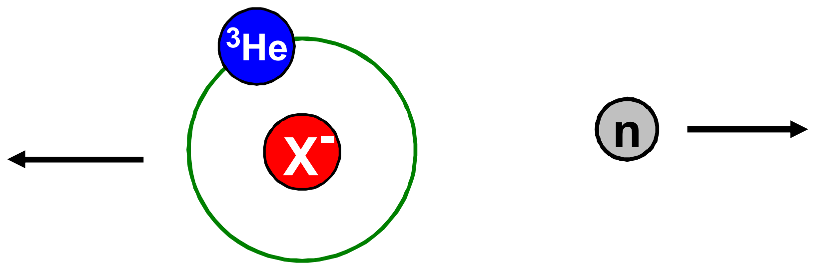


They discussed about

i) feasibility of stau-catalyzed d-d fusion

ii) possble production of stau particles.

In i), since the lifetime of X^- is sufficiently long (seconds to years), essincial issue is the probability of ($^3He - X^-$) sticking after fusion.



The authors estimated as

probability of X^- sticking to ^{3}He = 2×10^{-6} (cf. 0.12 in ddμ)

Therefore

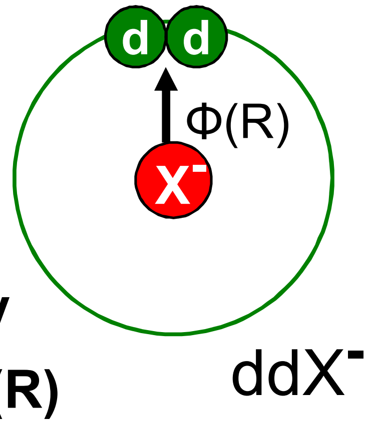
Energy production = 4 MeV / (2×10^{-6}) = 2000 GeV / X^-

per one fusion

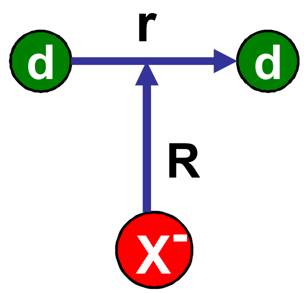
(dtμ : ~ 2 GeV /μ)

Amazing !

arXiv:hep-ph/0607256 (July, 2006)
K. Hamaguchi, T. Hatsuda and T. Yanagida

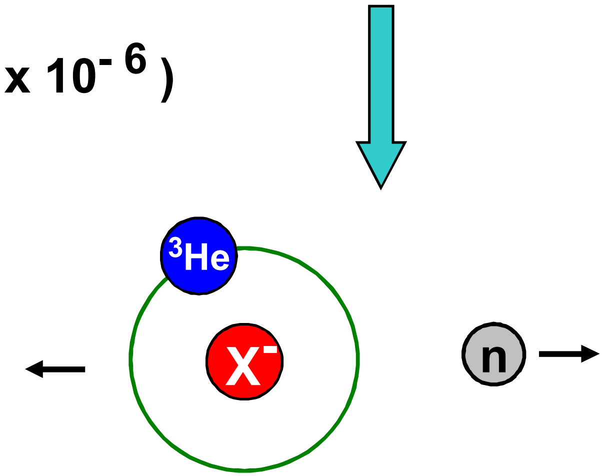


But,
I found that the authors did not calculate the ddX^- 3-body wave function, but assumed too naive wave function Φ(R) between d-d pair and X^- when d-d fusion takes place.



Kino and myself solved the 3-body problem of $d + d + X^-$ system accurately and calculated the sticking probability.

We obtained : X^- sticking to ^{3}He = 0.023 (not 2×10^{-6})



It is pity to conclude that

the stau-catalyzed d-d fusion in the (ddX^-) atom is not feasible.

