Calculation of Muonic Atom cascade Dynamics in D-T Mixtures

JINR, Dubna, Mucf07
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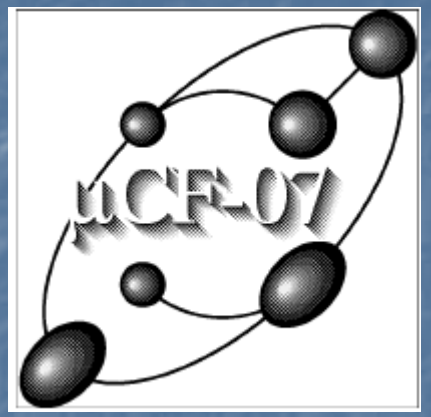


- Introduction
- Cascade processes of muonic atoms
- Kinetics of Cascade processes of muonic atoms
- Cascade Processes of Kaonic atoms
- Forthcoming Research

Introduction

- Why the calculations of kinetics of cascade processes of muonic atoms are important.

1. Cascade processes of muonic atoms affect the muon catalyzed fusion.

2. It can help to realize the kinetics of cascade processes of the other Exotic atoms.

Introduction

$\pi^- \longrightarrow \mu^-(4.3\ \text{MeV}) + \bar{\nu}$

I) Ionization $\mu + (x)_{gs} \longrightarrow \mu + e + x^+$

II) Inelastic collision $\mu + (x)_{gs} \longrightarrow \mu + (x)_{nl}$

III) Captureing $\mu + (x)_{gs} \longrightarrow (\mu x)_{nl}\ (1eV \sim 10eV) + e$ if $V_\mu \sim V_e$

$x = D, D_2$

In high energy with the Born approximation, the predicted time for stoping muon is in the order of : 10-12 s to 10-11 s

Introduction

$$r_n = \frac{n^2\hbar^2}{\bar{m}_\mu z e^2} \qquad E_n = -\frac{ze^2}{2r_n} = -\frac{\bar{m}_\mu z^2 e^4}{2n^2\hbar^2} \qquad V_n = \frac{ze^2}{n\hbar}$$

$\bar{m}_\mu , z$ are Muon reduced mass and charge of nuclues, Resp.

They have a peak around the following values. (J.S. Cohen, Phys. Rev. ,27(1983)167.

$$n_{peak} = m_\mu^{\frac{1}{2}} \approx 12 \qquad l_{peak} = (2m_\mu)\left(1 - \frac{m_\mu}{2n^2}\right)^{\frac{1}{2}}$$

Cascade processes of muonic atoms

- Cascade processes

Mechanism	Process	Energy Dependence	Density Dependence
Chemical dissociation	$(x\mu)_{n_i} + x_2' \rightarrow x'^{+} + x^{-} + (x\mu)_{n_f}, n_f < n_i, l_f = l_i$	Weak	Linear
Radiative	$(x\mu)_{n_i} \rightarrow (x\mu)_{n_f} + \gamma$	None	None
Auger	$(x\mu)_{n_i} + x_2' \rightarrow x_2'^{+} + e + (x\mu)_{n_f}$	Weak	Linear
Stark mixing	$(x\mu)_{nl} + X \rightarrow (x\mu)_{nl'} + X$	Moderate	Linear
Scattering	$(x\mu)_n + X \rightarrow (x\mu)_n + X'$	Strong	Linear
Coulomb	$(x\mu)_{n_i} + x' \rightarrow (x\mu)_{n_f} + x'$	Strong	Linear
Transfer	$(x\mu)_n + x' \rightarrow (x'\mu)_n + x$	Strong	Linear

Cascade processes of muonic atoms

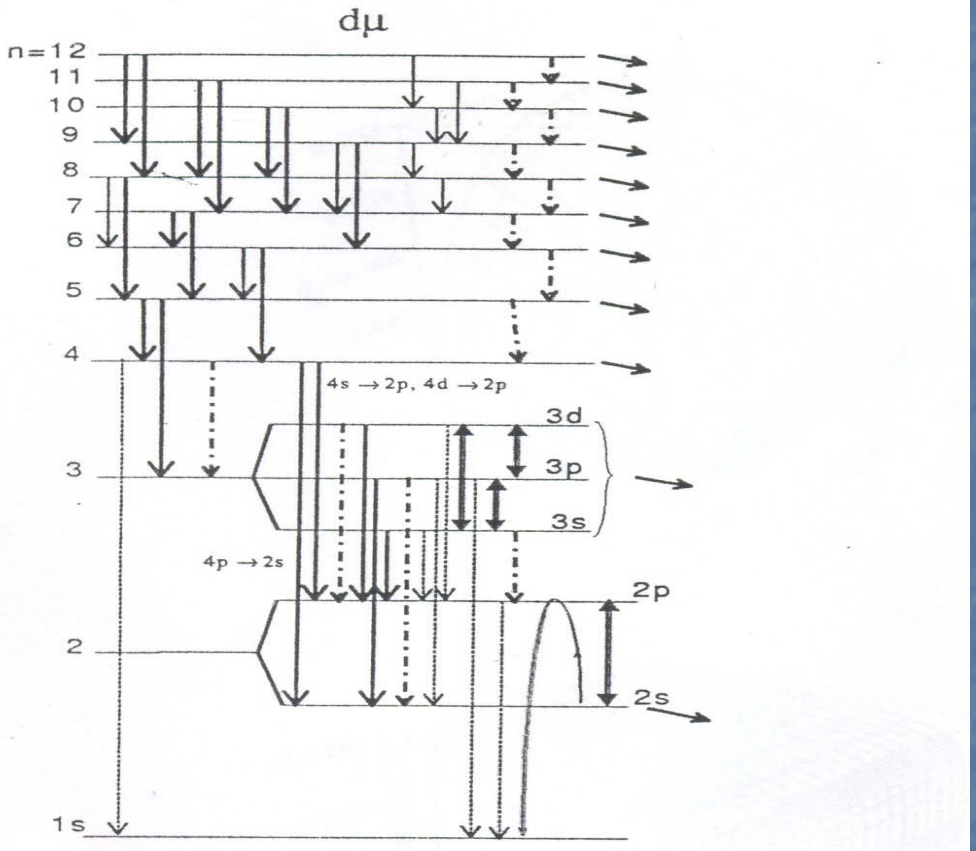


Fig. 1. Scheme of the muonic atom cascade for muonic deuterium.

Cascade processes of muonic atoms

I) Chemical Dissociation

$$(x\mu)_{n_i} + x_2' \rightarrow x'^{+} + x^{-} + (x\mu)_{n_f}, n_f < n_i, l_f = l_i$$

$$\Gamma_{nn'} = \frac{N}{2} v \pi a_n^{\,2} (s^{-1}), \Delta E_{nn'} = E_n - E_{n'}$$

II) Radiative Transition

$(\mu x)_n \longrightarrow (\mu x)_{n'} + \gamma$ In low excited State is dominated

$$\Gamma_{nn'} = \frac{4}{3} (\Delta E)^3 \left| R_n^{n'} \right|^2 \overline{M}^{-2} \times 1.60 \times 10^{-10} (s^{-1})$$

Average over l-value of initial state and summed over the final state.
Bethe H.A. and Salpeter E.E.1975,Quantum Mechanics of one and two electron atoms

Cascade processes of muonic atoms

III) External Auger effect

$$\lambda_{if} = N v_i \sigma_{if} = 4.3\times10^{15}(\sec^{-1})(R^{n'l'}_{\ nl})^2 \overline{M}^{-2}(2\Delta E + 1.39)^{-1/2}$$

$$R_{nl} = (\frac{z}{a_0})^{\frac{2}{3}} e^{zr/na_0} f_{nl}(\frac{zr}{a_0}) \qquad a_0 = \frac{1}{\overline{M}}$$

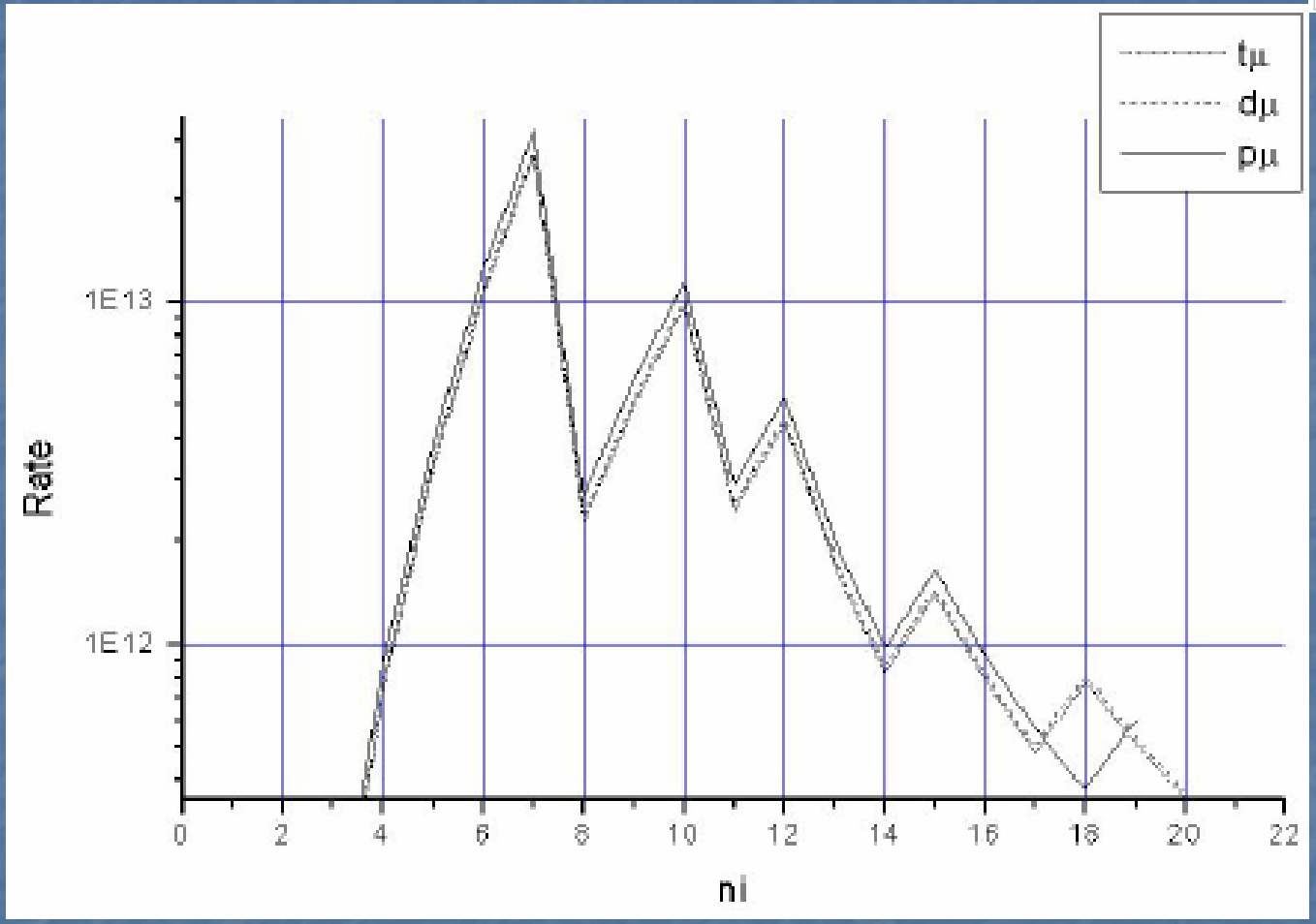
$$\Delta E = \Delta E_{x\mu} - \delta_H$$