Section 3.2

stau-catalyzed d-α fusion

in Big-Bang nucleosynthesis

In the Big-Bang nucleosynthesis,

^{6}Li nucleus is produced mainly by

$$d + \alpha \rightarrow {}^{6}\mathrm{Li} + \gamma$$

$$\frac{\text{mass}(d)}{\text{mass}(\alpha)} \approx \frac{\text{charge}(d)}{\text{charge}(\alpha)}$$

because

Since, in this capture-γreaction, E1 transition is heavily suppressed,

$$\sigma(d + \alpha \rightarrow {}^{6}\mathrm{Li} + \gamma)^{E2} \sim 10^{-5} \times \sigma(t + \alpha \rightarrow {}^{7}\mathrm{Li} + \gamma)^{E1}$$

Producton of ^{6}Li is much smaller than that of ^{7}Li.

This is one of the key points of

the beautiful success of the Big-Bang nucleosynthesis scenario.

(H, D, T, ^{3}He, ^{4}He, ^{6}Li, ^{7}Li)

But, an exciting idea by Pospelov (2006):

Stau-catalyzed nuclear fusion might destroy the success.

A very exciting paper:

hep-ph/0605215 (May, 2006) (v4, Mar 2007) (PRL, June 2007)

“ Particle-physics catalysis of thermal Big-Bang nucleosynthesis “

M. Pospelov

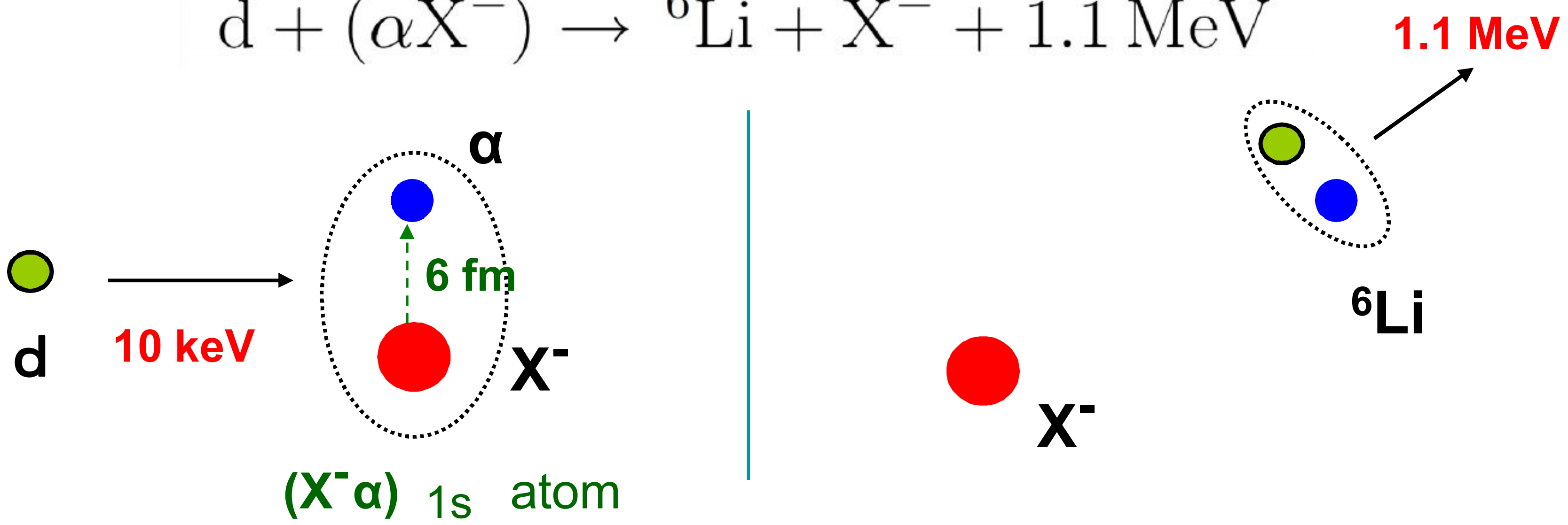
(According to the SUSY physics, **X⁻** is generated just after the inflation of the univers)

If the lifetime of X⁻ is long enough (10^3– 10^4 sec)

- **X⁻ survives until the Big Bang time**
- **X⁻ forms a bound state such as (αX⁻)**
- **X⁻ couses a dangerous reaction such as**

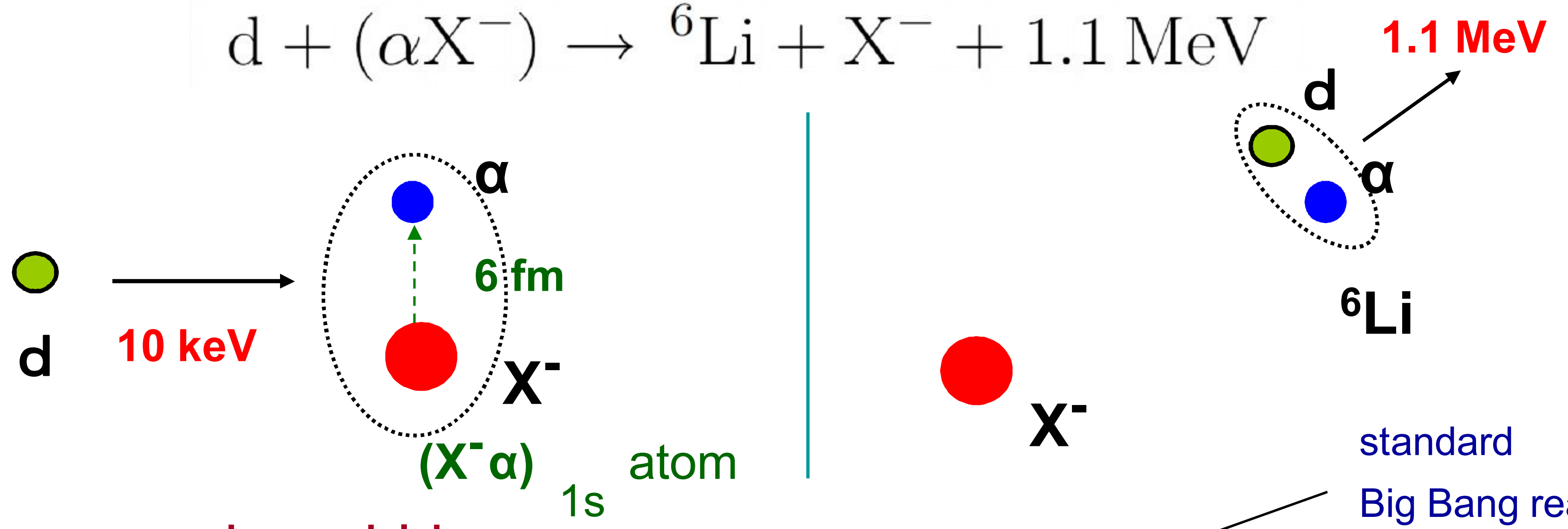
(X⁻ catalyzed fusion in flight)

$$d + (\alpha X^-) \rightarrow {}^6\mathrm{Li} + X^- + 1.1\,\mathrm{MeV}$$



M. Pospelov (2006) : stau-catalyzed d-α fusion

$$\mathrm{d} + (\alpha X^-) \to {}^6\mathrm{Li} + X^- + 1.1\,\mathrm{MeV}$$



standard
Big Bang reaction
to produce ^{6}Li

Using a very naive model, he gave

$$\sigma(\mathrm{d} + (X^-\alpha) \to {}^6\mathrm{Li} + X^-) \sim 10^8 \times \sigma(\mathrm{d} + \alpha \to {}^6\mathrm{Li} + \gamma)$$

Serious problem : production of too much ^{6}Li

This distroys the success of the Big-Bang-nucleosynthesis scenario.

If this estimation is correct, we are enforced to assume

i) very short life time of X^- to disappear before the nucleosynthesis time,

ii) very small density of X^- at the nucleosynthesis time,

which strongly confines the property of X^- .

Therefore, this is one of fassionable subjects in an overlap region of cosmology, elementary particle physics and nuclear astrophysics.

Many people wanted to know wherether the Pospelov's naive estimation

$$\sigma(\ d + (X^{-}\alpha) \rightarrow {}^{6}Li + X^{-}\ ) \sim 10^{8} \times \sigma(\ d + \alpha \rightarrow {}^{6}Li + \gamma\ )$$

is valid or not.