$\Delta E_{x\mu}$ **Is the excitation energy of** xμ atom

$\delta_H = 15.6eV$ Is the Ionization energy of hydrogen atom

Leon M. and Bethe H., Phys. Rev.,127(1962)636

Cascade processes of muonic atoms

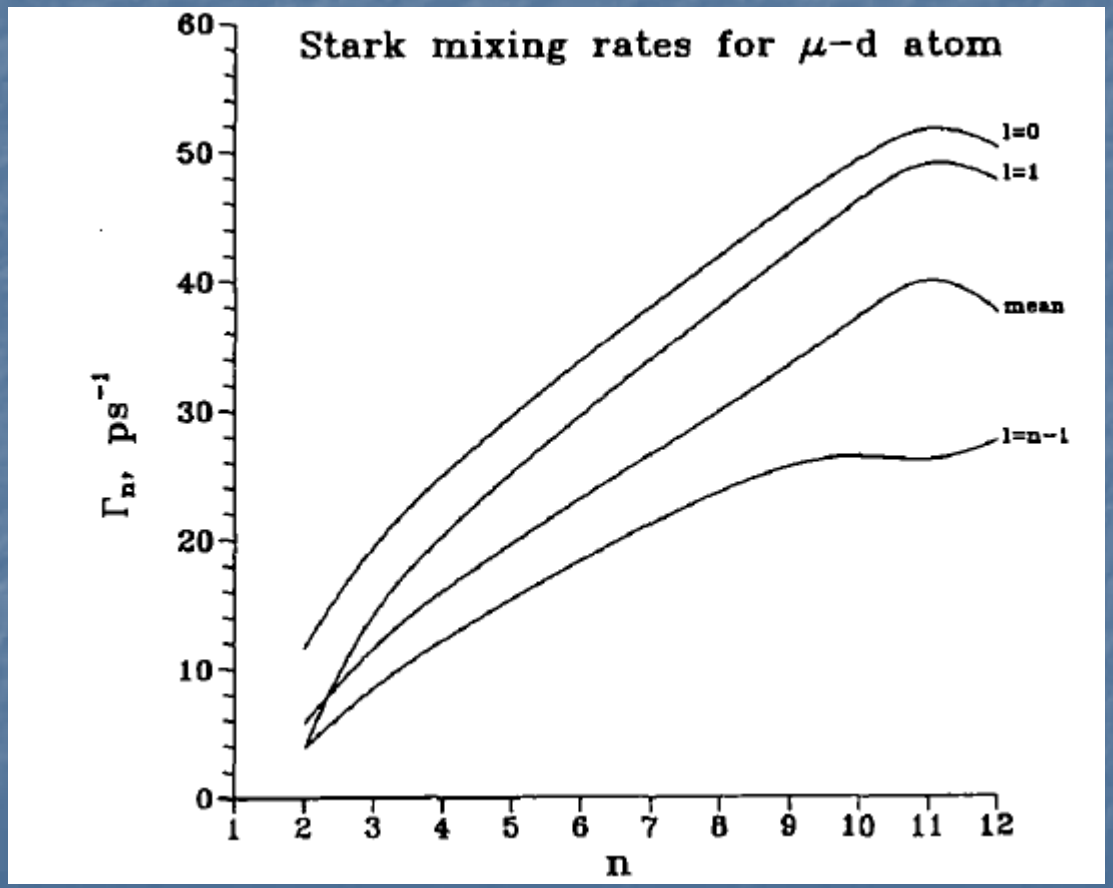


External Auger effect rates as a function of initial state.

Cascade processes of muonic atoms

IV) Stark Mixing

$$(\mu x)_{nl} + x' \longrightarrow (\mu x)_{nl'} + x'$$



V.P. Popov and V.N. Pomerantsev, Hyper. Intract. 101/102(1996)133

Cascade processes of muonic atoms

V) Deceleration (elastic scattering)

$$(\mu x)_n + x' \longrightarrow (\mu x)_n + x'$$

$$\lambda_n^{dec} = N_0 v \frac{2M_\mu M_x}{(M_\mu + M_x)^2} \sigma_n^{el}(E) \qquad \sigma_n^{el} = \frac{\pi \hbar^2 (n^2 - 1)}{4 M_\mu E} (cm^2)$$

L.I. Menshikov and L.I. Ponomarev, Z. Phys. D 2(1986)1

VI) Coulomb deexcitation

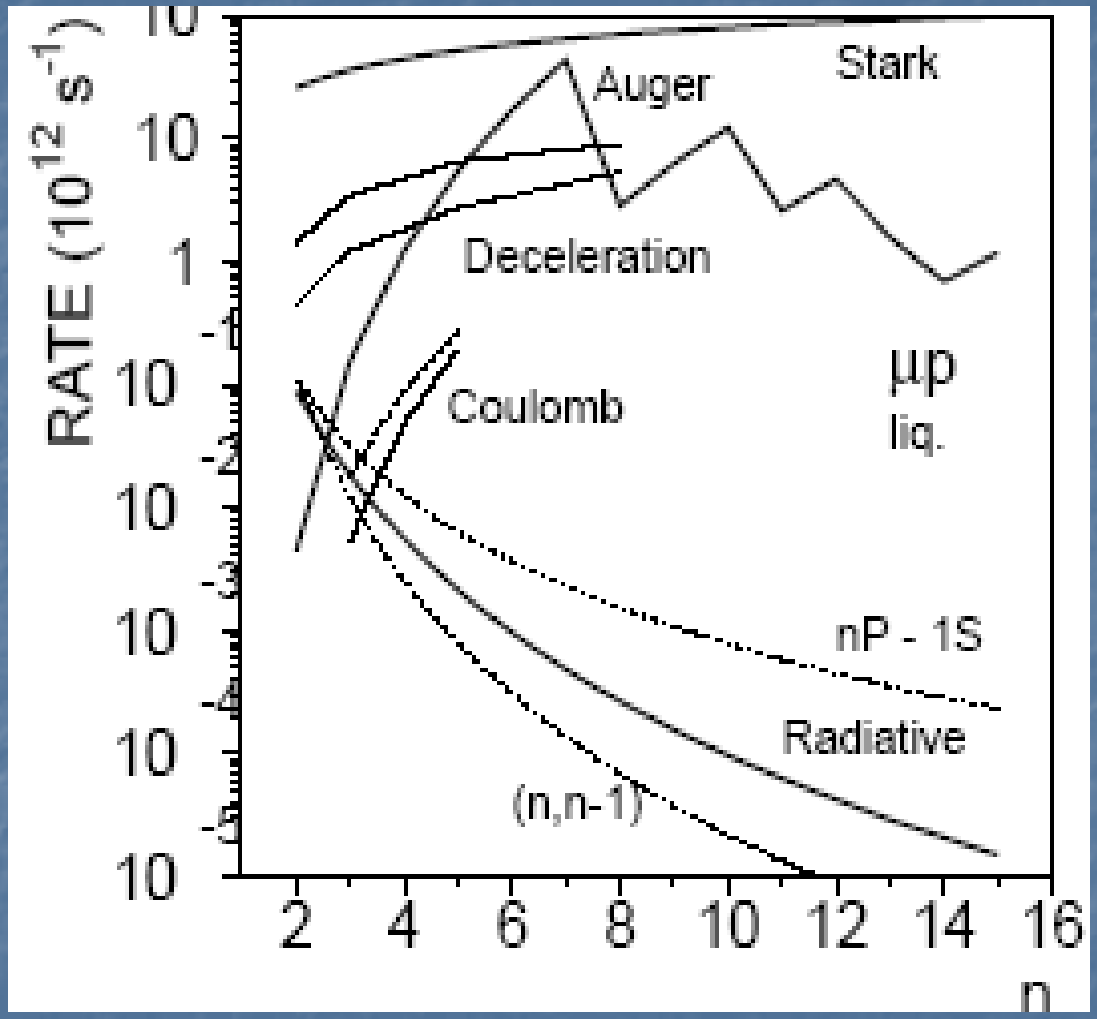
$$(\mu x)_n + x' \longrightarrow (\mu x)_{n-1} + x'$$

The released energy in this process is about 5keV/n(n-1)^2.

$$\Gamma^{coul}_{n \to n'} = \frac{Nv}{20} \pi a_n^{\ 2} \left(1 + \frac{m_{\mu d}}{E n^2}\right), n' = n - 1$$

Barcci L.I.,et al,Nuovo Cimento A,43(1978)9.