I was asked, 6 month ago, by these 3 authors, to examine this estimation.

arXiv:hep-ph/0607256 (July, 2006)

" Stau-catalyzed Nuclear Fusion "

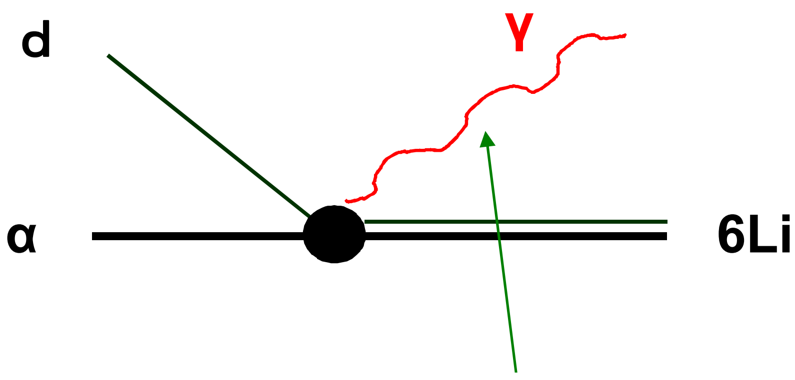
K. Hamaguchi, T. Hatsuda and T. Yanagida

(University of Tokyo)

SBBN (Standard B.B. Nucleosyntheis) **CBBN** (Catalyzed …….)

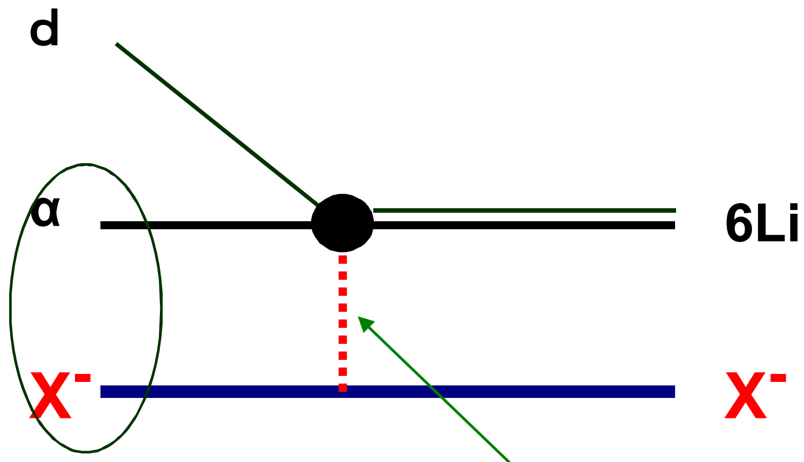
$d + \alpha \rightarrow {}^6Li + \gamma$

$$\mathrm{d} + (\alpha X^-) \rightarrow {}^6\mathrm{Li} + X^- + 1.1\,\mathrm{MeV}$$



E2 real photon

wave length 130 fm



$(\alpha X^-)_{1s}$

Bohr radius a_0 = 3.6 fm

E2 virtual photon

wave length a_0= 3.6 fm

Essential part of the

Pospelov's estimation

$$\text{Enhance factor} = \frac{\text{CBBN}}{\text{SBBN}} \sim \left(\frac{130\ \text{fm}}{3.6\ \text{fm}}\right)^{\overset{\text{E2}}{5}} \sim 5 \times 10^7$$

(No consideration on nuclear α-d potential and on angular momentum between αand X)

Kino and myself did precise 3-body reaction calculation of

$$\mathrm{d} + (\alpha X^-) \rightarrow {}^6\mathrm{Li} + X^- + 1.1\,\mathrm{MeV}$$

and published it together with the 3 authors

(including an additional discussions from the Particle Physics models):

" Stau catalyzed ^{6}Li production in Big-Bang nucleosynthesis "

K. Hamaguchi, T. Hatsuda, M. Kamimura, Y. Kino and T.T Yanagida

Phys. Lett. B (2007), in press; hep-ph/0702274 (Feb, 2007)