Cascade processes of muonic atoms



Comparison of the rates of competetive Cascade processes of muonic atoms.

Kinetics of cascade processes of muonic atoms

We have suggested two method for Kinetics calculattions

1. Find the kinetics equations and solve them by neumerical methods

2. Simulate the kinetics of cascade processes by Monte-Carlo method

Kinetics of cascade processes of muonic atoms

1. Kinetics equations

***Calculation of the kinetic energy of muonic atoms after collision**

$$\vec{v}'_a = \vec{v}_{cm} + \gamma_b \vec{v}'_{rel} \qquad \vec{v}'_b = \vec{v}_{cm} - \gamma_a \vec{v}'_{rel}$$

a and b denotes the muonic and the kind of target isotopes, respectively

$$\gamma_b = \frac{m_b}{m_a + m_b} \qquad \gamma_a = \frac{m_a}{m_a + m_b}$$

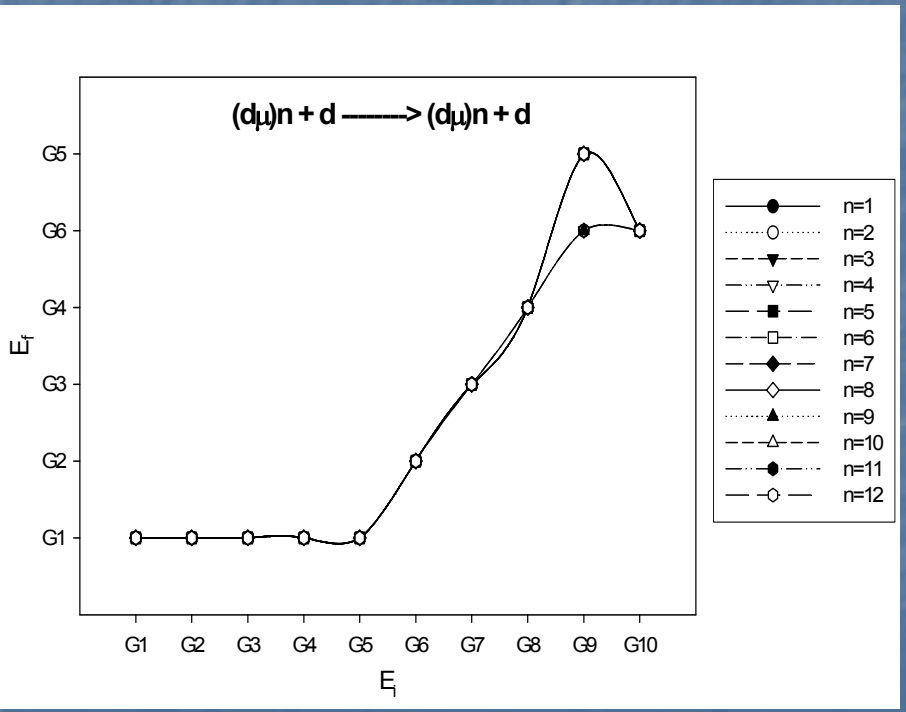
$$E_a + E_b = E'_a + E'_b + \Delta u \qquad \left|\vec{v}'_{rel}\right| = \left|\vec{v}_{rel}\right| \sqrt{1 + \frac{\Delta u}{E}}$$

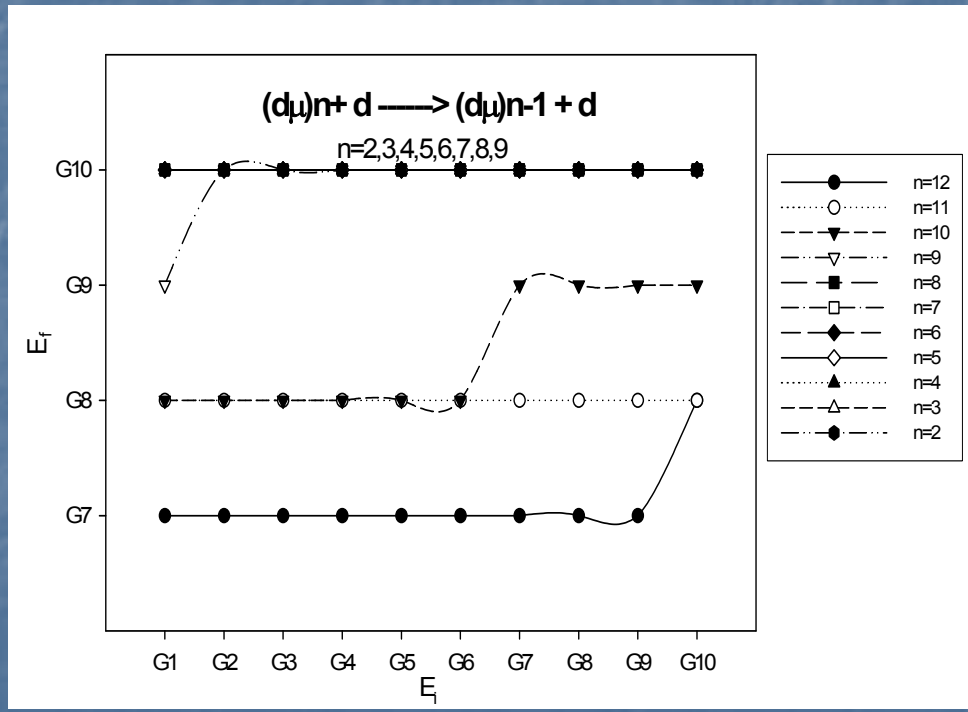
Δu and E are the released energy in reaction and relative kinetic energy in collision, respectively.

Kinetics of cascade processes of muonic atoms

1. Kinetics equations

*For consideration of the energy dependency of the cross sections we have used a multi group method.





Elastic Collision **Coulomb de-excitation**

Determination of the number of energy group after collision

Kinetics of cascade processes of muonic atoms

1. Kinetics equations

$$\frac{dP_{(n,m)_{d\mu}}}{dt} = \sum_{i\,(i>n)} \lambda_{ra}^{\begin{cases} i\to n \\ m\to m \end{cases}} P_{(i,m)_{d\mu}} - \sum_{i\,(i<n)} \lambda_{ra}^{\begin{cases} n\to i \\ m\to m \end{cases}} P_{(n,m)_{d\mu}} + \sum_{i(i>n)} \phi\lambda_{au}^{\begin{cases} i\to n \\ m\to m \end{cases}} P_{(i,m)_{d\mu}} - \sum_{i(i<n)} \phi\lambda_{au}^{\begin{cases} n\to i \\ m\to m \end{cases}} P_{(n,m)_{d\mu}}$$

$$+ \sum_{l} \phi(c_d)\lambda_{coul}^{\begin{cases} n+1\to n \\ l\to m \end{cases}} P_{(n+1,l')_{d\mu}} - \sum_{l} \phi(c_d)\lambda_{coul}^{\begin{cases} n\to n-1 \\ m\to l \end{cases}} P_{(n,m)_{d\mu}} + \sum_{l} \phi(c_t)\lambda_{coul}^{\begin{cases} n+1\to n \\ l\to m \end{cases}} P_{(n+1,l)_{d\mu}} - \sum_{l} \phi(c_t)\lambda_{coul}^{\begin{cases} n\to n-1 \\ m\to l \end{cases}} P_{(n,m)_{d\mu}}$$

$$+ \sum_{l} \phi(c_d)\lambda_{tr}^{\begin{cases} n\to n \\ l\to m \end{cases}} P_{(n,l)_{t\mu}} - \sum_{l} \phi(c_t)\lambda_{tr}^{\begin{cases} n\to n \\ m\to l \end{cases}} P_{(n,m)_{d\mu}} + \sum_{l} \phi(c_d)\lambda_{tr}^{\begin{cases} n+1\to n \\ l\to m \end{cases}} P_{(n,l)_{t\mu}} - \sum_{l} \phi(c_t)\lambda_{tr}^{\begin{cases} n\to n-1 \\ m\to l \end{cases}} P_{(n,m)_{d\mu}}$$

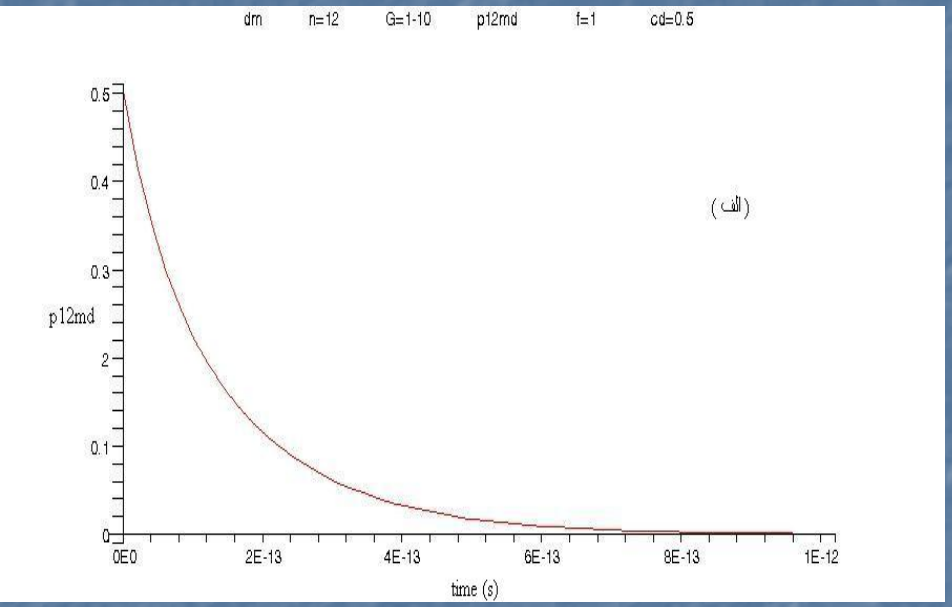
$$+ \sum_{l(l>m)} \phi(c_d)\lambda_{dec}^{\begin{cases} n\to n \\ l\to m \end{cases}} P_{(n,l)_{d\mu}} - \sum_{l(l<m)} \phi(c_d)\lambda_{dec}^{\begin{cases} n\to n \\ m\to l \end{cases}} P_{(n,m)_{d\mu}} + \sum_{l(l>m)} \phi(c_t)\lambda_{dec}^{\begin{cases} n\to n \\ l\to m \end{cases}} P_{(n,l)_{d\mu}} - \sum_{l(l<m)} \phi(c_t)\lambda_{dec}^{\begin{cases} n\to n \\ m\to l \end{cases}} P_{(n,m)_{d\mu}}$$