We simply applied the same calculational method of

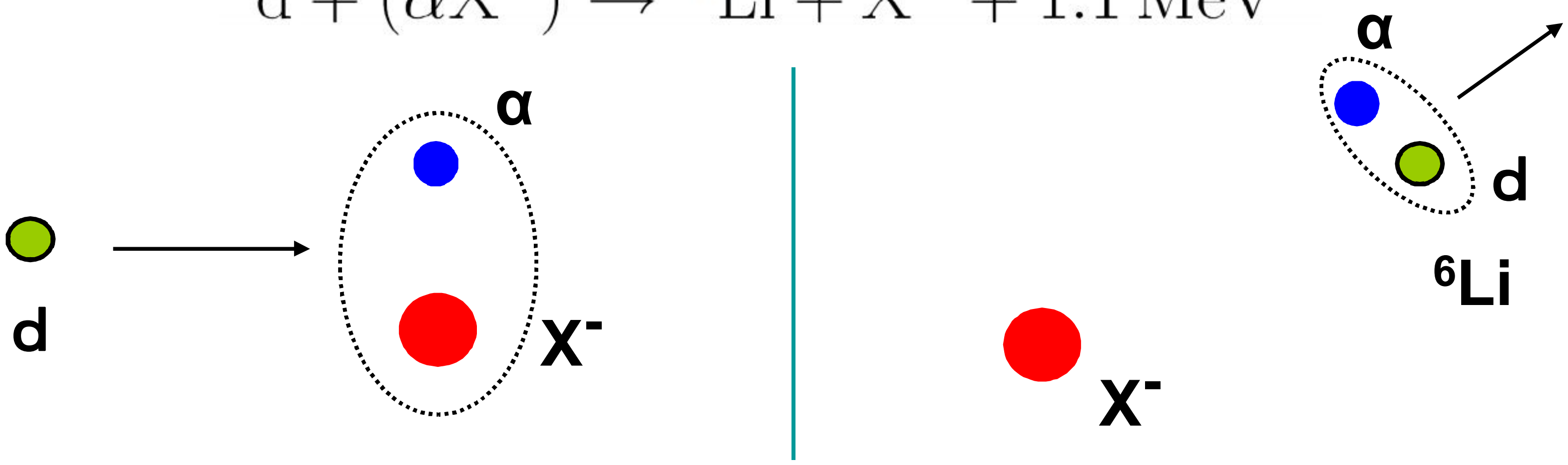
our muon-transfer-reaction calculation

to this stau-catalyzed nuclear fusion reaction

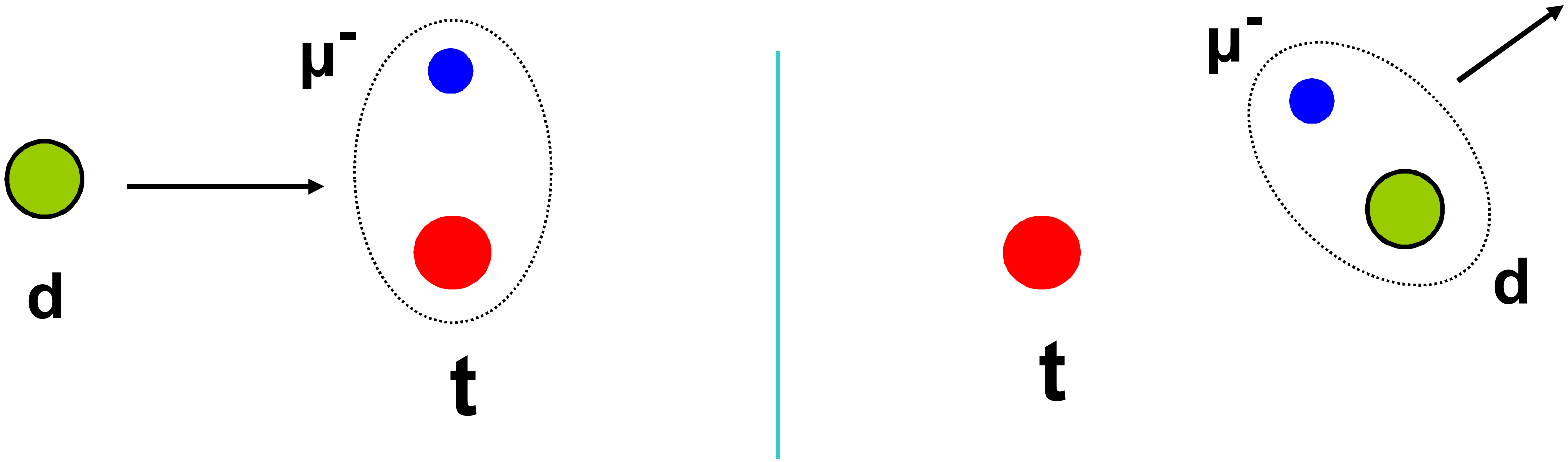
because the two types of the reactions have the same structure