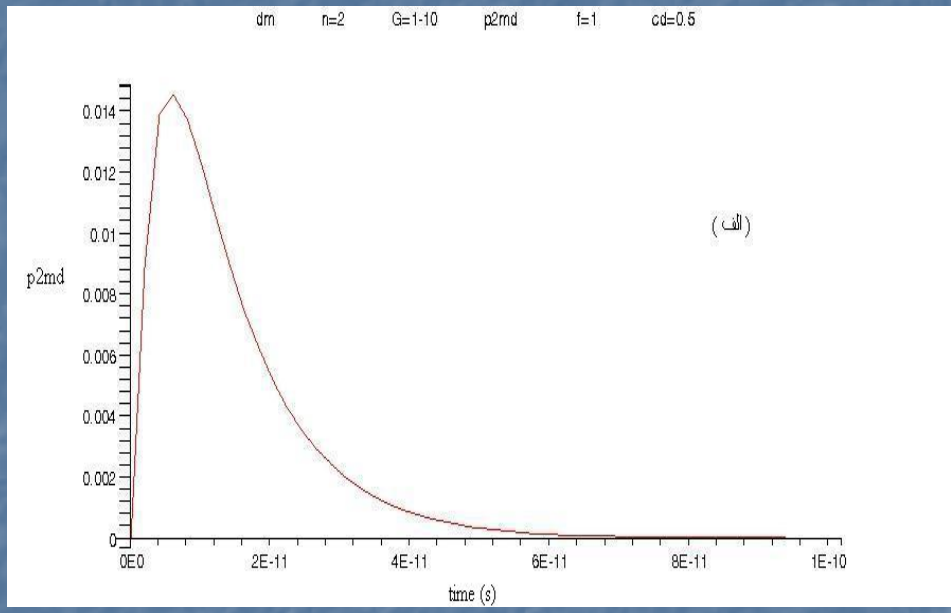
Principal quantum number: n=12
Number of energy group: m=10

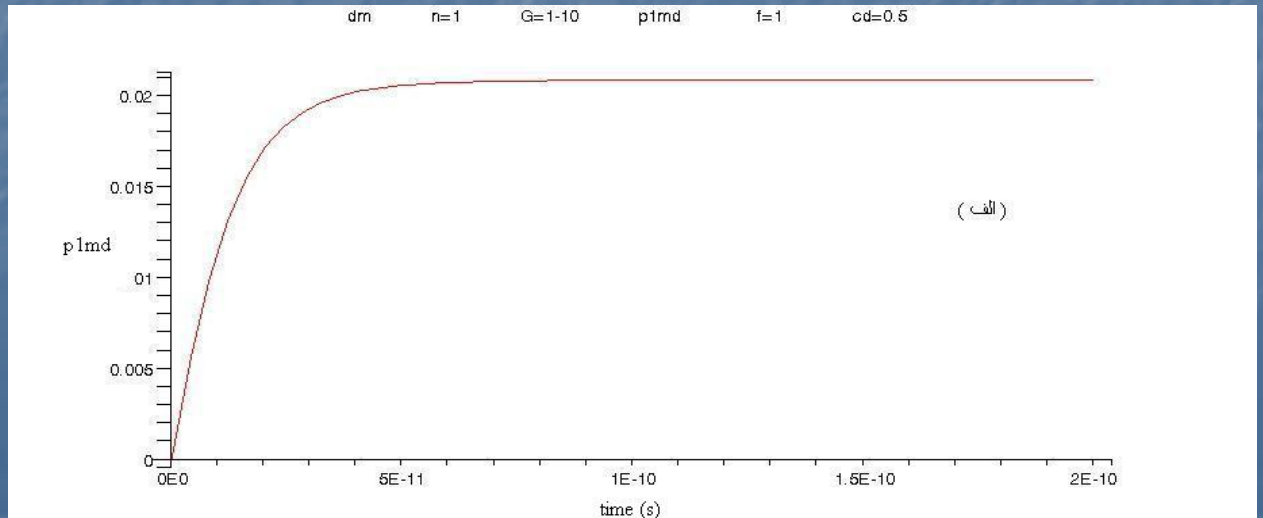
These 240 equations are coupled linear differential equations. To solve them we use the Runge-Kutta method in the fourth order.