as is seen in the next figure:

$$\mathrm{d} + (\alpha X^-) \rightarrow {}^6\mathrm{Li} + X^- + 1.1\,\mathrm{MeV}$$



$$t + (d\mu^-) \longrightarrow (t\mu^-) + d + 48\ \mathrm{eV}$$



Coulomb barrier $\longrightarrow$ $\mathrm{d} + (\alpha X^-) \rightarrow {}^6\mathrm{Li} + X^- + 1.1\,\mathrm{MeV}$

No Coulomb barrier $\longrightarrow$ $t + (d\mu^-) \longrightarrow (t\mu^-) + d + 48\ \mathrm{eV}$

However, large difference between two types of reactions:

1) Stau-catalyzed nuclear fusion takes place much below the Coulomb barrier.

incoming energy = 10 – 100 keV (Temperature=10 keV)
Coulomb barrier = 500 keV

2) We have to treat simultaneously both the long-range Coulomb potential and the short-range nuclear potential which is the the driving force of the fusion reaction.

This 3-body reaction calculation is much more tedious than in the muon transfer reaction.

Therefore, for safety, Kino and myself solved
the same 3-body Schroedinger equation,
using quite different 2 methods to each other and
compared the calculated S-matrix elements.

We were so careful about this exciting problem.

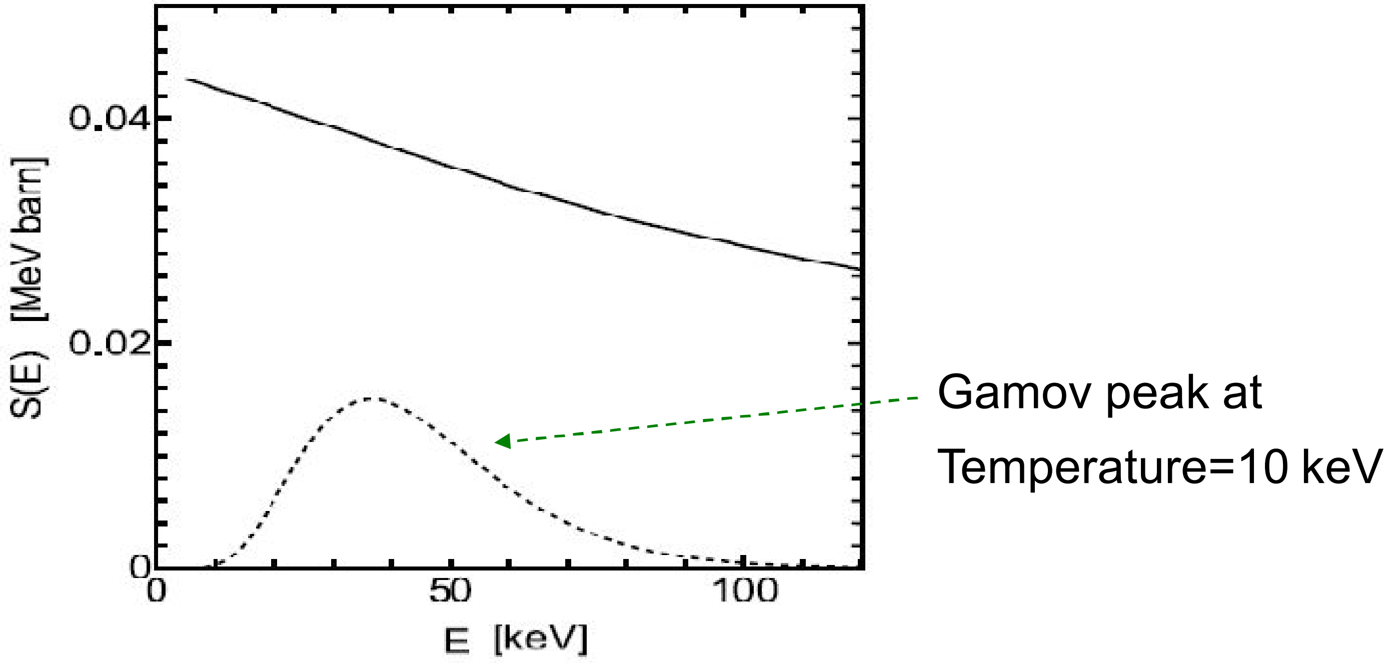
We found

1) Kino's result with the direct numerical (finite-difference) method
2) Kamimura's result with the Kohn-type variational method

agree very well to each other.

$$\mathrm{d} + (\alpha X^-) \rightarrow {}^6\mathrm{Li} + X^- + 1.1\,\mathrm{MeV}$$

Calculated astrophysical S-factor



Reaction rate

$$N_A \langle \sigma v \rangle = 2.37 \times 10^8 (1 - 0.34\, T_9)\, T_9^{-2/3} \exp(-5.33\, T_9^{-1/3}) \quad \mathrm{cm^3\, s^{-1}\, mol^{-1}}. \qquad ($$

PLB (in press, 2007)

Comparison in reaction cross section

	Pospelov (2006) too naive model	Kino & Kamimura 3-body calculation
$d + (X^{-}\alpha) \rightarrow {}^{6}Li + X^{-}$	10	1
$t + (X^{-}\alpha) \rightarrow {}^{7}Li + X^{-}$	1000	1

Reason of the difference :

Pospelov's virtual photon model pays no attention

i) to the angular momenta between particle pairs

ii) to the nuclear potential

(so simple model).

But, the Pospelov's idea (stau-catalyzed nuclear fusion) itself is very much interesting and appreciated.

His estimation of the ^{6}Li production at the Big Bang time

$$\sigma(\ d + (X^-\alpha) \rightarrow {}^6\mathrm{Li} + X^-\ ) \sim 10^8 \times \sigma(\ d + \alpha \rightarrow {}^6\mathrm{Li} + \gamma\ )$$

is reduced to

$$\sigma(\ d + (X^-\alpha) \rightarrow {}^6\mathrm{Li} + X^-\ ) \sim 10^7 \times \sigma(\ d + \alpha \rightarrow {}^6\mathrm{Li} + \gamma\ )$$

by our calculation,

but still enough large to destroy the success of the standard Big-Bang scenario.

By the way, I was surprized to see the following thing:

within only 2 weeks after our preprint was posted on the arXive,

5 new preprints appeared in the arXive citing our result.

So busy the community is.

$$\sigma(\ d + (X^{-}\alpha) \rightarrow {}^{6}\mathrm{Li} + X^{-}\ ) \sim 10^{7} \times \sigma(\ d + \alpha \rightarrow {}^{6}\mathrm{Li} + \gamma\ )$$

Standard Big-Bang reaction

One of those 5 arXive preprints says :

" this factor of 10^7 is very severe from the viewpoint of the compatibility between particle physics models and Big-Bang nucleosynthesis".

We are now calculating all the possible cases of the stau-catalyzed nuclear reactions in Big-Bang nucleosynthesis and studying its influence on particle physics models.

It is my pleasure to see that developments of calculational methods which have been stimulated by μCF are now very useful to studies in other fields.

Let me skip the summary.

Thank you.