Kinetics of cascade processes of muonic atoms

1. Kinetics equations: Some of our results



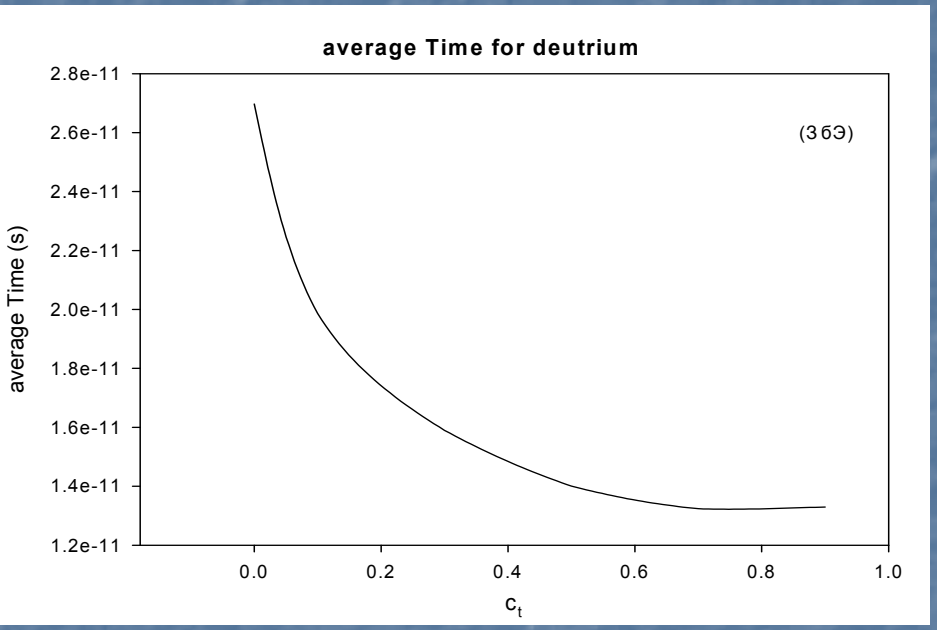


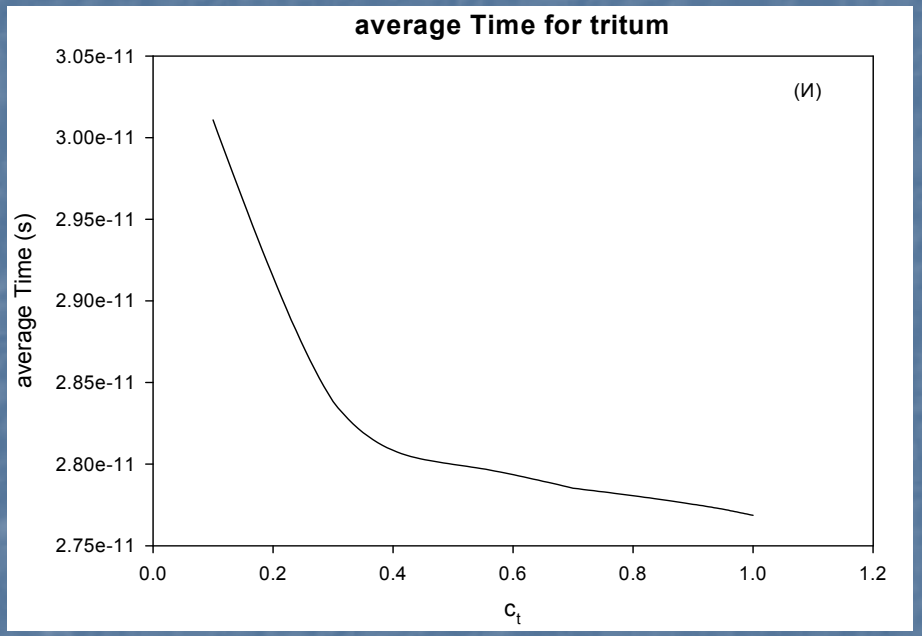


Populations of muonic atoms

Kinetics of cascade processes of muonic atoms

1. Kinetics equations: Some of our results

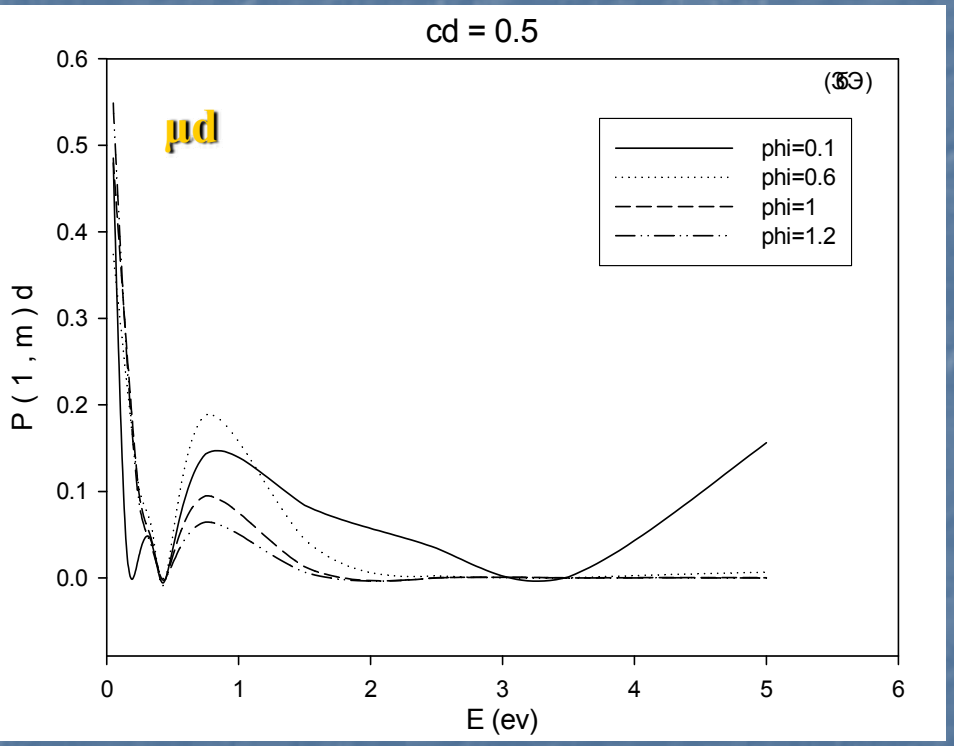


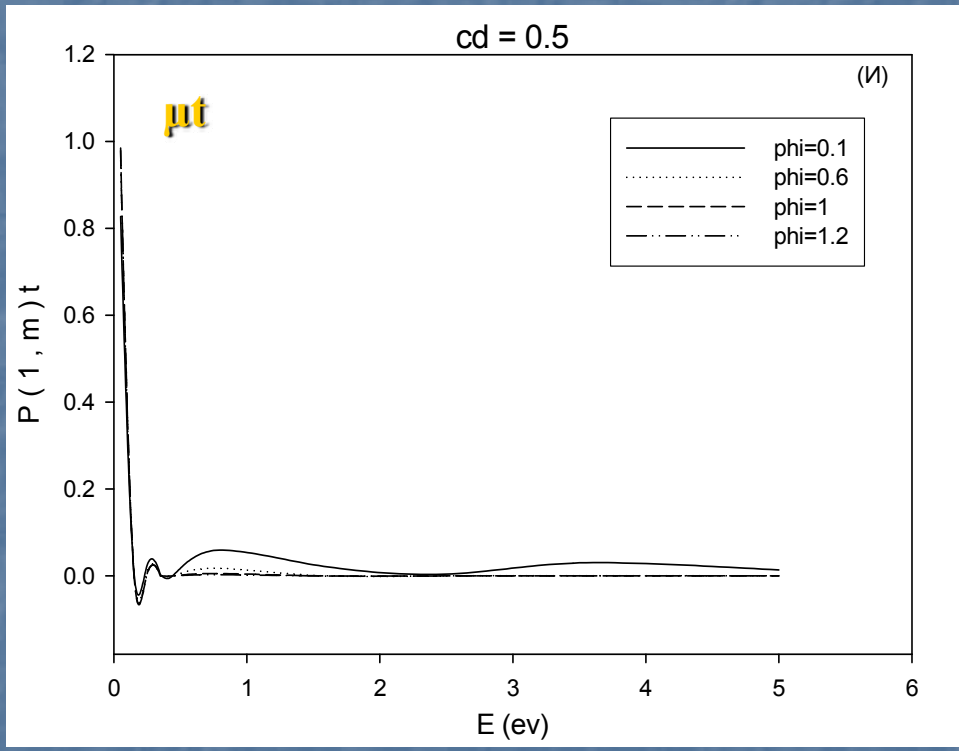


Average time of cascade processes

Kinetics of cascade processes of muonic atoms

1. Kinetics equations: Some of our results





Kinetic Energy distribution of muonic atoms in the 1s state. It is not Maxwelian.

Kinetics of cascade processes of muonic atoms

1. Kinetics equations

Advantages of this method:

1. We can consider the energy dependency of the cross sections.
2. We can solve the problem as a function of time.
3. We can solve the problem in different physical conditions.

Disadvantages:

We should write many kinetics equations especially if the number of groups is chosen larg.

Kinetics of cascade processes of muonic atoms

We have suggested two method for Kinetics calculations

1. Find the kinetics equations and solve them by numerical methods

2. Simulate the kinetics of cascade processes by Monte-Carlo method

Kinetics of cascade processes of muonic atoms

2. Monte-Carlo method

* **Total transition rate from excited state n is:**

$$\lambda^{n}_{total}(E) = \sum_{n'=1}^{n-1} \lambda^{n\to n'}_{columb}(E) + \sum_{n'=1}^{n-1} \lambda^{n\to n'}_{radiation} + \lambda^{n\to n}_{scattering}(E) + \sum_{n'=n-1}^{n} \lambda^{n(x\mu)\to n'(x'\mu)}_{transfer}(E)$$

$$+ \lambda^{(n,l\to n,l')_{Ave}}_{stark}(E) + \sum_{n'=1}^{n-1} \lambda^{n\to n'}_{auger}(E)$$

* **Transition probability from $n \longrightarrow n'$ ($n' < n$) due to ith mechanism in a kinetic energy E is :**

$$P_i^{n\to n'}(E) = \lambda_i^{n\to n'}(E) / \lambda^{n}_{total}(E)$$

* **Cumulative distribution function:**

$$PDF(n, j, E) = \sum_{i=1}^{j} \sum_{n'=1}^{n-1} P_i^{n\to n'}(E)$$

Kinetics of cascade processes of muonic atoms

2. Monte-Carlo method

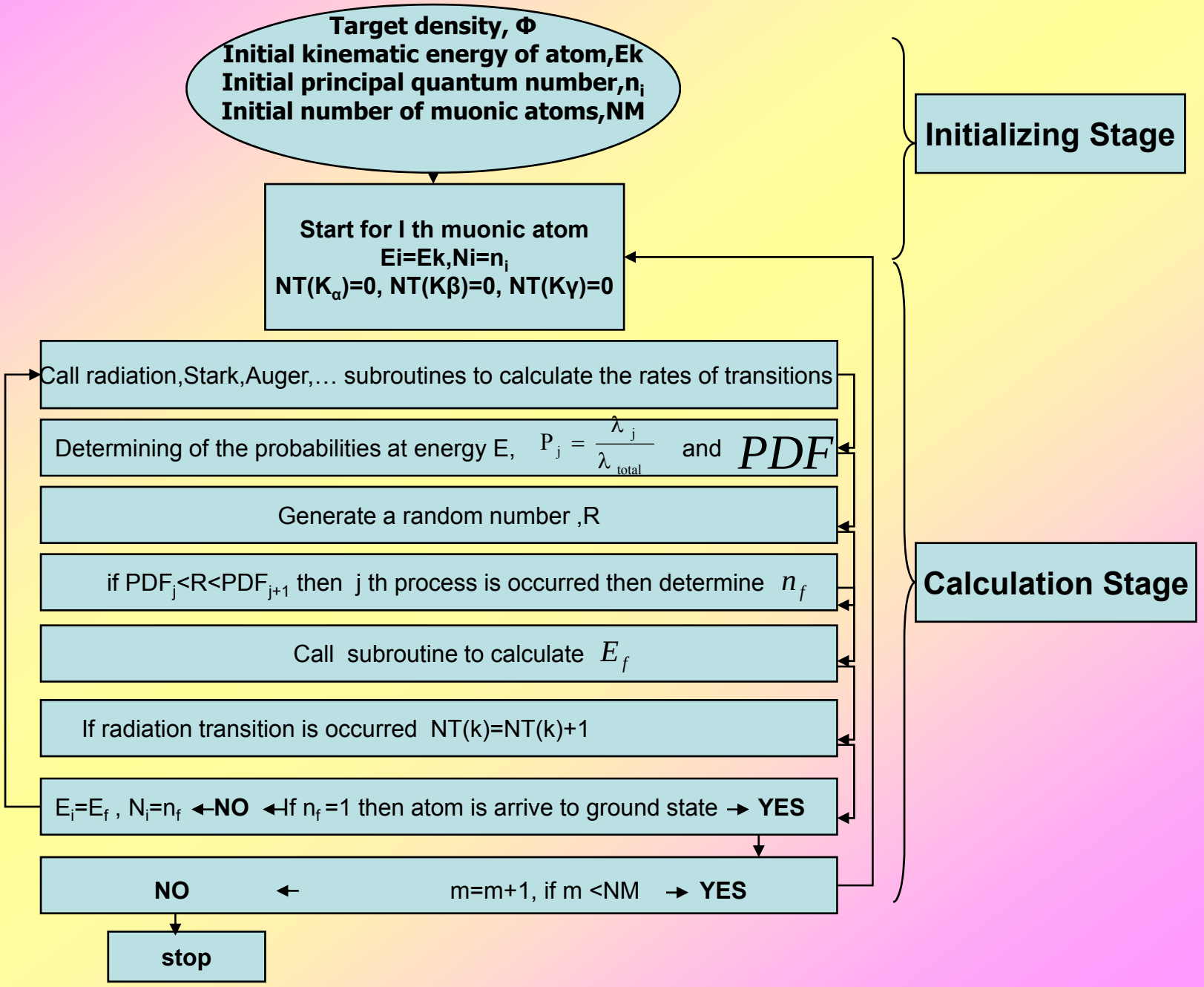
I) Calculate the cumulative distribution function at specific energy PDF(n,j,E).

II) Generate a random number R in the Interval (0,1) .

III) If PDF(n,j,E)< R <PDF(n,j+1, E) then j th mechanism transition is accepted. Then by the same proceed we can determine n'

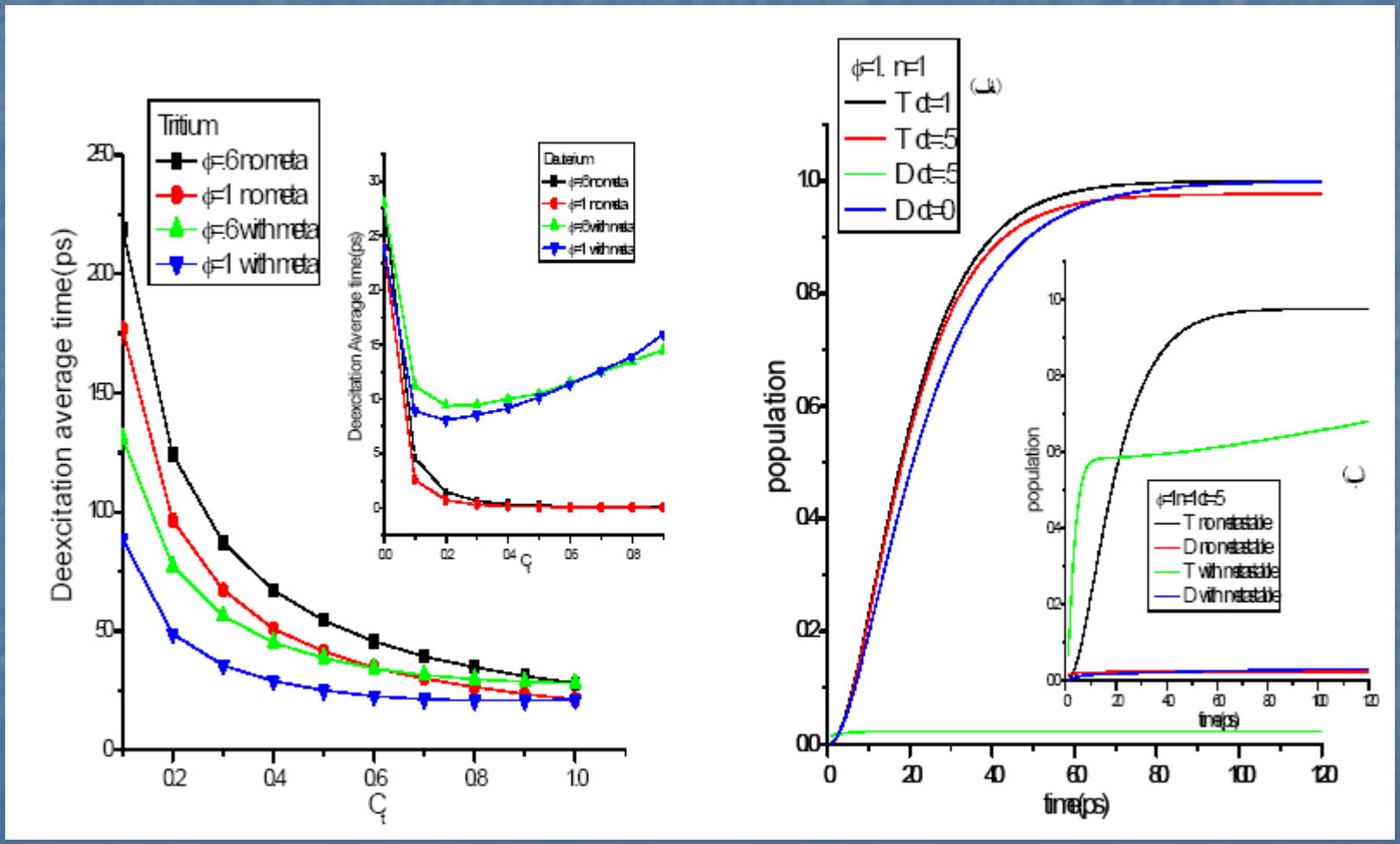
If the collisional process is every one of the scattering, Coulomb deexcitation or transfer, We should determine the kinetic energy of muonic atom after collision. Otherwise, continue from step (III) for the new n state.
We should also determine the time of transition.

Monte-Carlo Simulation Flow Chart



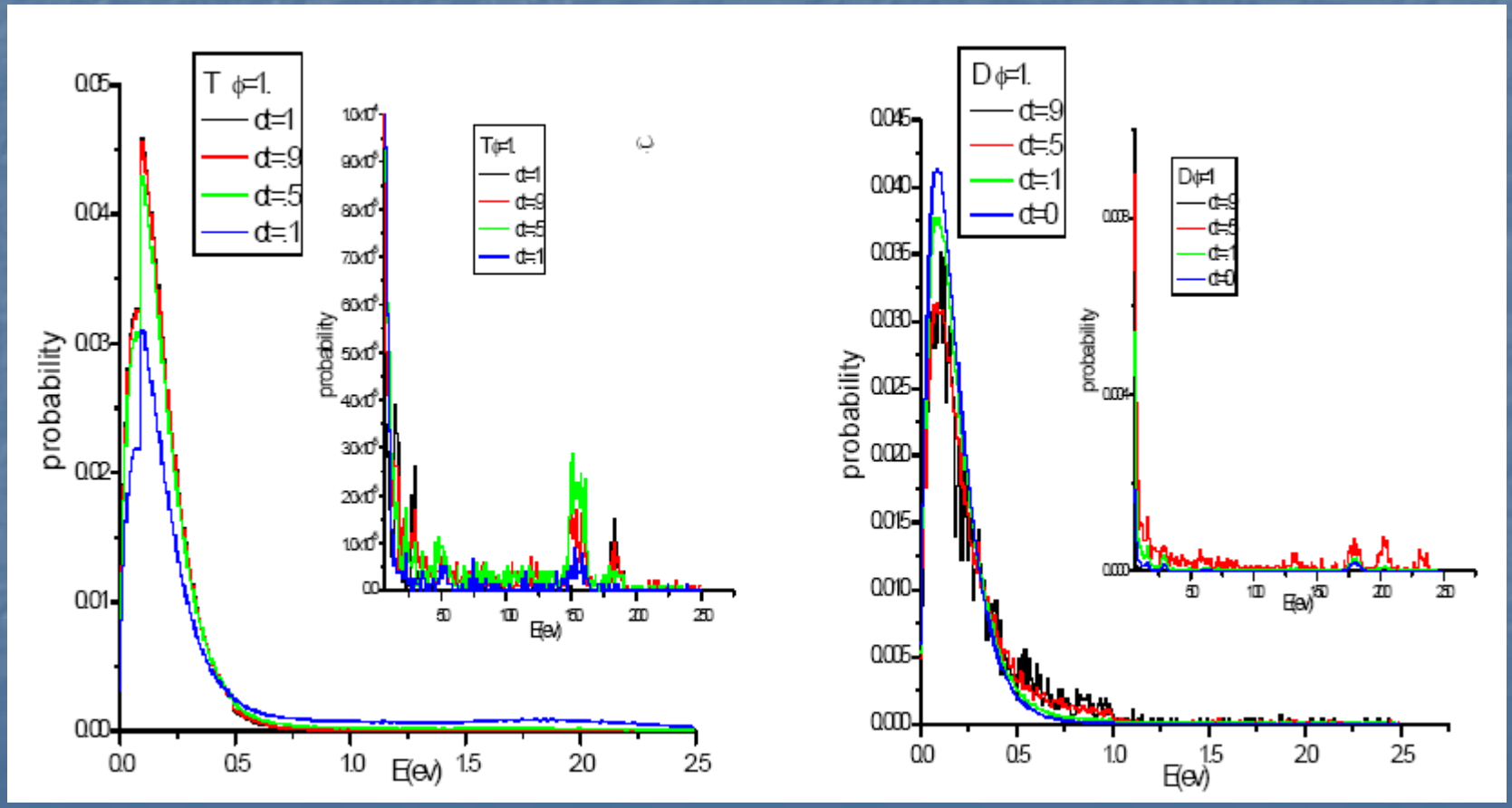
Kinetics of cascade processes of muonic atoms

2. Monte-Carlo method: Some of our results



Kinetics of cascade processes of muonic atoms

2. Monte-Carlo method: Some of our results

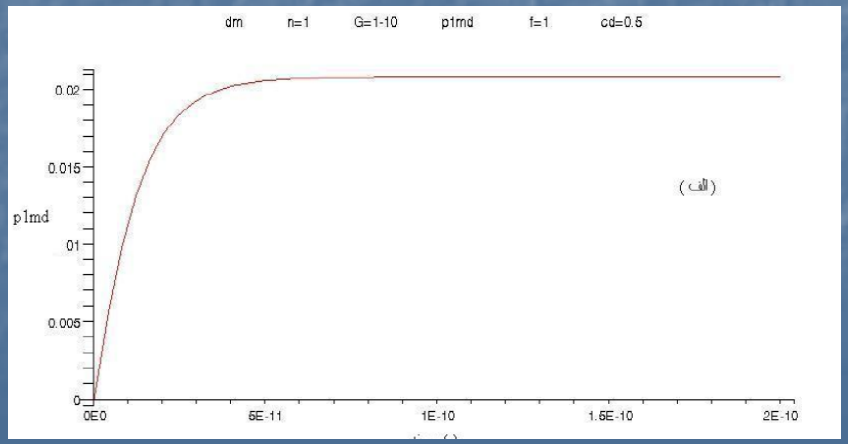


Kinetic energy distribution of muonic atoms

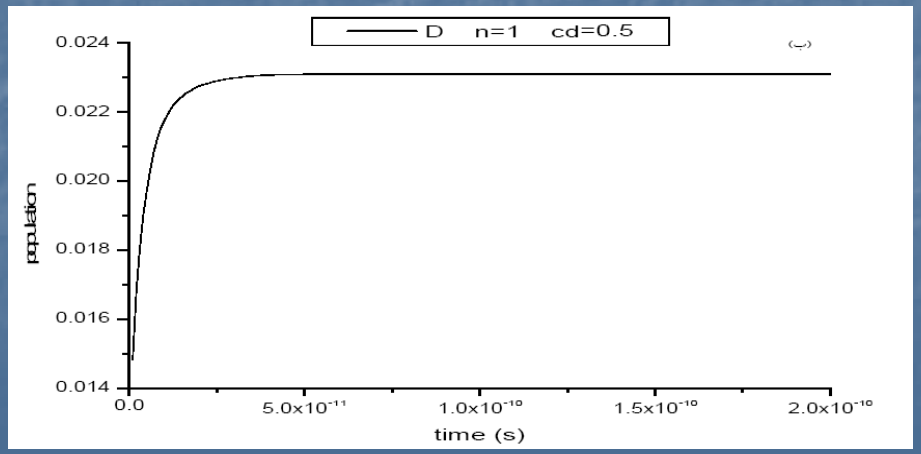
Kinetics of cascade processes of muonic atoms

Comparison of the results from two method

Average deexcitation time	μd	μt
Solve of kinetics equations	2.7×10^{-11}s	2.8×10^{-11}s
Monte-Carlo simulation	2.4×10^{-11}s	2.1×10^{-11}s



Kinetics method

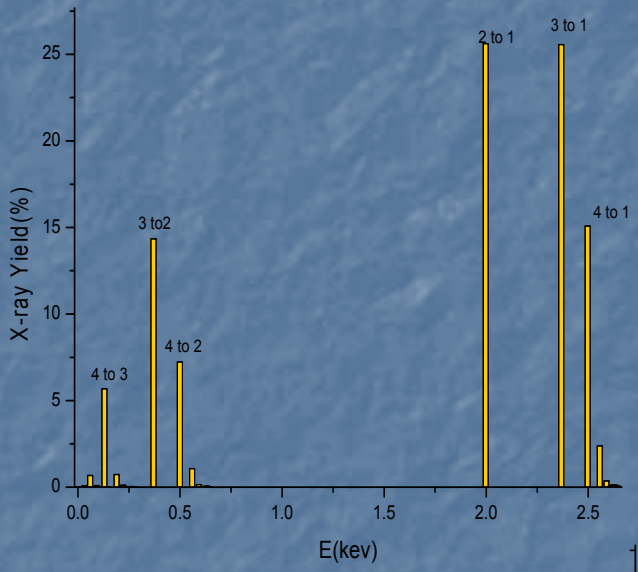


Mont Carlo method

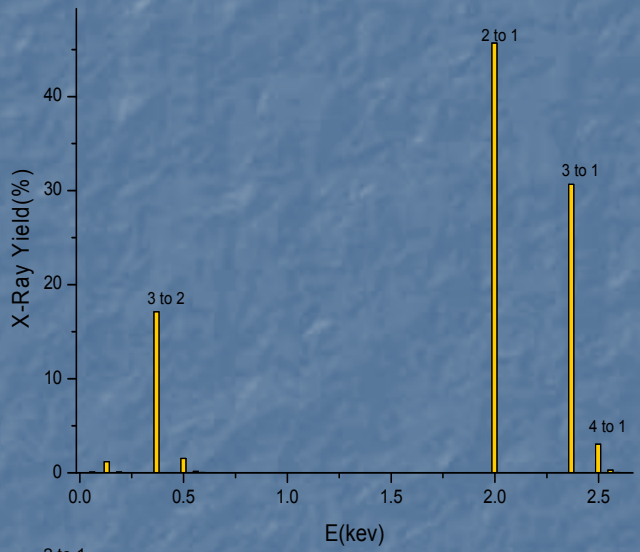
Kinetics of cascade processes of muonic atoms

X-ray yields from muonic atoms by Monte-Carlo method

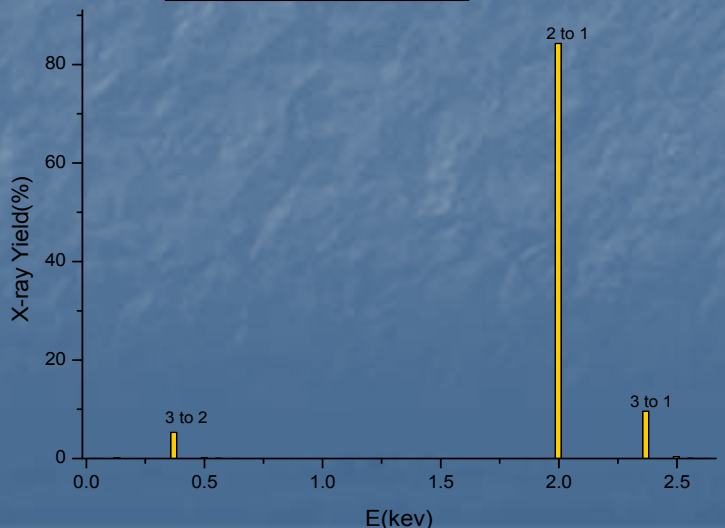
Muonic Deuterium ϕ=.01



Muonic Deuterium ϕ=.1

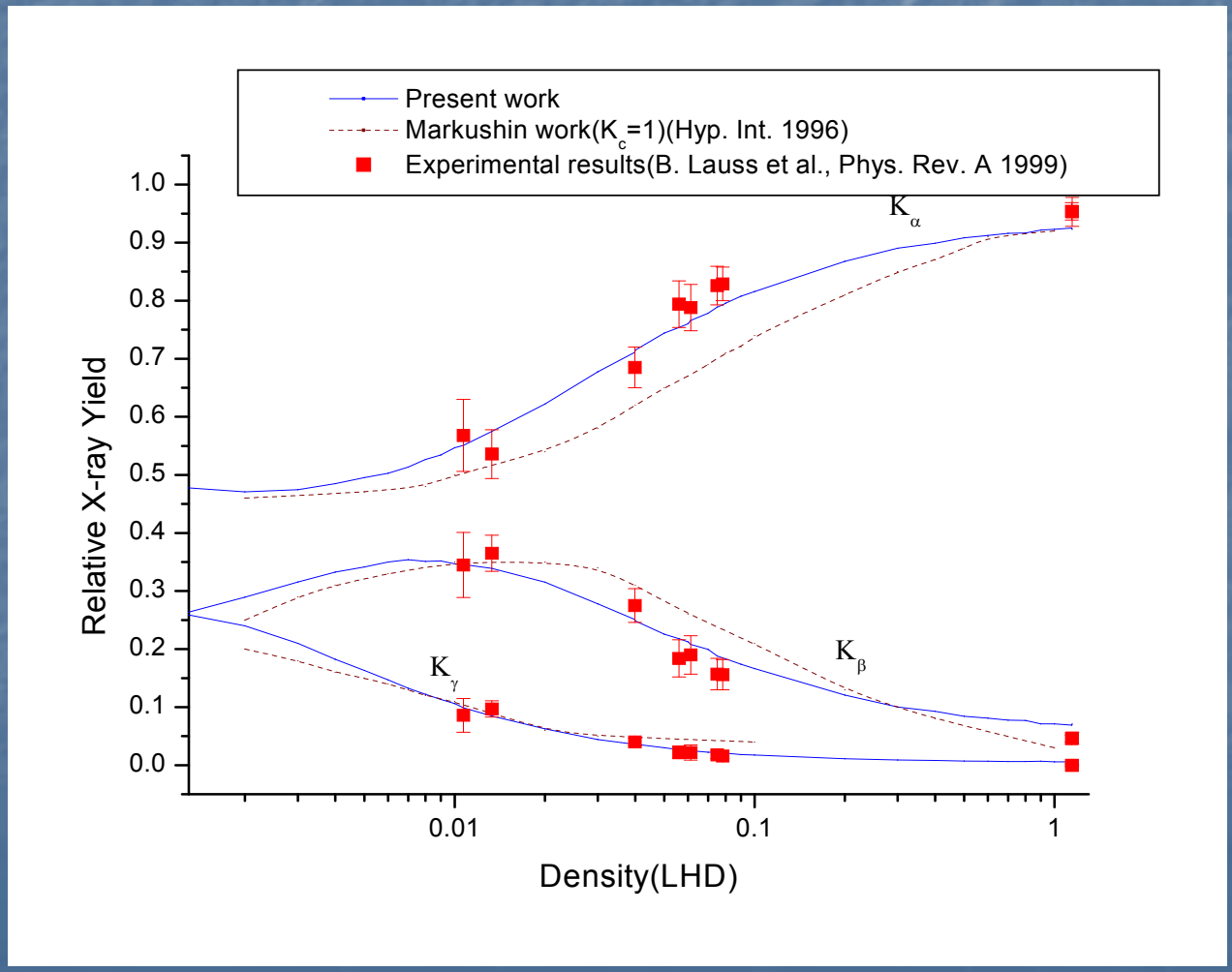


Muonic Deuterium ϕ=1.



Kinetics of cascade processes of muonic atoms

Comparison our results with the other results



Cascade processes of Kaonic atoms

- Cascade processes

Mechanism	Process	Energy Dependence	Density Dependence
Radiative	$(xK)_{n_i} \rightarrow (xK)_{n_f} + \gamma$	None	None
Auger	$(xK)_{n_i} + x_2' \rightarrow x_2'^{+} + e + (xK)_{n_f}$	Weak	Linear
Stark mixing	$(xK)_{nl} + X \rightarrow (xK)_{nl'} + X$	Moderate	Linear
Scattering	$(xK)_n + X \rightarrow (xK)_n + X'$	Strong	Linear
Coulomb	$(xK)_{n_i} + x' \rightarrow (xK)_{n_f} + x'$	Strong	Linear
Transfer	$(xK)_n + x' \rightarrow (x'K)_n + x$	Strong	Linear
Nuclear absorption	$(xK)_n + X \rightarrow \pi^0 + \Lambda + X$	?	?
Nuclear reaction	$(xK)_n \rightarrow \pi^0 + \Lambda$	None	None
Weak decay	$K^- \rightarrow \mu^- + \bar{\nu}_\mu$	None	None

Simulation of cascade processes of K^-p atoms

X-ray yields from cascade of kaonic atoms is important to test of strong interaction and the theory of QCD in low energies.

As the kaon interacts strongly with the nucleus the 1s energy level is shifted and broadened

$$\varepsilon_1 = (E_{K_\alpha})_{\exp} - (E_{K_\alpha})_{em}$$

Density of the target can affect the X-ray yields.

Because the X-ray yields are rare therefore we should find an optimum condition of the target for the experiment.

Shift of energy in 1s state and width of 1s and 2p and … may change the X-ray yields. We can use them in our calculation as a free parameter.

Simulation of cascade processes of K^-p atoms

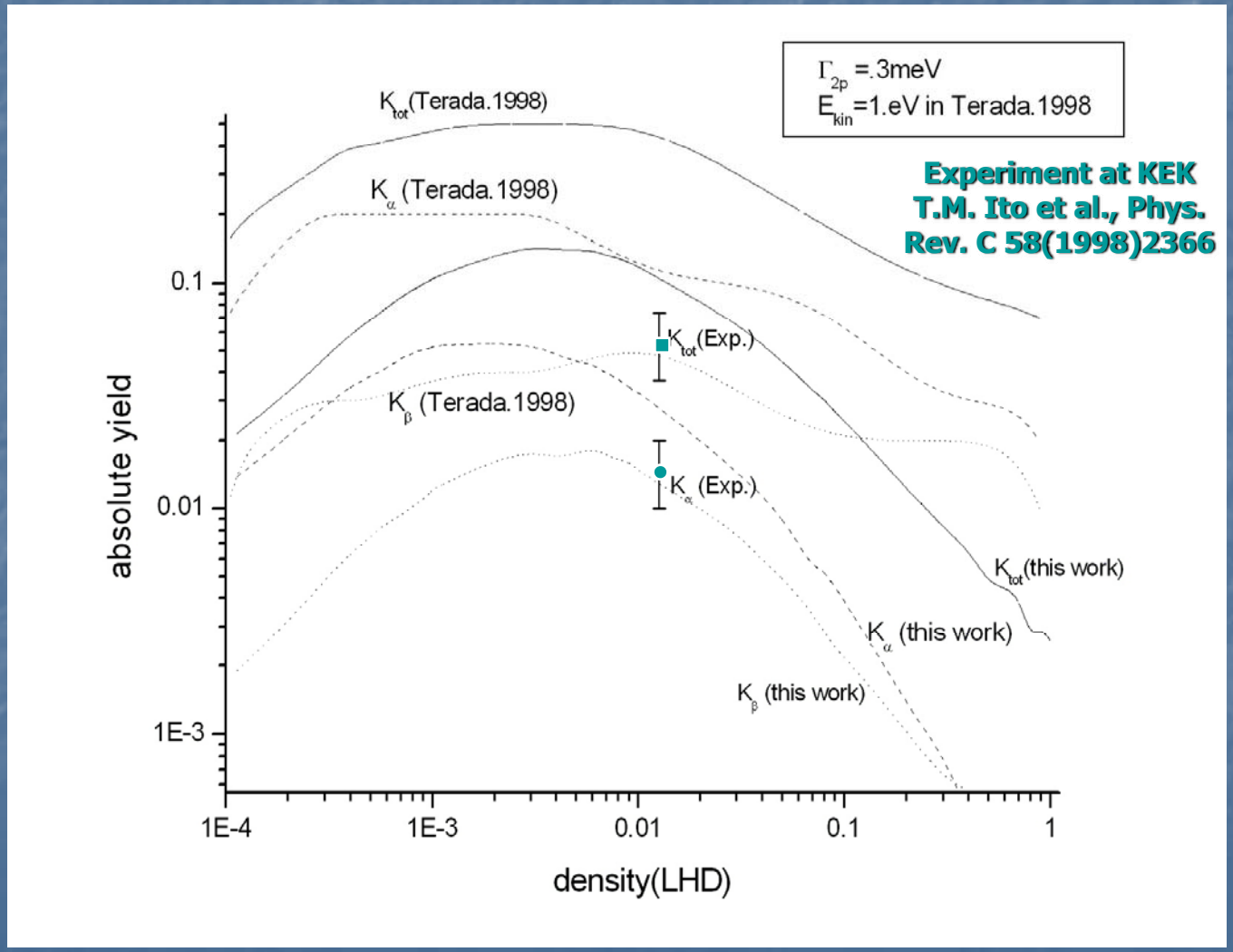
Our plan to do the simulation

I. Determining of the cross sections or rates of the Stark transitions, external Auger effect, Coulomb deexcitation, elastic scattering and absorption as a function of kinetic energy of kaonic atoms and the rates of radiative transition and nuclear reactions of kaonic atoms in any excited states.

II. Preparing a computer code based on the Monte-Carlo method to simulate the cascade processes in kaonic atoms. The program begins by a highly excited kaonic atom and calculates its life history as they cascade down trough the ***nl*** states. The program determines the kinetic energy of the kaonic atom after any collisional processes such as Coulomb deexcitation and elastic scattering in each step of time. The program repeat for 10^6 kaonic atoms and take an average of any quantity.

IV. If during the simulation any radiative transition is take place, the energy and time of emitted X-ray is recorded. Finally the X-ray spectrum can be calculated in any time. These calculations can be done for any physical conditions such as density target.

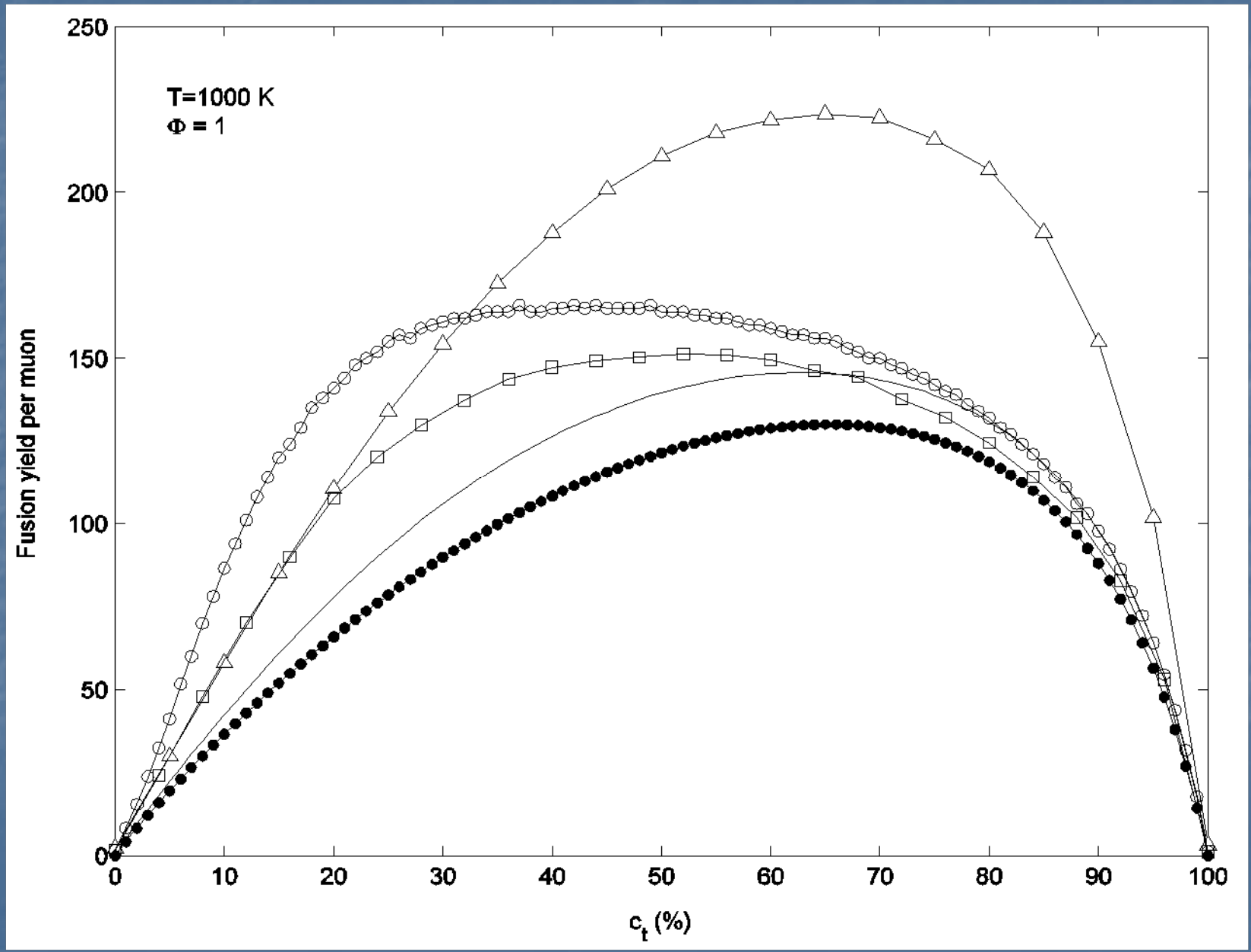
Simulation of cascade processes of K^-p atoms



Simulation of cascade processes of K^-d atoms in SIDDHARTA

We hope to determine an optimum condition for the SIDDHARTA experiment (For K^-d atoms at LNF) to achieve the highest X-ray yields.

Then In order to study the strong Interactionour results will be compared with the experiment.


T=1000 K
Φ = 1
Fusion yield per muon
250
200
150
100
50
0
0
10
20
30
40
50
60
70
80
90
100
c_t (%)